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Sraffa on the monetary theory of distribution and inequality[*]

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Abstract

Sraffa's book *Production of Commodities by Means of Commodities* (Sraffa, 1960) is, as the subtitle indicates, a *Prelude to a critique of economic theory.* This critique is based on the notion of self-replacing system which has as a condition for its realization that the prices must be such that the revenues and the expenditures of the producers and of the workers allow the system to replicate and, concurrently, market clearing occurs. Normally this accounting equilibrium excludes the possibility that exchanges take place wit the use of deferred means of payments such as financial papers or fiat money. But money is essential for the functioning of any evolved economic system.

In this paper fiat money, credit and debt are inserted into Sraffian schemes.

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Contents

1 Towards a Sraffian Monetary Theory of Production and Distribution.

Sraffa's book *Production of Commodities by Means of Commodities* (Sraffa, 1960, from now on PCMC) is, as the subtitle indicates, a *Prelude to a Critique of Economic Theory.* This critique is based on the notion of self-replacing system and the associated notions of self-replacing prices, wages and profit rates. Where self–replacing is the situation where production is taking place as it did during the previous production cycle.

This condition is important because it allows focus on prices and distribution of the surplus also in the case where the produced physical quantities are not changing.

Sraffa has shown that the commodities' *natural prices* are not unique, i.e. they are not independent from distribution, but are a function of the rate of profits or of the wage rate[1] (PCMC, Chs. III-V) and that the relative prices would be (or could be) not a monotonic function of the embodied labour values (PCMC, Ch. VI) .

In other words the values of the gross or net output and of the means of production (the value of capital) are themselves a function of the distribution or/and prices of the surplus to workers and producers. This is so for the micro level (firms) or the macro level (industries and the whole economy). This critique is to be addressed to both the marginalist theories of value and the incorporated labour theories.

The critique was discussed in the 60's during the two Cambridges capital controversy (Pasinetti, 1966; Samuelson, 1966; Garegnani, 1966; Harcourt, 1972; Cohen and Harcourt, 2003; Pasinetti, 2003; Felipe and McCombie, 2003), but it has disappeared from current economic theory debates.

These problems have been ignored by mainstream economists by postulating that the dependence of the value of capital and output of distribution (i.e., prices) is problematic only for very special cases. These cases have been considered by the majority of economists as if they were anomalies or perversities (Pasinetti and Scazzieri, 1987). Recent work has confirmed that the dependence on distribution is the general case (Zambelli, 2004, 2018a,c). If any, it is the very special neoclassical case, where prices of the factors of production are determined by marginal productivity, which should be doomed as a perversity because it is very unlikely it might occur.

Zambelli (2018c) extends the Sraffian schemes to the general case where the rates of profits are not uniform. In this paper we aim at extending Sraffian schemes also to the cases where deferred means of payments are generated and used for the exchanges. We consider the cases in which when prices alone are not enough to allow for self-replacing there is the possibility that commodities may be exchanged thanks to the issuing of new loans (credit and debt) or throughout the transfers of fiat money.

This is not the first time that money is inserted in the Sraffian schemes. This has done before, but it is the first time, as far as I know, that money is not treated as a commodity produced inside the system. That money is not a commodity is an essential characteristic of the capitalistic mode of production.

2 Sraffa on Money.

Sraffa's research may be divided into two apparently disjointed parts. The first part is the one where his research interests are closely related with monetary and banking theory (Sraffa, 1920b, 1922a, 1932a). The second part is the one concerned with the measurement

[1] Where the rate of profits - the wage rate - are distributional variables of the surplus produced.

and distribution of the physical surplus produced by an economy (Sraffa, 1925b, 1926, 1951, 1960).

Here it is claimed, also following Panico (1988b)[2], that the efforts Sraffa put in these two parts are coherent with respect to a focused research agenda which has been substantially the same throughout the whole of his life. We stress the conjecture that Sraffa had his research agenda well set during the years that went from 1920 to 1930. His subsequent research work was about furthering this research agenda. Which through time was only, if any, marginally modified.

Sraffa's research may be characterized as a contribution towards precision in theory. This is well encapsulated in his intervention at the 1958 conference of the International Economic Association at Corfú on the *'Theory of Capital'*.

> [O]ne should emphasize the distinction between two types of measurement. First, there was the one in which the statisticians were mainly interested. Second, there was measurement in theory. The statisticians' measures were only approximate and [provide] a suitable field for work in solving index number problems. The theoretical measures required absolute precision. Any imperfections in these theoretical measures were not merely upsetting, but knocked down the whole theoretical basis.
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> The work of J. B. Clark, Bohm-Bawerk and others was intended to produce pure definitions of capital, as required by their theories, not as a guide to actual measurement. If we found contradictions, then these pointed to defects in the theory, and an inability to define measures of capital accurately. It was on this - the chief failing of capital theory - that we should concentrate rather than on problems of measurement. " – Piero Sraffa, Interventions in the debate at the Corfú Conference on the "Theory of Capital", 4-11 September 1958 (Lutz and Hague, 1961).

We might place Sraffa's work on a segment where on one extreme we find the applied (and necessarily imprecise measurements and) contributions and on the other the theoretical (and hence precise measurements and) theoretical contributions(see Fig.2.1).

With respect to Sraffa contributions on monetary matters we find:

a) in the proximity of the applied extreme an analysis of the Italian institutional and power structure surrounding and including the banking system (Sraffa, 1920a, 1922a,b);

b) in the proximity of the theoretical extreme we find the need of eliminating imprecisions from pure theory (the critique of Hayek's 'Prices and Production' made side by side with Keynes (Keynes, 1931; von Hayek, 1931, 1932; Sraffa, 1932a,b))[3].

[2]In his excellent paper discussing the early work of Sraffa (1920b, 1922a) on money and banking (Panico, 1988b, p.7) underlines that:

> Sraffa started his research working on applied monetary and banking problems, making some relevant contributions to the area. The theoretical problems that made his fame as an outstanding theoretician came to dominate his interests only at a later stage.

Panico also argues that:

> ... there is a close link between [Sraffa] earlier writings on applied problems and his later theoretical works, a link which clarifies the origins of his theoretical interests, showing the error of considering his contributions as 'merely abstract exercise in pure theory'.

[3]There is evidence of Sraffa's involvement in the corrections of Keynes (1930) 'Treatise on Money' And

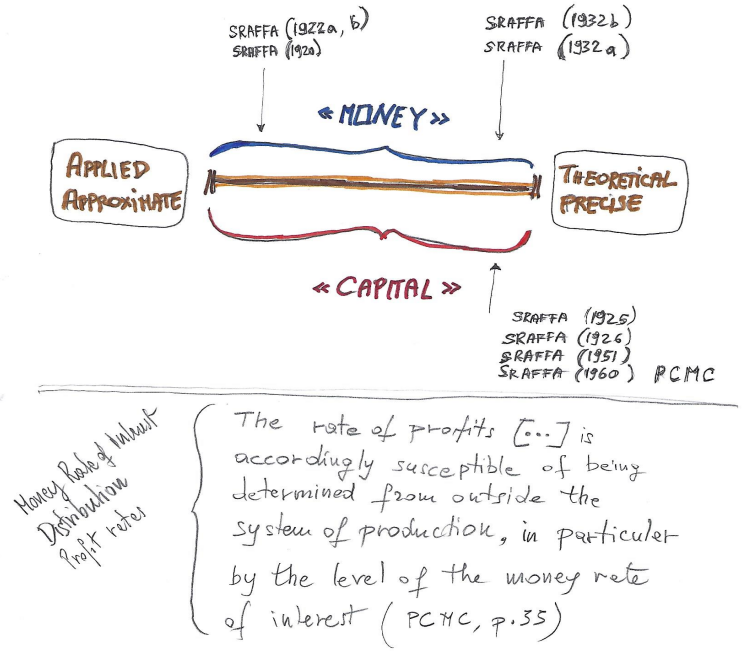

Fig. 2.1: ***Characterization of Sraffa's work seen as a whole.*** *The criteria is the one put forward at Corfú's conference (see quote at page 5).*

Sraffa's contributions in pure theory dealing with the problem of measurement of capital at the firm's level (Sraffa, 1925b, 1926) or at the aggregate level (Sraffa, 1951, 1960) are all to be placed at the precise extreme.

It is important to point out that *Production of Commodities by Means of Commodities* (Sraffa, 1960, from now on PCMC) has elements which are clearly related with money and credit through the importance put, explicitly or implicitly, on the relevance of the money interest rate for the development of the critique towards economic theory.

There are two major points where the monetary interest rate is particularly relevant in PCMC, one explicitly mentioned and the other which is implicit.

Although these reference points to be found in PCMC seem to be marginal for a *critique of economic theory*, they are in our view central starting points for a rigorous or precise reconstruction of economic theory.

2.1 PCMC. Explicit reference to the importance of the money rate of interest.

The explicit point is in the well known quotation:

here one should also keep in mind that Sraffa did translate into Italian (Keynes, 1923, 'Tract on Monetary Reform') and his involvement with some discussions surrounding the writing of Keynes 'General Theory'. Chapter 17 (*The Essential Properties of Interest and Money*) in Keynes (1936)' *General Theory* may be interpreted as addressing the relation between interest and money using also some of the arguments put forward by Sraffa in his review to Hayek And in fact Keynes (1936, 223, fn.1), as it is well known, does acknowledge that the idea of the own rate of interest was taken from Sraffa (1932a)'s critique to Hayek's 'Prices and Production'. Furthermore, it is also well known that a first title for the 'General Theory', was 'A Monetary Theory of Production'.

> The rate of profits [...] is accordingly susceptible of being determined from outside the system of production, in particular by the level of the money rate of interest (PCMC, p.33).

The relationship between the rate of profits and the money rate of interest is one of the central themes of economic theory. In particular, the central issue is whether there exists a (unique) *natural* rate of profits which is related to the productivity of physical capital towards which the monetary interest rate would converge or whether the sectoral rates of profits converge towards the (unique) monetary rate of interest. Representative of the debates centred on these issues is the work of Fisher (1906, 1907, 1911) and Wicksell (1898).

If the money rate of interest may determine the commodities rate of profits and the commodities rate of profits determine the distribution of the surplus, it must also be the case that the forces that determine the monetary interest rate determine distribution as well[4].

That the monetary interest rate is an important factor (but not the only one) for the determination of the distribution of the surplus is stated in a response to Garegnani (Sraffa Papers D3_12_111_144-146), asking explanations with respect to the above quotation[5]:

> ...I am convinced that the maintenance of the rate of interest by the bank or the stock exchange has played its part in determining the distribution of income among the social classes: because it is an obligatory passage for those who give and those who borrow loans ...I did not mean anything very binding (*impegnativo*), and in general I only wanted to send some signals to avoid anyone to believe that the system [of PCMC] is presented [by me] as a "foundation" for a theory of relative supplies of capital and labor! It is the negation that seems important to me[6]: as for the affirmative I have no intention of putting forward yet another mechanical theory which, in one form or another, reinforces the idea that distribution is determined by natural, or technical, or perhaps accidental circumstances, but such as to render anyway futile any

[4]This was an important point already put forward in Sraffa (1922a) and subsequently in his debate with Hayek (Sraffa, 1932a). Chapter 17 (*The Essential Properties of Interest and Money*) in Keynes (1936)' *General Theory* may be interpreted as addressing this problem using also the arguments put forward by Sraffa. The reading of textual evidence taken from Sraffa (1920b, 1921c,b,a, 1922a,b, 1925a, 1932a,b) and some conjectures on the participation of Sraffa to 'Keynes Club' (This is the way in which Sraffa would sometimes indicate in his diaries the meetings with Keynes and the members of his group., and the contribution of Sraffa in the making of Keynes (1930), 'A Treatise on Money' and of Keynes (1936) 'General Theory'Sraffa was also the translator of Keynes (1923), 'Tract on Monetary Reform'. linked with Sraffa's statement in PCMC. Keynes' General Theory (Keynes, 1936) is a monetary theory of production because fiat money, credit and debt relations are both central in allowing production to take place and are also the cause of a mismatch between production and demand. Sraffa's book has elements that lead to focus primarily on the production side, while Keynes' book is devoted to the understanding of the role that a monetary system may have in generating imbalances between supply and demand. In both worlds the role of prices is essential.

[5]Garegnani was writing a review of PCMC and writes to Sraffa in order to have clarifications about some points.

[6]In this private correspondence Sraffa emphasises the *negation* which I interpret as a way for Sraffa to stress to Garegnani that PCMC is a *Prelude to a Critique of Economic Theory*. Garegnani in his letter was in fact providing explanations concerning a possible functioning of an economic system where the elasticity of demand of loans depends on the rate of interest which is around the rate of profits (Sraffa Papers (Garegnani's letter D3_12_111_149). In the remaining part of this quotation Sraffa does in fact presents a warning with respect to interpreting PCMC as a positive or mechanical theory of the functioning of an actual economic system

> action, from one side or the other, aimed to modify it [distridution]. (Sraffa Papers, D3_12_111_149)[7]

Clearly, Sraffa sees the banks and the stock market, which have relevance to the setting of the monetary rate of interest, as being important in the determination of distribution. But the point here is very subtle. Sraffa does in PCMC show that the determination of prices and hence of distribution cannot be determined exclusively inside the economic system. There is one degree of freedom and he picks, for the importance it had and has for dominant economic theory, the profit rate to be the variable which is determined from outside the economic system. Once the profit rate is given, the distribution of the surplus is also given. This is the *negative*, i.e. the fact that the system is undetermined. But if on the contrary it were the case that the there was a theory that would *mechanically* explain the determination of the profit rate through the determination of the monetary interest rate, that in turn would make the system determined by the mechanical forces of the functioning of the banking system or the stock market. Also in this case any attempt to modify distribution would be *futile*.

The point is that the decisions with respect to the allocation of new loans depend also by the power and hence decisions of the banks that provide the *necessary financial backing* to powerful groups of companies controlled by a few individuals.

But this is not an explanation that can be understood by the mechanics of economic forces alone. This is the point that Sraffa is making when writing *I did not mean anything very binding (*impegnativo*), and in general I only wanted to send some signals to avoid anyone to believe that the system [of PCMC] is presented [by Sraffa] as a "foundation" for a theory of relative supplies of capital and labor!*[8]

As early as 1922 Sraffa did write this very clearly:

> The large industries are stimulated on their part to make themselves independent by acquiring control of a bank so as to obtain from it, without undergoing heavy impositions, **the necessary financial backing**. As a result of this opposition, however, it cannot be said, generally speaking, that either of the two opposite tendencies has the absolute upper hand over the other. The general tendency seems to be towards the elimination of this opposition by the formation of large "groups" of companies of the most varied kinds concentrated

[7] This is my own translation from Italian

> ... sono convinto che il mantenimento del saggio d'interesse da parte della banca o della borsa abbia avuto la sua parte nel determinare la distribuzione del reddito fra le clasdsi sociali: perch un passaggio obbligato per chi d e per chi prende a prestito. ... io non ho inteso dir niente di molto impegnativo, e in generale ho solo voluto metter fuori qualche segnale per evitare che si creda che il sistema viene presentato come "fondamenta" per una teoria delle offerte relative di capitale e lavoro! E' la negazione che mi sembra importante: quanto alla affermativa non ho nessuna intenzione di mettere avanti un'altra teoria meccanica che, in una forma o nell'altra, ribadisca l'idea che la distribuzione sia determinata da circostanze naturali, o tecniche, o magari accidentali ma comunque tali da rendere futile qualsiasi azione, da una parte o dall'altra, per modificarla.

[8] If I interpret this correctly it is puzzling that many so–called Sraffians have put a strong emphasis on the mechanics of the economic determination of the long term uniform profit rate. +The emphasis on mechanical forces is to be found in:

- Gravitation Garegnani
- Long-term equilibrium Neri-Kurz
- Ahit

round one or more banks, mutually related by the exchange of shares and by the appointment of Directors common to them. Within these "groups" the various interests are all equally **subject to the interests of a few individuals** who control the whole group, possessing on their own only a very few shares of the various companies. Very little is known and very little can be generalised about these groups, on account of the undetermined state of their structure, of their **unofficial character**, of the variety of the various groups, and of the continual shifting of the elements which compose them. What the public knows and feels-not only when disasters take place, fatal to the existence to some of them, or when hostilities break out between one group and another-is **the enormous financial and political power which they have and the frequent use they make of it to influence both the foreign and home policy of the Government in favour of their own interests**. Each group keeps several press organs which support its policy, and some of the accusations made against certain Ministries of being actuated by the interests not of a class, but of private concerns, and of favouring one financial group against another, have no doubt a basis of truth (Sraffa, 1922a, p.196, emphasis added).

2.2 PCMC. Implicit influence of the money rate of interest on the measurement of capital and distribution

The implicit influence of the money rate of interest to be found in PCMC is in the critique that Sraffa develops about the possibility of having a measurement of aggregate capital with the use of the Austrian notion of the 'period of production' or 'roundaboutness of capital', where as the interest rate decreases the length of the period of production increases.

Sraffa shows in PCMC that the length of 'the period of production' (Ch.VI 'Reduction to dated quantites of labour' in PCMC) cannot be independent of distribution or the rate of profits. The view put forward by authors like Jevons and Böhm-Bawerk was that as the money rate of interest would fall, the 'period of production' (roundaboutness of capital) would increase[9].

Sraffa writes:

> The reduction to dated labour terms has some bearing on the attempts that have been made to find in the **'period of production'** an independent measure of the quantity of capital which could be used, without arguing in a circle, for the determination of prices and of the shares in distribution. [What demonstrated in this book] seems to be **conclusive** in showing the impossibility of aggregating the **'periods'** belonging to the several quantities of labour into a single magnitude which could be regarded as representing the quantity of capital. The reversal in the direction of the movement of relative prices, . . . , cannot be reconciled with any notion of capital as a measurable quantity independent of distribution and prices (PCMC, p. 38, emphasis added).

The critique that Sraffa makes with respect to the measurement of capital in terms of the Austrian notion of the 'period of production' is also an implicit reference on the lack

[9] On the notion of the *period of production* and on Sraffa's personal notes on it, see Sinha (2016, Chap. 5, pp. 111–151). On Sraffa's critique of the period of production as being independent of the rate of profits, see PCMC, Chapter 6, and the harsh comment of Sraffa (1962) on the review Harrod (1961) of PCMC.

of any foundation on the monotonic inverse relation between the 'period of production' (i.e. capital) and the money interest rate.

Sraffa intervention at the 1958 Corfú International Economic Associaiton conference on the Theory of Capital is part of a discussion of Hicks (1961a). Hicks' article has the title *The Measurement of Capital in Relation to the Measurement of other Economic Aggregates*. The concluding phrase in Hicks' article is: *The Marginal Productivity of Capital is the Marginal Productivity of Roundaboutness [period of production], after all.* (Hicks, 1961a, p.31).

And it is here tha Hicks economic theory clashes against Sraffa's critique, which was published two years later. If Sraffa is right, Hicks is wrong, or the other way about.

But Sraffa was right and hence a monotonic relation between the value of capital (period of production) and the profit rate cannot be established (see also (Zambelli, 2018a)). Below we are going to show that also any monotonic relation between capital and the monetary interest rate may not be established.

3 Motivations for the inclusion of money and the monetary interest rate inside Sraffain Schemes

3.1 Neo-Walrasian General Equilibrium and Money. Exchanges between Hicks and Sraffa

One of Sraffa's objectives is to determine the

> *. . . set of exchange-values which if adopted by the market restores the original distribution of the products and makes it possible for the process to be repeated; (PCMC, p.3, emphasis added)*

so that the system is in a *self-replacing state*.

Sraffian self-replacing prices are market clearing prices. Sraffian schemes can be interpreted as budget constraints of a general economic system where the producers at the end of the production cycle and at the opening of the market have to exchange their products (endowments) with the quantities necessary for production to take place (repeating the production of the previous period) during the next production cycle[10].

There are several similarities between the production structure set by Hicks in his *Value and Capital* Hicks (1939)[11] and PCMC. Hicks does separate the production cycle (the week) and the market day (Monday) in the same way in which Sraffa does separate the production cycle (the year) with the market day (occurring after the harvest).

That there has been discussions about these matters is certified indirectly by Hicks himself when in the *Preface to First Edition* writes explicitly "I have had some very useful criticism from Mr. Sraffa" (Hicks, 1939, p.vi)[12].

[10]Hahn (1982), Hahn and Petri (2002), Fratini (2018), Garegnani (2000), Mandler (1999b,a, 2002b,a, 2005), (Negishi, 2014, p.7), Parrinello (2008), Schefold (2005, 2008), Sinha and Dupertuis (2009) are examples where Sraffian schemes are studied in relation (or as a special case) of Walrasian (intertemporal) general equilibrium. In these contributions there is a clear acknowledgment that Sraffian schemes are equivalent to the aggregate budget constraints of the producers, workers and consumers that compose the economic system.

[11]Which is considered to be a foundation for Neo–Walrasian Equilibrium

[12]From the archives, one can see that Sraffa wrote the 1927 notes in London. Hicks, immediately after graduation had a position at the London School of Economics (from 1927 to 1935). From 1935 to 1938 Hicks had a University Lecturership at Cambridge (and Fellowship of Gonville and Caius College).
"My years at Cambridge (1935-38) were mainly occupied in writing Value and Capital (Hicks, 1992)". .

Value and Capital Hicks (1939) is recognized as a foundation of Neo-Walrasian General Equilibrium. But differently from other authors Hicks aimed also at providing foundations to the notion of monetary interest rates:

> ... it is evident that any treatment which pretends to deal with the economic system as a whole ... cannot possibly regard the rate of interest in isolation. It is a price, like other prices, and must be determined with them as part of a mutually interdependent system. The problem is not one of determining the rate of interest *in vacuo*, but is really the general problem of price–determination in an economy where borrowing and lending are practised, and in which the rate of interest is therefore a constituent part of the general price-system (Hicks, 1939, Ch.XII, *The Determination of the Rate of Interest*, p.154).

Sraffa's response to Garegnani's query (see above p.7) together with the content found in PCMC is very illuminating. Sraffa's considers that the rates of profits and/or the monetary interest rates may not be determined mechanically by the working of economic forces alone. Therefore, attempts to determine distribution through a change in the power structure or policy will not be *futile*. In Hicks's world attempts to change the distribution of the surplus produced would be *futile* because of the mechanics of the working of the system that would determine distribution at full employment. For Sraffa, to repeat, the General Equilibrium prices are not determined by the mechanical forces of the markets alone and hence any attempt to influence the distribution of the surplus is not *futile*[13].

It is quite puzzling that Hicks never commented in public Sraffa's critique to economic theory, which was, clearly, also a critique addressed towards his work on general equilibrium[14].

There is no evidence of meetings occurred at the London School of Economics. But one can conjecture that Sraffa went to LSE. He had several meetings with Robbins and there is the 1932 Hayek-Sraffa-Keynes debate. There is evidence from Sraffa's diaries that they met frequently while Hicks was in Cambridge (from 1935 to 1938).

There is also a very interesting draft of a Letter from Sraffa to Hicks. It is a non dated (n.d.) document from Sraffa Papers (document C133). Most likely the letter was written after 1932 I am able to locate the writing of the letter during a period going from 1932 to 1939. The reason being that Sraffa makes a precise quote relative to Hicks' "The Theory of Wages", First Edition, Hicks (1932). Massimo Di Matteo at the University of Siena has been a student of Hicks. In a personal conversation Di Matteo has told me that he asked what Sraffa's "useful criticisms" were. He recalls that Hicks answer was that they discussed matters related with Walrasian General Equilibrium and Pareto.

[13]In support of this interpretation there is also the work of Panico (1988b, p.26):

> A well-defined thread, going throughout his writings up to Production of Commodities by Means of Commodities, can thus be found in Sraffa's work. His theoretical interests were enhanced by the need to provide firm analytical foundations for his original view of the working of the economic system. This view emphasised the role of state intervention in determining income distribution and the equilibrium position of the economy, and the fact that policy decisions are influenced by the pressures of the dominant groups, and are not the result of purely technical considerations. His earlier analyses on the influence of monetary policies on income distribution were closer to the classical and the Marxist tradition than to the neoclassical one. The study of these analyses, together with that of his contacts with Keynes, thus provides useful insights in the evolution of Sraffa's thought, underlining that his later work is not 'merely abstract exercise in pure theory'.

[14]In 1960 Sraffa sent a copy of PCMC to Hicks. Hicks wrote to Sraffa (Sraffa Papers, D3/12/111: 267268) concluding his 4 pages letter by writing *Economic theory (teachable economic theory, at least) was getting just a bit boring lately; for the second time in your life you have livened it up again.*. Hicks was preparing an article for the Review of Economic Studies and wrote *I shall certainly add a reference to your work, which is so clearly to the point.* But he did not (Hicks, 1961b).

The prices in Walrasian general equilibrium are the prices that are associated with equilibrium exchanges: no trade is allowed outside equilibrium[15]. Within Sraffian schemes the endowments are the quantities in the possession of the agents after the harvest and before the market day begins.

Evidently the self-replacing prices are market clearing prices where production (and consumption) plans are implemented due to the fact that producers and workers have the necessary purchasing power to buy the necessary means of production and the surplus, i.e. the commodities not used in production. That is, if deferred means of payments do not exist or are not generated, it has to be the case that during the market day the prices must be such that the revenues of the producers and workers match exactly their expenditures (i.e., the excess supply and the excess demand functions are zero).

The necessity of money for the exchanges is well explained by Arrow and Hahn:

> The terms in which contracts are made matter. In particular, if money is the good in terms of which contracts are made, then the prices of goods in terms of money are of special significance. This is not the case if we consider an economy without the past and the future. ... If a serious monetary theory comes to be written, the fact that contracts are indeed made in terms of money will be of considerable importance (Arrow and Hahn, 1971, pp.365-7).

The relevance of Sraffa's results have been questioned by Hahn (1982) on the ground that the uniform rate of profits assumption present in PCMC reduces Sraffian schemes to be considered just a special case of the more general Walrasian General Equilibrium. In the companion paper Zambelli (2018c) the Sraffa's assumption of the uniform rate of profits has been removed and the properties of the system have been studied under more general conditions. It is shown that there exists an enumerable infinity of price vectors and wage rates that would ensure self-reproduction of the economic system and to each of these price vectors and wage rate there is a unique vector of (almost always non uniform) profit rates and hence a unique distribution of the surplus and a different value of capital and of output (for the firms, industries and the whole aggregate). The conclusions to be

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At the beginning of the letter Hicks mentions that he was back from a few months visit to Morishima. It is quite interesting to note that both Morishima and Hicks after the publication of PCMC wrote several books and article that were dealing with issues that were central to PCMC, but both never addressed directly (or quote) the critique present in PCMC. (They both quoted only Sraffa for his editorial work on Ricardo). In particular it is in *Capital and Growth* (Hicks, 1965) and in *Capital and Time* (Hicks, 1973) that one would expect a clear response to Sraffa's critique.

This is very strange. Either Hicks (and Morishima) i) never really understood the critique and have considered Sraffa's contribution just a variant of von Neumann growth model or of Leontief input-output tables or ii) they understood the critique and decided to ignore it.

I leave to the reader the choice between these two possibilities or the finding of others.

What it is clear is that it is either Hicks that was right when he wrote in 1958 at the Corfú conference

[15] After twelve chapter defining and proving theorems about Walrasian general equilibrium Arrow and Hahn (1971) state this clearly:

> Of course, our model is in no shape to give a satisfactory formal account of the role of money. In particular it would be hard to "explain" the holding of money or why it mediates in most acts of exchange (p. 338).

> In the preciding chapter [XII, Stability with Recontracting] we concerned ourselves with the infvestigation of an extreamely artificail formulation of the "price mechanism" . In particular, we insisted that no trade takes place out of equilibrium. This restriction. strictly interpreted, is not only obviously unrealistic, but also seems to carry the logical implication that trade never takes place (p.324)

derived from Sraffian schemes do not change when the uniform rate of profits assumption is dropped.

Hence the remark made by Hahn (1982) that Sraffian schemes do to the uniform rate of profits assumption are just a very special and unique case of Walrasian general equilibrium does not or cannot hold when considering differential rates of profits.

As it was indicated in the subtitle of his work Sraffa (1960) contribution serves as a *Prelude to a critique of economic theory*. As prelude to a critique the assumption of uniform rate of profits and the absence of monetary means of exchange (which imply accounting equilibrium prices) are more than justified. In Zambelli (2018c) the assumption of uniform rate of profits has been removed and here we add by extension the possibility of having exchanges of real goods or services against deferred means of payments (money, credit and debt). The insertion of deferred means of payments inside Sraffian schemes renders the schemes a good starting point for the attempt of introducing money in the general equilibrium approach.

3.2 Marx and the Money-Commodity-Money' capitalistic mode of production.

The objective of this paper, expanding and extending Zambelli (2018c), is to introduce money, credit and debt inside Sraffian schemes. Using the words of Nuti (1971, p.33) reported in the introduction of Panico (1988a, p.7) this is done being convinced that:

> The most appropriate way of approaching the theory of distribution, reintroducing the reality of class struggle into this important branch of Political Economy, seems therefore that of combining the Sraffian relation between wage and profit rates with the little we know - not least from Marx- about the interaction of real and monetary phenomena.

Although Sraffa's work has been widely interpreted as a critique of marginalism, but it is also a critique to the embodied labour theory of value. In Chapter 6, *Reduction of Dated Quantities of Labour* Sraffa is very clear about this:

> The reduction to dated labour terms has some bearing on the attempts that have been made to find in the 'period of production' an independent measure of the quantity of capital which could be used, without arguing in a circle, for the determination of prices and of the shares in distribution. But the case just considered seems to be **conclusive in showing the impossibility of aggregating the 'periods' belonging to the several quantities of labour into a single magnitude which could be regarded as representing the quantity of capital**. The reversal in the direction of the movement of relative prices, in the face of unchanged methods of production, cannot be reconciled with any notion of capital as a measurable quantity independent of distribution and prices (PCMC, p. 38, emphasis added).

Sraffa did demonstrate that the embodied labour terms[16] when summed up cannot be used as an invariable measure of capital (and this can be seen as a critique of Ricardo

[16]The quotation is made at the end of section 47 *Pattern of the movements of individual terms with changes in distribution*. Fig.2 at page 36 of Sraffa's book shows the value of different 'labour terms' which when summed up make the value in terms of 'embodied labour' of capital, which varies as the profit rate (i.e. the distribution of the surplus) varies. This result is also stressed with the aid of Fig. 3 which is found in the same page where there is the above quotation.

labour theory of value and of the use of it made by Marx) and, most importantly, it cannot be reconciled with 'period of production' as an independent measure of of the quantity of capital[17].

Sraffian schemes are viewed as a critique but also as a solution of Ricardo's embodied labour value problem and a way to address Marx transformation problem (Marx, 1894, Capital, volume III, Ch. XII) between values and competitive prices or between values and the profit levels[18].

Marx characterizes economic systems with two circular processes: the Commodity-Money-Commodity, *C–M–C*, and the Money-Commodity-Money, *M–C–M*.

> In the circuit *C–M–C*, the money is in the end converted into a commodity, that serves as a use-value; it is spent once for all. In the inverted form, *M–C–M*, on the contrary, the buyer lays out money in order that, as a seller, he may recover money. ... He lets the money go, but only with the sly intention of getting it back again. The money, therefore, is not spent, it is merely advanced (Marx, 1867, vol. 1, p.148)

There are those that interpret Sraffa's contribution as *"antagonistic"* view with respect to Marx's work (Pilling, 1972; Lebowitz, 1973; de Brunhoff, 1975, 1990; Rowthorn, 1974; Nicholas, 2014) and those that see it as an *"harmonistic"* one (Dobb, 1975; Eatwell, 1974; Laibman, 1975; Garegnani, 1978; Lippi, 1979; Hodgson, 1982; Steedman, 1977).

The *'antagonists'* do recognize that Marx had in his analysis some *unsolved* issues (i.e., the relation between commodity, money and capital when defined as social necessities or determined)[19], but consider Sraffian solution of the transformation problem and the associated solution related to value theory only a solution of Ricardo's search for an *"invariable measure of value"*, but not a solution of Marx's value problem.

The major reason would be that commodity prices, money and capital can be studied in the form they may be observed from a pure economic theory point of view (as in Ricardo), but when discussing value, according to Marx, one has to consider the inescapable role of money in commodity circulation. In the capitalistic mode of production the change from the *use value* of the commodities into their *exchange value* means a change in focus from the *C-M-C* circuit to the capitalistic circuit *M-C-M*. Hence Sraffian schemes may shed light to the *C-M-C* circuit, but, as they stand, cannot shed light on the *M–C–M* circuit, which, according to Marx, is what characterizes the capitalistic mode of production, because *it is money which is the form that the commodities take*[20].

[17] A first reading may lead to the conclusion that the critique to economic theory based on the impossibility of measuring capital or the values in general is confined to the adherence to the labour theory of value. This would be incorrect. Zambelli (2018c), for example, has shown that the values, even when the prices and values are determined having as *numéraire* the surplus, are surely a function of distribution. This is also shown below, in this article. Furthermore, Zambelli (2018a) presents the results of an empirical investigation where the *numéraire* is the agricultural sector and the conclusion is that there is NOT a simple and unique measure of capital which could be considered to be independent from distribution. This is a 'modern' critique of modern neoclassical macro and microeconomics which does not depend on the labour theory of value and on the Austrian notion of the 'period of production'.

[18] For a contemporary discussion of the transformation problem see also, among many, Dobb (1967), Laibman (1973), Samuelson (1971), Seton (1957), Shaikh (1984), Steedman (1977), Winternitz (1948)

[19] Pilling (1972); de Brunhoff (1973, 1975) discuss the difference between Ricardo (and a Ricardian interpretation of the work of Sraffa made by the neo-Ricardians) and Marxian theory of value as it is dealt with in Marx's Capital. *"For Marx did not attempt to construct a pure economic theory, his field of problems and his point of departure is quite different from that of Ricardo and that of pure economics"* (de Brunhoff, 1973, p.423). Crucial points are i) Marx's distinction between "labour-time", "abstract labour", "labour force power" and socially determined labour and ii) the different meaning given to the notion of money.

[20] Marx's critique to Ricardo on this point may be summarized by the following:

Clearly a formal treatment of money in the context of Sraffian schemes may be necessary if one aims at shedding light to the mechanics of the economic aspects of the C-M-C circuit and or the M-C-M one.

3.3 Empirical short-run and long-run foundations for Sraffian schemes

Recent work has attempted to use Sraffian schemes also empirically: Han and Schefold (2006); Degasperi and Fredholm (2010); Mariolis and Tsoulfidis (2011); Shaikh (2016); Mariolis and Tsoulfidis (2016); Boglioni and Zambelli (2017); Zambelli et al. (2017); Zambelli (2018a); Boglioni and Zambelli (2018a).

These are all contributions that imply a uniform rate of profit and hence market clearing (i.e., long-run equilibrium).

There is the necessity of modifying Sraffian schemes so that they can be used for short-run as well long-run analysis of the functioning of the economic systems.

A first modification is to allow for the rates of profits to be non-uniform and this has been achieved in Zambelli (2018c).

A second modification is to introduce deferred means of payments (money, credit and debt) inside Sraffian schemes. Extending the approach present in Zambelli (2018c), this is achieved in this paper.

There are other companion papers where modifications of Sraffian schemes are made so as to allow empirical uses for analysis and policy prescriptions. But this requires that the non-uniform rate of profits and the market clearing assumptions are removed.

In the following companion papers, based on the results of this paper, further modifications are introduced:

i Zambelli (2018d) treats the issue presented here in more depth;

ii Zambelli (2018b) does study Sraffian schemes when the self-replacing assumption is removed;

iii Bracci and Zambelli (2018a) Does addresses the issues associated with basic and non-basic commodities. It is shown that for empirical and theoretical work the distinction is important when interpreted as Sraffa did, but it is important and useful for empirical applications;

> With him [Ricardo], however, wage labour and capital are again conceived as a natural, not as a historically specific social form [*Gesellschaftsform*] for the creation of wealth as use value; i.e. their form as such, precisely because it is natural, is *irrelevant*, and is not conceived in its *specific* relation to the form of wealth, just as wealth itself, in its exchange-value form, appears as a merely formal mediation of its material composition; thus the specific character of bourgeois wealth is not grasped precisely because it appears there as the adequate form of wealth as such, and thus, although *exchange value* is the point of departure, the specific economic forms of exchange themselves play no role at all in his economics. Instead, he always speaks about distribution of the general product of labour and of the soil among the three classes, as if the form of wealth based on exchange value were concerned only with use value, and as if exchange value were merely a ceremonial form, which vanishes in Ricardo just as money as medium of circulation vanishes in exchange. Therefore, in order to bring out the true laws of economics, he likes to refer to this relation of money as a merely formal one. This does not mean that commodities do not have *use values*, *but it is money which is the form that the commodities take.* Hence also his weakness in the doctrine of money proper (de Brunhoff, 1973, p.429, author translation from Marx's *Grundrisse*).

iv Bracci and Zambelli (2018b) does use the methods and theoretical results present in Zambelli (2018b) to formalize the Hayek-Keynes-Sraffa debate on the importance of money;

v Boglioni and Zambelli (2018b) proposes a measure of the surplus and of its distribution which is world-wide. This approach is based on the definition of distribution presented in Zambelli (2018c) and below in eqs. 4.21, 4.22, 4.24. The used databank is the same as in Boglioni and Zambelli (2017), Zambelli et al. (2017), Zambelli (2018a),Boglioni and Zambelli (2018a);

vi Zambelli (2018f) shows that the results present in PCMC and in all the works based on it do not depend on the use of fixed coefficient production functions;

vii Zambelli et al. (2018) shows that when using the standard mainstream aggregate production function to measure productivity serious mistakes are made, These mistakes may be avoided or reduces when using the modified Sraffian schemes - as it is done in the above contributions.

4 Modified Sraffian schemes with money, credit and debt.

4.1 Production, consumption and distribution.

4.1.1 Production.

Following Sraffa's method of investigation here we

> *... consider an extremely simple society .... Commodities are produced by separate industries and are exchanged for one another at a market held after the harvest (PCMC, p.3).*

and will search for the

> *... set of exchange-values which <u>if adopted</u> by the market restores the original distribution of the products and makes it possible for the process to be <u>repeated</u> (PCMC, p.3).*

These exchange values are to be distinguished from the values and the exchanges that would actually take place during the coming *market* day, but are elements to be used inside thought experiments aiming at studying the conditions that might allow the economic system to repeat a previously observed production cycle. Following the tradition of the classics we can call these values *natural prices*[21]

A privileged observer, after the *harvest* and at the beginning of the *market day*, may know the *methods of production* used during the previous production cycle (let us say a year), the quantity of labour used and available and the actual produced b-quantities *brought* to the market:

$$[b_1, b_2, \dots b_i, \dots, b_n]^T \tag{4.1}$$

$i = 1, 2, \dots, n$.

[21] These prices can be seen as bookkeeping values to ensure allocation of purchasing power among the producers and the workers that is necessary for the system to be in a self-replacing condition.

The system is self-replacing when the exchanges during the market day are (or would be) at the end of the market day the producers would have the means of production necessary to replicate the production process of the previous year.

$$\overbrace{b_i \xrightarrow{exchange} a_i^1, a_i^2, \ldots, a_i^j, \ldots a_i^{n-1}, a_i^n, \ell_i}^{Market\ Day} \xrightarrow{production} b_i \tag{4.2}$$

where a_i^j is the mean of production j used in the production of good i, the quantity b_i. For the whole system this circularity is summarized with the following standard notation

$$\begin{array}{ccccccccc}
b_1 & \xrightarrow{exchange} & a_1^1 & \ldots & a_1^j & \ldots & a_1^n & \ell_1 & \xrightarrow{production} & b_1 \\
b_2 & \xrightarrow{exchange} & a_2^1 & \ldots & a_2^j & \ldots & a_2^n & \ell_2 & \xrightarrow{production} & b_2 \\
\vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \xrightarrow{production} & \vdots \\
b_i & \xrightarrow{exchange} & a_i^1 & \ldots & a_i^j & \ldots & a_i^n & \ell_i & \xrightarrow{production} & b_i \\
\vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \xrightarrow{production} & \vdots \\
b_n & \xrightarrow{exchange} & a_n^1 & \ldots & a_n^j & \ldots & a_n^n & \ell_n & \xrightarrow{production} & b_n
\end{array} \tag{4.3}$$

These quantities may be written in compact matrix notation in the following way:

$$\mathbf{b} \xrightarrow{exchange} \mathbf{A}, \mathbf{L} \xrightarrow{production} \mathbf{b} \tag{4.4}$$

where: $\mathbf{A}$ is an $n \times n$ matrix whose components are the used means of production $\{a_i^j\}$; $\mathbf{L}$ is a $n \times 1$ vector whose elements $\{\ell_i\}$ is the labour used in production; $\mathbf{b}$ is $n \times 1$ vector whose element $\{b_i\}$ is the harvest of good i[22].

4.1.2 Surplus available for distribution or consumption

Note that once we have the observed and hence known quantities of eq. 4.3 or eq. 4.4 we know also the Physical Surplus that was produced during the production period. That is

$$\begin{array}{ccccc}
s_1 & = & b_1 & - & \sum_{i=1}^n a_i^1 = b_1 - \mathbf{e}^T\mathbf{a}^1 \\
s_2 & = & b_2 & - & \sum_{i=1}^n a_i^2 = b_2 - \mathbf{e}^T\mathbf{a}^2 \\
\vdots & = & \vdots & - & \vdots \\
s_j & = & b_j & - & \sum_{i=1}^n a_i^j = b_j - \mathbf{e}^T\mathbf{a}^j \\
\vdots & = & \vdots & - & \vdots \\
s_n & = & b_n & - & \sum_{i=1}^n a_i^n = b_n - \mathbf{e}^T\mathbf{a}^n
\end{array} \tag{4.5}$$

where: s_j is the surplus of commodity j available for distribution after the quantities $\{a_i^j\}$ have been put aside for the next year production or, alternatively, is the quantity b_j produced in the previous period which is left once the inputs used in production have been removed. In compact matrix notation we have:

$$\mathbf{S} = (\mathbf{B} - \mathbf{A})^T\mathbf{e} \tag{4.6}$$

where: $\mathbf{e}$ is an $n \times 1$ unit or summation vector (each element is 1); T is the transpose operator; $\mathbf{S}$ is the $n \times 1$ Physical Surplus vector or Physical *Net National Product*; $\mathbf{B}$ is an $n \times n$ diagonal matrix having as elements in the diagonal the elements of gross production $\mathbf{b}$ and the other elements are $0s$.

Given the self-replacing condition in this paper the surplus is also consumption.

[22]In the sequel we will indicate with bold lowercase letters the vectors, with the exception of the labour vector $\mathbf{L}$ and the physical surplus vector $\mathbf{S}$, and uppercase letter matrices. To simplify notation the the row i of a matrix, let us take as example matrix $\mathbf{A}$, would be written in bold in the following way, $\mathbf{a}_i$, while the column j will be written as $\mathbf{a}^j$.

4.2 Bookkeeping self–replacing prices.

> *The significance of the equations is simply this: that if a man fell from the moon on the earth, and noted the amount of things consumed in each factory and the amount produced by each factory during a year, he would deduce at which values the commodities must be sold, [. . .] and the process of production repeated.* Sraffa(1927 or 1928, D3/12/7, emphasis added)[23]

Let us now search for the prices and wage rate that would allow self-reproduction of the system and would be associated with a distribution of the surplus $\mathbf{S}$, i.e., matrix $\mathbf{C}$[24]. This is a matrix with $n \times 1$ rows and n columns. The rows from 1 to n are the aggregate physical consumption of the owners of the industries, i.e. each industry has final consumption $\mathbf{c}_i$ with $i = 1, \ldots, n$. The $n \times 1$ row is the aggregate consumption of the workers, ${\mathbf{c}_{n+1}}$[25].

In synthesis the prices would have to be such that the exchange process during the market day would lead to the following distribution of the gross product and employment available: $\mathbf{b} \xrightarrow{exchange} \mathbf{A}, \mathbf{L}, \mathbf{S}$. The quantity $\mathbf{S}$, distributed as in $\mathbf{C}$, is consumed while the means of production $\mathbf{A}$ and the $\mathbf{L}$ is transformed into the output $(\mathbf{A}, \mathbf{L} \xrightarrow{production} \mathbf{b})$.

Given an arbitrary vector of prices and a wage rate we would have the following:

$$\begin{array}{rcl} a_1^1 p_1 + \ldots + a_1^j p_j + \ldots + a_1^n p_n + \ell_1 w & \lessgtr & b_1 p_1 \\ a_2^1 p_1 + \ldots + a_2^j p_j + \ldots + a_2^n p_n + \ell_2 w & \lessgtr & b_2 p_2 \\ \vdots + \vdots + \vdots + \vdots + \vdots + \vdots & \lessgtr & \vdots \\ a_i^1 p_1 + \ldots + a_i^j p_j + \ldots + a_i^n p_n + \ell_i w & \lessgtr & b_i p_i \\ \vdots + \vdots + \vdots + \vdots + \vdots + \vdots & \lessgtr & \vdots \\ a_n^1 p_1 + \ldots + a_1^j p_j + \ldots + a_n^n p_n + \ell_n w & \lessgtr & b_n p_n \end{array} \tag{4.7}$$

which in matrix notation could be written as:

$$\mathbf{A}\mathbf{p} + \mathbf{L}w \lessgtr \mathbf{B}\mathbf{p} \tag{4.8}$$

[23] Catalogue of Sraffa Papers, Wren Library, Trinity College, Cambridge, edited by Jonathan Smith. This quotation is from the archives of the Wren Library. It is dated between 1927 and 1928. See Gilibert (2006, p.28) and Gilibert (2003). From the quotation above I have removed the phrase (to be inserted instead of the dots, *[...]*) *" if the rate of interest must be uniform"*. The reason being that in this paper the uniform rate of profits condition is considered an assumption that was fundamental for Sraffa's critique of economic theory, but that it is not necessary for the use of Sraffa's schemes and method as tools for a reconstruction of economic theory. In the companion article Zambelli (2018c) a case is made for the removal of this assumption. As we will see below our task is here that of searching for the prices and deferred means of payments that will allow the system to replicate. The idea of a *man from the moon* that has to search for the prices that would allow the system to replicate is powerful and can be used in different context to pose attention to the role of an external observer. In the sequel we will adopt the framework present in PCMC and search for the prices and the credit and debt relations that would allow the system to reproduce. The *new man from the moon* will have to:

> *a) note the amount of things consumed in each factory; b) note the amount produced by each factory during the previous year; c) note the standing deferred payments contracts. He would have to deduce i) at which values the commodities must be sold and ii) the old deferred payments may eventually be written off, ii) new one issued, iv) and the process of production repeated.*

[24] A detailed description of this matrix is to be found in discussion on the properties of this matrix

[25] For this paper we mean by consumption both final consumption and eventual *net investment*. If the thought experiment concerns the computation of the prices that would allow the system to replicate, net-investment (and dis-investment) would have to be zero. The possibility of net investment to be positive (or negative) is considered in the companion paper, Zambelli (2018e).

The left hand sides of eqs. 4.7 and 4.8 would be the individual industries production expenditures (or costs of production) while the right hand side would be the industries production revenues.

A general definition of profit rates is the one for which the following equation holds:

$$(\mathbf{I} + \mathbf{R})\mathbf{A}\mathbf{p} + \mathbf{L}w = \mathbf{B}\mathbf{p} \tag{4.9}$$

where $\mathbf{R} = diag(\mathbf{r})$, with $\mathbf{r} = [r_1, r_2, \ldots r_i, \ldots, r_n]^T$[26,27] .

4.3 Exchanges and deferred means of payments

> In the world in which we live, however, most acts of exchange are exchanges of goods for money and money for goods, A real household, if constrained to the mediation of money, may be willing to exchange something of one good for money on the supposition that the money so acquired will be used in the exchange of some other good (Arrow and Hahn, 1971, p.338).

When we discuss an economic system, whether real or virtual, we operate in a world of exchanges where it is tautological to say that if someone buys there is another one that sells.

The prices and the wage rate in the above eq. 4.7 and eq. 4.8 are to be seen as virtual or bookkeeping prices that may or may not be actual exchange prices.

They cannot be actual exchange prices when there is at least one industry for which the bookkeeping expenditures (left hand side) would be greater than bookkeeping revenues (right hand side).

In this case there would not be enough purchasing power for the exchanges to take place and some industries would be left with unsold commodities which means that most industries would not be in the self-replacing condition.

This is true if we exclude the possibility of exchanges where actual physical quantities are sold in exchange of deferred means of payments. With the idea of been as general as possible, we may call these deferred means of payments *I Owe Yous (IOUs)*. It includes all forms of financial contracts where there is a promises to return goods in the future. These obligations may be with an explicit delivery date, as in the case of the *forward* contracts associated with real goods *forward* markets, or with a relatively loose delivery date as in the case of standard means. This last category of includes cash, checks, debt and credit accounts, bonds and so on.

4.4 The taxonomy of exchanges

Here two types of exchanges will be considered;

Barter exchanges or equivalents where a commodity is sold in exchange with another commodity without the use of means of payments or only through a temporary use

[26]The *accounting* implicit in eq. 4.9 is consistent with the choice made in PCMC. We think that the alternative choice of computing the profit rates as including also labour costs would be simpler and more appropriate. Nevertheless the qualitative conclusions would not change. The difference is that, for example, eq. 4.9 would have to be written as $(\mathbf{I} + \mathbf{R})(\mathbf{A}\mathbf{p} + \mathbf{L}w) = \mathbf{B}\mathbf{p}$. The proposition made in the sequel of the article may be appropriately modified to consider this different accounting. We leave an analysis of the consequences of this alternative to another article.

[27]Eq. 4.9 is a system of n equations with $2n + 1$ variables: n prices $\mathbf{p}$; n profit rates $\mathbf{r}$; the wage rate, w. With the addition of a *numéraire* (eq.4.23) the number of equations is $n + 1$ and if the prices are given the number of unknowns reduces to $n + 1$, i.e, the profit rates $\mathbf{r}$ and the wage rate w.

of means of payments. Given an accounting period (a day, a week, a month, a year ...) an equivalent to a barter exchange occurs also when means of exchanges are temporarily used to buy or sell during the period quantities of commodities without involving deferred payments to occur in the future periods. In other words, an exchange of commodities between two persons may be considered to be a barter exchange if it takes place without direct or indirect use of deferred means of payments, as if it were carried through triangular trade (PCMC, p.4);

Credit and debt exchanges where a commodity is exchanged with a promise to pay back at a future point in time (deferred payment). The person selling the commodity would see its credit increase and the person buying the commodity would have its debt increase. This might take place by the writing off of previously generated means of payments or by issuing new means of payments. This could take place through an institution, like the banking system, of by a direct writing of contracts.

Paraphrasing Irwin Fisher we may classify exchanges into three groups[28]:

i) **Barter exchanges** or equivalents. The exchange of goods against goods. The exchanges take place so that those buying the necessary commodities are selling the commodities in their possession. We may consider also as barter equivalents those exchanges that take place during the period as if they were barter exchanges where means of payments are used temporarily. In other words barter exchanges are those exchanges where goods are exchanged without observing, in the hand of the individual participating to these exchanges, changes in the financial positions;

ii) **Credit-debt exchanges** the exchange of I Owe You (IOUs) against goods, or purchase and sale (this occurs because those buying have the possibility of conducting the exchange because those in the possession of the commodities to be bought accept future promises to pay, deferred means of payments);

iii) **Pure financial exchanges**. Exchange of one type of IOUs against another type of IOUs, or changing IOUs. These types of exchange are financial contracts or promises to pay with real goods or services at a deferred point in time. Clearly, there can be many types of different financial contracts that can be generated and exchanged. These contracts might change the future obligations, but do not imply exchanges of type i) or ii) as defined above. In this paper we are going to classify only the generation of deferred means of payments used for payments of type ii) and the future interest payments on these deferred means of payments (see below section 4.8);

4.5 Barter exchanges.

Associated to each industry i and the workers the possible barter revenues are:

$$\begin{array}{rcll} \textit{Barter Revenues}(i) & = & b_i^{Barter} p_i & i = 1, \ldots, n \\ \textit{Barter Revenues}(n+1) & = & \mathbf{e}^T \mathbf{L}^{Barter} w & \end{array} \tag{4.10}$$

[28]The precise quote is the following:

> ... we may classify exchanges into three groups: the exchange of goods against goods, or barter; the exchange of money against money, or changing money; and the exchange of money against goods, or purchase and sale ((Fisher, 1911, p.13).

This quotation is taken from Chapter 2, *The equation of exchange"* of his work *"Purchasing Power of Money"*. Money is there seen, in my view, as a deferred mean of payment as it is the case for all the IOUs.

While the barter expenditures are:

$$\begin{array}{rcl} \textit{Barter Expenditures}(i) & = & \mathbf{a}_i^{Barter}\mathbf{p} + \ell_i^{Barter} w + \mathbf{c}_i{}^{Barter}\mathbf{p} \quad i = 1, \ldots, n \\ \textit{Barter Expenditures}(n+1) & = & \mathbf{c}_{(n+1)}^{Barter}\mathbf{p} \end{array} \tag{4.11}$$

Therefore the whole system, for given prices and wages, may be constrained in the following way:

$$\begin{bmatrix} \mathbf{B}^{Barter} & \mathbf{0}_{n\times 1} \\ \mathbf{0}_{1\times n} & \mathbf{e}^T\mathbf{L}^{Barter} \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ w \end{bmatrix} = \begin{bmatrix} \mathbf{A}^{Barter} & \mathbf{L}^{Barter} \\ \mathbf{0}_{1\times n} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ w \end{bmatrix} + \mathbf{C}^{Barter}\mathbf{p} \tag{4.12}$$

where $\mathbf{B}^{Barter}$ is the quantities actually sold in exchange of real quantities $\mathbf{A}^{Barter}$, labour $\mathbf{L}^{Barter}$ and consumption $\mathbf{C}^{Barter}$.

When the prices are not self-replacing prices we have that $\mathbf{B}^{Barter} < \mathbf{B}$, $\mathbf{A}^{Barter} < \mathbf{A}$, $\mathbf{L}^{Barter} < \mathbf{L}$, $\mathbf{C}^{Barter} < \mathbf{C}$.

4.6 Barter and Credit–Debt exchanges.

Clearly those industries that are in the condition for which the expenditures would be higher than the revenues would not have the purchasing power to buy the means of production necessary to replicate the production of the previous period. These industries are in a condition of potential financial deficit. Concurrently there would be industries which would not be able to sell all of their product. The industries in "financial deficit" would be able to purchase the necessary means of production only by agreeing to a deferred payment to take place during the years to follow and at the same time the industries in potential "financial surplus" would be able to sell all of their product by agreeing to deferred payments by the borrowers.

4.6.1 A *virtual* bank as clearing house.

If the quantities **A**,**L**, **S** (and **C**) have to be restored independently from any given prices we must have that in general the inequalities of eq. 4.7 may be *"eliminated"*. This is possible only if there are possibilities for deferred payments which might take the form of *I Owe Yous (IOUs)* or debt and credit relations (i.e. selling of real quantities or labour now by an agent with the promise of another agent of paying back at a future point of time). Therefore a producer may sell a part of the physical quantities produced in return of other physical quantities (*Barter Revenues*)[29] or in exchange of a future promise to pay (Δ*Credit*$(\cdot)$). In the case of the workers the same would apply, in the sense that they could sell a part of labour in exchange of liquid means of payments and another part in exchange of future promises to pay by the employer.

Whether this takes place bilaterally of through a clearing house is here not important. For the simplicity of the argument we might presume here that there is a bank or clearing house: a central bank which takes care of the good functioning of the paying system.

For each industry and for the workers we have the following:

$$\textit{Revenues}(\cdot) = \textit{Barter Revenues}(\cdot) + \Delta\textit{Credit}(\cdot) \tag{4.13}$$

[29]Whether these exchanges take place triangularly or by use of a common accepted mean of payments is here not relevant. What is relevant is that during the *market day* the quantity sold with the use of means of payments is used to buy produced goods by others. We consider these exchanges as if they were barter exchanges.

The same would apply for the expenditures where for each industry and the workers we have the following:

For each industry and for the workers we have the following:

$$Expenditures(\cdot) = Barter\ Expenditures(\cdot) + \Delta Debt(\cdot) \tag{4.14}$$

Recall that the current exercise is to study the conditions that would allow the economic system to be in a self-replacing state as described above and summarized in the relation **??**. Associated to each industry i and the workers the revenues are:

$$\begin{array}{rcccc} Revenues(i) & = & b_i p_i & = & b_i^{Barter} p_i + b_i^{Credit} p_i \\ Revenues(n+1) & = & \mathbf{e}^T \mathbf{L} w & = & \mathbf{e}^T \mathbf{L}^{Barter} w + \mathbf{e}^T \mathbf{L}^{Credit} w \end{array} \tag{4.15}$$

While the expenditures are:

$$\begin{array}{rcl} Expenditures(i) & = & \mathbf{a}_i \mathbf{p} + \ell_i w + \mathbf{c}_i \mathbf{p} \quad = \\ & = & \mathbf{a}_i^{Barter} \mathbf{p} + \ell_i^{Barter} w + \mathbf{c}_i{}^{Barter} \mathbf{p} \quad + \\ & + & \mathbf{a}_i^{Debt} \mathbf{p} + \ell_i^{Debt} w + \mathbf{c}_i{}^{Debt} \mathbf{p} \end{array} \tag{4.16}$$

$$Expenditures(n+1) = \mathbf{c}_{(n+1)} \mathbf{p} = \mathbf{c}_{(n+1)}^{Barter} \mathbf{p} + \mathbf{c}_{(n+1)}^{Debt} \mathbf{p}$$

In matrix notation we have:

$$\begin{bmatrix} \mathbf{B}^{Barter} & \mathbf{0}_{n\times 1} \\ \mathbf{0}_{1\times n} & \mathbf{e}^T \mathbf{L}^{Barter} \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ w \end{bmatrix} + \begin{bmatrix} \mathbf{B}^{Credit} & \mathbf{0}_{n\times 1} \\ \mathbf{0}_{1\times n} & \mathbf{e}^T \mathbf{L}^{Credit} \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ w \end{bmatrix} =$$

$$= \begin{bmatrix} \mathbf{A}^{Barter} & \mathbf{L}^{Barter} \\ \mathbf{0}_{1\times n} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ w \end{bmatrix} + \mathbf{C}_{(n+1)\times n}^{Barter} \mathbf{p} + \begin{bmatrix} \mathbf{A}^{Debt} \mathbf{p} & \mathbf{L}^{Debt} w \\ \mathbf{0}_{1\times n} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ w \end{bmatrix} + \mathbf{C}_{(n+1)\times n}^{Debt} \mathbf{p} \tag{4.17}$$

The left-hand side are the revenues for the whole system, while the right-hand side are the expenditures[30].

The value of the issued IOUs, i.e lending, that would allow self-replacing is given by:

$$\mathbf{\Delta Credit} = \begin{bmatrix} \mathbf{B}^{Credit} & \mathbf{0}_{n\times 1} \\ \mathbf{0}_{1\times n} & \mathbf{e}^T \mathbf{L}^{Credit} \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ w \end{bmatrix} \tag{4.18}$$

The borrowing would be given by:

$$\mathbf{\Delta Debt} = \underbrace{\begin{bmatrix} \mathbf{A}^{Debt} & \mathbf{L}^{Debt} \\ \mathbf{0}_{1\times n} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ w \end{bmatrix}}_{Production\ cost\ financed\ with\ borrowing} + \overbrace{\mathbf{C}_{(n+1)\times n}^{Debt} \mathbf{p}}^{Consumption\ with\ borrowing} \tag{4.19}$$

The general set of exchanges that allow for self-replacing condition

[30]Note that the Barter values are uniquely determined by the prices and the wage rate. In the absence of the exchanges taking place with the use of deferred means of payments the system would shrink to a lower level of production. Here we consider the prices, wage rate and IOUs that would allow exchanges to be such that the self-replacing condition is potentially fulfilled. This means that the total amount bought of the means of production using IOUs is uniquely determined. Therefore the is means that that the values of the revenues purchasing capacity of consumption goods is also uniquelly detrmined and so is the split the between $\mathbf{C}_{(n+1)\times n}^{Credit}$ and $\mathbf{C}_{(n+1)\times n}^{Barter}$. If prices are uniform and given, as explained above in section A, $\mathbf{C}_{(n+1)\times n}$ is equivalent to any feasible different allocations $\bar{\mathbf{C}}$ among the consumers of the surplus $\mathbf{S}$.

$$\overbrace{\overbrace{\begin{bmatrix} \mathbf{B}^{Barter} & \mathbf{0}_{n\times 1} \\ \mathbf{0}_{1\times n} & \mathbf{e}^T\mathbf{L}^{Barter} \end{bmatrix}\begin{bmatrix} \mathbf{p} \\ w \end{bmatrix}}^{Solt\ with\ barter\ exchanges,\ value} + \overbrace{\mathbf{\Delta Credit}}^{Sold\ with\ IOUs}}^{Total\ quantity\ sold,\ value} =$$

$$= \underbrace{\underbrace{\begin{bmatrix} \mathbf{A}^{Barter} & \mathbf{L}^{Barter}_{n\times 1} \\ \mathbf{0}_{1\times n} & 0 \end{bmatrix}\begin{bmatrix} \mathbf{p} \\ w \end{bmatrix} + \mathbf{C}^{Barter}_{(n+1)\times n}\mathbf{p}}_{Bought\ with\ barter\ exchanges,\ value} + \underbrace{\mathbf{\Delta Debt}}_{Bought\ with\ IOUs}}_{Total\ quantity\ bought,\ value} \tag{4.20}$$

Clearly, the quantities of means of production to be sold in order to have self-replacying are the crops after the *harvest* $\mathbf{b}$ and total employed labour $\mathbf{e}^T\mathbf{L}$.

During the market day, as we have seen above, for self-replacing to take place we might have *barter* exchanges or *credit* exchanges.

It must always be the case that:

i) $\mathbf{B} = \mathbf{B}^{Barter} + \mathbf{B}^{Credit}$, the total quantity produced must be sold either through barter exchanges or through deferred forms of payments, lending-credit.

ii) $\mathbf{A} = \mathbf{A}^{Barter} + \mathbf{A}^{Debt}$, the means of production might be bought through *barter* exchanges or through deferred forms of payments, borrowing-debt.

iii) $\mathbf{L} = \mathbf{L}^{Barter} + \mathbf{L}^{Credit}$, labour is sold with *barter* exchanges or through credit exchanges.

iv) $\mathbf{C} = \mathbf{C}^{Barter} + \mathbf{C}^{Debt}$, the produced surplus $\mathbf{S}$ is sold (or bought) either with *barter* exchanges or credit exchanges.

v) $\mathbf{e}^T(\mathbf{\Delta Credit} - \mathbf{\Delta Debt}) = 0$, when the vectors $\mathbf{\Delta Credit}$ and $\mathbf{\Delta Debt}$ are not $\mathbf{0}$, the sum of their differences would always, obviously, be equal to zero.

4.7 Financial balances and feasible distributions.

Each producer i can buy the consumption vector $\bar{\mathbf{c}}_i$ if he has the purchasing power to do it. The purchasing power necessary to buy a given composite vector a given share of the total Physical Surplus is $\mathbf{dS}^T\mathbf{p} = \mathbf{Cp}$.

The agents would have this purchasing power if the prices, the wage rate and *Lending and Borrowing* are such that:

$$\mathbf{dS}^T\mathbf{p} = \overbrace{\begin{bmatrix} d_1\mathbf{S}^T\mathbf{p} \\ d_2\mathbf{S}^T\mathbf{p} \\ \vdots \\ d_n\mathbf{S}^T\mathbf{p} \\ d_w\mathbf{S}^T\mathbf{p} \end{bmatrix}}^{\substack{Value\ of \\ Effective\ Demands}} = \overbrace{\begin{bmatrix} b_1p_1 - \mathbf{a}_1\mathbf{p} - \ell_1 w - (\Delta Credit(1) - \Delta Debt(1)) \\ b_2p_2 - \mathbf{a}_2\mathbf{p} - \ell_2 w - (\Delta Credit(2) - \Delta Debt(2)) \\ \vdots \\ b_np_n - \mathbf{a}_n\mathbf{p} - \ell_n w - (\Delta Credit(n) - \Delta Debt(n)) \\ \mathbf{e}^T\mathbf{L}w - (\Delta Credit(n+1) - \Delta Debt(n+1)) \end{bmatrix}}^{Purchasing\ Capacity} \tag{4.21}$$

Written in a more compact notational form we have:

$$\mathbf{dS}^T\mathbf{p} = \begin{bmatrix} (\mathbf{B}-\mathbf{A})\mathbf{p} - \mathbf{L}w \\ \mathbf{e}^T\mathbf{L}w \end{bmatrix} - (\Delta\mathbf{Credit} - \Delta\mathbf{Debt}) \tag{4.22}$$

If the prices and the wage rate are expressed in terms of the surplus we have that $\mathbf{S}^T\mathbf{p}$=1, see eq.4.23[31]. If we had a different *numéraire* by dividing left and right sides of the above equation we would obtain the equation below, where the above accounting identity eq. 4.21, may be rewritten as:

$$\mathbf{d} = \overbrace{\begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \\ d_w \end{bmatrix}}^{\substack{Physical \\ and \\ Value\ Distribution}} = \overbrace{\underbrace{\begin{bmatrix} (\mathbf{B}-\mathbf{A}) & -\mathbf{L} \\ \mathbf{0}_{1\times n} & \mathbf{e}^T\mathbf{L} \end{bmatrix}\begin{bmatrix} \mathbf{p} \\ w \end{bmatrix}}_{Income} - \underbrace{\begin{bmatrix} \Delta\mathbf{Credit} - \Delta\mathbf{Debt} \end{bmatrix}}_{Lending\ and\ Borrowing}}^{Purchasing\ Capacity} \tag{4.24}$$

The distributional vector $\mathbf{d}$ is the distribution of the produced surplus $\mathbf{S}$ among producers and workers both in value terms and as fraction of the physical surplus $\mathbf{S}$.

Eq. 4.24 is the core of the analysis. Once the prices and wage rates are given the system is in self-replacing state if the vectors $\Delta\mathbf{Credit}$ and $\Delta\mathbf{Debt}$ are equal to $\mathbf{0}$ and the vector $\mathbf{d}$ has each element greater or equal 0 and its sum equal to 1. The domain of all possible vectors $[\mathbf{p}, w]^T$ for which the system is in a self-replacing condition can be found by trying all possible combinations of $\mathbf{d}$ (Zambelli, 2018c). The knowledge of the distribution vector and of the lending and borrowing among the producers and among the workers are enough information for the computation of prices, wage rates and profit rates that would allow the system to be in the self-replacing state.

Here the problem is reversed: once the prices and the wage rate are given the problem is to find the vectors $\mathbf{\Delta Credit}$ and $\mathbf{\Delta Debt}$ so that $\mathbf{d} \geq \mathbf{0}$ and $\mathbf{e}^T\mathbf{d} = 1$.

This is captured by the following:

$$\begin{bmatrix} \mathbf{p} \\ w \end{bmatrix} = \begin{bmatrix} (\mathbf{B}-\mathbf{A}) & -\mathbf{L} \\ \mathbf{0}_{1\times n} & \mathbf{e}^T\mathbf{L} \end{bmatrix}^{-1} \left(\mathbf{d} + \mathbf{\Delta Credit} - \mathbf{\Delta Debt}\right) \tag{4.25}$$

Once the inverted matrix is expanded we have the following:

$$\begin{bmatrix} \mathbf{p_d} \\ w_\mathbf{d} \end{bmatrix} = \begin{bmatrix} (\mathbf{B}-\mathbf{A})^{-1} & -\frac{\mathbf{L}}{\mathbf{e}^T\mathbf{L}} \\ \mathbf{0}_{1\times n} & \frac{1}{\mathbf{e}^T\mathbf{L}} \end{bmatrix} \left(\mathbf{d} + \mathbf{\Delta Credit} - \mathbf{\Delta Debt}\right) \tag{4.26}$$

[31]It is convenient to measure prices, $\mathbf{p}$ in terms of the purchasing power of the Physical Surplus or Physical Net National Product. Once the *numéraire* is picked to be the Surplus $\mathbf{S}$ we have by definition that the following relation should hold:

$$\mathbf{S}^T\mathbf{p} = \mathbf{e}^T(\mathbf{B}-\mathbf{A})\mathbf{p} = 1 \tag{4.23}$$

This simplifies the analysis without changing the substance of the argument. Relative price ratios do not change as the *numéraire* changes. Therefore one can shift from one *numéraire* to another without having to change the accounting relation or other things. But most importantly with this particular choice for the *numéraire* we have that the wage rate w, under certain conditions, could also be interpreted as the share of the physical surplus that goes to workers. Furthermore, as it will be shown in the next section because the value of the surplus would be 1, we also have that the Share of the Surplus has the same numerical value as the measured quantities. Last, the financial magnitudes are too in units of the Net National Product and this simplifies further the analysis by making apparent what could otherwise remain hidden.

Which can be written explicitly in terms of the distribution and financial balances of the industries and of the workers:

$$\begin{aligned}\mathbf{p_d} = (\mathbf{B}-\mathbf{A})^{-1}\big(\mathbf{d}_{n\times 1} + \mathbf{\Delta Credit}_{n\times 1} - \mathbf{\Delta Debt}_{n\times 1}\big) - \\ -\frac{\mathbf{L}}{\mathbf{e}^T\mathbf{L}}\big(d_w + \Delta Credit(n+1) - \Delta Debt(n+1)\big)\end{aligned} \tag{4.27}$$

where: $\mathbf{p_d}$ is the vector of prices consistent or determined by the distribution $\mathbf{d}$ of the surplus $\mathbf{S}$; $\mathbf{d}_{n\times 1}$ is the distribution among the n industries; d_w is the distribution of the surplus to the workers – given our choice of

$$w_{\mathbf{d}} = \frac{1}{\mathbf{e}^T\mathbf{L}}\big(d_w + \Delta Credit(n+1) - \Delta Debt(n+1)\big) \tag{4.28}$$

4.8 Deferred payments and sequences of market days

The existence of deferred means of payments implies the definition of a point in the future in which the deferred payments are written off (and eventually new contracts emerge).

In this paper we aim at identifying the set of prices that would allow self-replacing. Therefore it is important to study the effects that already existing future promises to pay already in existence may have to the determination of the set of self-replacing prices. IOUs may be generated and transferred to one period or another and this might influence, periods after period, the set of self-replacing prices. On the contrary the existing prices do have an impact on the generation on the set of feasible self replacing vectors of self replacing credit and debt vectors.

The evolution of the IOUs would be the following:

$$\begin{aligned}\mathbf{F}_t^{Assets} &= \mathbf{\Delta Credit}_t + (1+i_t^F)\mathbf{F}_{t-1}^{Assets} \\ \mathbf{F}_t^{Liabilities} &= \mathbf{\Delta Debt}_t + (1+i_t^F)\mathbf{F}_{t-1}^{Liabilities} \\ \mathbf{F}_t^{Balances} &= \mathbf{F}_t^{Assets} - \mathbf{F}_t^{Liabilities} = \\ &= \mathbf{\Delta Credit}_t - \mathbf{\Delta Debt}_t + (1+i_t^F)(\mathbf{F}_{t-1}^{Assets} - \mathbf{F}_{t-1}^{Liabilities})\end{aligned} \tag{4.29}$$

where: t is the index associated with the periodization of time; $\mathbf{\Delta Credit}_t$ and $\mathbf{\Delta Debt}_t$, dimension $(n+1)\times 1$, are variations of credit and debt as defined above in section 4.6 or in eqs. 4.21 and 4.22; $\mathbf{F}_t^{Assets}$ and $\mathbf{F}_t^{Liabilities}$, dimension $(n+1)\times 1$, are the stock of deferred means of payments present in the system at the end of period t or beginning of period $t+1$; i_t^f is an exogenous interest rate on financial contracts[32].

It is important to point out that both $\mathbf{\Delta Credit}_t$ and $\mathbf{\Delta Debt}_t$ are defined as exchanges of a financial mean of payment against real quantities, as described above in eqs. 4.18 and 4.19 respectively. That is, the total variations of credit and debt position would depend also on the interest rate on financial contracts.

Therefore for each industry and for the workers we have to consider the variations in financial positions due to the revenues and expenditures made with the use of IOUs and the payment of interest rates. These are:

$$\begin{aligned}\Delta\mathbf{F}_t^{Assets} &= \mathbf{F}_t^{Assets} - \mathbf{F}_{t-1}^{Assets} &= \mathbf{\Delta Credit}_t + i_t^F\mathbf{F}_{t-1}^{Assets} \\ \Delta\mathbf{F}_t^{Liabilities} &= \mathbf{F}_t^{Liabilities} - \mathbf{F}_{t-1}^{Liabilities} &= \mathbf{\Delta Debt}_t + i_t^f\mathbf{F}_{t-1}^{Liabilities} \\ \Delta\mathbf{F}_t^{Balances} &= \mathbf{F}_t^{Balances} - \mathbf{F}_{t-1}^{Balances}\end{aligned} \tag{4.30}$$

[32]It is conceivable to consider cases where the interest rate is different in accordance to the sectors involved. In that case i_F would have to be a vector. For the simplicity of the exposition the monetary or financial interest rate i_t^F is here assumed to be uniform.

Clearly $\Delta\mathbf{F}_t^{Assets}$ is the total variation of *Positive Financial Balances* and $\Delta\mathbf{F}_t^{Liabilities}$ is the total variations of *Negative Financial Balances*.

The temporal evolution of debit and credit can be represented in the following way:

$$
\begin{aligned}
&\textbf{Period 1}\\
&\overbrace{\xrightarrow{production}\mathbf{A}_0,\mathbf{L}_0,\mathbf{F}_0^{Balances},\ \overbrace{\mathbf{b}_1}^{first\ harvest}\ \xrightarrow{exchange}}^{first\ production\ cycle}\\
&\xrightarrow{exchange}\{\mathbf{p}_1,w_1,i_1^F\}\rightarrow\overbrace{\mathbf{A}_1,\mathbf{L}_1,\mathbf{S}_1,\mathbf{d}_1,\mathbf{C}_1,\Delta\mathbf{F}_1^{Balances}}^{first\ market\ day}\xrightarrow{production}\\
&\textbf{Period 2}\\
&\overbrace{\xrightarrow{production}\mathbf{A}_1,\mathbf{L}_1,\mathbf{F}_1^{Balances},\ \overbrace{\mathbf{b}_2}^{second\ harvest}\ \xrightarrow{exchange}}^{second\ production\ cycle}\\
&\xrightarrow{exchange}\{\mathbf{p}_2,w_2,i_2^F\}\rightarrow\overbrace{\mathbf{A}_2,\mathbf{L}_2,\mathbf{S}_2,\mathbf{d}_2,\mathbf{C}_2,\mathbf{\Delta}\mathbf{F}_2^{Balances}}^{second\ market\ day}\xrightarrow{production}\\
&\rightarrow\cdots\rightarrow\cdots\rightarrow\cdots\rightarrow\cdots\rightarrow\cdots\rightarrow\cdots\rightarrow\cdots\rightarrow\\
&\textbf{Period t}\\
&\overbrace{\xrightarrow{production}\mathbf{A}_{t-1},\mathbf{L}_{t-1},\mathbf{F}_{t-1}^{Balances}\xrightarrow{exchange}\ \overbrace{\mathbf{b}_t}^{t\text{-}th\ harvest}\ \xrightarrow{exchange}}^{t\text{-}th\ production\ cycle}\\
&\xrightarrow{exchange}\{\mathbf{p}_t,w_t,i_t^F\}\rightarrow\overbrace{\mathbf{A}_t,\mathbf{L}_t,\mathbf{S}_t,\mathbf{d}_t,\mathbf{C}_t,\mathbf{\Delta}\mathbf{F}_t^{Balances}}^{t\text{-}th\ market\ day}\xrightarrow{production}\\
&\rightarrow\cdots\rightarrow\cdots\rightarrow\cdots\rightarrow\cdots\rightarrow\cdots\rightarrow\cdots\rightarrow\cdots\rightarrow\\
&\textbf{Period } t_f\\
&\overbrace{\xrightarrow{production}\mathbf{A}_{t_f-1},\mathbf{L}_{t_f-1},\mathbf{F}_{t_f-1}^{Balances}\ \overbrace{\mathbf{b}_{t_f}}^{t_f\ harvest}\ \xrightarrow{exchange}}^{t_f\ production\ cycle}\\
&\xrightarrow{exchange}\{\mathbf{p}_{t_f},w_{t_f},i_{t_f}^F\}\rightarrow\overbrace{\mathbf{A}_{t_f},\mathbf{L}_{t_f},\mathbf{S}_{t_f},\mathbf{d}_{t_f},\mathbf{C}_{t_f},\mathbf{\Delta}\mathbf{F}_{t_f}^{Balances}}^{t_f\ market\ day}
\end{aligned}
\tag{4.31}
$$

where t is the index for the periodization and t_f is a final time, $t\in[0,t_f]$.

The above shows or indicates that once the initial conditions
$\mathbf{A}_0,\mathbf{L}_0,\mathbf{F}_0^{Balances},\mathbf{b}_1$ and the sequences of prices
$\{(\mathbf{p}_1,w_1,i_1^F),(\mathbf{p}_2,w_2,i_2^F),\ldots,(\mathbf{p}_t,w_t,i_t^F),\ldots,(\mathbf{p}_{t_f},w_{t_f},i_{t_f}^F)\}$.

Eq. 4.31 is not restricted to the self-replacing condition. We have kept this more general notation to emphasize the generality of the approach[33]. The self-replacing condition

[33]In the companion paper Zambelli (2018e) the self-replacing assumption is removed and the implications are presented

is a special case of the above sequence. It is the one where:

$$\begin{array}{ccccccccccccccc} \mathbf{b} & = & \mathbf{b}_0 & = & \mathbf{b}_1 & = & \mathbf{b}_2 & = & \ldots & = & \mathbf{b}_t & = & \ldots & = & \mathbf{b}_{t_f} & = & \ldots; \\ \mathbf{A} & = & \mathbf{A}_0 & = & \mathbf{A}_1 & = & \mathbf{A}_2 & = & \ldots & = & \mathbf{A}_t & = & \ldots & = & \mathbf{A}_{t_f} & = & \ldots; \\ \mathbf{L} & = & \mathbf{L}_0 & = & \mathbf{L}_1 & = & \mathbf{L}_2 & = & \ldots & = & \mathbf{L}_t & = & \ldots & = & \mathbf{L}_{t_f} & = & \ldots; \\ \mathbf{S} & = & \mathbf{S}_0 & = & \mathbf{S}_1 & = & \mathbf{S}_2 & = & \ldots & = & \mathbf{S}_t & = & \ldots & = & \mathbf{S}_{t_f} & = & \ldots; \end{array} \tag{4.32}$$

In the case of self-replacing the composition of surplus or net national product, $\mathbf{S}$ is constant, but this does not imply at all that the distribution of the surplus, $\mathbf{d}_t$ or $\mathbf{C}_t$ is constant as well, it depends on prices and wage rate relative to the period considered and on the existing and newly generated deferred means of payments, which in turn will also depend on the interest rate on financial contracts, i (see eq.4.29).

As in Sraffa's PCMC here we do not provide a theory of prices and distribution, but determine, for given past methods of production, use of labour and output, the set of prices and the possible financial conditions that would allow the system to reproduce itself. In other words, what is provided here are the necessary conditions

When deferred means of payments and rules concerning the form of which there repayments would have to take place we have to conclude that the set of self-replacing prices is path-dependent and hence that credit and debt is important because it has effects to distribution.

5 Example using Sraffa's numbers from PCMC (p.19): self-replacing without and with borrowing and lending

If we had excluded the possibility of borrowing and lending we would be in the conventional and traditional world of the Sraffian Schemes as in the uniform rate of profits case as in PCMC or in the non-uniform rates of profits case as in Zambelli (2018c).

These would be the cases in which the exchanges are all barter exchanges as defined in eq.4.12 and distribution is computed with eq.4.24 for the case in which there are no deferred payments contracts, $\Delta\mathbf{Credit} = \mathbf{0}$ and $\Delta\mathbf{Debt} = \mathbf{0}$.

Here we aim at identifying a general set of prices and wage rates that would allow the system to reproduce itself with and without the use of deferred payments. Once a particular element belonging to this set is given or (virtually observed, let us say a couple $\{\bar{\mathbf{p}}, \bar{w}\}$ there are two possibilities (or a combination of them) that would allow self-replacing (see eqs. 4.21, 4.22, 4.24).

The first possibility is that the the distribution (i.e., consumption of the surplus) will be such that the purchasing power of the goods sold by the owners of the means of production is exactly equal to the values of the quantities bought. In terms of the definitions of the above equation 4.13 we have that the revenues are all barter revenues and the credit revenues are 0, i.e. *Revenues*$(\cdot)$ = *Barter Revenues*$(\cdot)$. Therefore also the expenditures must be only barter expenditures. In terms of eq. 4.14 this is the case where *Expenditures*$(\cdot)$ = *Barter Expenditures*$(\cdot)$.

In this case there is not a change in the credit and debt positions due to lack of purchasing power of the owners of the means of production, i.e., $(\mathbf{\Delta Credit} - \mathbf{\Delta Debt}) = \mathbf{0}_{(n+1)\times 1}$[34].

[34]Clearly if previously issued deferred means of payments exists and the interest rate on financial contracts i_i^F is different from zero the evolution of credit and debt positions follows the dynamics of eqs. 4.29 and 4.30. When the expenditures are exclusively barter expenditures this means that there is not a

The second possibility is that these prices are not such that the revenues from the selling of the means of production (including labour) is not matched exactly with the values of the effective demands. In that case credit and debt exchanges as defined above must occur or must have occurred.

If at the beginning of the market day there were not already existing IOUs which match exactly, but with the opposite sign, the emergence of new credit and debt flows it must follow that after the next year and at the opening of the next year market day the producers and the workers would have the means of production to be exchanged and debts to be paid back and credits to be cashed in. This might happen with a reduction of consumption by those that have to pay back the debt or, alternatively, this might also happen if there is a new set of prices $\{\bar{\mathbf{p}}, \bar{w}\}$ that allow the producers and the workers to pay back the debt and cash in the credit and keep the same level of consumption as in the previous period.

Let us consider these possibilities by using as example of self-replacing condition the numbers in PCMC, p. 19.

$$\mathbf{A} = \begin{bmatrix} 90 & 120 & 60 \\ 50 & 125 & 150 \\ 40 & 40 & 200 \end{bmatrix}; \mathbf{L} = \begin{bmatrix} \frac{3}{16} \\ \frac{5}{16} \\ \frac{8}{16} \end{bmatrix}; \mathbf{b} = \begin{bmatrix} 180 \\ 450 \\ 480 \end{bmatrix}; \tag{5.1}$$

The first row is the iron industry, the second the coal industry, and the third the wheat industry. The columns of $\mathbf{A}$ indicate the means of production used as inputs by the industries. The first column is iron, the second is coal, and the third is wheat. At the end of the production period, producers have produced quantities $\mathbf{b} = [180, 450, 480]^T$ which have to be exchanged to organize production. We do not know what would happen during the market day, but if things have to be done like the previous year, we know that at the end of the market day the surplus to be distributed would be $\mathbf{S} = [0, 165, 70]^T$, derived as the difference between the gross output $\mathbf{b}$ and the means of production used to produce it (the column sum of $\mathbf{A}$, $\mathbf{eA^T}$.

5.1 Determination of prices, wage rates, profit rates and credit and debt positions that allow self-replacing

Following Sraffa's method, we do not know what would happen during the market day, but if production has to be repeated the gross production $\mathbf{b}_i$ has to be exchanged in such a way that producers i can buy the means of production $\mathbf{a}_i$ and labour ℓ_i.

In the following we will identify the set of prices and wage rate that would allow self-replacing. Credit and debt relations requires the existence of obligations that link different points in time. It is in the nature of credit to postpone exchanges to the future (deferred payments).

As shown above (see eq. 4.25), given any triple $\mathbf{A}, \mathbf{L}, \mathbf{b}$, the prices that would allow the system to be in a self-replacing state are all the combinations for which: (i) each element of the vector of distribution in shares is greater than or equal to zero, $\big(d_z \geq 0 \quad z \in [1, n+1]\big)$; (ii) the sum of all the elements is 1, $\big(\sum_{z=1}^{n+1} d_z = 1\big)$; (iii) the vector $\mathbf{\Delta Credit} - \mathbf{\Delta Debt}$ is such as to provide the necessary deferred means of payments allowing the exchanges to

change in the credit and debt position due to imbalances in the generation of purchasing power.

> But each commodity, which initially was distributed between the industries according to their needs, is found at the end of the year to be entirely concentrated in the hands of its producer (Sraffa, 1960, p.3).

take place also when barter exchanges (as defined above) are not self-replacing. The case in which $\mathbf{\Delta Credit - \Delta Debt} = \mathbf{0}_{(n+1)\times 1}$ is the case studied in Zambelli (2018c).

From the computational point of view, it is also the case that for any given distribution $\mathbf{d}$, there is a cloud of price vectors, $\mathbf{p_d}$, wage rate $w_\mathbf{d}$ and variations in $\mathbf{\Delta Credit - \Delta Debt}$.

Figure 5.1 is the pairwise domain of self-replacing prices. When individual prices fall outside the domain, the economic system cannot replicate. The larger grey area identifies values of prices that would require the emergence of new credit debt relations. The green region is the cases in which the system would be self-replacing without the need of the emergence of new credit and debt relations.

Figure 5.2 is the domain of the rates of profits. As in Figure 5.1, in order for the system to be in a self-replacing condition, the rates of profits have to fall inside the grey area. The case in which the rate of profits is uniform is identified by the black line. The domain for the case of a uniform rate of profits is the case presented in PCMC. As we can see, this is a very small subset of all the possible combinations. The green area identifies the cases in which there is no emergence of credit and debt

Figure 5.3 is a graphical representation of the major critique of economic theory presented in PCMC. It is the value of the aggregate capital used for the production of the same output and surplus as occurred during the previous production cycle. The physical quantities of the means of production and of production do not change, but the distribution may. As is clear from the figure, the value of the aggregate capital would be different with different distributions or prices. As in the previous figures the grey area is the wider self-replacying domain where financing is endogenously generated. This figure captures Sraffa's main critique of economic theory presented in PCMC (p. 38): '...the movement of relative prices, in the face of unchanged methods of production, cannot be reconciled with any notion of capital as a measurable quantity independent of distribution and prices.'

It is quite puzzling that almost 70 years after the publication of PCMC, this important critique has not yet been widely incorporated in economic theory[35]. It is also important to point out that this is not only a critique, but is also a constructive result because it indicates how to link values with distribution, where distribution is not computed as an index or in nominal terms, but is computed as a fraction of the physical surplus, or a bundle of commodities and services, available to the whole society.

Obviously the dependence of the value of capital on the distribution is not to be considered a result pertaining exclusively to the macro-level of the whole economy, but it has also to be seen in connection with the values of the means of production used by the individual industries (and the firms composing it).

Figure 5.4 shows the capital/output ratio in relation with prices (and hence with distribution) for each industry (industries are organized in columns). It is Figure 5.3 extended to the value of capital of the different industries. We can see that there is no regularity between the output/capital ratios. For a given price, let us say wheat, there is a range of different output/capital ratios associated with it[36], the only exception being

[35] See Zambelli (2018a) for a theoretical and empirical discussion and demonstration of the impossibility of a measurement of aggregate capital which would be independent of distribution and prices. This impossibility implies the total in-utility of the notion of the neoclassical aggregate production function and of the associated notion of marginal productivity of labour as a decreasing function of the labour employed. Note that here, the notion of aggregate production function is general because is extended to all the cases where capital is composite: firms, industries, and the whole economy.

[36] The industry is composed of firms. The standard microeconomic theory of the firm is based on the notion of the output produced per unit of capital: the output/capital ratio (as in the first chapter on the theory of the firm to be found in almost all textbooks on microeconomics). The explicit assumption is that this ratio is not a function of prices. The figure shows that for a given value of the quantity of

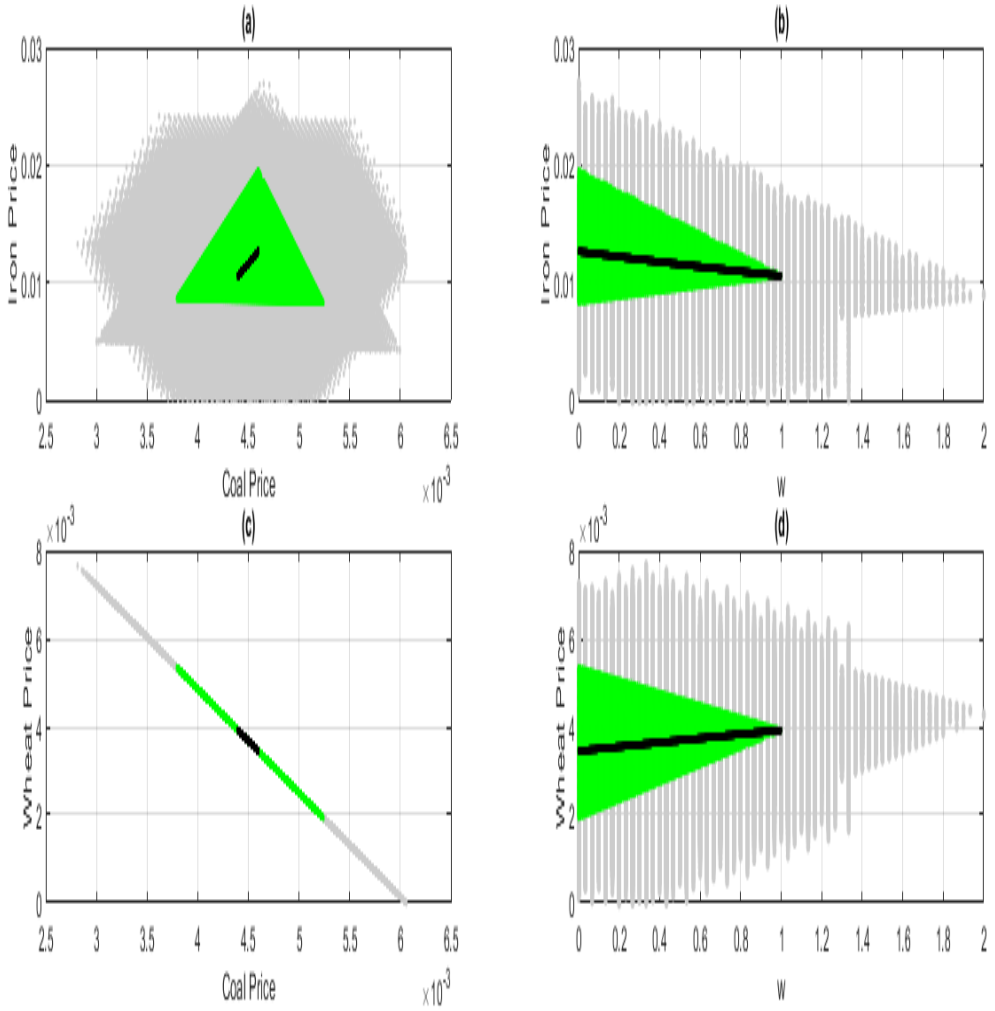

Fig. 5.1: ***Domain of the self-replacing prices and wage rate.***

Domain of the prices and the wage rate associated to all possible feasible self-replacing distribution vectors.
Grey area: set of all the prices that allow the system to be in a self-replacing state.
Green area: set of all the prices that allow the system to be in a self-replacing state without the emergencew of credit and debt. **Black line**: subset of the self-replacing prices associated with a uniform rate of profits.
(a) Domain for the prices of coal and iron.
(b) Domain for the wage rate (or share to workers of the surplus) and price of iron.
(c) Domain for the prices of coal and wheat.
(d) Domain for the wage rate (or share to workers of the surplus) and price of wheat.
The triple **A**, **L**, **b** used for the computations of the triangles was given in 5.1 (or PCMC, p. 19).
Numéraire: surplus vector, $\mathbf{S} = [0$ t.iron, 165 t.coal, 70 qr.wheat$]^T$.
Nota bene I. Given the particular choice of the *numéraire*, the numerical value of the wage rate (w) is also the share of the surplus going to the workers (d_w).
Nota bene II. The elements of prices and wage rate assuring self-replacement are not independent. They are determined by eq. 4.25.

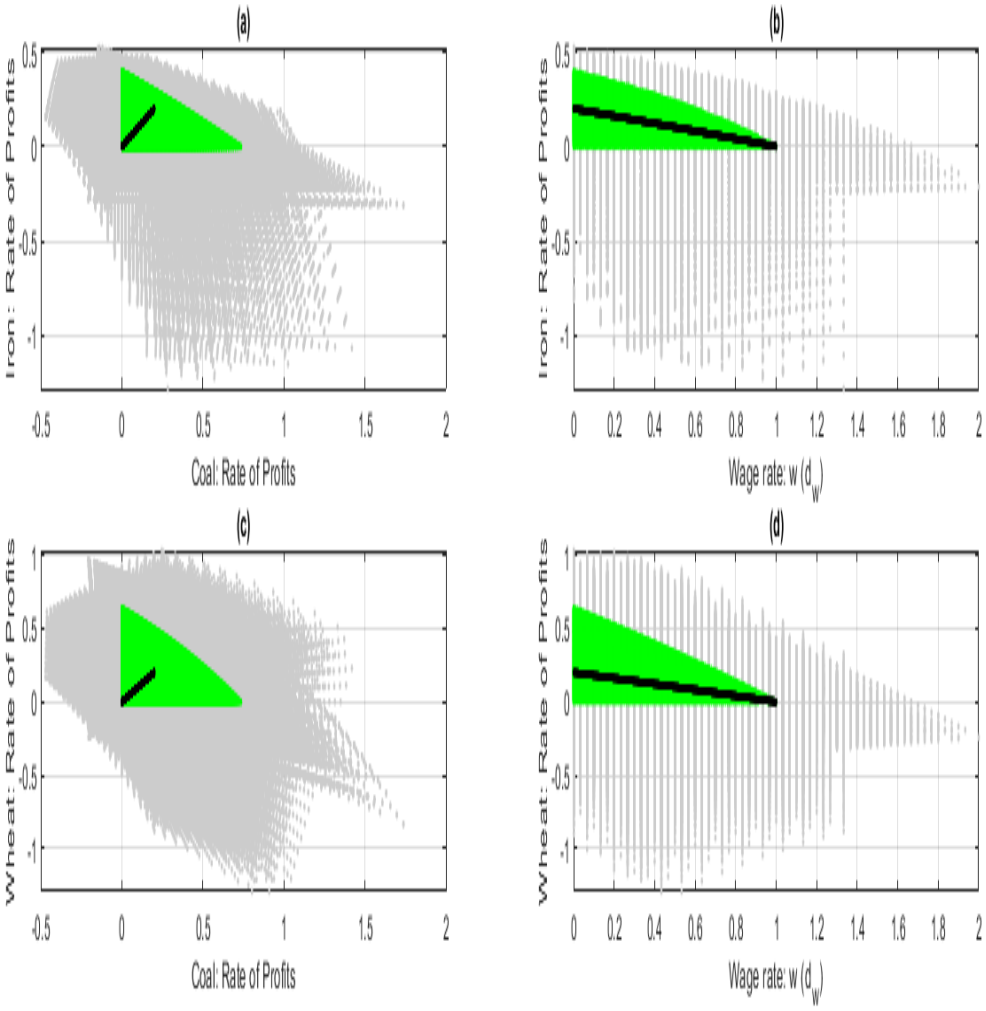

Fig. 5.2: **Domain: rates of profits and wage rates.**

Domain of rates of profits and wage rate associated to all possible feasible self-replacing distribution vectors.
Grey area: set of all the prices that allow the system to be in a self-replacing state.
Green area: set of all the prices that allow the system to be in a self-replacing state without the emergencew of credit and debt. **Black line**: the subset of the self-replacing domain associated with a uniform rate of profits.
(a) Domain for the rates of profits of coal and iron.
(b) Domain for the wage rate (or share to workers of the surplus) and rates of profits of iron.
(c) Domain for the rates of profits of coal and wheat.
(d) Domain for the wage rate (or share to workers of the surplus) and rates of profits for wheat.
The triple **A**, **L**, **b** used for the computations of the triangles was given in 5.1 (or PCMC, p. 19).
Numéraire: surplus vector, $\mathbf{S} = [0$ t.iron, 165 t.coal, 70 qr.wheat$]^T$.
Nota bene I. Given the particular choice of the *numéraire*, the numerical value of the wage rate (w) is also the share of the surplus going to the workers (d_w).
Nota bene II. The elements of the quadruple of rates of profits and wage rate assuring self-replacing are not independent. Their rates of profits are computed with eq. 4.28 and the wage rate with eq. 4.28.

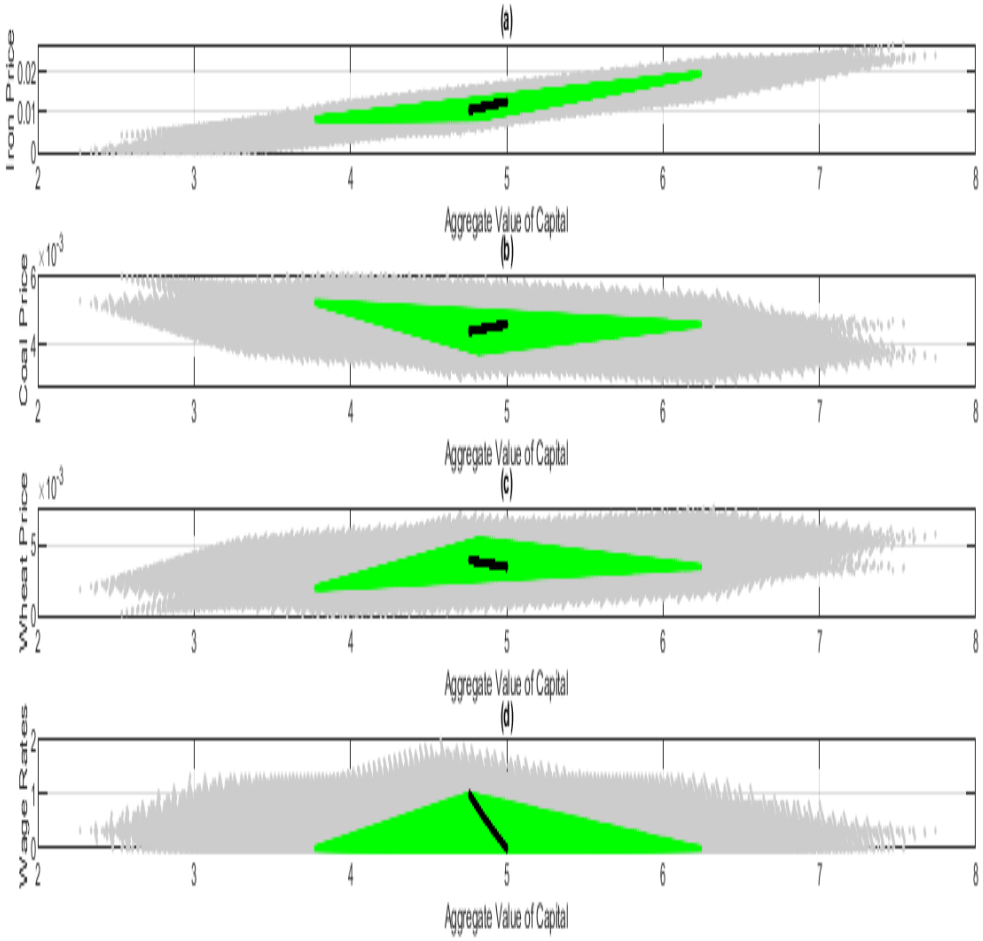

Fig. 5.3: **Domain: aggregate value of capital, prices and wage rate.**

Aggregate value of capital (the sum of the values of the physical means of production over all industries).
Grey area: set of all the prices that allow the system to be in a self-replacing state.
Green area: set of all the prices that allow the system to be in a self-replacing state without the emergencew of credit and debt. **Black line**: subset of the self-replacing domain (prices, wage and the value of capital) associated with a uniform rate of profits.
(a) Domain for the value of aggregate capital and price of iron.
(b) Domain for the value of aggregate capital and price of coal.
(c) Domain for the value of aggregate capital and price of wheat.
(d) Domain for the value of aggregate capital and wage rate.
The triple **A**, **L**, **b** used for the computations of the triangles was given in 5.1 (or PCMC, p. 19).
Numéraire: surplus vector, $\mathbf{S} = [0 \text{ t.iron}, 165 \text{ t.coal}, 70 \text{ qr.wheat}]^T$.
Nota bene I. Given the particular choice of the *numéraire* the numerical value of the wage rate (w) is also the share of the surplus going to the workers (d_w).
Nota bene II. Once the value of capital is picked, the subset of self-replacing prices is determined. Alternatively, once a vector of prices and wage rate is fixed as in Figure 5.1, the value of capital is determined.
Nota bene III. The value of the aggregate output would be the value of capital plus the value of the physical net product or surplus (which is the *numéraire* and therefore is by definition equal to 1).

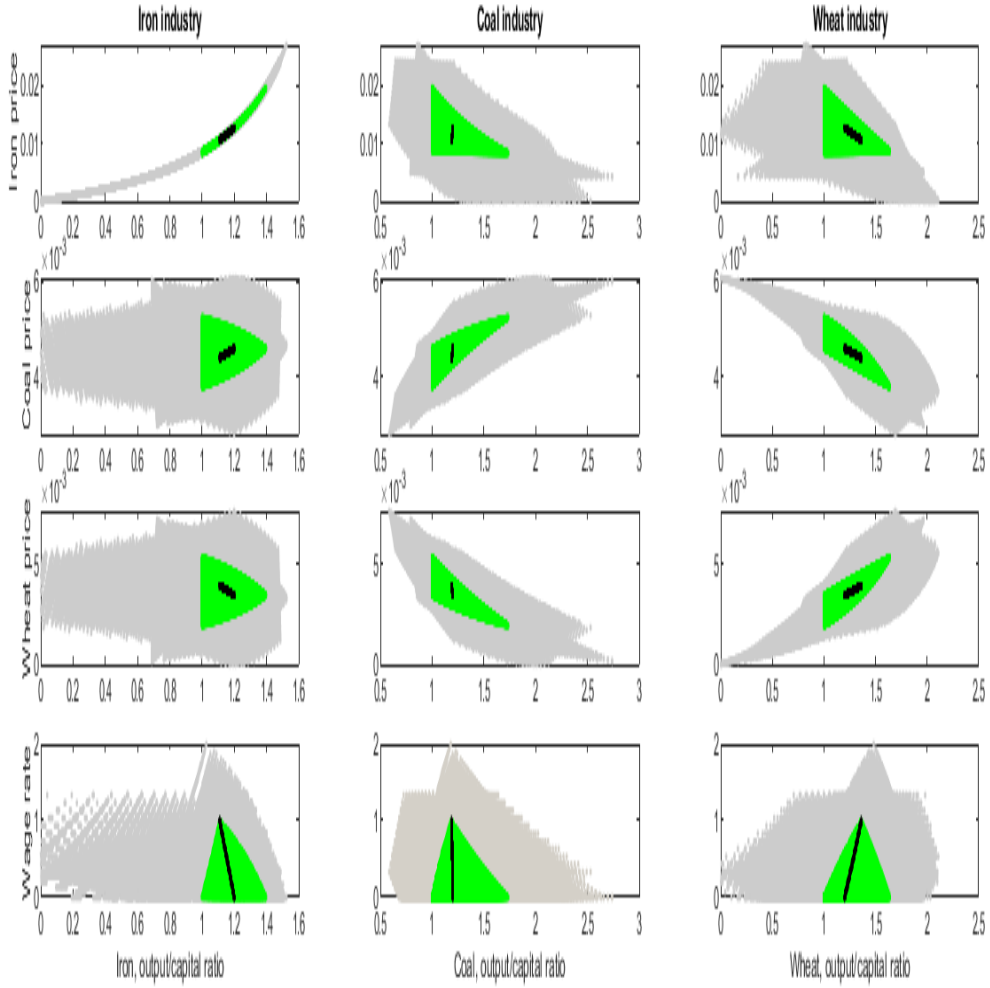

Fig. 5.4: **Domain: industry output/capital ratios, self-replacing prices and wage rate.**

Individual industry output/capital ratios, i.e. the value of the output produced divided by the value of the capital ratios, in relation with self-replacing prices.
Organized by columns are the values associated with individual industries: iron, coal and wheat.
Grey area: set of all the prices that allow the system to be in a self-replacing state.
Green area: set of all the prices that allow the system to be in a self-replacing state without the emergencew of credit and debt. **Black line**: subset of the self-replacing domain (prices, wage and the value of capital) associated with a uniform rate of profits.
The triple **A**, **L**, **b** used for the computations of the triangles was given in 5.1 (or PCMC, p. 19).
Numéraire: surplus vector, $\mathbf{S} = [0$ t.iron, 165 t.coal, 70 qr.wheat$]^T$.
Nota bene I. When the *numéraire* is the surplus vector, the numerical value of the wage rate (w) is also the share of the surplus going to the workers (d_w).
Nota bene II. Once the output/capital ratios per industry is picked, the subset of self-replacing prices is determined. Alternatively, once a vector of prices and wage rate is given, as in Figure 5.1, the value of capital per industry is determined.

Table 5.1: **Period 0 (t=0).** ***Initial values: Self-replacing without credit and debt positions and uniform rates of profits.***

	(1) **d** Distribution	(2) Δ**CD** Flow	(3) $\mathbf{F}^{Balances}$ Stock	(4) $\mathbf{p}, w$ Prices	(5) $\mathbf{r}, w$ Rates	(6) Exp. Barter	(7) Exp. Credit	(8) Rev. Barter	(9) Rev. Credit
Iron	0.18	0.00	0.00	11.52*	0.10	2.07	0.00	2.07	0.00
Coal	0.17	0.00	0.00	4.49*	0.10	2.02	0.00	2.02	0.00
Wheat	0.14	0.00	0.00	3.70*	0.10	1.77	0.00	1.77	0.00
Labour	0.51	0.00	0.00	0.51	0.51	0.51	0.00	0.51	0.00
Total	1.00	0.00	0.00	–	–	6.38	0.00	6.38	0.00

The triple **A**, **L**, **b** used for the computations was given in 5.1 (or PCMC, p. 19).
Numéraire: surplus vector, $\mathbf{S} = [0$ t.iron, 165 t.coal, 70 qr.wheat$]^T$.
Columns: (1) Distribution to industries and to the workers of the surplus **S**; (2) Variations in credit and debt positions for the industries and for the workers (flows); (3) Credit and debt positions for the industries and for the workers at the end of the period (stocks); (4) Prices for the commodities produced by the individual industries and wage rate of the workers; (5) Profit rates and wage rates ; (6) Expenditures taking place as if they were barter exchanges (physical goods with physical goods, see above section 4.6); (7) Expenditures taking place as if they were credit exchanges(physical goods exchanged now against a deferred payment in the future, IOUs, see above section 4.6); (8) Revenues taking place as if they were barter exchanges (physical goods with physical goods, see above section 4.6); (9) Expenditures taking place as if they were credit exchanges(physical goods exchanged now against a deferred payment in the future, IOUs, see above section 4.6)
(*) Multiplied by $\times 10^{-3}$

the case in which the price is the highest.

5.2 Debt extinction

Let us now consider the case where exchanges have taken place thanks to the use of deferred means of payments. Let us take as a starting point the values of table 5.1 (Period 0). If the prices and the distribution are those of 5.2 we have that for the system to be in self replacing the issuing of deferred means of payments is necessary. The values of the new credit and debt contracts are reported in table 5.2. We can see that both the iron industry as well as the coal industry are in debt and the wheat industry as well as the workers are in credit (see columns 2 and 3 - and columns 7 and 9 - where the values, differently from table 5.1 are not any longer zeroes).

There are several ways in which the debt extinction might be possible (while always keeping the condition of self-replacing).

5.2.1 Debt extinction thanks to prices changes at constant consumption

Clearly there is the possibility of having debt extinction if in period 2 new prices different from the previous ones (and moving in the opposite direction) would take place. The prices would have to be exactly those prices that would allow self-replacing, repayment of the debt and eventually the keeping of the same distribution of the surplus. That is, the same consumption. The prices that would allow the this possibility may not always exists and for the debt to be repaid there might take several periods.

[**** INSERT HERE EXAMPLE TABLES SHOWING THIS POSSIBILITY ****]

capital used, there is no unique output/capital ratio independent of the prices. Conversely, for a given price, there are a great variety of output/capital ratios. But in the simple exercise put forward here, the real quantities do not change at all. The only thing changing, as prices change, is the distribution of the physical net output, the surplus. This critique of the Marshallian or mainstream economic theory of the firm, which is basically a critique of the notion of partial equilibrium analysis, was already put forward in the 1920s by Sraffa (1925b, 1926).

Table 5.2: **Period 1 (t=1).** ***Self-replacing with changed prices with respect to Period 0. Same distribution of the surplus as in period 0 and consequent emergence of credit and debt positions.***

	(1) **d** Distribution	(2) Δ**CD** Flow	(3) $\mathbf{F}^{Balances}$ Balances	(4) $\mathbf{p}, w$ Prices	(5) $\mathbf{r}, w$ Rates	(6) Exp. Barter	(7) Exp. Credit	(8) Rev. Barter	(9) Rev. Credit
Iron	0.18	-0.15	-0.15	10.21*	0.02	1.34	0.65	1.34	0.50
Coal	0.17	-0.10	-0.10	4.32*	0.04	1.51	0.53	1.51	0.43
Wheat	0.14	0.10	0.10	4.11*	0.17	1.56	0.31	1.56	0.41
Labour	0.51	0.15	0.15	0.66	0.66	0.51	0.00	0.51	0.15
Total	1.00	0.00	0.00	—	—	4.92	1.49	4.92	1.49

The triple **A**, **L**, **b** used for the computations was given in 5.1 (or PCMC, p. 19).
Numéraire: surplus vector, $\mathbf{S}$ = [0 t.iron, 165 t.coal, 70 qr.wheat]^T.
Columns: (1) Distribution to industries and to the workers of the surplus **S**; (2) Variations in credit and debt positions for the industries and for the workers (flows); (3) Financial balances of the the industries and of the workers at the end of the period (stocks); (4) Prices for the commodities produced by the individual industries and wage rate of the workers; (5) Profit rates and wage rates ; (6) Expenditures taking place as if they were barter exchanges (physical goods with physical goods, see above section 4.6); (7) Expenditures taking place as if they were credit exchanges(physical goods exchanged now against a deferred payment in the future, IOUs, see above section 4.6); (8) Revenues taking place as if they were barter exchanges (physical goods with physical goods, see above section 4.6); (9) Expenditures taking place as if they were credit exchanges(physical goods exchanged now against a deferred payment in the future, IOUs, see above section 4.6)
(*) Multiplied by $\times 10^{-3}$

5.2.2 Debt extinction thanks to abstention from consumption

Once the situation described in table 5.2 has occurred there is the problem of repaying the debt. Here we consider the case that the prices and the wage rate remain the same as in column (4) of 5.2 and the debt is repaid. In this case, because the prices do not change the only possibility is to return the debt by abstaining from consumption.

The evolution in subsequent periods is reported in tables 5.3, 5.4, 5.5, 5.6, 5.7 before the whole debt is paid back. We can see that the iron producers are the last ones to repay the debt (after 7 periods, see table 5.8) while the coal producers repay the debt after period 3 and can have a positive share of the surplus from period 4 onward. Note that the rates of profits are, with the new prices, not uniform. The total

Table 5.8 extincted

5.3 Time to debt extinction

In the previous section 5.2.2 and in tables 5.2, 5.3, 5.4, 5.5, 5.6, 5.7, 5.8 we study the repayment path assuming that the financial interest rate is zero. Clearly the time to debt extinction increases as the exogenously given financial interest rate increases. To the point that, for sufficiently high financial interest rates, the debt extinction becomes impossible.

Figure 5.5 shows the relation between a positive financial interest rate and time to debt extinction. In the case of the example presented here the time to debt extinction increases more than proportionally having as asymptote 20% interest rate. Above this interest rate there would be an impossibility to repay the debt.

Table 5.3: **Period 2 (t=2).** ***Self-replacing with same prices as in Period 1 and partial repayment of the debt. Changed distribution of the surplus.***

	(1) **d** Distribution	(2) **ΔCD** Flow	(3) $\mathbf{F}^{Balances}$ Stock	(4) $\mathbf{p}, w$ Prices	(5) $\mathbf{r}, w$ Rates	(6) Exp. Barter	(7) Exp. Credit	(8) Rev. Barter	(9) Rev. Credit
Iron	0.00	0.03	-0.12	10.21*	0.02	1.81	0.00	1.81	0.03
Coal	0.00	0.07	-0.03	4.32*	0.04	1.85	0.03	1.85	0.10
Wheat	0.26	-0.02	0.08	4.11*	0.17	1.89	0.11	1.89	0.09
Labour	0.74	-0.07	0.08	0.66	0.66	0.64	0.10	0.64	0.02
Total	1.00	0.00	-0.00	—	—	6.18	0.23	6.18	0.23

The triple **A**, **L**, **b** used for the computations was given in 5.1 (or PCMC, p. 19).
Numéraire: surplus vector, $\mathbf{S} = [0$ t.iron, 165 t.coal, 70 qr.wheat$]^T$.
Columns: (1) Distribution to industries and to the workers of the surplus **S**; (2) Variations in credit and debt positions for the industries and for the workers (flows); (3) Financial balances of the the industries and of the workers at the end of the period (stocks); (4) Prices for the commodities produced by the individual industries and wage rate of the workers; (5) Profit rates and wage rates ; (6) Expenditures taking place as if they were barter exchanges (physical goods with physical goods, see above section 4.6); (7) Expenditures taking place as if they were credit exchanges(physical goods exchanged now against a deferred payment in the future, IOUs, see above section 4.6); (8) Revenues taking place as if they were barter exchanges (physical goods with physical goods, see above section 4.6); (9) Expenditures taking place as if they were credit exchanges(physical goods exchanged now against a deferred payment in the future, IOUs, see above section 4.6)
(*) Multiplied by $\times 10^{-3}$

Table 5.4: **Period 3 (t=3).** ***Self-replacing with same prices as in Period 1 and 2 and partial repayment of the debt. Changed distribution of the surplus.***

	(1) **d** Distribution	(2) **ΔCD** Flow	(3) $\mathbf{F}^{Balances}$ Stock	(4) $\mathbf{p}, w$ Prices	(5) $\mathbf{r}, w$ Rates	(6) Exp. Barter	(7) Exp. Credit	(8) Rev. Barter	(9) Rev. Credit
Iron	0.00	0.03	-0.09	10.21*	0.02	1.81	0.00	1.81	0.03
Coal	0.01	0.06	0.03	4.32*	0.04	1.87	0.02	1.87	0.08
Wheat	0.28	-0.05	0.03	4.11*	0.17	1.89	0.13	1.89	0.08
Labour	0.71	-0.05	0.03	0.66	0.66	0.64	0.07	0.64	0.02
Total	1.00	0.00	-0.00	—	—	6.21	0.21	6.21	0.21

The triple **A**, **L**, **b** used for the computations was given in 5.1 (or PCMC, p. 19).
Numéraire: surplus vector, $\mathbf{S} = [0$ t.iron, 165 t.coal, 70 qr.wheat$]^T$.
Columns: (1) Distribution to industries and to the workers of the surplus **S**; (2) Variations in credit and debt positions for the industries and for the workers (flows); (3) Financial balances of the the industries and of the workers at the end of the period (stocks); (4) Prices for the commodities produced by the individual industries and wage rate of the workers; (5) Profit rates and wage rates ; (6) Expenditures taking place as if they were barter exchanges (physical goods with physical goods, see above section 4.6); (7) Expenditures taking place as if they were credit exchanges(physical goods exchanged now against a deferred payment in the future, IOUs, see above section 4.6); (8) Revenues taking place as if they were barter exchanges (physical goods with physical goods, see above section 4.6); (9) Expenditures taking place as if they were credit exchanges(physical goods exchanged now against a deferred payment in the future, IOUs, see above section 4.6)
(*) Multiplied by $\times 10^{-3}$

Table 5.5: **Period 4 (t=4).** ***Self-replacing with same prices as in Period 1, 2 and 3 partial repayment of the debt. Changed distribution of the surplus.***

	(1) **d** Distribution	(2) **ΔCD** Flow	(3) $\mathbf{F}^{Balances}$ Stock	(4) $\mathbf{p}, w$ Prices	(5) $\mathbf{r}, w$ Rates	(6) Exp. Barter	(7) Exp. Credit	(8) Rev. Barter	(9) Rev. Credit
Iron	0.00	0.03	-0.06	10.21*	0.02	1.81	0.00	1.81	0.03
Coal	0.08	-0.01	0.02	4.32*	0.04	1.89	0.06	1.89	0.05
Wheat	0.25	-0.01	0.02	4.11*	0.17	1.91	0.07	1.91	0.06
Labour	0.67	-0.01	0.02	0.66	0.66	0.65	0.03	0.65	0.02
Total	1.00	0.00	-0.00	—	—	6.26	0.16	6.26	0.16

The triple **A**, **L**, **b** used for the computations was given in 5.1 (or PCMC, p. 19).
Numéraire: surplus vector, $\mathbf{S} = [0$ t.iron, 165 t.coal, 70 qr.wheat$]^T$.
Columns: (1) Distribution to industries and to the workers of the surplus **S**; (2) Variations in credit and debt positions for the industries and for the workers (flows); (3) Financial balances of the the industries and of the workers at the end of the period (stocks); (4) Prices for the commodities produced by the individual industries and wage rate of the workers; (5) Profit rates and wage rates ; (6) Expenditures taking place as if they were barter exchanges (physical goods with physical goods, see above section 4.6); (7) Expenditures taking place as if they were credit exchanges(physical goods exchanged now against a deferred payment in the future, IOUs, see above section 4.6); (8) Revenues taking place as if they were barter exchanges (physical goods with physical goods, see above section 4.6); (9) Expenditures taking place as if they were credit exchanges(physical goods exchanged now against a deferred payment in the future, IOUs, see above section 4.6)
(*) Multiplied by $\times 10^{-3}$

Table 5.6: **Period 5 (t=5).** ***Self-replacing with same prices as in Period 1, 2, 3 and 4 partial repayment of the debt. Changed distribution of the surplus.***

	(1) **d** Distribution	(2) **ΔCD** Flow	(3) $\mathbf{F}^{Balances}$ Stock	(4) $\mathbf{p}, w$ Prices	(5) $\mathbf{r}, w$ Rates	(6) Exp. Barter	(7) Exp. Credit	(8) Rev. Barter	(9) Rev. Credit
Iron	0.00	0.03	-0.03	10.21*	0.02	1.81	0.00	1.81	0.03
Coal	0.08	-0.01	0.01	4.32*	0.04	1.89	0.06	1.89	0.05
Wheat	0.25	-0.01	0.01	4.11*	0.17	1.91	0.07	1.91	0.06
Labour	0.67	-0.01	0.01	0.66	0.66	0.65	0.03	0.65	0.02
Total	1.00	0.00	0.00	—	—	6.26	0.16	6.26	0.16

The triple **A**, **L**, **b** used for the computations was given in 5.1 (or PCMC, p. 19).
Numéraire: surplus vector, $\mathbf{S} = [0$ t.iron, 165 t.coal, 70 qr.wheat$]^T$.
Columns: (1) Distribution to industries and to the workers of the surplus **S**; (2) Variations in credit and debt positions for the industries and for the workers (flows); (3) Financial balances of the the industries and of the workers at the end of the period (stocks); (4) Prices for the commodities produced by the individual industries and wage rate of the workers; (5) Profit rates and wage rates ; (6) Expenditures taking place as if they were barter exchanges (physical goods with physical goods, see above section 4.6); (7) Expenditures taking place as if they were credit exchanges(physical goods exchanged now against a deferred payment in the future, IOUs, see above section 4.6); (8) Revenues taking place as if they were barter exchanges (physical goods with physical goods, see above section 4.6); (9) Expenditures taking place as if they were credit exchanges(physical goods exchanged now against a deferred payment in the future, IOUs, see above section 4.6)
(*) Multiplied by $\times 10^{-3}$

Table 5.7: **Period 6 (t=6).** ***Self-replacing with same prices as in Period 1, 2, 3, 4 and 5 partial repayment of the debt. Changed distribution of the surplus.***

	(1) **d** Distribution	(2) **ΔCD** Flow	(3) $\mathbf{F}^{Balances}$ Stock	(4) $\mathbf{p}, w$ Prices	(5) $\mathbf{r}, w$ Rates	(6) Exp. Barter	(7) Exp. Credit	(8) Rev. Barter	(9) Rev. Credit
Iron	0.00	0.03	0.00	10.21*	0.02	1.81	0.00	1.81	0.03
Coal	0.08	-0.01	0.00	4.32*	0.04	1.89	0.06	1.89	0.05
Wheat	0.25	-0.01	0.00	4.11*	0.17	1.91	0.07	1.91	0.06
Labour	0.67	-0.01	0.00	0.66	0.66	0.65	0.03	0.65	0.02
Total	1.00	0.00	0.00	—	—	6.26	0.16	6.26	0.16

The triple **A**, **L**, **b** used for the computations was given in 5.1 (or PCMC, p. 19).
Numéraire: surplus vector, $\mathbf{S} = [0$ t.iron, 165 t.coal, 70 qr.wheat$]^T$.
Columns: (1) Distribution to industries and to the workers of the surplus **S**; (2) Variations in credit and debt positions for the industries and for the workers (flows); (3) Financial balances of the the industries and of the workers at the end of the period (stocks); (4) Prices for the commodities produced by the individual industries and wage rate of the workers; (5) Profit rates and wage rates ; (6) Expenditures taking place as if they were barter exchanges (physical goods with physical goods, see above section 4.6); (7) Expenditures taking place as if they were credit exchanges(physical goods exchanged now against a deferred payment in the future, IOUs, see above section 4.6); (8) Revenues taking place as if they were barter exchanges (physical goods with physical goods, see above section 4.6); (9) Expenditures taking place as if they were credit exchanges(physical goods exchanged now against a deferred payment in the future, IOUs, see above section 4.6)
(*) Multiplied by $\times 10^{-3}$

Table 5.8: **Period 7 (t=7).** ***Self-replacing with same prices as in Period 1, 2, 3, 4, 5 and 6 partial repayment of the debt. Changed distribution of the surplus.***

	(1) **d** Distribution	(2) **ΔCD** Flow	(3) $\mathbf{F}^{Balances}$ Stock	(4) $\mathbf{p}, w$ Prices	(5) $\mathbf{r}, w$ Rates	(6) Exp. Barter	(7) Exp. Credit	(8) Rev. Barter	(9) Rev. Credit
Iron	0.03	0.00	0.00	10.21*	0.02	1.84	0.00	1.84	0.00
Coal	0.07	0.00	0.00	4.32*	0.04	1.94	0.00	1.94	0.00
Wheat	0.24	0.00	0.00	4.11*	0.17	1.97	0.00	1.97	0.00
Labour	0.66	0.00	0.00	0.66	0.66	0.66	0.00	0.66	0.00
Total	1.00	0.00	0.00	—	—	6.42	0.00	6.42	0.00

The triple **A**, **L**, **b** used for the computations was given in 5.1 (or PCMC, p. 19).
Numéraire: surplus vector, $\mathbf{S} = [0$ t.iron, 165 t.coal, 70 qr.wheat$]^T$.
Columns: (1) Distribution to industries and to the workers of the surplus **S**; (2) Variations in credit and debt positions for the industries and for the workers (flows); (3) Financial balances of the the industries and of the workers at the end of the period (stocks); (4) Prices for the commodities produced by the individual industries and wage rate of the workers; (5) Profit rates and wage rates ; (6) Expenditures taking place as if they were barter exchanges (physical goods with physical goods, see above section 4.6); (7) Expenditures taking place as if they were credit exchanges(physical goods exchanged now against a deferred payment in the future, IOUs, see above section 4.6); (8) Revenues taking place as if they were barter exchanges (physical goods with physical goods, see above section 4.6); (9) Expenditures taking place as if they were credit exchanges(physical goods exchanged now against a deferred payment in the future, IOUs, see above section 4.6)
(*) Multiplied by $\times 10^{-3}$

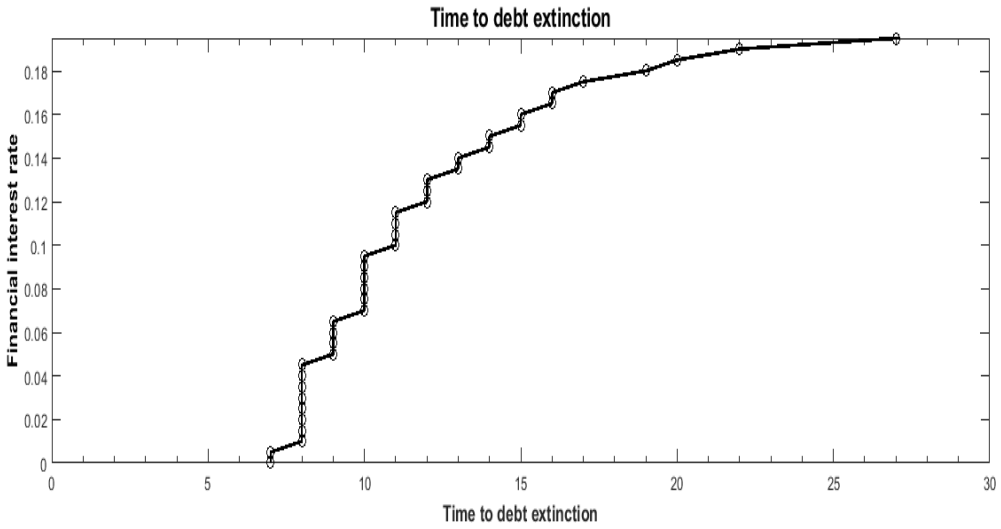

Fig. 5.5: ***Time to debt extinction and the financial interest rate.***

The time to debt extinction is different when the financial interest rate is different, given the debt and credit positions and the associated new financial balances described above when the prices change from those of table 5.1 to those of table 5.2.

6 Conclusions

In Zambelli (2018c) the assumption of the uniform rate of profits has been removed from the basic logical structure of Production of Commodities by Means of Commodities. It has been shown that the uniform rate of profits case is is a special case of the more general differential profit rates case.

Here we have studied the conditions that would allow the economic system to be in a self-replacing state also thanks to the creation of deferred means of payments, i.e. IOUs (money-credit-debt).

Sraffa's work in PCMC was a 'Prelude to a critique of economic theory'. Here the aim has been to show that this critique holds also when there exists a monetary interest rate and profit rates are not uniform.

Furthermore, we believe that the inclusion of IOUs inside Sraffian schemes may provide new foundations for theoretical and empirical economic theory.

Clearly distribution and prices are closely relates and hence the measures of any aggregates such as the Net National Product (as a scalar), capital (as a scalar), industry level aggregates and firm level aggregates vary as the distribution of the surplus vary.

But throughout this paper, as in PCMC, there has been no change in the surplus produced and in the means of production used. The self-replacing assumption has been maintained.

In order to explain what determines distribution one has to refer to some external explanation. And here a link with respect to the first works of Sraffa may be found:

> The large industries are stimulated on their part to make themselves independent by acquiring control of a bank so as to obtain from it, without undergoing heavy impositions, the necessary financial backing. As a result of this opposition, however, it cannot be said, generally speaking, that either of the two opposite tendencies has the absolute upper hand over the other. The general tendency seems to be towards the elimination of this opposition by the formation of large " groups " of companies of the most varied kinds concentrated round one or more banks, mutually related by the exchange of shares and by

> the appointment of Directors common to them. Within these " groups" the various interests are all equally subject to the interests of a few individuals who control the whole group, possessing on their own only a very few shares of the various companies. Very little is known and very little can be generalised about these groups, on account of the undetermined state of their structure, of their unofficial character, of the variety of the various groups, and of the continual shifting of the elements which compose them. What the public knows and feels-not only when disasters take place, fatal to the existence to some of them, or when hostilities break out between one group and another-is the enormous financial and political power which they have and the frequent use they make of it to influence both the foreign and home policy of the Government in favour of their own interests. Each group keeps several press organs which support its policy, and some of the accusations made against certain Ministries of being actuated by the interests not of a class, but of private concerns, and of favouring one financial group against another, have no doubt a basis of truth (Sraffa, 1922a, p.196).

If the self-replacing prices (and distribution of the surplus) are not determined by the economic sphere or economic forces alone, as it is demonstrated in PCMC and here, we have that other factors like the power structure and the existing institutions may be able to impose prices and to have access to credit. This view is well encapsulated in the quotation above. In PCMC Sraffa was aiming at "correcting" economic theory in very rigorous and precise way and in the quote above there is a description of the forces and of the role of the banking system in determining prices and distribution in the actual world.

A Actual and equivalent distribution of the surplus

The physical surplus $\mathbf{S}$ is obviously distributed among the members of the society (producers and workers). We can arrange the possible or observable per capita distribution of the physical surplus in a matrix, $\bar{\mathbf{C}}$, whose rows are the *actual* or *observed* individual physical distribution of the surplus[37].

$$\bar{\mathbf{C}} = \begin{bmatrix} \bar{\mathbf{c}}_1 \\ \bar{\mathbf{c}}_2 \\ \vdots \\ \bar{\mathbf{c}}_i \\ \vdots \\ \bar{\mathbf{c}}_n \\ \bar{\mathbf{c}}_w \end{bmatrix} = \begin{bmatrix} \bar{c}_1^1 & \bar{c}_1^2 & \dots & \bar{c}_1^j & \dots & \bar{c}_1^n \\ \bar{c}_2^1 & \bar{c}_2^2 & \dots & \bar{c}_2^j & \dots & \bar{c}_2^n \\ \vdots & & & & & \\ \bar{c}_i^1 & \bar{c}_i^2 & \dots & \bar{c}_i^j & \dots & \bar{c}_i^n \\ \vdots & & & & & \\ \bar{c}_n^1 & \bar{c}_n^2 & \dots & \bar{c}_n^j & \dots & \bar{c}_n^n \\ \bar{c}_w^1 & \bar{c}_w^2 & \dots & \bar{c}_w^j & \dots & \bar{c}_w^n \end{bmatrix} \tag{A.1}$$

$\bar{\mathbf{C}}$ is an $(n+1) \times n$ matrix. The first n rows are the distribution of the National Surplus to the n industries (or producers) and the last, the $n^{th}+1$ row, is the distribution of the surplus that goes to the workers. Clearly the column sum of the matrix has to give the surplus vector $\mathbf{S}^T$.

Among all possible physical distributions of the surplus there is a subset which will be particularly useful for the analysis developed here. A matrix belonging to this subset would be the following

$$\mathbf{C} = \mathbf{d}\mathbf{S}^T \tag{A.2}$$

where $\mathbf{d}$ is the Net National Product distribution vector with $\sum_i^{n+1} d_i = 1$, i.e., it is the share of the surplus distributed between producers and workers[38].

In the situation in which we assume that commodity prices are uniform we have that the purchasing power necessary to buy a consumption bundle $\bar{\mathbf{c}}_i$ is given by $\bar{\mathbf{c}}_i\mathbf{p}$ (where $\mathbf{p}$ is the $n \times 1$ commodities price vector). In value terms this is also equal to the values share d_i of the total Net National Product so that $\bar{\mathbf{c}}_i\mathbf{p} \equiv d_i\mathbf{S}^T\mathbf{p}$.

An important characteristic of the distribution $\mathbf{C}$ is that the composite physical distribution vector associated to each agent is a fraction of the total physical surplus generated. This being the case, the comparison between the consumption of two agents, say agent i and agent j, could be done simply by comparing the value of the two composite bundles[39].

[37] In order to avoid unnecessary complications we consider here distribution among the industry and the workers as a whole. Clearly the number of rows of $\bar{\mathbf{c}}$ could be as many as the individuals forming the society

[38] The dimension of the vector $\mathbf{d}$ could be very large. In the case in which we extend the analysis considering the distribution of the surplus to all the producers contributing to the industries and of all the workers the dimension of $\mathbf{d}$ would be the total amount of agents (population) belonging in the system. Just as an example, if the number of producers were 13 per industry and the industries were 7, while the workers were 3709 the number of columns of $\mathbf{c}$ or the dimension $\mathbf{d}$ would be $3800 = 13 \times 7 + 3709$. The trivial, but also very important observation, is that the sum by rows of $\mathbf{c}$ would give the total amount of Surplus to be distributed. In the case of the Physical Surplus to be distributed would the $\mathbf{S}$ vector, which has as element the surplus of the 7 commodities used by the system.

[39] The comparison of the consumption or surplus bundles of two agents may be problematic because the bundles of goods may have different proportions, $\bar{\mathbf{c}}_i \lesseqgtr \bar{\mathbf{c}}_j$. Hence the ratios of the individual goods composing the bundles would most likely be non uniform. That is $\frac{\bar{c}_i^1}{\bar{c}_j^1} \neq \frac{\bar{c}_i^2}{\bar{c}_j^2} \neq \ldots \neq \frac{\bar{c}_i^n}{\bar{c}_j^n}$. Once the prices are given we can compare the values of their bundles and we can compare the value of their bundles with respect to the value of the total surplus. The share in value of the surplus for agent i would be $d_i = \frac{\bar{\mathbf{c}}_i\mathbf{p}}{\mathbf{S}^T\mathbf{p}}$ and for agent j would be $d_j = \frac{\bar{\mathbf{c}}_j\mathbf{p}}{\mathbf{S}^T\mathbf{p}}$. Therefore we can compare the values of these bundles by comparing their shares of the total surplus: $\frac{d_i}{d_j} = \frac{\bar{\mathbf{c}}_i\mathbf{p}}{\bar{\mathbf{c}}_j\mathbf{p}}$. If we have that $d_i > d_j$ we can say that the value of the

We have an important equivalence relation which is given by the following:

$$\mathbf{dS}^T\mathbf{p} = \overbrace{\underbrace{\begin{bmatrix} d_1\mathbf{S}^T\mathbf{p} \\ d_2\mathbf{S}^T\mathbf{p} \\ \vdots \\ d_i\mathbf{S}^T\mathbf{p} \\ \vdots \\ d_n\mathbf{S}^T\mathbf{p} \\ d_w\mathbf{S}^T\mathbf{p} \end{bmatrix}}_{\mathbf{dS}^T\mathbf{p}}}^{\textit{Shares of Surplus}} = \overbrace{\underbrace{\begin{bmatrix} \bar{\mathbf{c}}_1\mathbf{p} \\ \bar{\mathbf{c}}_2\mathbf{p} \\ \vdots \\ \bar{\mathbf{c}}_i\mathbf{p} \\ \vdots \\ \bar{\mathbf{c}}_n\mathbf{p} \\ \bar{\mathbf{c}}_w\mathbf{p} \end{bmatrix}}_{\bar{\mathbf{c}}\mathbf{p}}}^{\textit{Effective Demands}} = \overbrace{\underbrace{\begin{bmatrix} \mathbf{c}_1\mathbf{p} \\ \mathbf{c}_2\mathbf{p} \\ \vdots \\ \mathbf{c}_i\mathbf{p} \\ \vdots \\ \mathbf{c}_n\mathbf{p} \\ \mathbf{c}_w\mathbf{p} \end{bmatrix}}_{\mathbf{cp}}}^{\substack{\textit{Effective Demands} \\ \textit{Equivalents}}} \tag{A.4}$$

Obviously if $\mathbf{p}$ is given also the price ratios are given. Hence to operate on the basis of the presumed actual physical distribution $\bar{\mathbf{C}}$ is equivalent with $\mathbf{C}$ (see also footnote 39, p.41). The matrix $\mathbf{C}$ is perfectly exchangeable with $\bar{\mathbf{C}}$[40].

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consumption output of agent i is greater that the value of the consumption bundle of agent j, but we cannot conclude in all cases that agent i has a consumption in physical terms which is greater than j because agent i may consume more of one good and less of another with respect to agent j.

Quite different would be the case in which the distribution was as in eq.A.2, $\mathbf{C} = \mathbf{dS}^T$. In this case we have that

$$\frac{c_i^1}{\bar{c}_j^1} = \frac{c_i^2}{c_j^2} = \ldots = \frac{c_i^n}{c_j^n} = \frac{d_i}{d_j} = \frac{d_i}{d_j} = \frac{\mathbf{c}_i\mathbf{p}}{\mathbf{c}_j\mathbf{p}} = \frac{\bar{\mathbf{c}}_i\mathbf{p}}{\bar{\mathbf{c}}_j\mathbf{p}} \tag{A.3}$$

For the whole system we have the following: $\bar{\mathbf{c}}\mathbf{p} = \mathbf{cp} = \mathbf{dS}^T\mathbf{p}$. These different distributions are equivalent in the sense that they belong to the same *budget constraint line* in the same meaning which is normally understood also in standard microeconomics textbooks. Therefore, for clarity and analytical purposes we will consider the distribution $\mathbf{c}$ because it is equivalent, for the reasons just explained to the observed or actual distribution $\bar{\mathbf{c}}$

[40]The notion of equivalence is used here in a very precise meaning. $\mathbf{C}$ is equivalent to $\bar{\mathbf{C}}$ if the propositions or computations do not depend on whether $\mathbf{C}$ is replaced by $\bar{\mathbf{C}}$ and viceversa. Some would say that $\mathbf{C}$ is *exchangeable* with $\bar{\mathbf{C}}$

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