# Why Selfish Politicians Reduce Manipulation?

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We model political manipulation of debt and sovereign wealth funds (SWFs) in a political budget cycle model. Assuming that a share of voters suffers from fiscal illusion the incumbent can increase her re-election chances by expanding government spending. However, the optimal manipulation may exceed the amount necessary to maximize re-election chances (overmanipulation), if the SWF (debt) does not have to be fully replenished (repaid). Then, more selfish politicians (with higher ego rents) reduce the over-manipulation. Furthermore, we find that punishment of government manipulation may or may not help to curb fiscal distortions.

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**Keywords:** debt; sovereign wealth fund; fiscal policy; political budget cycles; fiscal illusion; political economy.

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# 1 Introduction

Politicians have always been criticized as opportunistic; they have the incentive to increase their chances of being re-elected by manipulating economic policies prior to elections (for instance, Nordhaus 1975; Rogoff and Sibert 1988; Shi and Svensson 2006), for instance by reducing tax rates, by increasing the deficit, by expanding the money supply, or by delaying a currency appreciation. Besides these traditional monetary and fiscal policies, we argue that tapping into the sovereign wealth fund is another instrument for manipulation. Sovereign wealth funds play a big role in some countries, in particular, pension reserve funds, i.e., funds that were created for cushioning the effect of a decreasing population. Several countries have used their sovereign wealth fund to rescue the economy. Wang and Bohn (2019) suggest that the incumbent could gain an electoral advantage from tapping into an existing pension reserve fund. As far as we know, there is no empirical research on the role of sovereign wealth fund in political budget cycles. However, manipulating natural resource rents is like tapping into sovereign wealth funds. Neither is paid for by people's earnings; their exploitation is thus hard for individuals to detect, at least when only part of the funds is used. Klomp and de Haan (2016) find that incumbents also increase natural resource rents before elections (either by increasing the government's extraction or by increasing tax rates, tariffs and fees on the exploitation in case it is done by the private sector).

This paper takes two sources for financing the expansion of public spending into account, incurring debt and tapping into an existing sovereign wealth fund. Incumbents have an incentive to manipulate fiscal policy because a share of voters suffers from fiscal illusion. Fiscal illusion is a phenomenon which occurs when the taxpayer is incapable or unwilling to internalize the full cost of government spending (Dell'Anno and Mourao, 2012; Das and Omar, 2014; Mourao and Cabral, 2015; Bastida et al., 2017). We illustrate how the incumbent can take advantage of fiscal illusion and why this does not always mean that the incumbent increases her re-election chances.

Our paper rests on two pillars: the theory of political business or budget cycles and the fiscal illusion literature. The development of theoretical studies on political cycles could be grouped into three generations. The first generation is known as political business cycle models a là Nordhaus (1975). These adaptive expectations monetary policy models have been rejected empirically (Drazen, 2000; Faust and Irons, 1999). Both the second and the third generation models are rational expectations models. They focus on electoral cycles in fiscal policy, and emphasize the role of information asymmetries. Rogoff and Sibert's (1988) rational expectations political budget cycle (PBC) model marks the start of the second generation of models. However, these models are criticized by Shi et al. (2003) for being at odds with reality since they predict that only the more competent politicians distort the economy to win elections. Third generation models - pioneered by Shi and Svensson (2006) - are based on moral hazard. Neither politicians nor voters know the current competence level of the incumbent; so the incumbent cannot signal her type to the electorate. As a result, the incumbent always has incentives to exert a hidden effort to manipulate uninformed voters. We use this latter type to capture opportunistic government behaviour.

The crucial condition for the existence of PBCs is that "voters do not fully catch the debt instrument and underlie fiscal illusion at least to a certain degree" (Afflatet, 2015, p. 3). The discussion that the government intends to deceive taxpayers by using appropriate tax strategies dated back to the nineteenth century. However, the concept of "fiscal illusion" was first proposed by Puviani et al. (1960). It refers to a systematic misperception of fiscal parameters (Das and Omar, 2014; Dell'Anno and Dollery, 2014; Baekgaard et al., 2016; Buehn et al., 2018). The fiscal illusion literature concludes that fiscal illusion results from politicians' utility maximizing behaviour (Liu and Mikesell, 2019; Baekgaard et al., 2016; West and Winer, 1980). "Self-seeking" politicians design and manipulate fiscal systems to create fiscal illusion which makes voters underestimate the actual fiscal burden (Liu and Mikesell, 2019; Buchanan, 2014). The actual mechanism behind fiscal illusion may not necessarily be caused by imperfect information, but also by a lack of attention (Baekgaard et al., 2016).

The discussion about the relationship between the level of fiscal illusion and the magnitude of the public budget is still ongoing. Mourao and Cabral (2015) suggest that higher public budgets imply more fiscal illusion, but the relationship between the level of fiscal illusion and the magnitude of the public budget is nonlinear. The empirical evidence shows that consumers' consumption decisions are not affected by future tax implications of current government debt when debt levels are low (Dalamagas, 1993). West and Winer (1980, p. 617) suggest that there is "an optimal level of illusion for the public manager and median voter simultaneously". Oates (1985, p. 67) argues that "Fiscal illusion ... can only operate over a limited range". It cannot persist beyond a certain threshold. Actually, consumers tend to fully discount the future tax obligations when the debt-GNP ratio is high (Dalamagas, 1993; Nicoletti, 1988; Gobbin and Van Aarle, 2001). In other words, citizens in a country with a high debt-to-GDP ratio are less likely to suffer from fiscal illusion. In general, the level of fiscal illusion increases with the magnitude of the public budget at first, reaches a threshold which depends on the debt-to-GDP ratio, and then decreases with a further increasing public budget. In the model, we capture this by a continuous fiscal illusion function with the aforementioned properties and the debt level as its argument.

The possibility of tapping into the sovereign wealth fund and incurring debt provide two instruments for the incumbent to increase the public goods provision, and let the incumbent improve her performance and enhance the probability of re-election by exerting a hidden effort. As in Shi and Svensson (2006), we assume that politicians increase government debt to manipulate the outcome of elections. As in Wang and Bohn (2019), politicians may, however, also tap into an existing sovereign wealth fund. The two policy instruments, incurring debt and tapping into the sovereign wealth fund, are independent and need to be repaid, as least partially. Voters are separated into fiscally realistic voters and fiscal illusion suffering persons (henceforth FISPs). The fiscally realistic voters perceive the real level of debt and fund depletion. But voters who are suffering from fiscal illusion cannot fully perceive manipulations. FISPs underevaluate the debt level and the reduction of the sovereign wealth fund. This underevaluation increases with increasing the level of debt and fund depletion at first until it reaches a maximum, thereafter decreases with the level of debt and fund reduction.

Different from Shi and Svensson (2006), we allow the incumbent to increase her re-election chances by using both the sovereign wealth fund and by incurring additional debt simultaneously. There is no reason why the optimal manipulation by each one of the two instruments, debt and the sovereign wealth fund, should be directly dependent of one another. The assumption of a non-linear relationship between fiscal illusion and the magnitude of government debt (or the reduction of sovereign wealth fund) implies that there is a level of utilisation of each of the two instruments that maximises the winning probability for the incumbent. However, the optimal manipulation may exceed (overmanipulation), be equivalent or be lower (undermanipulation) than that value because spending does not only increase the incumbent's utility by increase the chance of re-election, but also because additional spending has, by itself, a beneficial effect on utility. It turns out that the optimal manipulation depends on whether full or partial or no replenishment (in case of the sovereign wealth fund) or repayment (in case of debt) are required. A second result is that, as the ego rent goes up, the degree of (both over and under) manipulation decreases. In other words, the higher ego rent makes the overmanipulation more costly, while the undermanipulation is too prudent. The optimal manipulation gets closer to the winning probability maximization point. Lastly, we also show that punishment may reduce manipulations in some circumstance, but may not in others.

The paper is structured as follows. In Section 2 and 3, we present our core PBC model with the sovereign wealth fund and debt manipulations and its general solution. Several propositions are discussed in Section 4. In Section 5, we extend our discussion to a model with punishment for manipulation. Details on both the core model and the extended model are presented in the appendix. Section 6 concludes.

## 2 Core Model

The model persented here is in the Shi and Svensson's (2006) tradition and a development of Wang and Bohn's (2019) work. The election is held every other period. Politicians, both incumbent $a$ and challenger $b$, are opportunistic and aim to win the election. The electorate votes for the party of the candidate who is expected to deliver higher utility.

Voter $i$'s utility function is:

$$U_t^i = \sum_{s=t}^{\infty} (\beta^i)^{s-t} E_s[u(c_s) + g_s + \phi\theta^i z_s], \qquad i = 1, ..., n; \tag{1}$$

where $\beta^i$ is a subjective discount factor; $E_s$ is the expectations operator; superscript $i$ denotes individual $i$; subscript $s$ denotes the time period; $u(c_s)$ is a concave function which represent the utility from private consumption; $g_s$ is public goods provision and $\theta^i z_s$ is the political component with relative weight parameter $\phi$.

Equation ((1)) shows that voters derive utility from two economic components (private consumption $c_t$ and public goods provision $g_t$) and a political components $\phi\theta^i z_t$. Voters share their preferences over private and public goods consumption, but have different political preferences. The political preferences $\theta^i$ are derived from the politicians' non-economic characteristics like trustworthiness or good looks. Parameter $\theta^i$ is uniformly distributed in the interval $[-1, 1]$; it is negative if voter $i$ is in favour of party $a$, or positive if party $b$ is preferred. Variable $z_t$ represents the party in power. When $a$ is elected, the value of $z_t$ is $-1/2$, otherwise $+1/2$. Together, $\theta^i z_t$ give voter $i$ positive utility when her favourite politician is elected, and negative utility when the opponent is in power.

Politicians, both the incumbent (henceforth referred to with superscript $a$ without limiting the general validity of the analysis) and challenger (hereinafter $b$), share the same preference. Politician $j$'s utility is composed of two economic components (private goods and public goods provision), which is similar to voters, and one political component, which is a political rent, $X_t > 0$, if the politician is in power in period $t$. This so-called ego rent could be either political income (Barro, 1973) or reputation. Politician $j$'s utility is:

$$V_t^j = \sum_{s=t}^{\infty} (\beta^j)^{s-t} E_s[u(c_s) + g_s + \mathbf{I}_s X_s], \qquad j = a, b; \tag{2}$$

$$\mathbf{I}_s = \begin{cases} 1 & \text{if in power in period } s; \\ 0 & \text{otherwise.} \end{cases}$$

The government is myopic, i.e., reelection prospects shorten the incumbent's time horizons (Buchanan and Wagner, 1977; Raveh and Tsur, 2017). More specifically, incumbents' subjective discount rate becomes larger relative to the interest rate at which they can borrow (Rieth, 2014; Raveh and Tsur, 2017). Following Aguiar et al. (2014), Rieth (2014) and Raveh and Tsur (2017), we assume that politicians' subjective discount factor is smaller than the discount factor. $\beta^j < 1/(1 + r_D)$ and $\beta^j < 1/(1 + r_W)$, where $r_D$ and $r_W$ are the interest rates on debt and sovereign wealth fund, respectively. Political myopia distorts incumbents' perception and makes them underestimate the cost of manipulation.

The expected consumption $c_t$ is given by the expected after-tax income (where $\tau$ is the tax rate and $y_t$ is income):

$$E_t^k[c_t] = E_t^k[(1 - \tau)y_t], \qquad k = i, \ j. \tag{3}$$

Expected public goods provision $g_t$, is financed by the government tax revenue, $\tau y_t$, and also affected by government competence, $\eta_t^j$. Additionally, the government could tap into the sovereign wealth fund $W_t$, and/or incur debt, $D_t$, to finance government spending, but may have to replenish the funds taken out of the SWF previously or repay previous debt:

$$E_t^k[g_t] = E_t^k[\tau y_t + \eta_t^j + \delta_t W_t + \zeta_t \overline{D} - \lambda_W(\delta_{t-1})(1 + r_W)W_{t-1} - \lambda_D(\zeta_{t-1})(1 + r_D)\overline{D}] \ , \tag{4}$$

where $\delta_t$ is the percentage of the dissolution of the sovereign wealth fund $W_t$ that exists at the begining of the period; $\overline{D}$ is a debt limit[1], and $\zeta_t \overline{D}$ is the amount of debt that is incurred in the period $t$. The two latter terms capture the replenishment and repayment obligations. $r_W$ and $r_D$ are interest rate on sovereign wealth fund and debt, $\lambda_W(\delta_{t-1})$ is the replenishment ratio of the repayment of fund reduction to the whole amount of the wealth fund, and $\lambda_D(\zeta_{t-1})$ is the repayment ratio of the debt incurred in the previous period to the debt limit.

Replenishment and repayment *obligations* are common knowledge. Both the incumbent and voters know that the fund reduction and debt that incurred in the election year need to be replenished and repaid, while may not entirely, in the following year. The more the fund has been used (debt has been incurred) in the previous year, the more needs to be replenished (repaid). Replenishment and repayment ratios in periot $t+1$ are

[1]For simplicity, we postulate an absolute debt limit. Debt limits have become more and more relevant in policy considerations. In particular, the ratio of sovereign debt-to-GDP ratio has received much attention. As we have discussed in Section 1, when the debt-to-GDP ratio is high, consumers tend to fully discount the future tax obligations. In our analysis, a debt-to-GDP ratio above $\frac{\overline{D}}{y}$ does no longer offer the government opportunities for manipulation and will since there is a fiscal illusion threshold, and fiscal illusion does not exist if the debt level beyond this debt limit. Therefore, for simplicity, be excluded from the analysis.

captured by functions of the ratio of the wealth fund dissolution of the previous period, $\delta_t$, and debt incurred in the previous period, $\zeta_t$. They have the following properties: $0 \leq \lambda_W(\delta_t) \leq \delta_t$ and $\lambda'_W(\delta_t) > 0$; and $\lambda_D(\zeta_t)$: $0 \leq \lambda_D(\zeta_t) \leq \zeta_t$, and $\lambda'_D(\zeta_t) > 0$. Besides, we assume that the repayment function is not very concave, which means that $\lambda''_W(\delta_t)$and $\lambda''_D(\zeta_t)$ are negative, and their magnitude will not be too large.

Politicians' competence $\eta_t^j$ follows an MA(1) process, i.e. it is determined by skills shocks $\mu$ for the current and previous periods:

$$\eta_t^j = \mu_t^j + \mu_{t-1}^j, \qquad j = a, b; \tag{5}$$

where $\mu_t$ is an i.i.d. random variable with mean 0, distribution function $F[\mu_t^j]$ and density function $f[\mu_t^j]$ with $f(0) > 0$. We assume that the past skills shocks are common knowledge, and current and future shocks are unknown to all agents. The current skills shocks can be deduced by agents once the public goods provision has materialized. Both politicians and fiscally realistic voters (share $(1-\psi)$) can deduce the current skills shock correctly, while, FISPs (fiscal illusion suffering persons, share $\psi$) have a distorted perception of the incumbent's skills which produces distorted perceptions of the wealth fund reduction and the level of debt.

In our model, we assume that FISPs underestimate the fund reduction $\delta_t W_t$ as $\alpha(\delta_t)W_t$ and the level of debt $\zeta_t\overline{D}$ as $\sigma(\zeta_t)\overline{D}$, and postulate the following simplifying properties for the perception of FISPs ($\alpha(\delta_t)$ and $\sigma(\zeta)$): for $0 \leq \delta_t \leq 1$, $0 \leq \alpha(\delta_t) \leq \delta_t \leq 1$, $\alpha'(0) = 0$, and $\alpha''(\delta_t) > 0$; analogously, for $0 \leq \zeta_t \leq 1$, $0 \leq \sigma(\zeta_t) \leq \zeta_t \leq 1$, $\sigma'(0) = 0$ and $\sigma''(\zeta_t) > 0$. The straight line in Figure 1 is a 45° line and depicts the fund reduction (or debt). The bent curves refer to the perception of the fund reduction (the black curve) and the perception of debt level by FISPs [2](the blue curve, or gray if in

[2]Figure 1 depicts the degree of fiscal illusion on the depletion of the sovereign wealth fund is less than the degree of fiscal illusion on the debt level. However, which policy instrument has a higher level fiscal

black and white). It indicates that FISPs have some awareness of the depletion of the sovereign wealth fund (and the level of debt), and their perception of the fund reduction (or the level of debt) is a monotonically increasing function of the fund reduction (or the level of debt) (Dalamagas, 1993; Nicoletti, 1988; Gobbin and Van Aarle, 2001).

In Figure 1, the difference between $\delta_t$ (on the 45° line) and $\alpha(\delta_t)$ captures the underestimation of the fund reduction. Analogously, the difference between $\zeta_t$ and $\sigma(\zeta_t)$ is the underestimation of the level of debts. It shows that when the level of fund reduction $\delta_t$ (and the level of debt $\zeta_t$) is small, FISPs hardly perceive it (Dalamagas, 1993; Nicoletti, 1988). As the fund reduction (and debt level) increases, this underestimation increases at first, and reaches a maximum at $\delta_t^W$ (and $\zeta_t^W$). Beyond this point, the underestimation decreases. While Oates (1985) suggests that fiscal illusion cannot persist above a threshold, we assume that the mistake FISPs made becomes smaller and smaller as in approach: The SWF and debt are used maximally and FISPs no longer make a mistake[3].

Under the assumptions that elections take place every other period and the competence of the politicians follows an MA(1) process[4], the problem can be divided into two-period

illusion, i.e., which manipulation is less visible, incurring debt or tapping into the sovereign wealth fund, does not affect the outcome since we assumed that incurring debt and tapping into the sovereign wealth fund is independent. The result is irrelevant to which curve is higher but is relevant to the curvature of curves.

[3]This implies that no group of voters would suffer from fiscal illusion and any government manipulation would no longer affect the probability of winning the election. The assumption facilitates the derivation of the solution as it guarantees an interior solution, but any other maximum value for $\alpha(1)$ would also be feasible.

[4]For instance, period $t$ is an election year, and period $t+1$ is an off-election year. The assumption of the MA(1) process for competence indicates that voters, even fiscally realistic voters, have no information for deducing the incumbent's (or the challenger's) competence in period $t+2$. Politicians have no policy instrument to improve their performance in period $t+2$. Hence, neither voters nor politicians take the utility in period $t+2$ into account. Another election takes place in period $t+2$. Period $t+2$ and $t+3$ form a new election cycle. The splitting into 2-period election cycles, see also (Bohn, 2019).

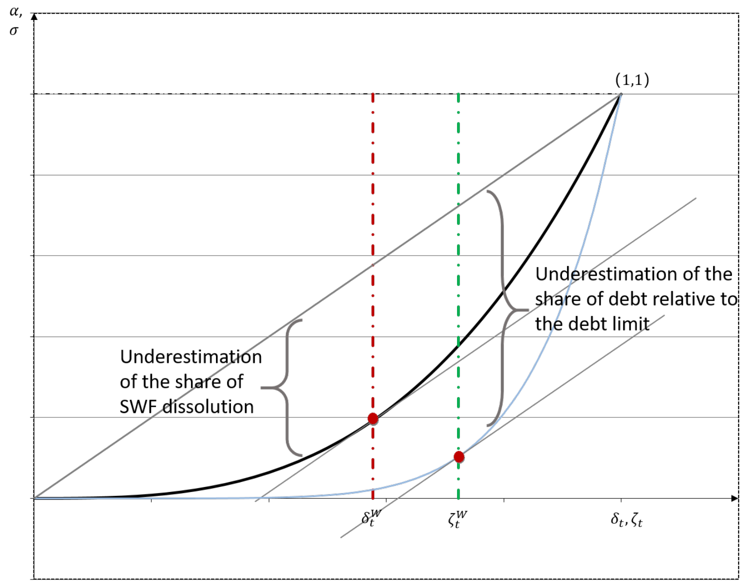



Figure 1: Perception of Debt and the Fund Reduction by Voters with Fiscal Illusion

maximization problems. The timing of events is as follows. Incumbent $a$ chooses the debt ratio $\zeta_t$ and a percentage of the fund reduction $\delta_t$ and uses the additional resources for public goods provision in the beginning of election period $t$. The level of public goods provision $g_t$ and the past skills shock $\mu_{t-1}$ are commen knowledge. But the current skill shock $\mu_t$ which occurs during period $t$ is unknown to all agents and can only be deduced. Fiscally realistic voters can deduce the competence of the incumbent correctly, but FISPs' perception of the incumbent's competence is affected by the debt level and the percentage of the fund reduction, $\delta_t$ and $\zeta_t$ respectively. The election takes place at the end of period $t$. The electorate votes for who they believe can provide a higher level of utility. In period $t+1$, the winner of the election receives an ego rent, repays debt and replenishes the fund depending on the degree of replenishment and repayment taken into account by the incumbent. Since there is no election in period $t+1$, the incumbent has no incentive to manipulate.

# 3 Model Solution

The model can be solved in three steps: first, we can derive the probability of voter votes for the incumbent; second, the incumbent's probability of winning can be obtained; and lastly, the incumbent's maximisation problem can be solved.

First, voter $i$ will vote for incumbent $a$ if she expects incumbent $a$ to deliver higher utility than challenger $b$:

$$\underbrace{E_t^i[u(c_{t+1}^a) + g_{t+1}^a + \phi\theta^i(-\frac{1}{2})]}_{\text{utility when } a \text{ in power}} > \underbrace{E_t^i[u(c_{t+1}^b) + g_{t+1}^b + \phi\theta^i(+\frac{1}{2})]}_{\text{utility when } b \text{ in power}}. \tag{6}$$

Opportunistic politicians share the same policy preferences and will implement the same policies in $t+1$, but may differ in terms of competence. No matter which party is in power, individuals' private good consumption is constrained by disposable income; the tax revenues, the repayment of debt and replenishment of wealth fund are the same. The only difference between the incumbent and the challenger is their competence. Both the skill shock of the incumbent in period $t+1$ ($\mu_{t+1}^a$) and the competence of the challenger ($\eta_{t+1}^b = \mu_t^b + \mu_{t+1}^b$) are unknown, and expected to be 0. Voters could deduce the skill shock of the incumbent in period $t$, $\mu_t^a$. Then voter $i$ would vote for the incumbent if she favours the incumbent ($\theta^i$ is negative) or she expects sufficient competence of the incumbent ($E_t[\mu_t^a]$ is large enough to compensate a positive value of $\theta^i$):

$$E_t^i[\mu_t^a] \quad > \quad \phi\theta^i. \tag{7}$$

Given that $\theta^i$ is uniformly distributed in the interval $[-1, 1]$, the probability of voting for the incumbent $a$ can be obtained:

$$\text{Prob}\ \left\{\ E_t^i[\mu_t^a] - \phi\theta^i\ \geq\ 0\ \right\} = \frac{E_t^i[\mu_t^a]}{2\phi} + \frac{1}{2}. \tag{8}$$

Second, the incumbent can win the election if she obtains at least 50% of the votes. The winning probability of the incumbent depends on voters' perceptions of the incumbent‘s current skill shocks ($E_t[\mu_t^a]$).

$$\text{Prob}^{win} == \{ \ (1-\psi)[\frac{E_t^{REAL}[\mu_t^a]}{2\phi} + \frac{1}{2}] + \psi[\frac{E_t^{FISP}[\mu_t^a]}{2\phi} + \frac{1}{2}] \geq \frac{1}{2} \ \}. \tag{9}$$

Fiscally realists (share $(1-\psi)$) perceive the composition of public goods production rationally and deduce $\mu_t^a$ from the budget constraint in period $t$,

$$E_t^{REAL}[\mu_t^a] \ = \ \mu_t^a \ = \ g_t - \tau y_t - \mu_{t-1}^a - \delta_t W_t - \zeta_t \overline{D}. \tag{10}$$

However, FISPs (fiscal illusion suffering persons, share $\psi$) underestimate the amount of debt and fund reduction required for benefiting for public goods spending. FISPs' expectations, $E_t^{FISP}[\mu_t^a]$ depends on their perception of the fund reduction, $\alpha(\delta_t)W_t$, and the level of debt, $\sigma(\zeta_t)\overline{D}$,

$$E_t^{FISP}[\mu_t^a] \ = \ \widehat{\mu_t^a} \ = \ \mu_t^a + [\ \delta_t - \alpha(\delta_t)\ ]\ W_t + [\ \zeta_t - \sigma(\zeta_t)\ ]\ \overline{D}\ . \tag{11}$$

As mentioned before, $\alpha(\delta_t) \leq \delta_t$ and $\sigma(\zeta_t) \leq \zeta_t$. Then obviously the incumbent's competence is overestimated by FISPs by $[\delta_t - \alpha(\delta_t)]W_t + [\zeta_t - \sigma(\zeta_t)]\overline{D}$. Based on this, we can derive:

$$\begin{aligned} & \text{Prob} \quad \{ \ \mu_t^a \geq \psi[\alpha(\delta_t) - \delta_t]W_t + \psi[\sigma(\zeta_t) - \zeta_t]\overline{D} \ \} \\ = \quad & 1 \quad - F[\ \psi W_t(\alpha(\delta_t) - \delta_t) + \psi\overline{D}(\sigma(\zeta_t) - \zeta_t)\ ]\ , \end{aligned} \tag{12}$$

where F[·] is the distribution function of the skills shock, which is monotonically increasing in the skills shock. It can be seen that both incurring debt and raidding the wealth fund could increase public goods production, individuals' utility, and thus increase the incumbent's competence as perceived by FISPs. Equation ((12)) shows that the winning chances of the incumbent are monotonically decreasing with the skill shocks. Without any manipulation, the winning probability would be $1 - \mathrm{F}[0]$. The incumbent is able to improve her re-election chances by manipulating FISPs' perception.

Equation ((12)) shows that the incumbents' winning probability is positively related to the degree of the underestimation by FISPs (fiscal illusion suffering persons), $(\alpha(\delta_t) - \delta_t)W_t + (\sigma(\zeta_t) - \zeta_t)\overline{D}$, which is determined by the incumbents' manipulation. Due to the properties of fiscal illusion, the degree of underestimation (both of the level of debt and fund reduction) by FISPs increases at first, reaches the maximal value at point $\delta_t^W$ and then declines as the level of manipulation goes beyond $\delta_t^W$ and $\zeta_t^W$. This relationship is shown in Figure 2.

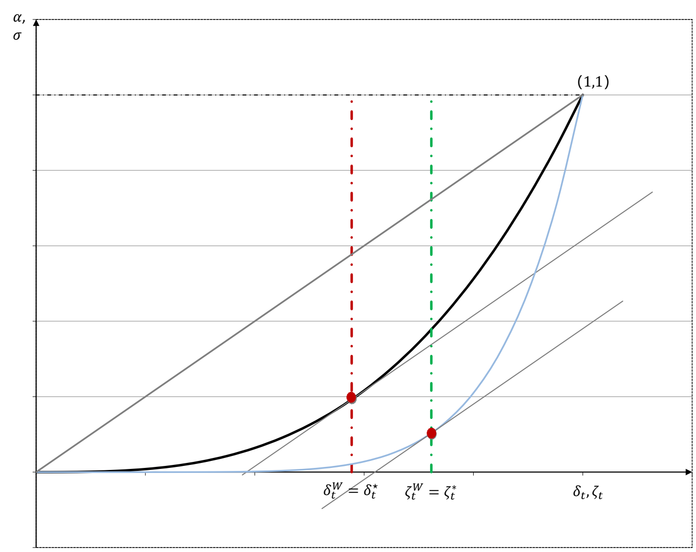



Figure 2: Winning probability maximization

Figure 2 shows that when both conditions, $\alpha'(\delta_t^W) = 1$ and $\sigma'(\zeta_t^W) = 1$, are satisfied, the degrees of underestimation by FISPs, both for the depletion of the sovereign wealth fund and the debt burden, are maximized. This means that the probability of winning is maximized.[5] In general, incumbents can enhance their re-election prospects by ma-

[5]The condition for this case will be discussed in Lemma 1 (a).

nipulating both the sovereign wealth fund and debt. Note, however, this does not mean that a larger public goods provision necessarily leads to a higher winning probability, since a public goods provision beyond the vote-maximizing points has, by itself, a utility-enhancing effect. The winning probability is maximized at the (red) points in Figure 2. Beyond levels represented by the red points, FISPs could still be manipulated, but they will realize more and more that there is manipulation.

Third, we can derive the maximization of the incumbent's expected utilities over 2 periods. It correspondent to the utility in period $t$ plus utility in period $t+1$ if the incumbent is elected times the probability of winning plus the utility in period $t+1$ if the incumbent loses times the probability of losing:

$$\begin{aligned} max_{\delta_t} V &= max_{\delta_t}\ V_t^a + \beta^a V_{t+1}^a \\ &= max_{\delta_t}\ E_t\{\ u(c_t) + g_t^a + X_t\ \} \\ &\quad +\beta^a E_t\{\ [1 - F[\cdot]]\,[u(c_{t+1}^a) + g_{t+1}^a + X_{t+1}]\ +\ F[\cdot]\,[u(c_{t+1}^b) + g_{t+1}^b]\ \}, \end{aligned} \tag{13}$$

where $F[\cdot] = F[\ \psi W_t(\alpha(\delta_t) - \delta_t) + \psi\overline{D}(\sigma(\zeta_t) - \zeta_t)\ ]$. It could be re-written as follows:

$$\begin{aligned} max_{\delta_t} &\quad u(c_t) + \tau y_t + \mu_{t-1}^a + \delta_t W_t + \zeta_t\overline{D} + X_t \\ + &\quad \beta^a[\ u(c_{t+1}) + \tau y_{t+1} - \lambda_W(\delta_t)(1 + r_W)W_t - \lambda_D(\zeta_t)(1 + r_D)\overline{D}\ ] \\ + &\quad \beta^a X_{t+1}[\ 1 - F[\cdot]\ ]\ . \end{aligned} \tag{14}$$

As long as the repayment function is not very concave (see discussion of its properties in Appendix 1), the second-order conditions $V_{\delta_t\delta_t} < 0$, and $V_{\delta_t\delta_t}V_{\zeta_t\zeta_t} - (V_{\delta_t\zeta_t})^2 > 0$ hold. The optimal percentage of fund dissolution, $\delta_t^\star$, and the optimal debt level, $\zeta_t^\star$, can be fully characterized by the first-order conditions:

$$V_{\delta_t} = \quad W_t - \beta^a \lambda'_W(\delta_t^\star)(1+r_W)W_t - \beta^a \psi W_t(\alpha'(\delta_t^\star)-1)F'[\cdot]\; X_{t+1} = 0\;; \tag{15}$$

$$V_{\zeta_t} = \quad \overline{D} - \beta^a \lambda'_D(\zeta_t^\star)(1+r_D)\overline{D} - \beta^a \psi \overline{D}(\sigma'(\zeta_t^\star)-1)F'[\cdot]\; X_{t+1} = 0\;. \tag{16}$$

Both equations look very similar. The first term both in equations ((15)) and ((16)) captures the marginal gain of increased public goods consumption by incurring debt or by using the wealth fund in the election year, respectively. The second term captures the marginal cost of incurring debt or using the wealth fund. The last term depicts the marginal effect on the expected ego rent, which is the marginal impact on the chance of re-election times the ego rent. Whether the marginal effect on the winning probability is positive or negative depends on the properties of repayment functions, $\lambda_W(\delta_t)$ and $\lambda_D(\zeta_t)$. Moreover, over- and under-manipulations of the sovereign wealth fund and debt are independent of one another. This is due to the assumption that the replenishment and repayment functions are independent of one another. These findings are crucial for Lemma 1 and Proposition 1.

## 4 Proposition and Discussion

**Lemma 1.** *-Overmanipulation and Undermanipulation.*

The optimal manipulations, both of the sovereign wealth fund and debt, are determined by the properties of the replenish function.[6]

(a) The incumbent's expected utility is maximized at the winning chances maximization point If and only if the subjective discounted marginal replenishment ratio of the

[6]The mechanism of the optimal manipulation of the sovereign wealth fund and debt are analogous. Hence, we only discuss one of them, namely, the sovereign wealth fund. The following optimal strategies are also applied to the manipulation of debt.

sovereign wealth fund satisfies that $\beta^a \lambda'_W(\delta_t^\star)\,(1+r_W) = 1$:

$$\alpha'(\delta_t^\star) = \alpha'(\delta_t^W) = 1 \quad , \tag{17}$$

Otherwise, the optimal manipulation deviates from the winning probability maximization point.

(b) The situation that the optimal value of $\delta_t$ is lower than the value for the maximal winning probability, we call undermanipulation. It is optimal for the incumbent to undermanipulate if and only if the subjective discounted marginal replenishment ratio of the sovereign wealth fund satisfies that $\beta^a \lambda'_W(\delta_t^\star)\,(1+r_W) > 1$ :

$$\alpha'(\delta_t^\star) < 1 \quad . \tag{18}$$

(c) The situation that the optimal value of $\delta_t$ goes beyond the value for the maximal winning probability, we call overmanipulation. It is optimal for the incumbent to overmanipulate if and only if the subjective discounted marginal replenishment ratio of the sovereign wealth fund satisfies that $\beta^a \lambda'_W(\delta_t^\star)\,(1+r_W) < 1$ :

$$\alpha'(\delta_t^\star) > 1 \quad . \tag{19}$$

*Proof.*See Appendix 1. ■

In general, the incumbent appears more competent by using debt and part of the wealth fund to raise the public goods provision in the election year. To maximize her utility, the incumbent faces a three-way trade-off between winning probability, public goods provision in the election year, and public goods provision in the following year. The fiscal illusion assumption determines the trade-off between the re-election chances and the public goods provision. Since the perception of FISPs (fiscal illusion suffering persons) is depicted by a convex function of the magnitude of manipulation, there exists a point where the incumbent's re-election chances are maximized. Lemma 1 (a) shows that only when the subjective discounted marginal sovereign wealth fund replenishment ratio perceived by the incumbent equals to the marginal benefit, then $\delta_t^W$ is the optimal manipulation. Analogously, the underestimation of fund reduction by FISPs and the incumbent's utility reach maximum, at point $\delta_t^W$. Otherwise, the maximized utility is achieved by overmanipulation (or undermanipulation) which delivers higher public goods provision to the incumbent in the current period (or in the future). And the trade-off between the public goods provision for now or for the future depends on the incumbent's subjective discounted marginal replenishment ratio.

Lemma 1 (b) and (c) illustrate two cases where the optimal manipulation deviates from the winning probability maximization point. Lemma 1 (b) suggests that if and only if the subjective discounted marginal replenishment exceeds the marginal benefit, then it is optimal to undermanipulate. As equation ((15)) shows, the marginal effect on the expected ego rent is positive when it is undermanipulated, which delivers positive utility to compensate for the disutility of replenishment. Hence, it is optimal to undermanipulate. Lemma 1 (c) suggests that if the marginal replenishment is lower than the marginal benefit, the optimal value of $\delta_t^\star$ goes beyond the winning probability maximization value. It indicates that the marginal replenishment cost is less than the marginal gain of increased public goods consumption, and the incumbent intends to sacrifice some share of votes for a higher public goods provision in the election year.

Lemma 1 is also suitable for the manipulation of debt. Figure 3 shows four possible combinations of the optimal manipulations of the sovereign wealth fund and debt[7]. The optimal manipulation of the wealth fund and debt are independent and depend on the properties of replenishment and repayment obligations. However, we assume that com-

[7]The remaining possible combinations are presented in Appendix 2.

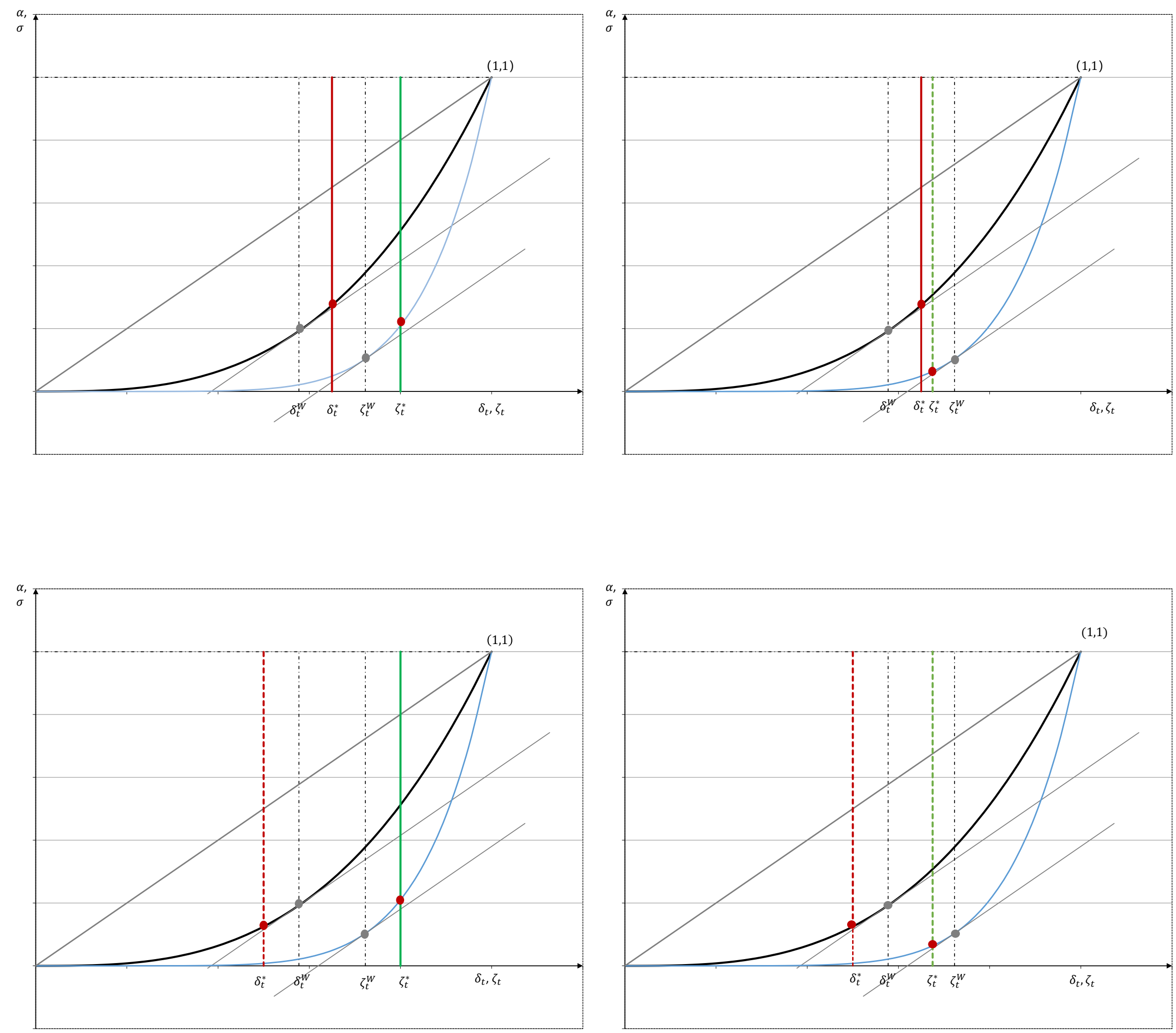



Figure 3: Maximization Problem

paring with tapping into the wealth fund, incurring debt is more costly, in other words, $r_D > r_W$, and more repayment than replenishment is likely, since that the sovereign wealth fund is a fund without any liability, financed either by fiscal surpluses or natural resources revenue and supposed to be used for macroeconomic purposes. These properties of the wealth fund suggest that it would not evoke too much attention and judgment to use it to financing public goods provision in the election year. Therefore, the incumbent prefers to use more wealth fund than debt, which indicates that it is more likely to overmanipulate with the wealth fund rather than debt. Under this assumption, the case that undermanipulating with the wealth fund and overmanipulating with debt is costly and less likely happen. Hence, our discussion mainly focuses on figures **??**, **??** and **??**. Figure **??** shows that it is optimal to overmanipulate with both sovereign wealth fund and debt. Figure **??** illustrates the situation that it is optimal to overmanipulate with the sovereign wealth fund and undermanipulate with debt. Figure **??** depicts the situation that it is optimal to undermanipulate with both sovereign wealth fund and debt.

**Proposition 1.** - *Ego Rent.*

As the ego rent increases, the incumbent becomes more eager to win. At the equilibrium, higher re-election chances can be obtained by reducing over/under-manipulations. The optimal manipulation gets closer to the value where the winning probability is maximized.

The most likely cases as discussed before are as follows (other cases are analogous):

(a) Figure **??** shows that it is optimal for the incumbent to overmanipulate with both the sovereign wealth fund and debt. At the equilibrium, as the ego rent increases, manipulations with both the wealth fund and debt go down:

$$\frac{\mathrm{d}\delta_t^\star}{\mathrm{d}X_{t+1}} < 0 \qquad \text{and} \qquad \frac{\mathrm{d}\zeta_t^\star}{\mathrm{d}X_{t+1}} < 0\,. \tag{20}$$

(b) Figure **??** shows that it is optimal to overmanipulate with the wealth fund and undermanipulate with debt. At the equilibrium, as the ego rent increases, manipulation with the wealth fund decreases, and manipulation with debt goes up:

$$\frac{\mathrm{d}\delta_t^\star}{\mathrm{d}X_{t+1}} < 0 \qquad \text{and} \qquad \frac{\mathrm{d}\zeta_t^\star}{\mathrm{d}X_{t+1}} > 0 \,. \tag{21}$$

(c) Figure **??** shows that it is optimal for the incumbent to undermanipulate with both the sovereign wealth fund and debt. At the equilibrium, as the ego rent increases, manipulations with both the wealth fund and debt go up:

$$\frac{\mathrm{d}\delta_t^\star}{\mathrm{d}X_{t+1}} > 0 \qquad \text{and} \qquad \frac{\mathrm{d}\zeta_t^\star}{\mathrm{d}X_{t+1}} > 0 \,. \tag{22}$$

*Proof.*See Appendix 2. ■

Figure 4 illustrates Proposition 1 graphically. A higher ego rent makes the incumbent adjust manipulations to increase her winning probability. Lemma 1 (b) and (c) show that the incumbent intends to sacrifice some share of votes for higher public goods provision, namely, overmanipulation or undermanipulation. It implies that a higher winning probability could be obtained if the government manipulates towards the winning probability maximization direction, that is reducing overmanipulation (or undermanipulation) with the wealth fund and debt. This results from the assumptions both of voters' perception and the properties of repayment. On the one hand, the degree of fiscal illusion is a convex function of manipulations; the winning probability is maximized at some point on the fiscal illusion curve. However, to maximize her utility, the incumbent is willing to sacrifice some share of votes and for a higher level of public goods provision in the election year (Lemma 1 (b)) or in the following year (Lemma 1 (c)). On the other hand, the properties of replenishment and repayment determine the outcome of the trade-off between public goods provision in period $t$ and $t + 1$. At the equilibrium, a higher ego rent increases the attraction of being re-elected, hence, the optimal manipulations get closer to the winning probability maximization values.

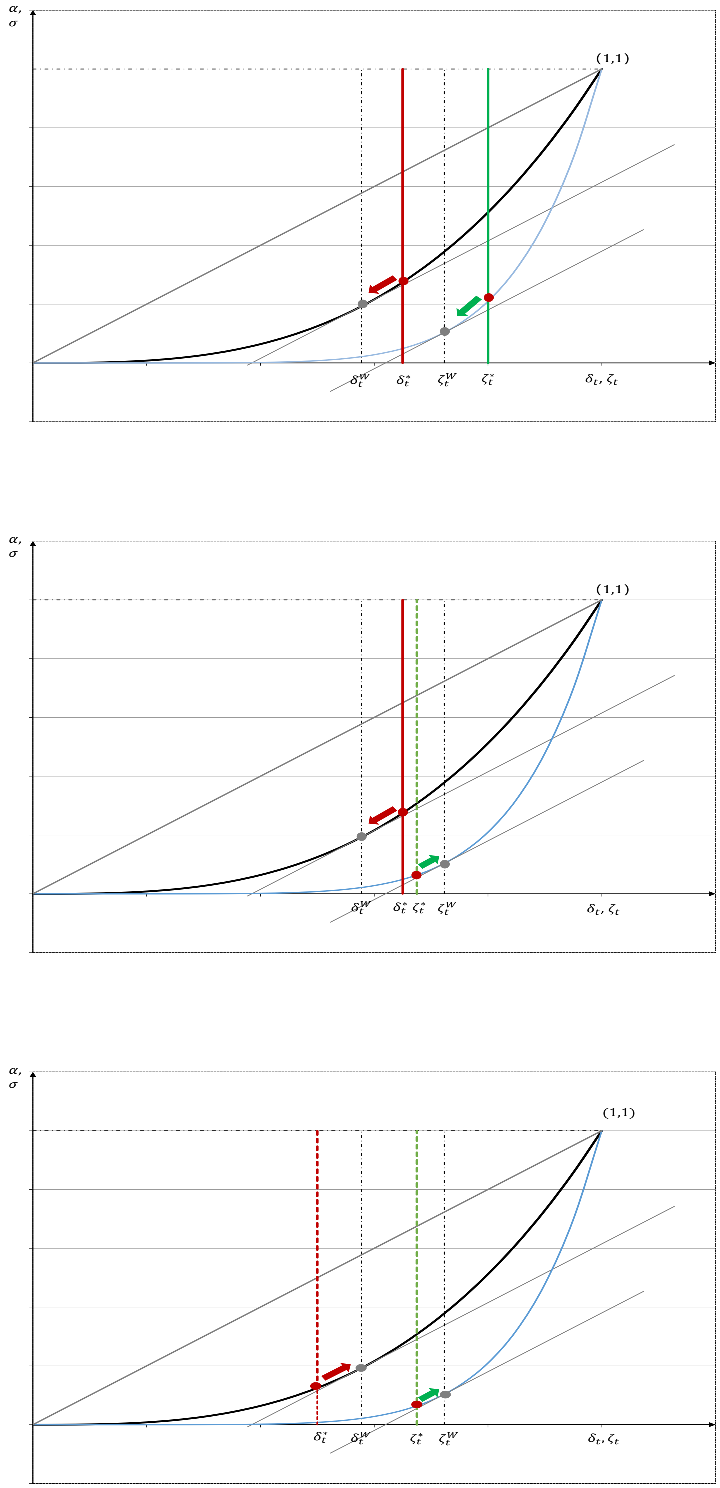



Figure 4: Ego rent

## 5 Model with punishment

Politicians are judged as opportunistic and selfish due to the manipulation in elections, which may incur a poor reputation. Hence, except for the economic costs of manipulation (replenishment and repayment), we also take the punishment, which could be reputation cost, into account. We assume that the incumbent will be punished in the off-election year if she has manipulated voters in the election year. The incumbent's decision is based on her expectation both of replenishment and punishment. Due to the fact that the manipulation of the sovereign wealth fund and debt are independent, we only analyze one of them, the sovereign wealth fund, here.

The politicians' utility function is:

$$V_t^j = \sum_{s=t}^{\infty} (\beta^j)^{s-t} E_s[u(c_s) + g_s - \mathbf{I}_{s-1} P(\delta_{s-1}) + \mathbf{I}_s X_s] \ , \tag{23}$$

$$j = \ a, b \ ; \quad \mathbf{I}_s = \begin{cases} 1 & \text{if in power in period } s; \\ 0 & \text{otherwise.} \end{cases}$$

where $P(\delta_{s-1})$ represents the punishment for manipulation. We assume the punishment depends on how much of the sovereign wealth fund has been used:

$$P(\delta_t) = m\delta_t W_t \quad , \tag{24}$$

where $m$ is a coefficient of punishment. Corresponding to the core model, the model with punishment can be solved by three steps. The detail of the solution is presented in the Appendix B. The second order conditions holds, the optimal ratio of fund reduction, $\delta_t^\star$, can be characterized by the first-order condition:

$$W_t - \beta^a m W_t - \beta^a \lambda_W'(\delta_t^\star)(1 + r_W)W_t - \beta^a \psi W_t(\alpha' - 1)F'[\cdot] \ X_{t+1} = 0 \tag{25}$$

It shows that the optimal manipulation not only depends on the properties of replenishment function, $\lambda_W(\delta_t)$, but also the magnitude of punishment, $m$.

**Proposition 2.** - *Optimal manipulation with punishment*

If the subjective discounted marginal punishment is relatively large, $\beta^a m W_t > W_t$, it is optimal to undermanipulate. Otherwise, the optimal manipulation depends on the relationship between the subjective discounted marginal replenishment ratio and the discounted net marginal benefit:

(a) If and only if the marginal benefit of using the pension reserve fund (or incurring debt) is offset entirely by the subjective discounted marginal replenishment ratio and subjective discounted marginal punishment, the incumbent's winning probability and utility are maximized simultaneously:

$$\text{If } \beta^a \lambda'_W(\delta_t^\star)\,(1 + r_W) \;+\; \beta^a m \;=\; 1\,, \quad \text{then } \alpha'(\delta_t^\star) \;=\; 1. \tag{26}$$

(b) If and only if the marginal benefit of using the pension reserve fund (or incurring debt) is larger than the subjective discounted marginal cost (which is composed by the subjective discounted marginal replenishment ratio and subjective discounted marginal punishment), it is optimal for the incumbent to overmanipulate:

$$\text{If } \beta^a \lambda'_W(\delta_t^\star)\,(1 + r_W) \;+ \beta^a m \;<\; 1\,, \quad \text{then } \alpha'(\delta_t^\star) \;>\; 1. \tag{27}$$

(c) If and only if the marginal benefit of using the pension reserve fund (or incurring debt) is not enough to compensate the subjective discounted marginal cost (which is composed by the subjective discounted marginal replenishment ratio and subjective discounted marginal punishment), it is optimal for the incumbent to undermanipulate:

$$\text{If } \beta^a \lambda'_W(\delta_t^\star)\,(1 + r_W) \;+ \beta^a m \;>\; 1\,, \quad \text{then } \alpha'(\delta_t^\star) \;<\; 1. \tag{28}$$

*Proof.*See Appendix 1. ■

The possibility of punishment increases the cost of manipulation, and hence, the incumbent's optimal manipulation changes accordingly. The optimal condition shows that in contrast to the optimal manipulation strategy in the core model, punishment reduces manipulations. The magnitude of reduction depends on the severity of the punishment. Lemma 1 shows that the incumbent's re-election chances and utility are maximized simultaneously if and only if the marginal benefit of using the pension reserve fund equals the subjective discounted marginal replenishment ratio, namely, $\beta^a \lambda'_W(\delta_t^\star)\,(1 + r_W) = 1$; otherwise, the optimal manipulation deviates from the maximized winning probability. However, if we take the punishment into account, this result does no longer hold. Proposition 2 shows that the marginal cost of manipulation (which is not only composed by the subjective discounted marginal replenishment ratio but also by the subjective discounted marginal punishment) increases by $\beta^a m$, which results in a decreasing threshold of the optimal manipulation.

Besides, there exists a possibility that the subjective discounted marginal replenishment ratio is smaller than the marginal benefit but larger than the difference between marginal benefit minus the subjective discounted punishment, $1-\beta^a m \ < \ \beta^a \lambda'_W(\delta_t^\star)\,(1+r_W) \ < \ 1$, which makes the optimal manipulation in the core model turns from overmanipulation into undermanipulation.

Otherwise, Proposition 1 holds when the punishment is take into account: A higher ego rent reduces both over and under manipulation.

We present another type of punishment in Appendix 2, which is a function of how much of the fund reduction (or debt level) has been underestimated.[8] An interesting finding is that a more severe punishment does not always reduce manipulations at the

[8]Comparing with the core model, the punishment does not affect the threshold of the optimal manipulation (over-/under-manipulation) but reverses the optimal manipulation from overmanipulation (or

equilibrium. A more severe punishment enlarger the political budget cycle when it is optimal to overmanipulate since the increasing overmanipulation reduces the marginal punishment in this case.

## 6 Conclusion

This paper studies how the incumbent increases her popularity during the election by using the sovereign wealth fund and incurring debt. We assume that the degree of fiscal illusion is a convex function of the depletion of the sovereign wealth fund and the level of debt, which means that as more of the sovereign wealth fund is used (or the more of debt incurred), the degree of fiscal illusion is increasing at first, reaching the threshold, and then decreasing. We argue that fiscal illusion suffering voters are unable or unwilling to find out the actual public spending (both the SWF and debt) and tend to underestimate the cost of public service. The model illustrates how politicians take advantage of fiscal illusion to obtain higher winning chances and a higher public goods provision.

The model illustrates that the property of fiscal illusion produces a point at which re-election chances is maximized. The properties of replenishment and repayment function determine whether over-manipulation or under-manipulation is optimal. Both overmanipulation and undermanipulation indicate that the incumbent is intend to sacrifice some votes for the economic gains in the election year or in the following year. When the subjective discounted marginal replenishment ratio (or repayment ratio) is smaller than the discount rate, higher public goods spending in the election year is more attractive, and it is optimal to over-manipulate. Otherwise, the incumbent perceives higher public goods spending in the following year is better and tends to undermanipulate. At the equi-

undermanipulation) to undermanipulation (or overmanipulation). This is due to the fact that marginal punishment is reducing as the manipulation increases. Lemma 1 holds: at the equilibrium, a higher ego rent reduces over-/under-manipulation.

librium, the overmanipulation (or undermanipulation) decreases when the ego rent gets larger. As the ego rent increases, the incumbent is more eager to win, and would like to give up the additional public goods for increasing her winning probability.

Moreover, we also take punishment into account. The punishment could be either a fine or a reputation loss. It is shown that higher punishment reduces manipulations if the punishment depends on how much of the sovereign wealth funds (or debt) has been used. For some cases, the optimal manipulation may even turn from overmanipulation into undermanipulation. However, a more severe punishment does not always reduce manipulations when the punishment depends on how much of the fund reduction (or debt level) has been underestimated.

Overall, the incumbent can increase her re-eection chances by either incuring debt or using an existing sovereign wealth fund. The obligation of repayment restrticts the level of manipulations; however, overmanipulation is still possible. A higher ego rent not necessarily increases manipulations, which means that selfish politicians are not always a threat to society.

# A The solution of the core model and perturbation results

## 1 The incumbent's maximisation problem

The model can be solved in three steps. First, we derive the probability of voters to vote for the incumbent. The logic corresponds to: voters vote for the incumbent if they expect the incumbent to deliver higher utility. Second, the incumbent's winning probability can be obtained. Fiscal illusion suffering persons (FISPs) underestimate both the level of debt and the sovereign wealth fund reduction. This faulty perception leads FISPs to overestimate the skills of the incumbent. They attribute part of the higher provision of public goods to the incumbent's competence. The incumbent's winning probability is

$$\begin{aligned} \text{Prob}^{win} &= \text{Prob}\ \{\ (1-\psi)\ [\frac{E_t^{REAL}[\mu_t^a]}{2\phi} + \frac{1}{2}] + \psi\ [\frac{E_t^{FISP}[\mu_t^a]}{2\phi} + \frac{1}{2}] \geq\ \frac{1}{2}\ \} \\ &= \text{Prob}\ \{\ \mu_t^a\ \geq\ \psi\ (\ \alpha(\delta_t) - \delta_t\ )\ W_t +\ \psi\ (\ \sigma(\zeta_t) - \zeta_t\ )\ \overline{D}\ \} \\ &= 1\ -\ F[\psi(\alpha(\delta_t) - \delta_t)W_t + \psi(\sigma(\zeta_t) - \zeta_t)\overline{D}], \end{aligned} \tag{1}$$

where $F[\psi(\alpha(\delta_t)-\delta_t)W_t+\psi(\sigma(\zeta_t)-\zeta_t)\overline{D}]$ is the distribution function of the skills shock. The maximum winning probability can be obtained when $F[\cdot]$ is minimized. As aforementioned, FISPs underestimate the degree of debt and the dissolution of the sovereign wealth fund, which means $\alpha(\delta_t) < \delta_t$ and $\sigma(\zeta_t) < \zeta_t$. So to minimize the monotonous function $F[\cdot]$, we need to minimize $(\alpha(\delta_t) - \delta_t)$ and $(\sigma(\zeta_t) - \zeta_t)$, which can be obtained when $\alpha'(\delta_t^W) = 1$ and $\sigma'(\zeta_t^W) = 1$.

All agents are utility maximizer. The incumbent's purpose is to maximize her utility over two-periods. The expected utility equals the utility in period $t$ plus the expected utility in period $t+1$ if she wins the election times the wining probability plus the expected utility if she loses the election times the probability of losing.

$$\begin{aligned}
max_{\delta_t}\, V^a = &\ max_{\delta_t} \quad V_t^a \ + \ \beta^a V_{t+1}^a \\
= &\ max_{\delta_t} \quad E_t^a \ \{ \ u((1-\tau)y_t) + g_t + X_t \ \} \\
&+ \qquad E_t^a \ \{ \ \underbrace{[\ 1 - F[\cdot]\ ]}_{\text{Prob. wins}} \beta^a [u((1-\tau)y_{t+1}) + g_{t+1}^a + X_{t+1}] \ \} \\
&+ \qquad E_t^a \ \{ \quad \underbrace{F[\cdot]}_{\text{Prob. loses}} \quad \beta^a [u((1-\tau)y_{t+1}) + g_{t+1}^b] \ \} \qquad (2) \\
= &\ max_{\delta_t} \quad u((1-\tau)y_t) + \tau y_t + \mu_{t-1}^a + \delta_t W_t + \zeta_t \overline{D} + X_t \\
&+ \qquad \beta^a \ \{ \ u((1-\tau)y_{t+1}) + \tau y_{t+1} - \lambda_W(\delta_t)(1+r_W)W_t \\
&\qquad -\lambda_D(\zeta_t)(1+r_D)\overline{D} + \beta^a X_{t+1}[\ 1 - F[\cdot]\ ]\ \}\ . \qquad (3)
\end{aligned}$$

The first order conditions (FOCs) are:

$$W_t - \beta^a \lambda_W'(\delta_t^\star)(1+r_W)W_t - \beta^a F'[\cdot] X_{t+1} \psi W_t (\alpha'(\delta_t^\star) - 1) = 0\ ; \qquad (4)$$

$$\overline{D} - \beta^a \lambda_D'(\zeta_t^\star)(1+r_D)\overline{D} - \beta^a F'[\cdot] X_{t+1} \psi \overline{D} (\sigma'(\zeta_t^\star) - 1) = 0\ . \qquad (5)$$

From equations ((4)) and ((5)), the second partial derivatives are found to be:

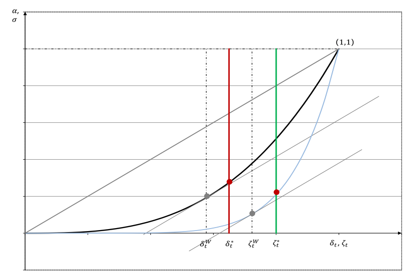
$$\begin{aligned}
V_{\delta\delta} \quad = &\quad -\beta^a \lambda_W''(\delta_t^\star)(1+r_W)W_t - \beta^a F''[\cdot] X_{t+1} \psi^2 W_t^2 (\alpha'(\delta_t^\star) - 1)^2 \\
&\quad -\beta^a F'[\cdot] X_{t+1} \psi W_t \alpha''(\delta_t^\star)\ ; \qquad (6) \\
V_{\delta\zeta} \quad = &\quad -\beta^a F''[\cdot] X_{t+1} \psi^2 W_t \overline{D} (\alpha'(\delta_t^\star) - 1)(\sigma'(\zeta_t^\star) - 1)\ ; \qquad (7) \\
V_{\zeta\zeta} \quad = &\quad -\beta^a \lambda_D''(\zeta_t^\star)(1+r_D)\overline{D} - \beta^a F''[\cdot] X_{t+1} \psi^2 \overline{D}^2 (\sigma\prime(\zeta_t^\star) - 1)^2 \\
&\quad -\beta^a F'[\cdot] X_{t+1} \psi \overline{D} \sigma''(\zeta_t^\star)\ . \qquad (8)
\end{aligned}$$

So that we have:

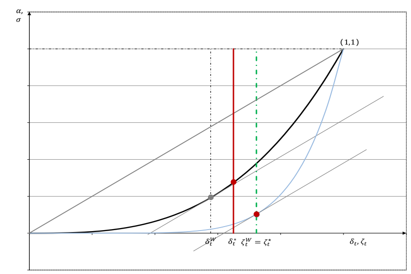
$$\left|H\right| = \begin{vmatrix} V_{\delta\delta} & V_{\delta\zeta} \\ V_{\delta\zeta} & V_{\zeta\zeta} \end{vmatrix}.$$

As we discussed, the curve of fiscal illusion perception is convex ($\sigma''(\zeta_t)$ and $\alpha''(\delta_t)$ are positive). With this condition, the second-order conditions hold if and only if the repayment functions are not too concave.

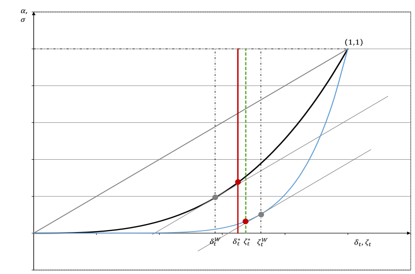
Then the optimal percentages of using the sovereign wealth fund, $\delta_t^\star$, and debt, $\zeta_t^\star$, are determined by the properties of the repayment functions. Therefore, Lemma 1 can be derived.



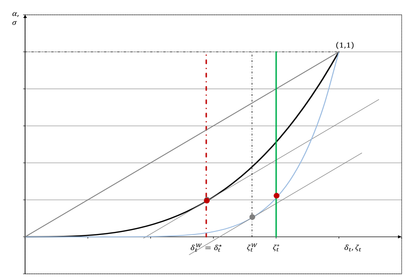
(a) SWF: over; Debt: over



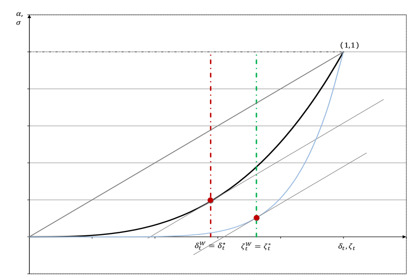
(b) SWF: over; Debt: neither nor



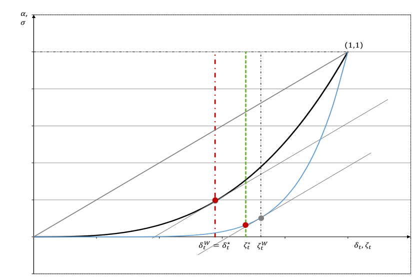
(c) SWF: over; Debt: under



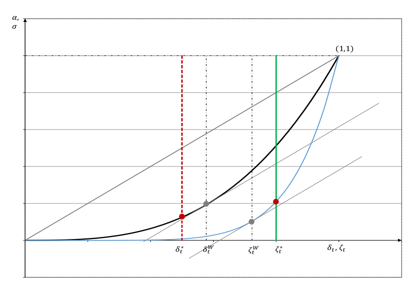
(d) SWF: neither nor; Debt: over



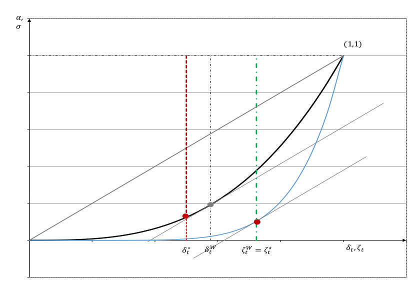
(e) SWF: neither nor; Debt: neither nor



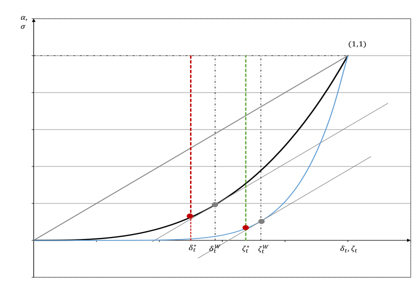
(f) SWF: neither nor; Debt: under



(g) SWF: under; Debt: over



(h) SWF: under; Debt: neither nor



(i) SWF: under; Debt: under

FIGURE 5: MAXIMIZATION PROBLEM

What Lemma 1 suggests is that the incumbent faces a three-way trade-off among re-elected chances, public good privision for now and for future. If and only if the subjective discounted marginal cost of manipulation (both incurring debt and tapping into the wealth fund) equals to the marginal benefit, the incumbent's utility is maximized at the value of winning probability maximization. Otherwise, the optimal manipulation deviates from the value of winning probability maximization. Both over and under manipulation are possible. Figure 5 shows all possible combinations of the optimal manipulations of the sovereign wealth fund and debt.

## 2 Perturbation results for the propositions

Indications on the results of Section 4

$$\frac{\mathrm{d}\delta_t^\star}{\mathrm{X_{t+1}}} = -\frac{1}{|H|}(V_{\zeta\zeta}\frac{\partial V_\delta}{\partial X_{t+1}} - V_{\delta\zeta}\frac{\partial V_\zeta}{\partial X_{t+1}}) \; ; \tag{9}$$

$$\frac{\mathrm{d}\zeta_t^\star}{\mathrm{X_{t+1}}} = -\frac{1}{|H|}(-V_{\delta\zeta}\frac{\partial V_\delta}{\partial X_{t+1}} + V_{\delta\delta}\frac{\partial V_\zeta}{\partial X_{t+1}}) \; . \tag{10}$$

And then the Proposition 1 obtained: At the equilibrium, a higher ego rent leads the optimal magnitude of manipulations get closer to the maximization winning probability point.

# B Punishment

A larger punishment reduces the optimal manipulations. Except repayments, we also assume that the incumbent will be punished in the off-election year if she has manipulate voters in the election year. As we have discussed in the main model, voters compare their expected utility in the following year and vote for the party which is expected to deliver higher utility. We assume that punishment for manipulation reduces politicians' utility but have no effect on the public goods provision. So taking the punishment into account, the probability of voter votes and the incumbent's probability of winning do not affected, but the incumbent's maximization problem is affected. The incumbent's expected utility over two-period becomes:

$$\begin{aligned} max_{\delta_t} V = \quad & max_{\delta_t} \quad V_t^a + \beta^a V_{t+1}^a \\ = \quad & max_{\delta_t} \quad u(c_t) + \tau y_t + \mu_{t-1}^a + \delta_t W_t + X_t \\ & + \quad \beta^a \; [\; u(c_{t+1}) + \tau y_{t+1} - \lambda_W(\delta_t)(1 + r_W) W_t - P(\delta_t) \;] \\ & + \quad \beta^a \; [\; 1 - F[\psi W_t(\alpha(\delta_t) - \delta_t)] \;] \; X_{t+1} \; . \end{aligned} \tag{1}$$

To maximize the incumbent's expected utility, we differentiate ((1)) with respect to $\delta_t$. The first order condition (FOC) is:

$$W_t - \beta^a P'(\delta_t) - \beta^a \lambda'_W(\delta_t^\star)(1 + r_W) W_t - \beta^a \psi W_t (\alpha' - 1) F'[\cdot] \; X_{t+1} = 0 \; . \tag{2}$$

The second order conditions holds, and the FOCs show that the optimal manipulation not only depends on the properties of repayment function, $\lambda_W(\delta_t)$, but also the properties of punishment function, $P(\delta_t)$.

# 1 Punishment 1

The first type of punishment is a function of how much sovereign wealth fund has been used in the previous year.

$$P(\delta_t) = m \delta_t W_t \; ; \tag{3}$$

The first-order condition and second-order condition are:

$$V_\delta = \quad W_t - \beta^a m W_t - \beta^a \lambda'_W(\delta_t^\star)(1+r_W)W_t - \beta^a \psi W_t(\alpha' - 1)F'[\cdot]\; X_{t+1} = 0\;; \tag{4}$$

$$V_{\delta\delta} = \quad -\beta^a \lambda''_W(\delta_t^\star)(1+r_W)W_t - \beta^a \psi W_t \alpha'' F'[\cdot] X_{t+1} - \beta^a \psi^2 W_t^2 (\alpha' - 1)^2 F''[\cdot]\; X_{t+1} \tag{5}$$

Similar to the core model, as long as the repayment function is not too concave, the second-order condition, $V_{\delta\delta} < 0$, holds. The optimal percentage of fund dissolution, $\delta_t^\star$, can be fully characterized by the first-order condition. If and only if the subjective discounted marginal costs (which is composed by both replenishment ratio and punishment) equals to the discounted marginal net benefit, the incumbent's expected utility is maximized at the winning probability maximization point. Otherwise, the optimal manipulation deviates from the winning probability maximization point. When the subjective discounted marginal cost is larger than the discounted subjective expected benefit of manipulation, then it is optimal to undermanipulate. Otherwise, it is optimal to overmanipulate.

For proposition 1: At the equilibrium, as the ego rent increases, the optimal manipulation gets closer to the winning probability maximization point.

$$V_{\delta, X_{t+1}} = \; - \; \beta^a \psi W_t(\alpha'(\delta_t^\star) - 1)F'[\cdot]\;; \tag{6}$$

$$\text{If } \alpha' \; > \; 1\;, \text{ then } V_{\delta_t, X_{t+1}} \; < \; 0\;, \text{ and } \frac{\mathrm{d}\delta_t^\star}{\mathrm{d}X_{t+1}} \; < \; 0\;; \tag{7}$$

$$\text{If } \alpha' \; < \; 1\;, \text{ then } V_{\delta_t, X_{t+1}} \; > \; 0\;, \text{ and } \frac{\mathrm{d}\delta_t^\star}{\mathrm{d}X_{t+1}} \; > \; 0\;. \tag{8}$$

Punishment parameter: At the equilbirum, as the punishment parameter increases, the optimal manipulation goes down.

$$V_{\delta_t,m} = -\beta^a W_t \ < \ 0 \ ; \tag{9}$$

$$\frac{\mathrm{d}\delta_t^\star}{\mathrm{d}m} = \ - \ \frac{V_{\delta_t,m}}{V_{\delta_t,\delta_t}} \ < \ 0 \ . \tag{10}$$

## 2 Punishment 2

We assume that the FISPs will realize that they were manipulated by the incumbent, and they will punish the incumbent for manipulation. Then the punishment in this cases is a function of how much of the fund reduction has been underestimated by FISPs:

$$P(\delta_t) = h\psi(\delta_t - \alpha(\delta_t))W_t \ . \tag{11}$$

The punishment $P_{t+1}$, in this case, is a concave function of the fund reduction in the previous year, $\delta_t$. Corresponding to the underestimation of the fund reduction by FISPs, the punishment increases with the fund reduction at first, and reaches the maximum at $\delta_t^W$. Beyond this point, the punishment decreases. The marginal punishment is decreasing in the dissolution of the wealth fund (debt).

The first-order condition and second-order condition are:

$$V_\delta = W_t - \beta^a \lambda_W'(\delta_t^\star)(1+r_W)W_t + \beta^a \psi W_t(\alpha'(\delta_t^\star) - 1) \ [ \ h - F'[\cdot]X_{t+1} \ ] \ = 0 \ ; \tag{12}$$

$$V_{\delta\delta} = -\beta^a \lambda_W''(\delta_t^\star)(1+r_W)W_t + \beta^a \psi W_t \alpha''(\delta_t^\star) \ [ \ h - F'[\cdot]X_{t+1} \ ]$$
$$-\beta^a \psi^2 W_t^2 (\alpha'(\delta_t^\star) - 1)^2 F''[\cdot]X_{t+1} \ . \tag{13}$$

We assume the punishment parameter, $h$, satisfies that $h < F'[\cdot]X_{t+1}$. It shows that the second-order condition, $V_{\delta\delta} < 0$, holds. Then the optimal manipulation could be characterized by the first-order condition, which is oppositely to the optimal manipulation in the core model:

$$\text{If } \beta^a \lambda'_W(\delta_t^\star)\ (1 + r_W) \ < \ 1\ ,\ \text{then } \alpha'(\delta_t^\star) \ < \ 1\ ; \tag{14}$$

$$\text{If } \beta^a \lambda'_W(\delta_t^\star)\ (1 + r_W) \ > \ 1\ ,\ \text{then } \alpha'(\delta_t^\star) \ > \ 1\ . \tag{15}$$

This outcome results by the property of the punishment function, more specifically, the marginal punishment being positive when the incumbent undermanipulates and negative when the incumbent overmanipulates. The property of the punishment increases the marginal cost in undermanipulation and decreases the marginal cost in overmanipulation.

Analogue to proposition 1: Although the optimal manipulation is reversed compared to the core model, Proposition 1 still holds: A higher ego rent reduces over-/under- manipulation at the equilibrium.

$$V_{\delta_t, X_{t+1}} = -\beta^a \psi W_t\ (\ \alpha'(\delta_t^\star) - 1\ )\ F'[\cdot]\ . \tag{16}$$

$$\text{If } \alpha' \ > \ 1\ ,\ \text{then } V_{\delta_t, X_{t+1}} \ < \ 0\ ,\ \text{and } \frac{\mathrm{d}\delta_t^\star}{\mathrm{d}X_{t+1}} \ < \ 0\ ; \tag{17}$$

$$\text{If } \alpha' \ < \ 1\ ,\ \text{then } V_{\delta_t, X_{t+1}} \ > \ 0\ ,\ \text{and } \frac{\mathrm{d}\delta_t^\star}{\mathrm{d}X_{t+1}} \ > \ 0\ . \tag{18}$$

Punishment parameter: When the punishment depends on the amount that has been underestimated, then a larger punishment cannot limit the opportunism which leads to overmanipulation: when overmanipulation is optimal for the incumbent, then, at the equilibrium, the budget cycle will be enlarged along with the punishment getting larger.

However, when it is optimal to undermanipulate, then manipulations decrease with an increase in the degree of punishment.

$$V_{\delta_t,h} = \beta^a \psi W_t \ ( \ \alpha'(\delta_t^\star) - 1 \ ) \ . \tag{19}$$

$$\text{If } \alpha' \ > \ 1 \ , \text{ then } V_{\delta_t,h} \ > \ 0 \ , \text{ and } \frac{\mathrm{d}\delta_t^\star}{\mathrm{d}h} \ > \ 0 \ ; \tag{20}$$

$$\text{If } \alpha' \ < \ 1 \ , \text{ then } V_{\delta_t,h} \ < \ 0 \ , \text{ and } \frac{\mathrm{d}\delta_t^\star}{\mathrm{d}h} \ < \ 0 \ . \tag{21}$$

It shows that if it is optimal for the incumbent to overmanipulate with the sovereign wealth fund, then as the punishment parameter increases, manipulation with the wealth fund goes up at the equilibrium. If it is optimal for the incumbent to undermanipulate with the wealth fund, then as the punishment parameter increases, manipulation with the wealth fund goes down. Again, this is due to the properties of the punishment.

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