[Provisional Draft]

Incomes from capital in alternative economic theories

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Abstract

The nature of incomes from capital is quite clear in practice: they are equity dividends, capital gains, interest on loans, managerial incentives. Much more complex is their analysis from the point of view of economic theory. In particular, as is clear, this analysis is closely linked to the notion of capital adopted.
The paper addresses the theoretical explanation of profit and interest within three different approaches: i) classical/Marxian; ii) marginalist; iii) Arrow-Debreu.
In the classical/Marxian approach incomes from capital are understood as a surplus whose amount depends on the class conflict. In the marginalist theory, the rate of interest is the price firms pay to households for the use of the particular factor of production named 'capital'. As is know from the 1960s, the view of capital as a factor of production is untenable. In the Arrow-Debreu equilibrium theory, interest rates and firms' profit are still there, but, as we shall try to show, they can hardly be interpreted as incomes from capital.

1. Introduction

Empirical studies claim that, in the most industrialized countries, the share of capital in the national incomes is in the interval 25-30% (see Piketty 2014, p. 222, figure 6.5). The nature of these incomes is quite clear in practice: they are equity dividends, capital gains, interest on loans, managerial incentives. Much more complex is their analysis from the point of view of economic theory. In particular, as is clear, this analysis is closely linked to the notion of capital adopted.

As is well-known, as far as income distribution is concerned, there two alternative views in economic theory. The first is the classical/Marxian one, according to which income distribution is a social phenomenon: it depends on the conflict among social classes. The second

is the neo-classical one and understands income distribution as a market phenomenon regulated by the equilibrium between supply and demand.

Here we shall try to address the theoretical explanation of incomes from capital from both the standpoint. We shall start, in section 2, from the classical/Marxian approach in which capital is the amount of value which allows producers to pay costs in advance, before revenues are obtained, ad profit is residual, it is a surplus-value. As for the neo-classical approach, we shall consider two different versions. In section 3 we shall deal with the traditional marginalist theory, in which capital is understood as a factor of production and the rate of interest as the price firms pay for its use. Then, in section 4, we shall discuss the Arrow-Debreu model, in which capital is neither a factor of production, nor an amount of value that allow firms to pay costs in advance. Some conclusions are drawn in section 5.

2. Capital and profit in the classical/Marxian approach

In order to introduce the conception of capital within the classical/Marxian approach, we need to start by a quick reconstruction of the basic features of the capitalist model of production. In particular, we can list the following important characteristics of this social organization:

i) Products are commodities. Production is intended for the market and not (directly) for consumption.

ii) Labour is performed by wage-earning workers. Workers are forced to sell their labour-power due to their separation from the means of production. Labour-power is a commodity.

iii) Natural resources (lands, mines, oilfields, ...) are of private property. Every natural resource has an owner and the class of landowners is distinct from the class of capitalists.

iv) Production processes are organized (directly or indirectly) by capitalists. They buy the inputs and sell the outputs.

The inputs of the production processes are: commodities (row materials, tools, machines, ...); labour-power performed by workers with different skills and the productive uses of natural resources. Points i-iii above imply that these inputs must be purchased on the market. Assuming, for the sake of simplicity, that they must be paid *ex-ante*, at the beginning of the production process, capital is necessary in order to finance the expenses of production and this is the reason why capitalists are the only possible organizers of the production processes (point iv).

In fact, since production takes time, in every single process, the employment of inputs must precede the production of outputs. Accordingly, inputs are generally purchased and employed before outputs are sold. Therefore, costs and revenues of the same process are not simultaneous, as the former generally precede the latter. As a result, the costs of a certain process cannot be financed by the revenues of the same process. Hence, capital is the amount of purchasing power that is required, for each process, to finance the costs.

The amount of capital invested is then recovered out of revenues when the outputs are sold. Moreover, revenues leave, in general, a surplus over and above costs. This surplus-value is profit. The profit per unit of capital invested (for a single process) is the rate of profit.

2.1 An example

Let us assume there are N commodities, A different kinds of labour services and B different sorts of natural resources. The production process of a generic commodity n, with n = 1, 2, ..., N, which starts in a certain period t, employs a vector of commodities $\mathbf{X}_t^n \in R_+^N$, a vector of labour services $\mathbf{L}_t^n \in R_+^A$ and a vector of productive uses of natural resources $\Lambda_t^n \in R_+^B$ in order to get an output C_{t+1}^n in the period $t + 1$.

Referring to the economy as a whole, the employment of inputs in period t is $\mathbf{X}_t = \sum_n \mathbf{X}_t^n$, $\mathbf{L}_t = \sum_n \mathbf{L}_t^n$ and $\Lambda_t = \sum_n \Lambda_t^n$ and the output of period $t + 1$ is a vector $\mathbf{C}_{t+1} = [C_{t+1}^1, C_{t+1}^2, ..., C_{t+1}^N]$. Let $\mathbf{p}_t \in R_+^N$, $\mathbf{w}_t \in R_+^A$ and $\rho_t \in R_+^B$ be the (row) vectors of commodity prices, wage rates and rent rate in period t, if wages and rents are paid *ex-ante*, then the total investment of capital in period t is:

$$K_t = \mathbf{p}_t \cdot \mathbf{X}_t + \mathbf{w}_t \cdot \mathbf{L}_t + \rho_t \cdot \Lambda_t \qquad (1)$$

The capital advanced in then recovered by revenues in period t +1. However, typically, $\mathbf{p}_{t+1} \cdot \mathbf{C}_{t+1} > K_t$. This means that the revenues leave a profit over and above the costs of production:

$$\Pi_{t+1} = \mathbf{p}_{t+1} \cdot \mathbf{C}_{t+1} - K_t \qquad (2)$$

The rate of profit is the amount of profit obtained for each unit of capital invested. Accordingly:

$$\pi_{t,t+1} = \frac{\Pi_{t+1}}{K_t} \qquad (3)$$

From this example, it should be clear that:

i) The amount of capital K_t is not the quantity of an input. The quantities of inputs employed are $\mathbf{X}_t$, $\mathbf{L}_t$ and $\mathbf{\Lambda}_t$.

ii) The rate of profit is not the price of capital. The amount of profit Π_{t+1} is a residuum and not the result of a price-times-quantity multiplication.

2.2 The ordinary rate of profit

Assuming free competition among producers, this approach focusses the attention of a theoretical (persistent) position characterized by the uniformity of the rate of profit on the capital invested in the different sectors of economic activity. Following Smith, we can call this uniform rate the "ordinary rate of profit" and the relative prices associated to it the "natural prices".[1]

Thanks to Sraffa's theory, we know that commodity natural prices and the ordinary rate of profit corresponding to a given wage rate must be determined simultaneously as the solution of a system of equations. In particular, starting from the same example of the previous sub-section, for the sake of simplicity, we can set aside natural resources and assume there is just one kind of labour service. In this case, if capital must get the same rate of profit π independently of the sector in which it is invested, then the following conditions must hold:

$$p^n C_{t+1}^n = (\mathbf{p} \cdot \mathbf{X}_t^n + w L_t^n)(1 + \pi) \qquad \forall\, n = 1, 2, ..., N \qquad (4)$$

Adopting as numéraire commodity the national net output—as Sraffa does—then:

$$\mathbf{p} \cdot (\mathbf{C}_{t+1} - \mathbf{X}_t) = 1 \qquad (5)$$

Given the wage rate w, equations (4) and (5) are able to determine the vector of natural prices $\mathbf{p}$ and the ordinary rate of profit π.

Finally, equations (4) allow us to define the price vector as a function of the distribution variables: $\mathbf{p} = \mathbf{p}(w, \pi)$. Substituting this function within equation (5), we get:

$$\mathbf{p}(w, \pi) \cdot (\mathbf{C}_{t+1} - \mathbf{X}_t) = 1 \qquad (6)$$

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[1] In Marx analysis, the same concepts correspond to the "general rate of profit" and the "prices of production".

Equation (6) is what Garegnani (1984) calls the "surplus equation." It is particularly useful for a comparative statics exercise. It shows how the rate of profit that solves the system of equations (4)-(5) varies as the wage rate level changes. In particular, equations (6) tells us that there is an inverse relation between w and π. Therefore—under normal conditions and *ceteris paribus*—incomes from capital tend to be high when the wage rate level is low. This fact provides the theoretical ground for the conflict between social classes that can be observed in the real world.

3. Capital as a factor of production

Within the classical/Marxian approach, the organization of the society in three classes—workers, landowners and capitalists—entails the division of the national income in three parts: wages, rents and profit. Within the neo-classical/marginalist theory, these three different incomes are understood as what firms pay to households for the employment of three 'factors of production': labour, land and capital.[2]

Once this standpoint is adopted, the existence of the social classes becomes inessentials. The working of the system depends on the decisions taken by two different sorts of economic agents: households and firms.

Firms organize the production processes. They demand factors of production and supply commodities. Factor prices—wage rate, rent rate and interest rate—are included into price vectors. Given a price vector, each firm decides its production plan in order to maximize its (net) profit, namely the difference between revenues and the costs for the employment of the production factors.

Households supply production factors—which are available in given quantities—and demand the commodities produced. Given a price vector, each household decides its consumption plan in order to maximize its utility subject to its budget constraint.

In this framework, workers, landowners and capitalists—provided that they can be distinguished by some special assumption—are on the same side: that of households. They are providers of factors of production. Then, wages, rents and interest paid by firms to households have exactly the same nature. They are incomes for households and costs for firms.

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[2] According to Marx, the factors of production are the result of the transformation of social classes into things. See Marx, *Capital III*, p. 830.

3.1 The equilibrium rate of interest

Once the distribution variables are understood as factor prices, they must be determined at the same time and by the same mechanism as all the other prices, namely by a system of general equilibrium conditions.

Focussing on the capital market, the rate of interest is thought to fall whenever the demand for capital by firms is smaller than the quantity of this factor made available by households, and to rise in the opposite case. The variation of the rate of interest with respect to the other factor prices should entail a change in the methods of production in use. In particular, a fall of the rate of interest should bring about the adoption of more capital-intensive methods so that, *ceteris paribus*, the demand for capital increases, leading to a reduction of the initial excess supply.

The fundamental idea behind this mechanism of substitutability between factors is well-known. The first order conditions for firms' (net) profit maximization impose the equality between factor prices and their respective marginal productivity. Hence, the principle of diminishing marginal productivity implies that, if the rate of interest falls in terms of the output commodity and the other factor prices are unchanged, the adoption of a more capital-intensive production plan is needed in order to restore the equality between the marginal product of capital and the rate of interest.

Therefore, the equilibrium rate of interest is typically conceived as the rate that makes capital demand for capital by firms equal to the supply provided by households—which in turns depends on their present and past saving decisions. In Marshall's words:

> interest, being the price paid for the use of capital in any market, tends towards an equilibrium level such that the aggregate demand for capital in that market, at that rate of interest, is equal to the aggregate stock forthcoming there at that rate (Marshall 1920, p. 534).

3.2 Interest and profit

In the marginalist approach, incomes from capital are what firms pay to household for the use of a factor of production. As a result, incomes from capital, transformed into the amount of interest on the capital employed, loses their residual nature. They are not the difference between revenues and costs. They are understood as the result of a price×quantity multiplication and enter into the costs.

Let us denote by $L^n, \Lambda^n \, K^n$ the quantities of the factors labour, land and capital employed in the production of a certain commodity n, with n = 1, 2, ... N. Given the technical conditions of

production, the amount of final output obtained[3] is expressed as a certain (differentiable) function of the quantities of the factors of production used:

$$C^n = F^n(L^n, \Lambda^n, K^n) \qquad (7)$$

Accordingly, let w, ρ and i be the wage rate, the rent rate and the interest rate respectively, the total costs of production of a final output C^n are: $wL^n + \rho\Lambda^n + iK^n$.

For a given price p^n of commodity n, the amount of profit earned from the production of a quantity C^n is:

$$\Pi^n = p^n C^n - (wL^n + \rho\Lambda^n + iK^n) \qquad (8)$$

Within this theory, the amount of profit Π^n is not part of the incomes from capital. Here, the income earned because of the employment of capital is the amount of interest iK^n that is included in the costs. The profit Π^n is understood an income earned by firms. In fact, as said at the beginning of this section, in the marginalist theory, firms decide their production plans in order to maximize their profit, namely the difference between revenues and the costs for the employment of the production factors.

Once it is clear that they are not understood as incomes form capital, the nature of firm profits is not very clear. It is not clear what their counterpart in the real world can be. However, this is not a serious problem because these profits disappear with their maximization.

In fact, substituting equation (7) into equation (8), the first order conditions for the maximization of the amount of profit Π^n are:

$$\begin{cases} p^n F_L^n(L^n, \Lambda^n, K^n) - w = 0 \\ p^n F_\Lambda^n(L^n, \Lambda^n, K^n) - \rho = 0 \\ p^n F_K^n(L^n, \Lambda^n, K^n) - i = 0 \end{cases} \qquad (9)$$

Therefore, if the production function $F^n(\cdot)$ exhibits constant returns to scale—i.e. it is homogeneous of degree one—and firms are using the factors in optimal quantities, then $\Pi^n = 0$.[4]

3.3 Capital as the value of the capital goods

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[3] Here we refer to vertically integrated processes that reproduced al the (circulating) capital goods employed and give a certain amount of final output.

[4] According to Euler's formula for homogeneous functions, if the production function $F^n(\cdot)$ is homogeneous of degree one, then: $C^n = F_l^n(L^n, \Lambda^n, K^n) \cdot L^n + F_\Lambda^n(L^n, \Lambda^n, K^n) \cdot \Lambda^n + F_K^n(L^n, \Lambda^n, K^n) \cdot K^n$. Therefore, equation (8) can be written as follows: $\Pi^n = [p^n F_l^n(L^n, \Lambda^n, K^n) - w]L^n + [F_\Lambda^n(L^n, \Lambda^n, K^n) - \rho]\Lambda^n + [F_K^n(L^n, \Lambda^n, K^n) - i]K^n$. It is now clear that the first order conditions (9) entail $\Pi^n = 0$.

It should be clear that the idea of the interest rate as a price represents the other side of the coin with respect to the idea of capital as a factor of production. That is, as something that firms materially employ in production, together with labour and land. Capital must in fact be substitutable to other factors and therefore must play the same role, it must satisfy the same need.

The conception of capital as a factor of production represents, no doubt, the biggest difficulty met by the marginalist theory of distribution. The idea of capital as something that is materially used in the production process is associated with its identification with the set of capital goods (commodities) employed.[5] Hence, the theory tries to present capital as an amalgam of capital goods.[6] At the cost of a drastic simplification, we can say that the basic idea it that of the existence of a special substance, a sort of "jelly", of which all capital goods are made and from which their productivity derives.

If this jelly existed, the form of the various types of capital goods that it actually takes would be a secondary aspect. In fact, first, to every possible set of heterogeneous capital goods one could associate the corresponding quantity of jelly. Secondly, sets of capital goods containing a greater quantity of gelatine would make it possible to obtain a larger product, all other things being equal.

Despite several attempts, the last of which was Samuelson's (1962), a substance or jelly with these extraordinary properties has never been found. Thus, in its absence, the employment of capital was generally identified with the value of capital goods used in production. It was therefore tried to create a hybrid between the (right) idea of capital as a value fund that advances costs and the (wrong) idea of capital as an amalgam of capital goods. However, considering the value of the capital goods as the quantity used of a factor of production can lead to paradoxical results.

First, as Samuelson (1966, p. 582) wrote, there is, in general, no unambiguous way to say that a process of production is more capital intensive than another—namely it entails a greater employment of capital per unit of labour. Since the prices of capital goods change as the

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[5] That is the reason why the problems met by the theory with reference to capital as a factor of production are often erroneously believe to be problems of "aggregation". Actually, no real problem exists for the aggregation of the capital goods into the corresponding amount of value by means of their prices. The problems arise if we want to regard this amount of value as the quantity of a factor of production and the rate of interest as the price for its use.

[6] The double presence of capital and capital goods generates a sort of schizophrenia in the marginalist theory. Taking Robert Solow as the personification of this theory, Paul Samuelson wrote that '[o]ne might almost say that there are two Solows': (a) 'the orthodox priest of the MIT school', who regards capital as 'a great variety of heterogeneous physical capital goods'; and (b) 'the busman on a holiday who operates brilliantly and without inhibitions in the rough-and-ready realm of empirical heuristics' and makes use of a 'Clark-like concept of aggregate "capital"' (Samuelson, 1962, p. 193).

interest rate changes, if we take two methods of production of the same commodity, it is possible that the method that is more capital-intensive for a certain level of the interest rate becomes the method that is less capital-intensive for a different level of the interest rate. The ranking of methods on the basis of capital intensity cannot be done independently of the level of the interest rate.

Second, given a certain interest rate, the method that employs more capital per unit of labour is not necessarily the one that enables the highest output per unit of labour to be obtained. This point is essentially an implication of the previous one. Suppose that, for a certain level of the rate of interest, method α is more capital-intensive than method β and also provides a greater output per unit of labour. If, for a different level of the interest rate, method β has become the most capital-intensive, then we have that, for the same amount of labour employed, the method that employs the most capital is the one that allows to obtain the lowest output.

Third, when an increase in the interest rate results in a change in the production method in use—i.e. the one that minimises the unit cost of production, the method that comes into use does not necessarily employ less capital per unit of labour than the method previously used. In other words, an increase in the interest rate may not make (relatively) more expensive methods that require more capital. This result, clearly, puts in doubt the idea that the interest rate can be seen as the price that companies pay for the use of the capital factor.

The point is that since there is no convincing way to understand the productive factor "capital", the rate of interest cannot be thought of as its price. This was in fact explicitly stated by Bliss in his authoritative book on capital theory:

> The value which accrues from a sale is the product of price and quantity sold. Hence if the rate of interest is the price of capital, the quantity of capital must be the wealth on which an interest yield is calculated. It will be shown shortly why this view is incorrect, but to cut a long story short, the conclusion may be announced at once. The rate of interest is not the price of capital. (Bliss 1975, pp. 6-7)

4. Interest and profit in the Arrow-Debreu theory

As seen above, in section 3.2, according to the marginalist theory, interest is the income earned by the owners of capital (households) whereas profit is the gain or loss made by firms. Once this distinction between interest and profit has been introduced, the latter can exist even in models without capital. This is the case of the Arrow-Debreu equilibrium theory.

Actually, the Arrow-Debreu model is a very peculiar one. In fact, the determination of rates of interest and amounts profit is addressed within this theory notwithstanding no form

of capital is there, neither the classical (an amount of value that allows the payment of costs at the beginning of the process), nor the marginalist (a factor of production).

4.1 *Own rates of interest*

In the Arrow-Debreu framework, production processes do not emply factors of production but Arrow-Debreu commodities, namely goods and services with a specific place and date of delivery. Accordingly, the rate of interest is not undestood as the price firms pay for the use of capital.

Actually, there is not just one rate of interest in the Arrow-Debreu models, but there are many "own rates of interest". Let p_t^n and p_{t+1}^n be the prices of commodity n (with n = 1, 2, …, N) delivered in period t and t+1 respectively, the own rate of interest of commodity n between the two periods $r_{t,t+1}^n$ is defined by the equation:

$$\frac{p_t^n}{p_{t+1}^n} = 1 + r_{t,t+1}^n \tag{10}$$

It is clear from equation (10), that this rate (or factor) of interest in neither the price of a factor of production, nor a source of income. It is just a relative price: $(1 + r_{t,t+1}^n)$ is the quantity of commodity n delivered in period t+1 that an agent must pay in order to have a unit of commodity n delivered in period t. Besids, since this quantity can be less than 1, the own rate of interest $r_{t,t+1}^n$ can be negative (but not smaller than -1). Hence, the own rates of interest are rather useless for the explanation of incomes from capital.

4.2 *Profits in a private ownership economy*

As far as profit is concerned, differently from the classical/Marxian approach, in the Arrow-Debreu theory it has no linkage with the investment of capital. If there are N different goods and services with T possible dates of delivery, putting aside the possibility of different places of delivery,[7] there are $L = N \times T$ Arrow-Debrey connodities. Let $\mathbf{p} \in \mathbb{R}_+^L$ be a price vector and $\mathbf{y}^f \in \mathbb{R}^L$ the production plan of firm f—i.e. i.e. a list of quantities of inputs, with negative sign, and outputs, with positive sign—then $\pi^f = \mathbf{p} \cdot \mathbf{y}^f$ is the firm's profit.[8] In a "private ownership economy" (Debreu 1959, pp. 78-80), this profit—which can be gains or losses—is divided

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[7] The existence of just one possible place of delivery and one stream of events can be assumed here for simplicity.
[8] In other words, let $\mathbf{y}^f = [y_1^f, y_2^f, …, y_L^f] \in \mathbb{R}^L$ be the production plan of a firm f. If $y_n^f < 0$, then it (taken in terms of absolute value) is the quantity of commodity n employed as input by firm f. If instead $y_n^f > 0$, then it is the quantity of commodity n obtained as output by that firm. As a result, $\mathbf{p} \cdot \mathbf{y}^f$ directly expresses the difference between revenues and costs.

amongst households, and enter into their budget constraints, in accordance with some exogenously given shares.

These shares cannot reflect the investment of capital (savings) households made for the very simple reason that saving and investment are unconceivable within this framework. As is known, in the Arrow-Debreu model, the L commodities, or rather the promises of their delivery, are traded simultaneously in a single instant, the initial moment of the first period. This assumption has a number of relevant implications, one of which will be focused on here, namely the impossibility of finding a role for the investment of capital.[9]

On the producers' side, if all the markets are open for one single instant only, then every firm can trade both inputs and outputs simultaneously, in the single instant in which markets are open. In other words, revenues and costs, in this model, are necessarily simultaneous and this makes the investment of capital impossible. In fact, as we have seen, capital is invested in order to finance the costs in advance, before revenues are obtained. As is known, if wages are paid in the same moment as output is sold, no capital is involved in the payment. In the Arrow-Debreu framework, this applies not only to wages but to expenditure on all the inputs (including capital goods), which must necessarily take place in the same instant as the outputs obtained with them are sold.

On the consumers' side, households cannot and do not need to move their purchasing capacity across time. Households' income arises and is entirely spent in the one instant in which the markets are open. Saving in order to transfer purchasing power to some future date would thus actually be impossible in the Arrow–Debreu model, as no further trade can take place after the initial instant of the first period.[10] As Currie and Steedman (1990, p. 147) pointed out, the idea of transferring wealth over time has no real meaning in this framework.

It is therefore clear that capital cannot exist in this framework and therefore no form of income from capital, be it profit or interest, can exist either.

Finally, we can stress that independently of any consideration about thies nature and the principle adopted for their distribution, the profit maximazied by firms is not a real source of households' incomes. In fact, let $\mathbf{p}^*$ and $\mathbf{y}^*$ be equilibrium price vector and aggregate

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[10] Some authors claim that the assumption that markets do not reopen after the initial instant is innocuous because even 'if markets were reopened at later dates for the same Arrow-Debreu commodities, then no additional trade would take place anyway' (Geanakoplos 1987, p. 122). This is not so. The question is not whether further trade will take place when the markets reopen, but rather whether agents will behave differently in the initial instant in the knowledge that markets will open again. In particular, the assumption that markets will reopen gives rise to all the problems connected with expectations and speculative trade, which are completely avoided in the Arrow-Debreu framework.

production plan, it can be easily proved[11] that $\pi^* = \mathbf{p}^* \cdot \mathbf{y}^* = 0$. This means that, in equilibrium, households' purchasing capacity depends on the value of their endowments of commodities only, there is no income coming from firms.

5. Conclusions

Three different approaches are considered in this paper. In the classical/Marxian approach (section 2), incomes from capital are understood as a surplus whose amount depends on the class conflict. At the beginning of the process of production, a certain amount of capital M is invested in order to advance the costs for the inputs employed. The inputs are transformed into a bundle of commodities C. When these commodities are sold on the market, an amount of revenues M' is obtained. It is the capitalist circuit M-C-M' and capitalists' incomes arise from the difference $M' - M$. This difference or surplus-value is the profit, i.e. the residual of the revenues over and above the costs of production.

By contrast, in the marginalist theory (section 3), the distribution variables—i.e. wage rate, rent rate and interest rate—are understood as the prices of three factors of production: "labour", "land" and "capital". According to this view, incomes from capital are what firms pay to households for the employment of the factor of production called "capital". In particular, the rate of interest is seen as the price for the use of capital and determined by means of a supply-and-demand equilibrium.

Thanks to the capital theory debates of the 1960s,[12] it became clear that the idea of capital as a factor of production and the rate of interest as the price for its use was untenable. This was explicitly admitted by authoritative neoclassical economists, such as Samuelson (1966), Hahn (1982) and Bliss (1975). However, at the same time, these scholars maintained that the modern general equilibrium theory is not affected by those problems since it does not rest on the idea that production processes employ the factors of production, but Arrow-Debreu commodities.

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[11] Let $\mathbf{z} : \mathbb{R}_+^L \to \mathbb{R}^L$ be the market excess-demand function from households. Because of the marke clearing condition, we know that $\mathbf{z}(\mathbf{p}^*) = \mathbf{y}^*$. Since the Walras's Law entails $\mathbf{p}^* \cdot \mathbf{z}(\mathbf{p}^*) = 0$, then $\mathbf{p}^* \cdot \mathbf{y}^* = 0$.
Moreover, since inactivity is a feasible production plan, no firm, in equilibrium, will abopt a production plan that entails losses. This, together with the absence of profit in the aggregate, means that, in equilibrium, each firm makes zero profit.

[12] For a survey of the results obtained during the "Two Cambriges debate", we can refer the reader to Harcourt (1969 and 1972).

Actually, in the Arrow-Debreu general equilibrium model (section 4), there is neither capital understood as the amount of value invested at the beginning of each process, nor capital as a factor of production. The problem is that in this model there is no idea or role for capital. As a result, incomes from capital can hardly find an explanation within the Arrow-Debreu theory.

Once capital is not understood as a factor of production, in the Arrow-Debreu model the rate of interest is not he rate of interest disappears and its place is taken by many commodity own interest rates, but they are essentially useless in the explanation of income distribution. Profits—intended as what firms maximize—are still there, but: i) they are distributed across households on the basis of shares arbitrarily assigned; ii) their equilibrium amount must be zero.

Therefore, on the basis of the analysis developed in these pages, we can conclude that neither the marginalist theory, nor the Arrow-Debreu one can provide us with a convincing theoretical explanation of incomes from capital. Thus, the old theory of the classical economists—submerged and forgotten because of the advent of the neoclassical approach—seems to be the only possible way to proceed at the moment.

References

Bliss, C.J. (1975) *Capital Theory and Distribution of Income*. Amsterdam/New York: North-Holland/Elsevier.

Debreu, G. (1959) *Theory of Value*. New York: Wiley.

Fratini, S.M. (2019) On the second stage of the Cambridge capital controversy. *Journal of Economic Surveys*. doi: 10.1111/joes.12312.

Garegnani, P. (1984) Value and Distribution in the Classical Economists and Marx. *Oxford Economic Papers* 36 (2): 291–325.

Garegnani, P. (2012) On the present state of the capital controversy. *Cambridge Journal of Economics* 36(6): 1417–1432.

Geanakoplos, J. (1987) Arrow-Debreu model of general equilibrium. In J. Eatwell, M. Milgate and P. Newman (eds.), *The New Palgrave Dictionary of Economics*, Vol. I (pp. 116–124). London: Macmillan.

Hahn, F.H. (1982) The neo-Ricardians. *Cambridge Journal of Economics* 6(4): 353–362.

Harcourt, G.C. (1969) Some Cambridge controversies in the theory of capital. *Journal of Economic Literature* 7(2): 369–405.

Harcourt, G.C. (1972) *Cambridge Controversies in the Theory of Capital*. Cambridge: Cambridge University Press.

Marx, K. Capital vol. 3.

Piketty, T. (2014) *Capital in the Twenty- First Century*. Cambridge (Mass.) and London: Harvard University Press.

Samuelson, P.A. (1962) Parable and realism in capital theory: the surrogate production function. *Review of Economic Studies* 29(3): 193–206.

Samuelson, P.A. (1966). 'A summing up.' *Quarterly Journal of Economics*, 80(4), pp. 568-583.