Alchemy of Financial Innovation: Securitization, Liquidity and Optimal Monetary Policy *

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Abstract

Securitization is shown to work as a tool to remove the "limited communication" problem. Securitization also functions as a mechanism through which agents can enjoy perfect risk-sharing. Using an overlapping generations model with random-relocation shocks, the effects of securitization are analyzed in three different hypothetical situations: 1. Only one region of the economy issues securities, 2. All regions issue securities with the same capital productivity, and 3. All regions issue securities, but capital productivity is disparate across regions. Optimal monetary policy follows the Friedman rule in cases 1. and 2. However, the rule does not apply in case 3.

Keywords: Securitization, Liquidity, Friedman Rule, Monetary Policy

JEL Codes: E52, G11, G21

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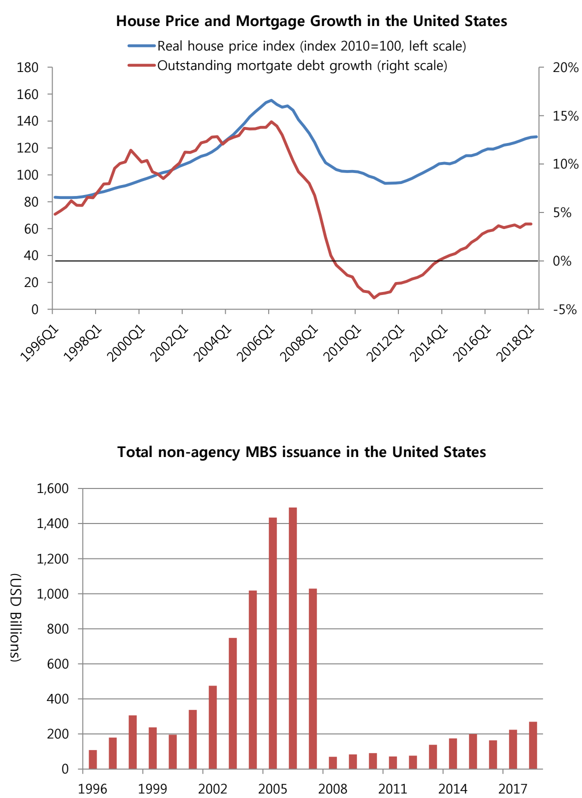
1 Introduction

Vilified as a key cause of the 2007-2008 global financial crisis, asset-backed securities (ABSes) dropped in popularity for a decade compared with their popularity during their peak. However, since around 2011 there have been recent signs that the issuance of U.S. mortgage-backed securities is on the rise again (see Figure 1).

Figure 1: House Price, Mortgage Growth, and MBS Issuance



Sources: IMF Global Financial Stability Report (2018); Federal Reserve Bank of St. Louis; SIFMA

While securitization has fundamentally changed the structure of capital markets and the role of financial intermediation, and raises critical questions about the existing theory of financial intermediation, there is a dearth of research into this topic (Gorton and Metrick, 2012a). Especially lacking in the literature are theoretical studies about the underlying mechanism as to dynamics and spillovers between two different regions (or countries) which determines how much ABSes they demand and supply ABSes in their own interest, and how monetary policy affects their decision making. This paper contributes to these issues by providing a model that explores the theoretical underpinning of the effects of securitization on an economy's need for liquidity, where the maturity structure trade-off between liquid and illiquid assets of financial intermediaries exists on the one hand, and where the liquidity preference resulting from uncertainty about the timing of consumption prevails on the other.

Following Townsend (1979), capital is regarded as an illiquid asset due to its immobility and "limited communication" problem.[1] Fiat money is, thus, regarded as useful means in this paper as in Townsend (1979). If the economy holds too many liquid but unproductive assets (e.g., fiat money) instead of less liquid, but more productive assets (e.g., capital), it may imply that the economy uses its resources in a less efficient way. Securitization is shown to work as a tool to remove the "limited communication" problem without using fiat money, and thus makes economic agents able to invest its resources more efficiently in high-yielding illiquid assets (capital) rather than liquid assets (fiat money). The welfare of an economy always increases by introducing securitization into the financial sector unless expansionary monetary intervention takes place. Securitization also functions as a mechanism through which agents can enjoy perfect risk-sharing. Moreover, it demonstrates that optimal monetary policy is present and viable, which can enhance an economys welfare, but takes a different form that depends on the structure of an economy.

Gorton and Souleles (2007) provide a theoretical model of the private securitization decision. Banks make decisions concerning how to finance their loans between demand deposits and newly issued ABSes to maximize equity holders' utility. The motivation of issuing ABSes in the current paper is different from that in Gorton and Souleles (2007), in that the interest of the banks is to find a way to reduce its investment in liquid assets, but also not to harm the welfare of depositors who have a liquidity preference. In other words,

[1] In this model, the direct trade in capital between agents living in different places cannot happen because the agents are separated in space, and too costly communication keeps agents living on separate locations from verifying the accuracy of the claims.

banks wish to discover a method to make the most efficient use of available resources.

Another important string of models on securitization involves security designs concerning asymmetric information, and the effect of securitization on financial stability, which issues are not discussed in the current paper.[2] By assuming the rate of return on illiquid assets is definitive and that no moral hazard or adverse selection problems arise, I simplified the model to focus more on other important issues, such as how two different places are interconnected through securitization, and how monetary policy affects their decision process, especially with regard to how securitization affects the liquidity needs of each economy and welfare. Incorporating information asymmetries and agency problems into the current paper is a challenging task which is left for future research.

Who wishes to hold ABSes as financial assets instead of other forms of assets, and for what reason? Pozsar (2013) asks why many cash pools are invested in deposit alternatives in the shadow banking system rather than in the traditional banking system. He notes that demand for ABSes has increased due to institutional investors. Another incentive to hold ABSes comes from the growing demand for collateral in real-time gross settlement systems (BIS, 2007) and in the repurchase agreement market.[3] The demand for ABSes in this paper comes from banks that have an incentive to hold assets that both satisfy the liquidity preferences and that prevent a maturity structure trade-off at the same time. This incentive is similar to that of economic agents in Bencivenga and Smith (1991), in which financial intermediaries are shown to arise endogenously in order to solve the conflicting problems between liquidity preference and maturity structure trade-off. Bencivenga and Smith (1991) showed that the introduction of financial intermediaries might affect the composition of savings toward more productive assets (capital), and that the intermediaries would thus enhance growth by reducing socially unnecessary capital liquidation. If securitization prompts an economy to use its resources more efficiently by increasing capital investment, then consumption and

[2]DeMarzo and Duffie (1999) and DeMarzo (2004) address important issues on security design when asymmetric information and agency problem are present.

[3]See Singh and Aitken (2009) and Gorton and Metrick (2012b).

welfare in the economy will both be enhanced.

To show how securitization affects an economy's overall liquidity and thus portfolio decisions, I adopt a two-period overlapping generations model (OLGs) in which the young generation faces random relocation shocks.[4] Banks make portfolio decisions on behalf of their depositors and hold both liquid and illiquid assets to maximize depositors' welfare. First, I set up a model in which securitization is not possible as a baseline model. Then, I assume three different hypothetical situations: 1. only one region of the economy issues securities, 2. all regions issue securities with the same capital productivity, and 3. all regions issue securities, but capital productivity is disparate across regions, and compare the results. The third case is similar to the situation of the euro area in that each member state has different productivity but is influenced by the monetary policy of the sole central bank, ECB. Although the effects of securitization on the portfolio decision and welfare are not the same and depend on the structure of the situation, one conclusion of this study is that, with securitization, liquid asset investments by banks are reduced compared to the case without securitization, as long as the rate of return on fiat money is greater than one. As the economy uses its resources more efficiently, agents' welfare is maximized as perfect risk-sharing is made possible.[5]

This paper also provides a theoretical explanation as to how monetary policy may affect the degree of securitization and the depth and width of ABS markets. Despite the importance of these issues, they are rarely discussed in the literature. Empirical studies concentrate primarily on explaining the impact of securitization on monetary policy,[6] and according to Gorton and Metrick (2012a), the main concerns of the literature lie either in 1. how

[4] In Champ et al. (1996), banks provide insurance to agents who have random needs for liquidity because of the possibility of relocation. This "relocation shock" takes the place of the "preference shock" in Diamond and Dybvig (1983)

[5] Loutskina (2011) shows the empirical evidence that the bank's ability to securitize loans reduces its liquid asset holdings. Maddaloni and Peydró (2011) illustrates the relationship between the short-term interest rate, securitization, and banks' asset choices using data from the euro area and the U.S.

[6] Estrella et al. (2002) and Loutskina (2011) analyze the effect of securitization on the efficacy of monetary policy using empirical macroeconomic models. Altunbas et al. (2009) argues that securitization alters the effectiveness of the bank lending channel and increases the capacity to supply new loans.

securitization affects the central banks influence on banks' lending channels, or in 2. how it changes the elasticity of interest rates on output.[7] In this paper, however, I show that the effect of securitization is also affected by inflation, which is controlled by the central bank and influences relative rates of return between different assets. The following natural question, then, is what the optimal monetary policy would be to achieve welfare maximization when securitization is introduced into the model. Friedman (1969, p.34) writes:

"Our final rule for the optimum quantity of money is that it will be attained by a rate of price deflation that makes the nominal rate of interest equal to zero."

The optimal monetary policy obtained in this paper in terms of Friedman's rule is that the rule does not apply when the economy is asymmetrical in capital productivity (situation 3), and does apply if the economy is symmetrical (situations of 1 and 2). The intuition behind this result is that if the economy is not symmetrical, the optimal deflation cannot be identical across different regions.[8] The central bank, which maximizes agents welfare in all regions of the economy combined as a social planner, cannot help but choose mediocre deflation by weighing two or more optimal deflations in each region. On the other hand, if all regions are identical in productivity, then the target rate is given as a single value, which is attainable by a central bank. This result has an important implication for the central bank, which implements a monetary policy with a single instrument, but the effects of which are applied to all places over which it has jurisdiction regardless of the degree of development in each region.

The rest of this paper is organized as follows. The environment and structure of the economy are described in Section 2. In Section 3, the equilibria without securitization are analyzed as a baseline model. In Section 4 the equilibria are examined when only one region of the economy issues ABSes. Then, Section 5 extends the model to one in which both regions

[7] See Estrella et al. (2002), Long et al. (2009), and Gambacorta and Marques-Ibanez (2011), to name just a few.

[8] This result implies that each region would want different levels of deflation. If they have a tool affecting deflation, then they would have an incentive to use it to raise or lower deflation depending on their situation. Sibert (1992) analyzed that a fiscal policy without cooperation in a monetary union leads to inflationary bias. Levine and Brociner (1994) and Beetsma and Bovenberg (1998) show similar results.

supply and demand ABSes at the same time, and I investigate two different situations to see whether or not the capital productivity remains the same. In Section 6, I investigate an optimal monetary policy using the three different situations discussed in Sections 4 and 5. Concluding remarks are in Section 7.

2 The Structure of the Economy

2.1 Random Relocation Model

The economy is made up of two distinct, geographically separated locations, Island A and Island B. Each island is populated by two age cohorts, the young and the old. A young person, the population of which on each island is normalized to one, is born on one of the two islands. The population of the initial old person is also normalized to unit mass. Each young agent, ex-ante identical, is born with a unit of endowment goods at date t and has nothing at a subsequent date. The goods can be used for consumption or investment, but are restricted from moving across to the other island. Each young person faces privately observable, stochastic relocation shocks in the middle of date t.

If a young agent, who is born on their home island, is subject to a relocation shock, they are called a "mover" and are relocated to the foreign island to spend their old age there. Due to limited communication, fiat money is valued by the movers as a medium of exchange for old-age consumption. Let λ $(0 < \lambda < 1)$ be the probability that a young person is subject to a relocation shock. We assume that the probability of being relocated is the same on both islands. If there were no aggregate relocation shocks, and there were large numbers of young agents, and their relocation shocks are assumed to be independent, then the fraction of movers is equal to the probability of being a mover, according to the Law of Large Numbers.

All agents are assumed to consume only in their old age and thus are uncertain about the precise place as to where to consume. The agent's problem born at date t will be that of maximizing the following von Neumann-Morgenstern utility function

$$E_t[U(c^{ij}_{m,t+1}, c^i_{n,t+1})] = \lambda u(c^{ij}_{m,t+1}) + (1-\lambda)u(c^i_{n,t+1}) \quad (1)$$

where $i, j = \{a, b\}$ and $i \neq j$. $c^{ij}_{m,t+1}$ denotes the consumption of a mover who is born at date t on Island i moves to Island j after the relocation shock and consumes at $t+1$, and $c^i_{n,t+1}$ is the consumption of non-movers born on Island i consuming on the same island at date $t+1$. The utility function satisfies the usual neoclassical properties: increasing, strictly concave, and twice continuously differentiable.

2.2 Portfolio Decision

Each person needs to save for their old age consumption. It is not possible for banks to make contingent deposit contracts with their depositors because the portfolio decision is made before the relocation shocks are revealed. There are two different assets that they are able to access: a liquid asset (fiat money) and an illiquid asset (capital). Let p_t be the nominal price of one unit of consumption goods at date t. Then one unit of the goods that is traded for fiat money at date t yields p_t/p_{t+1} units of consumption goods at date $t+1$. The illiquid asset (capital) takes only one period to mature, but is regarded as an illiquid asset due to its immobility. Moreover, we assume that limited communication problems exist in the economy. One unit of the goods invested in the current period are to be converted into $R > 1$ units of the goods at date $t+1$. There might exist a trade-off between an asset's liquidity and its return. The illiquid asset is immobile, but pays a higher return overall. Due to limited communication, however, claims against capital are assumed to be useless. The liquid asset is mobile, universally accepted, but generally yields a lower return.

2.3 Money Market Equilibrium

No one in future generations is born with fiat money. For young agents born at date t, to acquire fiat money they must trade with the initial old people who are endowed in total with M_t units of fiat money at date t. The supply of fiat money on each island in period t is $M_t/2$ because the population of old people is distributed equally on each island. The demand for fiat money comes from the young. Let q_t^i be the real demand for money from an individual young agent born on Island i at time t, so q_t^i denotes the number of goods for which each young agent chooses to sell their fraction of endowment for fiat money. The price of consumption goods at date t, p_t, is determined at the beginning of period t by the coincidence of the demand for fiat money by the young and the supply of it by the old.

The central bank controls a monetary instrument, the aggregate money supply, which enables it to choose the current price level, p_t. The stock of fiat money at date t increases according to the rule:

$$M_t = z_t M_{t-1},$$

where M_{t-1} is a predetermined variable and $z_t > 0$ denotes the gross rate of the expansion of the money supply, which depends on the central bank's discretion. Then new printed units of fiat money at date t are

$$M_t - M_{t-1} = (z_t - 1) M_{t-1}.$$

The newly printed (resp. withdrawn) money if $z_t > 1$ (resp. if $0 < z_t < 1$) is used to raise (resp. to decrease) government purchases, and thus does not affect directly the relative desirability of $c_{m,t+1}^{ij}$ or $c_{n,t+1}^{i}$.

The rate of return from holding q_t^i is $\frac{p_t}{p_{t+1}} q_t^i$, where p_{t+1} is the price level at date $t+1$ when the gross rate of money supply by the central bank is $z_{t+1} > 0$. The money market equilibrium condition at date t implies the following:

$$p_t = \frac{z_t M_{t-1}}{q_t^a + q_t^b}.$$

The denominator denotes the total demand for fiat money and the numerator is the total supply of fiat money at date t. Likewise, the expected future price level at date $t+1$ is similarly expressed as

$$p_{t+1} = \frac{z_{t+1} M_t}{q_{t+1}^a + q_{t+1}^b}.$$

Then, the rate of return on fiat money is given as:

$$\frac{p_t}{p_{t+1}} = \frac{q_{t+1}^a + q_{t+1}^b}{q_t^a + q_t^b} \frac{1}{z_{t+1}}.$$

In a stationary equilibrium, we have $q_t^i = q_{t+1}^i = q^i$, and the rate of return on fiat money (and thus inflation) depends only on expected gross money growth rate by the central bank $1/z_{t+1}$, or expected inflation z_{t+1}.

Assumption 1 *Relationship between capital and fiat money returns is assumed to take the following form:*

$$z_{t+1} R \geq \lambda.$$

In the literature, the expected rate of return on capital is usually assumed to be greater than or equal to that of fiat money, i.e., $z_{t+1} R \geq 1$. In the relationship between capital returns and money returns, this assumption takes a weaker form than the traditional assumption about capital return. Therefore, according to the assumption, the rate of return for fiat money may be higher than that of capital, and thus the central bank can choose z_{t+1} such that $R \leq 1/z_{t+1}$, as long as $\lambda \leq z_{t+1} R$. The size of λ, given R, serves as a limitation to a central bank's monetary policy, i.e., the rate of return on fiat money, z_{t+1}, cannot be greater than R/λ. This assumption is satisfied unless R is very low and/or the rate of return on fiat money, $1/z_{t+1}$ is not too high.

3 Equilibrium without Securitization

3.1 Efficient Solution

In this section we examine an efficient solution by a social planner and identify how the solution can be achieved via financial intermediaries. Securitization is not introduced yet, and the results derived in this section are compared to the allocations in Sections 4 and 5 in which securitization is introduced. The social planner maximizes (1) for agents born at t, subject to the budget constraint,

$$q_t^i + k_t^i = 1, \tag{2}$$

the feasibility conditions,

$$\lambda c_{m,t+1}^{ij} \leq \frac{q_t^i}{z_{t+1}}, \tag{3}$$

$$\lambda c_{m,t+1}^{ij} + (1-\lambda) c_{n,t+1}^{i} = \frac{q_t^i}{z_{t+1}} + k_t^i R, \tag{4}$$

and the incentive constraint,

$$c_{m,t+1}^{ij} \leq c_{n,t+1}^{i}. \tag{5}$$

First, the social planner determines how to divide its members' endowment goods between real money and capital at date t. The feasibility conditions show how much the social planner should pay to movers who need to make a withdrawal at the end of date t. The social planner enters into a contract with its members, which promises to give a fixed amount of fiat money at date t through the paying out of all available liquid assets, divided equally among those withdrawing (constraint (3)). Non-movers are paid whatever is available at their final periods, as is shown in constraint (4). Constraint (4) says that the consumption by non-movers is limited by the total value of the illiquid asset, capital, plus the amount of fiat money left over, if any, after the movers have been paid. Constraint (5), the incentive compatibility constraint, says that the consumption by non-movers must be at least as much as the real money promised to movers. Since whether a certain agent is relocated or not is private information, and, even a social planner can thus not identify who is a mover and who is a non-mover, non-movers have an incentive to pretend to be movers unless this constraint holds. Each will reveal their true type if and only if constraint (5) is satisfied. The efficient solution satisfies the following:

$$u'\left(c^{ij}_{m,t+1}\right)\frac{1}{z_{t+1}} = u'\left(c^{i}_{n,t+1}\right)R \tag{6}$$

Assuming $u(c) = \ln c$, the optimal values of variables are given as $q^i = \lambda$, $c^{ij}_{m,t+1} = 1/z_{t+1}$ and $c^{i}_{n,t+1} = R$.

3.2 Decentralized Solution

Now, suppose that young agents born at date t can organize financial intermediaries called banks.[9] They deposit all their endowment goods in a bank, and the bank then uses the proceeds to acquire assets in favor of its members. Banks function as liquidity providers. Banks maximize the expected utility of the consumers by offering deposit contracts due to free entry into the banking industry and competition.

At the beginning of each period, financial intermediaries located on Island i take deposits from typical individuals and provide a standard deposit contract, promising consumers a fixed amount of fiat money if they withdraw at date t, and consumption goods, $c^{i}_{n,t+1}$, if they withdraw at date $t+1$. More specifically, the arrangement is comprised of giving them d^i units of fiat money if they withdraw at the end of period t, irrespective of the occurrence of the state of nature if it has enough liquid assets, or $c^{i}_{n,t+1}$ units of the consumption goods

[9]Financial intermediaries are defined as voluntary coalitions of agents. Agents born in each period form coalitions respectively for their own benefit.

if they withdraw at date $t+1$. d is the face value of the deposit at t, thus movers will consume $c^{ij}_{m,t+1} = d^i/p_{t+1}$ in period $t+1$.

The bank's problem is to maximize (1), subject to a budget constraint, (2), the feasibility conditions,

$$c^{ij}_{m,t+1} \leq \frac{d^i}{p_{t+1}},$$
$$c^{ij}_{m,t+1} + (1-\lambda)c^i_{n,t+1} = \frac{d^i}{p_{t+1}} + k^i R,$$

and the incentive constraint (5).

This maximization problem is exactly the same as that in the efficient solution if the bank chooses d^i such that $d^i = \frac{p_t q^i_t}{\lambda}$.

3.3 Equilibrium with Securitization

Securitization is generally regarded as the process of taking illiquid assets and transforming them into liquid assets, and thus it allows banks to liquidate illiquid assets to finance their liquidity needs. One of the goals of this paper is to study how financial innovation, represented by securitization,[10] changes the financial intermediarys portfolio decisions and how it alters the aggregate needs for liquid assets used for the consumption of agents facing liquidity shocks. Also, I show that securitization not only allows banks to transform illiquid assets into liquid assets, but it also provides an alternative source for the investment in liquid assets.

Suppose that some new technology, through which capital is securitized and liquid financial assets can be generated, is introduced in one or both islands as a result of some financial development. Future returns that capital generates are used as collateral for these securities. From now on, let us call these securities ABSes. The banks that are able to issue ABSes

[10] Financial innovation is a term widely used in relation to the act of creating new financial institutions, markets and instruments, as well as inventing a new process for distributing financial services. Among the many items related to financial innovation, this section explores securitization in particular and investigates how securitization affects the liquidity needs of banks.

trade these in financial markets for fiat money, which will be ultimately used for consumption by movers.[11] The demand for and supply of ABSes are equated in the securities markets, and the equilibrium of each island is affected by the decision made by the other island, and thus there exists spillover effects. The effects of securitization on economic decisions depend on how many regions can issue ABSes, and whether capital productivity is identical across regions or not, which are discussed in detail in Sections 4 and 5.

4 Only One Island Issues ABSes (Case I)

Suppose that only banks on Island A have the ability to produce liquid financial assets (ABSes) appealing to banks on Island B, some of the depositors at which face liquidity shocks. The reason for making this assumption is to reflect the reality that not all regions of a country or all countries in the world can supply financial assets.

4.1 Securitization Markets Equilibrium

Banks located on Island A (originators) begin the securitization process by gathering a series of illiquid assets (capital) and issue securities (ABSes), which are backed by the assets they hold. They then sell these ABSes to the banks located on Island B. The originator will have received proceeds from the securitization, and the proceeds (fiat money) are used for the consumption of the movers. The returns generated by the underlying assets (capital on Island A) are then transferred to the investors who purchase (banks on Island B) and hold ABSes (movers located to Island A from Island B). During the process, Island A banks can increase their overall liquidity by securitizing illiquid assets into liquid assets and to generate immediate proceeds from their assets. These proceeds give the bank the potential to reduce

[11] There are many potential benefits both to the issuer and investors when illiquid assets are securitized. For example, the issuer may manage interest rate risk better, improve their liquidity level, diversify their funding sources, and so forth. I do not outline the detailed process and various benefits of securitization, nor cover the important features regarding the theoretical issues of security design happening in an asymmetric information situation. For these matters, see Gorton and Metrick (2012a).

the amount invested in liquid assets.[12]

Let k_t^a be the investment in capital which the bank on Island A chooses, and assume that the bank assigns α fraction of k_t^a to be securitized. Let B_t be the (nominal) price of an ABS that promises to provide R units of consumption goods per unit of securities at date $t+1$. The total supply of ABSes is $\alpha k_t^a R$.

Banks on Island B invest in securities with the fiat money that they hold. Suppose that a bank on Island B uses β percentage of its total holdings of fiat money, $p_t q_t^b$, to purchase securities. Then, the supply of fiat money for buying the securities is $\beta p_t q_t^b$. It is assumed that neither short selling nor interbank lending are allowed, which implies $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$. The price of an ABS, B_t, is determined so that the demand for and supply of securities match.

$$b_t = \frac{B_t}{p_t} = \frac{\beta q_t^b}{\alpha k_t^a R}, \tag{7}$$

where b_t denotes the real price of an ABS. The price of the securities depends on the bank from Island A's decisions on k_t^a and α, and the bank from Island B's portfolio decisions about q_t^b and β. If R is a random variable, so too is b_t, but throughout this paper, R is assumed to take a definite value so that we can concentrate on an analysis as to the way in which securitization affects the amount of liquid assets and welfare in an economy.

4.2 Supply of Securitized Bonds: Island A

The bank on Island A determines how to construct a portfolio (q_t^a, k_t^a) and how much capital to securitize in order to maximize its depositors' expected utility. Since banks on Island A securitize their capital investment at the rate of α, the total amount of capital to be

[12]Typically, there exist several players in the securitization process, such as originators, special purpose vehicles, credit rating agencies, investors, and so forth. I simplified the process by assuming the securities issued by originators (banks on Island A) are sold directly to investors (banks on Island B) and no uncertainties arise during the process since the rate of return on the securities is certain. This simplification is intended to reveal more clearly the relationship between securitization and a bank's liquidity needs, without considering irrelevant details. See Gorton and Metrick (2012a) for a rigorous explanation of the securitization process and related issues.

securitized is αk_t^a. The consumption of a non-mover is

$$c_{n,t+1}^a = \frac{(1-\alpha)k_t^a R}{1-\lambda}. \tag{8}$$

Total investment in liquid assets of the bank on Island A is $p_t q_t^a$, and the proceeds that the bank on Island A will have received from the securitization (fiat money) from the bank on Island B are $\beta p_t q_t^b$. The aggregate consumption of movers who are born on Island A and are relocated to Island B is

$$\lambda c_{m,t+1}^{ab} = \frac{q_t^a + \beta q_t^b}{z_{t+1}}.$$

The per capita consumption of movers is rewritten from the ABS market equilibrium condition (7) as

$$c_{m,t+1}^{ab} = \frac{q_t^a + \alpha k_t^a b_t R}{\lambda z_{t+1}}. \tag{9}$$

The preceding analysis shows that securitization can reduce the total amount of liquid asset holdings at the bank on Island A in its portfolio by $\alpha k_t^a b_t R$ to a number that is lower than before the capital is securitized. If the right-hand side of (9) is higher than the value of d^i/p_{t+1} in Section 3.2, then the consumption of movers increases by the introduction of securitization, which implies that the bank on Island A now has more margin to invest its resources in a more efficient place, i.e., in capital by relatively reducing q_t^a, and thus producing more output in total than before securitization. In other words, the feasible set of banks on Island A now is expanded due to securitization, given the budget constraint of agents, (2). Now we can construct the maximization problem of banks on Island A as follows. Those banks maximize their depositors' expected utility as

$$\max_{c_{m,t+1}^{ab}, c_{n,t+1}^a} E_t[U(c_{m,t+1}^{ab}, c_{n,t+1}^a)] = \lambda c(c_{m,t+1}^{ab}) + (1-\lambda)u(c_{n,t+1}^a), \tag{10}$$

subject to (8), (9), the budget constraint,

$$q_t^a + k_t^a = 1, \tag{11}$$

the feasibility conditions,

$$\lambda c_{m,t+1}^{ab} \le \frac{q_t^a + \alpha k_t^a b_t R}{z_{t+1}}, \tag{12}$$

$$\lambda c_{m,t+1}^{ab} + (1-\lambda) c_{n,t+1}^a = \frac{q_t^a + \alpha k_t^a b_t R}{z_{t+1}} + (1-\alpha) k_t^a R, \tag{13}$$

and the incentive constraint

$$c_{n,t+1}^a \ge c_{m,t+1}^{ab}. \tag{14}$$

The Lagrangian function of this optimization model is

$$\begin{aligned}\mathcal{L} =& \lambda u\left(c_{m,t+1}^{ab}\right) + (1-\lambda) u\left(c_{n,t+1}^a\right) \\ &+ \mu_1 \left[\frac{q_t^a + \alpha k_t^a b_t R}{z_{t+1}}\right] \\ &+ \mu_2 \left[\frac{q_t^a + \alpha k_t^a b_t R}{z_{t+1}} + (1-\alpha) k_t^a R - \lambda c_{m,t+1}^{ab} - (1-\lambda) c_{n,t+1}^a\right].\end{aligned}$$

The first order conditions with respect to consumptions are given

$$\begin{aligned}\left(c_{m,t+1}^{ab}\right) \quad & u'\left(c_{m,t+1}^{ab}\right) = \mu_1 + \mu_2, \\ \left(c_{n,t+1}^a\right) \quad & u'\left(c_{n,t+1}^a\right) = \mu_2.\end{aligned}$$

The non-binding feasibility condition (12) implies $c_{m,t+1}^{ab} = c_{n,t+1}^a$, because $\mu_1 = 0$. When (12) is binding, the consumption becomes $c_{m,t+1}^{ab} < c_{n,t+1}^a$ because $\mu_1 > 0$. Therefore, the incentive constraint is always satisfied. First order conditions with respect to q_t^a and α yield the following:

$$(\mu_1 + \mu_2) \left(\frac{1 - \alpha b_t R}{z_{t+1}} \right) = \mu_2 (1 - \alpha) R,$$
$$(\mu_1 + \mu_2) \frac{(1 - q_t^a) b_t R}{z_{t+1}} = \mu_2 (1 - q_t^a) R.$$

Combining all these first order conditions, we have

$$u' \left(c_{m,t+1}^{ab} \right) \left(\frac{1 - \alpha b_t R}{z_{t+1}} \right) = (1 - \alpha) u' \left(c_{n,t+1}^{a} \right) R, \tag{15}$$

$$u' \left(c_{m,t+1}^{ab} \right) \frac{b_t}{z_{t+1}} = u' \left(c_{n,t+1}^{a} \right). \tag{16}$$

Comparing (15) with (6) we can see that the marginal costs incurred by increasing q_t^i and k_t^i respectively are both decreased by $\left(\frac{1 - \alpha b_t R}{z_{t+1}} \right)$ and $(1 - \alpha) R$ with securitization, which is less than $\frac{1}{z_{t+1}}$ and R as long as $\alpha > 0$. Solving (15) and (16) gives us an optimal value of q_t^{a*} and α^*, given the price of the ABSes as b_t.

4.3 Demand for Securitized Bonds: Island B

Total investment in liquid assets on Island B is $p_t q_t^b$. Among fiat money obtained, Island B trades β fraction of the money for ABSes issued on Island A. The amount of money transferred to Island A is $\beta p_t q_t^b$. Then, the aggregate consumption of movers born on Island B is

$$\lambda c_{m,t+1}^{ba} = \frac{(1 - \beta) q_t^b}{z_{t+1}} + \alpha k_t^a b_t R,$$

The first term on the right-hand side denotes the consumption from fiat money holdings, excluding the β rate used for purchasing ABSes. The second term is returns generated by the underlying assets (capital on Island A). The per capita consumption of movers is obtained by substituting the ABS market equilibrium condition (7) as

$$c^{ba}_{m,t+1} = \frac{1}{\lambda}\left[\frac{(1-\beta)q^b_t}{z_{t+1}} + \frac{\beta q^b_t}{b_t}\right], \tag{17}$$

and the consumption of non-movers is

$$c^b_{n,t+1} = \frac{k^b_t R}{1-\lambda}. \tag{18}$$

The bank's problem is to choose (q^b_t, k^b_t) and β to maximize its depositors' expected utility

$$\max_{c^{ba}_{m,t+1}, c^b_{n,t+1}} E_t[U(c^{ba}_{m,t+1}, c^b_{n,t+1})] = \lambda u(c^{ba}_{m,t+1}) + (1-\lambda)u(c^b_{n,t+1}), \tag{19}$$

subject to subject to (17), (18), the budget constraint,

$$q^b_t + k^b_t = 1,$$

the feasibility conditions,

$$\lambda c^{ba}_{m,t+1} \leq \left[\frac{(1-\beta)q^b_t}{z_{t+1}} + \frac{\beta q^b_t}{b_t}\right],$$

$$\lambda c^{ba}_{m,t+1} + (1-\lambda)c^b_{n,t+1} \leq \left[\frac{(1-\beta)q^b_t}{z_{t+1}} + \frac{\beta q^b_t}{b_t}\right] + k^b_t R,$$

and the incentive constraint,

$$c^b_{n,t+1} \geq c^{ba}_{m,t+1}.$$

The first order conditions with respect to q_t^b and β yields the following:

$$u'\left(c_{m,t+1}^{ba}\right)\left(\frac{1-\beta}{z_{t+1}}+\frac{\beta}{b_t}\right)=u'\left(c_{n,t+1}^{b}\right)R, \text{ and} \tag{20}$$

$$\frac{1}{z_{t+1}}=\frac{1}{b_t}. \tag{21}$$

Solving the equations (20) and (21) gives us the optimal value of q_t^{b*} and β^*. Real bond prices are set such that b_t equals the expected inflation. Substituting (21) into (16), we get $c_{m,t+1}^{ab}=c_{n,t+1}^{a}$. Perfect risk sharing is made for agents born on Island A. However, note that (20) takes the same form as (6) since $b_t = z_{t+1}$. Agents born on Island B are thus not affected by ABSes, and the benefits of securitization are attributed to the Island A which issues the securities.

4.4 Comparative Statics

Solving first order conditions of the banks from both islands A and B together, we get the optimal value of $q_t^{a^*}$ and $q_t^{b^*}$ taking the following forms:

$$q_t^{a^*}=\frac{\lambda(z_{t+1}R-1)}{z_{t+1}R\lambda-2\lambda+1}, \text{ and} \tag{22}$$

$$q_t^{b^*}=\lambda. \tag{23}$$

Before securitization is introduced, the bank on Island A invested in liquid assets amounting to $q_t^a=\lambda$. The difference in liquid asset investment before and after securitization is given as:

$$q_t^{a^*}-\lambda=\frac{\lambda(1-\lambda)(z_{t+1}R-2)}{\lambda z_{t+1}R-2\lambda+1}.$$

Given R and λ, the sign of $q_t^{a^*}-\lambda$ depends on the value of z_{t+1}. Following Assumption 1, the denominator is always positive. Also, as long as $z_{t+1}R\leq 2$, the sign of numeration is not positive, which implies banks on Island A hold less liquid assets by issuing ABSes than the case without ABSes. Therefore, if the rate of return on fiat money, $1/z_{t+1}$, is at least as high as or higher than half of the capital returns, $R/2$, then Island A would invest less in liquid assets than would be the case without issuing ABSes. However, if $z_{t+1}R > 2$, which may occur when z_{t+1} is high, given R, then the bank on Island A will hold more liquid assets. In sum, total liquid asset investments of the whole economy are confined by $q_t^{a^*} - \lambda$ if $z_{t+1}R \leq 2$.[13]

Figure 2 illustrates a numerical example in cases without and with securitization. The utility function is assumed to be $u(c) = \ln c$, and the parameter values are given as $\lambda = 0.5$ and $R = 2$. The top figure compares liquid asset investment of Island A with and without securitization. As explained above, $q_t^{a^*}$ is less than λ when $z_{t+1} < 1$.

To compensate for the consumption loss due to reduced liquid asset investments amounting to $(q_t^{a^*} - \lambda)$, banks on Island A issues ABSes backed by the return from the capital investment, $k_t^a R$, and trade them for fiat money of banks on Island B, $(1-\beta)q_t^b$. Capital investment increases by $(\lambda - q_t^{a^*})$, if $z_{t+1}R \leq 2$, and the economy enjoys more output. The ratio of capital that is mobilized by securitization and sold to the bank on the island B is given as:

$$\alpha^* = \frac{\lambda}{z_{t+1}R}. \tag{24}$$

The ratio of securitization is lower, as the rate of return on capital, R, is bigger, because the marginal cost incurred from securitization becomes huge as more capital is securitized. Also, α decreases as expected inflation, z_{t+1}, is higher. Since the real bond price is set such that $b_t = z_{t+1}$, higher inflation implies more expensive real bond prices, which enhances movers' consumption.

The banks on Island B purchase ABSes issued by banks on Island A as fraction β of $q_t^{b^*}$

[13] Also, note that Island A will hold fewer and fewer liquid assets as the magnitude of the liquidity shock becomes bigger $(\partial(q_t^{a^*} - \lambda)/\partial\lambda > 0)$.

as follows

$$\beta^* = \frac{1-\lambda}{\lambda z_{t+1} R - 2\lambda + 1}, \tag{25}$$

which is always positive following Assumption 1. From (21), the equilibrium bond price is determined such that

$$b_t^* = z_{t+1}. \tag{26}$$

The consumption of movers born on Island A is given as follows after substituting (22), (24) and (26) into (8) and (9)

$$c_{m,t+1}^{ab^*} = c_{n,t+1}^{a^*} = \frac{z_{t+1}R - \lambda}{z_{t+1}(\lambda z_{t+1} R - 2\lambda + 1)}. \tag{27}$$

The consumption of movers from Island A is

$$\begin{aligned} c_{m,t+1}^{ab^*} - c_{m,t+1}^{ab} &= \frac{z_{t+1}R - \lambda}{z_{t+1}(\lambda z_{t+1} R - 2\lambda + 1)} - \frac{1}{z_{t+1}} \\ &= \frac{(z_{t+1}R - 1)(1-\lambda)}{z_{t+1}(\lambda z_{t+1} R - 2\lambda + 1)} \geq 0. \end{aligned}$$

Movers' consumption increases with securitization as long as capital return is greater than that of fiat money, i.e., when $R > 1/z_{t+1}$. The consumption of the non-movers born on Island A, on the other hand, decreases with securitization as follows.

$$\begin{aligned} c_{n,t+1}^{a^*} - c_{n,t+1}^{a} &= \frac{z_{t+1}R - \lambda}{z_{t+1}(\lambda z_{t+1} R - 2\lambda + 1)} - R \\ &= -\frac{\lambda (z_{t+1}R - 1)^2}{z_{t+1}(\lambda z_{t+1} R - 2\lambda + 1)} \leq 0. \end{aligned}$$

The gap is minimized when $z_{t+1}R = 1$, i.e., when the rates of returns on capital and fiat money are equalized. The consumption of the movers on Island B is obtained by substituting (23), (25) and (26) into (17)

$$c_{m,t+1}^{ba^*} = \frac{1}{z_{t+1}}. \tag{28}$$

Since the bond price is set in the markets such that $b_t^* = z_{t+1}$, the gains from securitization are not vested in the movers from Island B. Finally, replacing q_t^{b*} in (18) with (23), the consumption of non-movers is calculated as:

$$c_{n,t+1}^{b*} = R. \tag{29}$$

Numerical examples of these results are shown in Figure 7. The middle figure compares the consumptions of agents born on Islands A and B, and the bottom figure illustrates the expected utility level of the agents. The expected utility is affected by inflation, but agents born on Island A experience higher utility overall as perfect risk-sharing is made with securitization.

5 Both Regions Issue Securities

Now let us suppose that both regions can issue ABSes. What would happens in at the level of overall liquidity and with consumers welfare if both countries started to issue ABSes? A more interesting question is, if the productivity in capital is different across islands, and still both regions can issue ABSes (Case III), how will the new equilibrium be different from the results obtained in Case II, in which the productivity is was the same across islands?

5.1 Same Productivity (Case II)

Now, a bank located on Island i assign α^i fraction of k_t^i to be securitized and purchases ABSes issued by j bank at the rate of β^i of holding fiat money, $p_t q_t^i$. We get the real ABS price, $\hat{b}_t$, from the ABS market equilibrium condition, as follows:

$$\hat{b}_t = \frac{\hat{B}_t}{p_t} = \frac{\beta^a q_t^a + \beta^b q_t^b}{\alpha^a k_t^a R + \alpha^b k_t^b R} \tag{30}$$

The numerator denotes total demand for ABSes and the denominator total supply of them. As both regions issue ABSes at the same time, the depth and breadth of the ABS markets are intensified. The consumption of movers and non-movers born on Island i, where $i, j = \{a, b\}$ and $i \neq j$, takes the following forms:

$$c^{i}_{n,t+1} = \frac{(1-\alpha^{i})k^{i}_{t}R}{1-\lambda}, \tag{31}$$

$$c^{ij}_{m,t+1} = \frac{1}{\lambda}\left[\frac{(1-\beta^{i})q^{i}_{t}}{z_{t+1}} + \frac{\beta^{i}q^{i}_{t}}{\hat{b}_{t}} + \frac{\alpha^{i}k^{i}_{t}\hat{b}_{t}R}{z_{t+1}}\right]. \tag{32}$$

The consumption of movers is made up of three different parts. The first term in the brackets denotes the consumption from holding fiat money, except fiat money used to buy ABSes issued by banks on Island j. The second term stands for the consumption from the proceeds of capital by holding ABSes issued by banks on Island j. The final term expresses the fiat money traded for the ABSes issued by banks on Island i.

The problem facing banks on Island i is that it has to choose (q^{i}_{t}, k^{i}_{t}), α^{i} and β^{i} to maximize its depositors' expected utility, (1), subject to the budget constraint, (2), the feasibility conditions,

$$\lambda c^{ij}_{m,t+1} \leq \frac{(1-\beta^{i})q^{i}_{t}}{z_{t+1}} + \frac{\beta^{i}q^{i}_{t}}{\hat{b}_{t}} + \frac{\alpha^{i}k^{i}_{t}\hat{b}_{t}R}{z_{t+1}},$$

$$\lambda c^{ij}_{m,t+1} + (1-\lambda)c^{i}_{n,t+1} \leq \frac{(1-\beta^{i})q^{i}_{t}}{z_{t+1}} + \frac{\beta^{i}q^{i}_{t}}{\hat{b}_{t}} + \frac{\alpha^{i}k^{i}_{t}\hat{b}_{t}R}{z_{t+1}} + k^{i}_{t}R,$$

(31), (32), and the incentive constraint, (5).

Solving this maximization problem yields the following three first order conditions.

$$u'\left(c^{ij}_{m,t+1}\right)\left(\frac{1-\beta^{i}}{z_{t+1}} + \frac{\beta^{i}}{\hat{b}_{t}} - \frac{\alpha^{i}\hat{b}_{t}R}{z_{t+1}}\right) = (1-\alpha^{i})u'\left(c^{i}_{n,t+1}\right)R, \tag{33}$$

$$u'\left(c^{ij}_{m,t+1}\right)\frac{\hat{b}_{t}}{z_{t+1}} = u'\left(c^{i}_{n,t+1}\right), \tag{34}$$

$$\frac{1}{z_{t+1}} = \frac{1}{\hat{b}_t}. \tag{35}$$

Real ABS prices are set such that $\hat{b}_t = z_{t+1}$ as in Case I. From (34) and (35), we get $c^{ij}_{m,t+1} = c^i_{n,t+1}$. Therefore, if both islands are symmetric and trades both fiat money and ABSes at the same time, perfect risk-sharing is made possible between movers and non-movers.

5.1.1 Comparative Statics

The ratio of capital securitized on Island A does not change compared with Case I, i.e.,

$$\alpha^{i'} = \frac{\lambda}{z_{t+1}R} = \alpha^* \tag{36}$$

On the other hand, the value of β of Island B is different from in Case I, which takes the following form

$$\beta^{i'} = \frac{1-\lambda}{z_{t+1}R-1}. \tag{37}$$

$\beta^{i'}$ is greater than $\beta^* = \frac{1-\lambda}{\lambda z_{t+1}R-2\lambda+1}$ as long as $z_{t+1}R < 2$. The size of β in Case II is affected more by inflation than in Case I.[14]

The optimal value of the liquid asset investment $q^{i'}_t$ is given as:

$$q^{i'}_t = \frac{\lambda(z_{t+1}R-1)}{z_{t+1}R\lambda-2\lambda+1} \tag{38}$$

The investment in liquid assets is a function of inflation z_{t+1}. Differentiating $q^{i'}_t$ by z_{t+1}, we get $\frac{\partial q^{i'}_t}{\partial z_{t+1}} = \frac{R\lambda(1-\lambda)}{(R\lambda E[z_{t+1}]^2-2\lambda+1)^2} > 0$. As higher inflation is expected, the bank substitutes liquid asset investments for illiquid asset investments to secure the consumption of movers. Given R and λ, the gap between $\sum(q^{i'}_t - q^{i*}_t)$ is a function of the future inflation rate, as well. The

[14] $\frac{\partial \beta^{i'}}{\partial z_{t+1}} - \frac{\partial \beta^*}{\partial z_{t+1}} > 0$. For a numerical example, see Figure 3.

gap is calculated as

$$\sum(q_t^{i'} - q_t^{i*}) = \frac{2\lambda(E[z_{t+1}]R-1)}{E[z_{t+1}]R\lambda - 2\lambda + 1} - \left(\frac{1-\lambda}{\lambda E[z_{t+1}]R - 2\lambda + 1} + \lambda\right)$$
$$= \frac{\lambda(1-\lambda)E[z_{t+1}]}{(\lambda E[z_{t+1}]R - 2\lambda + 1)^2},$$

which is positive. When both regions issue ABSes at the same time, total liquid asset investments are higher than if only one region issues ABSes. Note that this does not necessarily mean that the economy inefficiently invests its resources, because only $(1-\beta^i)q_t^i$ among q_t^i actually takes the form of fiat money, which is used for consumption of movers (see (32)). Figure 6 illustrates actual fiat money used for the consumption of movers. The top figure indicates the fiat money holding ratio for each island in three different cases, and the bottom figure shows the sum of Islands A and B. As the bottom figure illustrates, consumption from holding fiat money is lower in Case II than Case I.

The combined magnitude of α and β may be used as a proxy for a financial market's breadth and depth. As we can see in (36) and (37), α and β decrease when higher inflation is expected, and trade in financial markets is thus lessened.

The optimal level of consumption for movers and non-movers is given as:

$$c_{m,t+1}^{ij'} = c_{n,t+1}^{i'} = \frac{z_{t+1}R - \lambda}{z_{t+1}(z_{t+1}R\lambda - 2\lambda + 1)} \tag{39}$$

5.2 Different Technology (Case III)

This section deals with the situation where 1. both regions issue ABSes, but 2. the productivity of capital on each island is not identical. This situation symbolizes countries which are comprised of different regions having disparate productivity levels respectively, but being all under the control of the same central bank. For modeling this, it is assumed that the capital productivity on Island i and j is not identical and $R^a > R^b \geq \lambda/z_{t+1}$. Due to the different productivity levels, the yield from ABSes that are mobilized from capital investments is not the same, which implies that two different assets are traded in the ABS markets. The demand for ABSes issued by Island i comes from Island j, and the equilibrium ABS price is determined such that

$$\tilde{b}_t^i = \frac{\beta^j q_t^j}{\alpha^i k_t^i R^i}, \tag{40}$$

where $i, j = \{a, b\}$, and $i \neq j$. Consumption of mover and non-movers on Island i is given as:

$$c_{m,t+1}^{ij} = \frac{1}{\lambda}\left[\frac{(1-\beta^i)q_t^i}{z_{t+1}} + \frac{\beta^i q_t^i}{\tilde{b}_t^j} + \frac{\alpha^i k_t^i \tilde{b}_t^i R^i}{z_{t+1}}\right], \tag{41}$$

$$c_{n,t+1}^i = \frac{(1-\alpha^i)k_t^i R^i}{1-\lambda}. \tag{42}$$

Note that movers' consumption is now affected by both $\tilde{b}_t^i$ and $\tilde{b}_t^j$. The problem faced by the bank on Island i is to maximize (1) subject to (2), the feasibility conditions, (5).

$$\lambda c_{m,t+1}^{ij} \leq \frac{(1-\beta^i)q_t^i}{z_{t+1}} + \frac{\beta^i q_t^i}{\tilde{b}_t^j} + \frac{\alpha^i k_t^i \tilde{b}_t^i R^i}{z_{t+1}},$$

$$\lambda c_{m,t+1}^{ij} - (1-\lambda)c_{n,t+1}^i = \frac{(1-\beta^i)q_t^i}{z_{t+1}} + \frac{\beta^i q_t^i}{\tilde{b}_t^j} + \frac{\alpha^i k_t^i \tilde{b}_t^i R^i}{z_{t+1}} + (1-\alpha^i)(1-q_t^i)R^i,$$

(40), (41), (42), and the incentive constraint. The Lagrangian function take the following form:

$$\begin{aligned}\mathcal{L} =& \lambda u\left(c_{m,t+1}^{ij}\right) + (1-\lambda)u\left(c_{n,t+1}^i\right) \\ &+ \mu_1\left[\frac{(1-\beta^i)q_t^i}{z_{t+1}} + \frac{\beta^i q_t^i}{\tilde{b}_t^j} + \frac{\alpha^i k_t^i \tilde{b}_t^i R^i}{z_{t+1}} - \lambda c_{m,t+1}^{ij}\right] \\ &+ \mu_2\left[\frac{(1-\beta^i)q_t^i}{z_{t+1}} + \frac{\beta^i q_t^i}{\tilde{b}_t^j} + \frac{\alpha^i k_t^i \tilde{b}_t^i R^i}{z_{t+1}} + (1-\alpha^i)(1-q_t^i)R^i - \lambda c_{m,t+1}^{ij} - (1-\lambda)c_{n,t+1}^i\right].\end{aligned}$$

Solving the above Lagrangian function with respect to q_t^i, α^i and β^i, we have the following three first order conditions.

$$u'\left(c_{m,t+1}^{ij}\right)\left(\frac{1-\beta^i}{z_{t+1}}+\frac{\beta^i}{\tilde{b}_t^j}-\frac{\alpha^i\tilde{b}_t^iR^i}{z_{t+1}}\right)=(1-\alpha^i)u'\left(c_{n,t+1}^i\right)R^i, \tag{43}$$

$$u'\left(c_{m,t+1}^{ij}\right)\frac{\tilde{b}_t^i}{z_{t+1}}=u'\left(c_{n,t+1}^i\right), \tag{44}$$

$$\frac{1}{z_{t+1}}=\frac{1}{\tilde{b}_t^i}. \tag{45}$$

Note that the ABS prices on both islands is set to be equal to the inflation, and thus the same across islands, even though capital productivity is different from each other, i.e.,

$$\tilde{b}^{i''}=z_{t+1}. \tag{46}$$

α^i and β^i are adjusted such that $\tilde{b}_t^a=\tilde{b}_t^b=z_{t+1}$. In other words, the following relation is satisfied.

$$\tilde{b}_t^a=\frac{\beta^bq_t^b}{\alpha^ak_t^aR^a}=\frac{\beta^aq_t^a}{\alpha^bk_t^bR^b}=\tilde{b}_t^b. \tag{47}$$

Given $R^a>R^b$, this implies that $\frac{\beta^bq_t^b}{\alpha^ak_t^a}>\frac{\beta^aq_t^a}{\alpha^bk_t^b}$. The above equation can be rewritten using the capital productivity ratio as

$$\frac{R^a}{R^b}=\frac{\beta^bq_t^b}{\beta^aq_t^a}\frac{\alpha^bk_t^b}{\alpha^ak_t^a}=\frac{\alpha^b}{\alpha^a}\frac{\beta^b}{\beta^a}. \tag{48}$$

The product of α^i of β^i of Island B is always greater than that of Island A. The island having lower productivity participate in ABS markets more actively than the island with higher productivity. Also, perfect risk-sharing among movers and non-movers is made again, which can be identified from (44) and (45).

5.2.1 Comparative Statics

Solving (43), (44) and (45) simultaneously, we get the optimal amount of liquid asset investment, which takes the following form:

$$q_t^{i''} = \frac{\lambda(z_{t+1}R^i - 1)}{z_{t+1}R^i\lambda - 2\lambda + 1}. \tag{49}$$

Differentiating $q_t^{i''}$ with respect to R^i yields

$$\frac{\partial q_t^{i''}}{\partial R^i} = \frac{\lambda(1-\lambda)z_{t+1}}{(z_{t+1}R^i\lambda - 2\lambda + 1)^2} > 0, \tag{50}$$

which implies that $q_t^{a''} > q_t^{b''}$.[15] The higher R is, the higher portion of the endowment is devoted to consumption for movers due to the consumption smoothing motive.

The decision as to how much capital should be securitized and how much should be used to purchase ABSes issued by the other island is made such that:

$$\alpha^{i''} = \frac{\lambda}{z_{t+1}R^i}, \tag{51}$$

$$\beta^{i''} = \frac{(1-\lambda)(\lambda z_{t+1}R^i - 2\lambda + 1)}{(z_{t+1}R^i - 1)(\lambda z_{t+1}R^j - 2\lambda + 1)}. \tag{52}$$

The determination of the ratio $\alpha^{i''}$ depends only on the capital productivity of the island i, and $\alpha^{i''}$ decreases in R^i. $\alpha^{a''}$ is thus smaller than $\alpha^{b''}$.[16] The more productive Island A issues a smaller amount of ABSes than the less productive Island B. Two different ABSes issued by Island A and B have the same price with future inflation, irrespective of capital productivity. For that the supply of ABSes should be adjusted. On the other hand, $\beta^{i''}$ depends on the productivity of both islands. $\beta^{i''}$ decreases in R^i[17], and does in R^j as well

[15] See the middle figure of Figure 4 for a numerical example. In Case II q^a is equal to q^b, and thus the gap between q^b(III) and q^b(II) denotes $q_t^{a''} - q_t^{b''}$.

[16] See the top figure of Figure 5 for a numerical example.

[17] $\frac{\partial \beta^{i''}}{\partial R^i} = -\frac{z_{t+1}(1-\lambda)^2}{(z_{t+1}R^i - 1)^2(\lambda z_{t+1}R^j - 2\lambda + 1)} < 0.$

as long as $z_{t+1}R^i > 1$.[18] Banks on Island i wish to purchase fewer ABSes from Island j when the productivity of Island i and j is lower than otherwise. The relative size of $\beta^{a''}$ and $\beta^{b''}$ depends on parameter values, and given that $\lambda = 0.5$, two βs are equalized when $z_{t+1} = \frac{1}{R^a} + \frac{1}{R^b}$. Figure 5 illustrates α and β in Cases II and III, with $R^a = 3$ and $R^b = 1.5$. z_{t+1} that makes $\beta^{a''}$ and $\beta^{b''}$ equal to each other is 1. The combined market participation ratio of Island B, $\alpha^{b''} + \beta^{b''}$, is greater than that of Island A, $\alpha^{a''} + \beta^{a''}$. Consumptions by each type is

$$c^{ij''}_{m,t+1} = c^{i''}_{n,t+1} = \frac{z_{t+1}R^i - \lambda}{z_{t+1}(\lambda z_{t+1}R^i - 2\lambda + 1)}, \tag{53}$$

which increases in R^i. Agents born on Island A consume more than agents on Island B.

Let us compare real liquid asset investments $q^{i''}_t$ with $q^{i'}_t$ in Case II. The difference in the amount of liquid asset investments is a function of inflation, and is given as

$$\sum_i (q^{i''}_t - q^{i'}_t) = \sum_i \left(\frac{\lambda(z_{t+1}R^i - 1)}{z_{t+1}R^i\lambda - 2\lambda + 1} \right) - \frac{2\lambda(z_{t+1}R - 1)}{z_{t+1}R\lambda - 2\lambda + 1}, \tag{54}$$

which is negative because $q^{b''}_t < q^{b'}_t$, given $R = R^a > R^b$ (see (50)). The bottom figure of Figure 4 shows the total liquid asset investments combined of Islands A and B with the different values of z_{t+1} and how the gap changes as z_{t+1} shifts.

The size of $q^{i'}_t$ or $q^{i''}_t$ themselves may not always matter because only $(1-\beta)q$ is actually used to hold fiat money. Figure 6 shows how much money is actually used for consumption by movers (fiat money holding ration), $(1-\beta)q$, in all cases. The market participation ratio is the sum of α and β as shown in Figure 5. The ABS market participation rate is higher when two islands have disparate technology.

[18] $\frac{\partial \beta^{i''}}{\partial R^j} = -\frac{(1-\lambda)\lambda z_{t+1}(\lambda z_{t+1}R^i - 2\lambda + 1)}{(z_{t+1}R^i - 1)(\lambda z_{t+1}R^j - 2\lambda + 1)^2} < 0.$

6 Optimal Monetary Policy

6.1 Case I

Proposition 1 *Perfect risk sharing is made both on Island A and B when the Friedman rule is implemented such that* $z_{t+1}R = 1$ *on both Islands A and B. The liquid asset investment (capital investment) is reduced (increased). Banks from Island A invest all their deposits in capital and the fiat money holding ratio of banks on Island B,* $(1-\beta^*)q_t^{b^*}$*, is reduced to zero.*

Proof. Combining FOCs (15), (16) and (21) in Section 4, $1/z_{t+1} = R$ is obtained. Inserting this result into (20), we have $c_{n,t+1}^{ba^*} = c_{n,t+1}^{b^*}$, i.e., perfect risk-sharing is made on Island B as well. Substituting $z_{t+1}R = 1$ into the optimal value of q_t^{a*} and α^*, we obtain:

$$q_t^{a*} = \frac{\lambda(z_{t+1}R - 1)}{z_{t+1}R\lambda - 2\lambda + 1} = 0, \text{ and}$$

$$\alpha^* = \frac{\lambda}{z_{t+1}R} = \lambda.$$

Banks on Island A invest all their deposits into capital and securitizes λ fraction among them, and sell it all to banks located on Island B. The investment from banks on Island B into liquid assets is not affected by the monetary policy, i.e., $q_t^{b^*} = \lambda$. Among $q_t^{b^*}$, B banks uses β^* ratio among $q_t^{b^*}$ for purchase of ABSes issued by banks from Island A, which takes the following value:

$$\beta^* = \frac{1 - \lambda}{\lambda z_{t+1}R - 2\lambda + 1} = 1.$$

Banks from Island B spend all their cash holdings to buy ABSes, and thus the fiat money holding ratio becomes zero, i.e., $(1-\beta^*)q_t^{b^*} = 0$. The consumptions are given as:

$$c_{m,t+1}^{ab^*} = c_{n,t+1}^{a^*} = \frac{z_{t+1}R - \lambda}{z_{t+1}(\lambda z_{t+1}R - 2\lambda + 1)} = R,$$

$$c_{m,t+1}^{ba*} = \frac{1}{z_{t+1}} = R,$$

$$c_{n,t+1}^{b*} = R.$$

Even though the $(1-\lambda)$ fraction of endowments of Island B is not utilized for capital investment,[19] all agents born at date t are able to consume R. These consumption streams are possible because all the fiat money of the economy combined is held by movers born on Island A as β becomes 1. Therefore, the consumption of the movers born on Island A is given as

$$c_{m,t+1}^{ab} = \frac{(1+\beta)M_t}{2\lambda p_{t+1}} = \frac{M_t}{\lambda p_{t+1}}.$$

p_{t+1} should be adjusted so that $\frac{M_t}{\lambda p_{t+1}} = R$. As the purchasing power of fiat money increases at date $t+1$, movers from Island A can consume R. See Figure 7 for a numerical example of how consumption and expected utility of agents change with different values of z_{t+1}.

■

6.2 Case II

Proposition 2 *Optimal monetary policy is following the Friedman rule, such that* $z_{t+1}R = 1$*. The liquid asset investments (capital investments) are minimized (maximized). Banks on both islands invest all their deposits in capital, and agents' welfare is thus maximized.*

Proof.

From (33), we get $z_{t+1}R = 1$. Therefore, the optimal monetary policy by a central bank should be implemented such that $z_{t+1} = 1/R$. If the central bank implements the monetary policy such that $z_{t+1} = 1/R$ and economic agents believes so, then

[19] Therefore, the total consumption of goods produced by capital investment by the young generation born at date t is constrained by $(2-\lambda)R$.

$$q_t^{i'} = \frac{\lambda(z_{t+1}R - 1)}{z_{t+1}R\lambda - 2\lambda + 1} = 0, \tag{55}$$

$$\alpha^{i'} = \frac{\lambda}{z_{t+1}R} = \lambda, \tag{56}$$

$$\beta^{i'} = \frac{1-\lambda}{z_{t+1}R - 1} \to 1.^{20} \tag{57}$$

The banks on both islands invest all their deposits into capital. Banks on Island i assigns λ fraction of k_t^i to be securitized respectively. They exchange ABSes for fiat money from the old agents living on the same island.[21] With those proceeds, banks from Island i purchase ABSes issued by banks on Island j.

How much money should be withdrawn to achieve the optimal result specifically? If the return on fiat money, $1/z_{t+1}$, is equal to the return to capital, R, then the real bond price, $\hat{b}_t$, is equal to $1/R$. Substituting optimal values calculated above into (30), we get

$$\hat{b}_t' = z_{t+1} = \frac{1}{R} = \frac{M_t/p_t}{2\lambda R}. \tag{58}$$

The money supply needs to be reduced at a rate of $\frac{M_t/p_t}{2\lambda R}$, which implies that the total money supply at $t+1$ is $\frac{(M_t)^2}{2\lambda p_t R}$.

Finally, by substituting the optimal values into the consumption of agents, (32) and (31), we get the following:

$$c_{m,t+1}^{ij'} = \frac{1}{\lambda}\left[\frac{(1-\beta^i)q_t^i}{z_{t+1}} + \frac{\beta^i q_t^i}{\hat{b}_t} + \frac{\alpha^i k_t^i \hat{b}_t R}{z_{t+1}}\right] = R, \tag{59}$$

$$c_{n,t+1}^{i'} = \frac{(1-\alpha^i)k_t^i R}{1-\lambda} = R. \tag{60}$$

Unlike Case I, banks do not invest in liquid assets at all, and all resources are provided with

[20] β is upper-bounded by 1.

[21] Note that capital is assumed to be illiquid due to its immobility and limited communication. In this model, capital investments need only one period of time before they are transformed into consumption goods, and the old can consume before they die by holding ABSes issued when capital is harvested.

capital. ■

6.3 Case III

Unlike Case I and II, the optimal inflation rate is not given as a simple form from solving the first order conditions due to disparate capital productivity. Combining FOCs (43), (44) and (45), we get $1/z_{t+1} = R^i$. Single z_{t+1} is not obtained due to disparate R^i. The central bank has only one policy instrument (controlling z_{t+1}), but has two different policy objectives (R^a and R^b). This situation forces the central bank to solve the following problem to find the optimal value of z_{t+1}.

$$\max_{z_{t+1}} \sum_{i,j=\{a,b\},i\neq j} \big(\lambda u(c^{ij}_{m,t+1}) + (1-\lambda)u(c^{i}_{n,t+1})\big) \tag{61}$$

subject to (53). Substituting (53) into (61) and differentiating with respect to z_{t+1}, we get the optimal inflation rate, $z^{''}_{t+1}$.

The optimal value of z_{t+1} takes a complex form, the form of which is not shown here, and is given as a function of the parameter values of R^a, R^b and λ. For each island the optimal deflation is $z^{''}_{t+1}$ such that $z^{''}_{t+1}R^i = 1$. Therefore, given R^i, $z^{''}_{t+1}$ should increase as R^j decreases. The top (resp. bottom) figure of Figure 8 illustrates the consumption of agents when $R^a = 2$ and $R^b = 1.5$ (resp. $R^a = 2$ and $R^b = 1.1$). When $R^b = 1.5$, the optimal deflation for agents born on Island A is $z_{t+1} = 1/R^a = 0.5$ and for consumers on Island B it is $z_{t+1} = 1/R^b = 2/3$. The central banks have no choice but to decide $z^{''}_{t+1}$ between two different values. In this example, $z^{''}_{t+1}$ is determined to be around 0.59. When R^b is given as 1.1, the $z^{''}_{t+1}$ is determined to be around 0.76.

7 Conclusion

In this paper, I investigated how securitization affects portfolio decisions and the welfare of an economy in three different situations, and what would be the optimal monetary policy in each case. The outcomes of introducing securitization into an economy can be summarized as follows: 1. securitization makes perfect risk sharing possible, 2. securitization "generally" increases welfare by allowing resources to be used more efficiently, and 3. the development of asset-backed security markets is affected by monetary policy. The reason why I use a quotation marks in No. 2. is that the outcome depends on the monetary policy.

The effects of securitization are disparate according to 1. either one or both regions that issue ABSes, and 2. whether the economies are symmetrical or not in terms of capital productivity. The monetary policy affects equilibrium in a different way, depending on the structure of the economy. The optimal monetary policy obtained in this paper does not follow the Friedman rule if the economy is asymmetrical in terms of capital productivity, but it is effective if the productivity of the economies are symmetrical. The optimal monetary policy obtained in this paper concerning the Friedman rule is that the rule does not apply when the economies are asymmetrical in terms of capital productivity (situation in (3)), and it does apply if the economies are symmetrical (situations in (1) and (2)).

In this paper, capital is assumed to be completely depreciated after production. If we relax this assumption, then the dynamic effects of securitization will be studied, and the relationship between securitization and growth can be investigated along with the development of financial markets. Moreover, capital return, R, is assumed to be deterministic in this paper, and perfect information between economic agents is supposed. These assumptions make the model simple but restrictive, unable to explain interesting issues like security design and the relationship between securitization and financial stability. These issues will be dealt with in further research.

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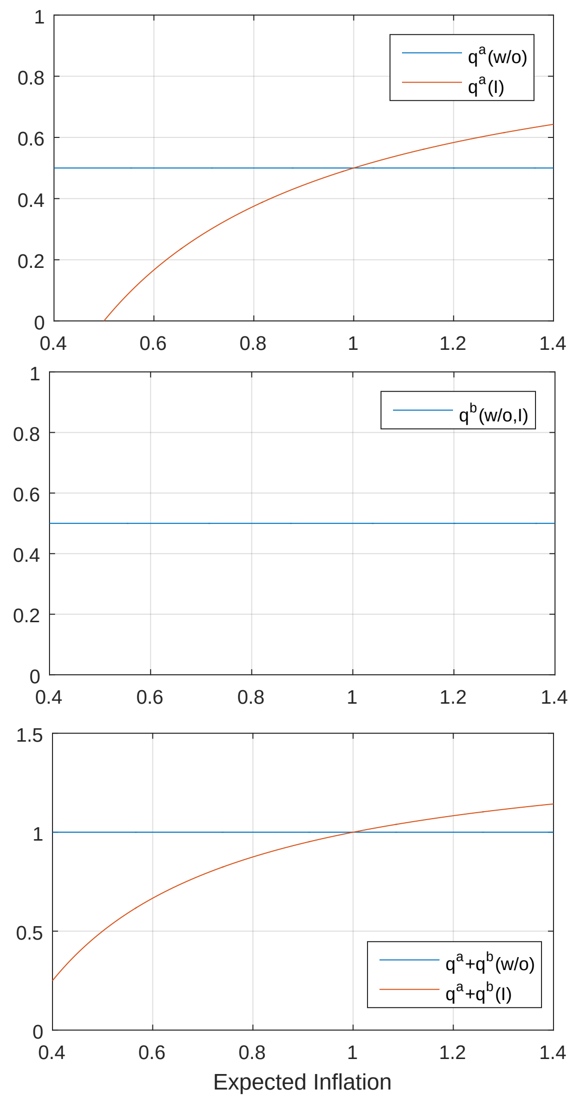
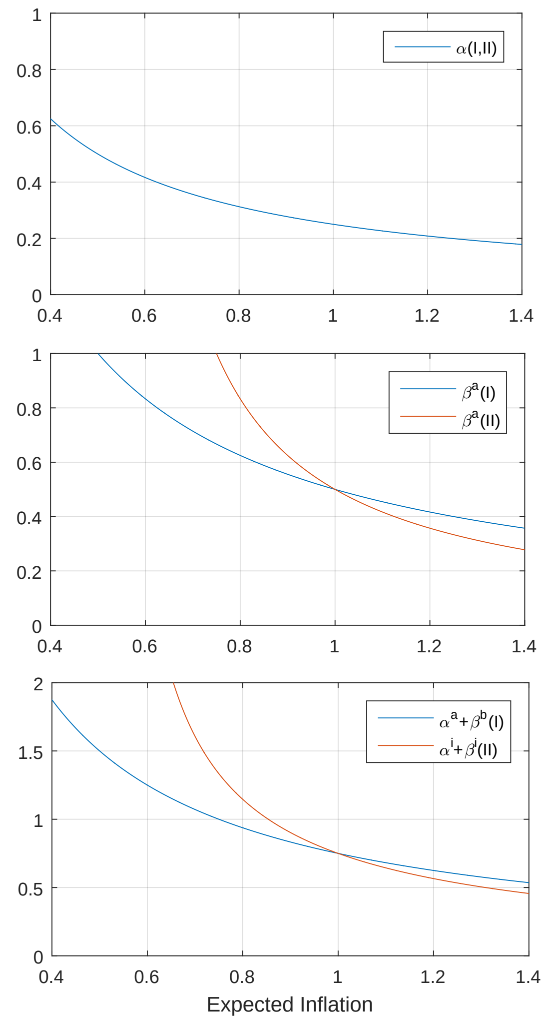
Figure 2: Effects of Inflation on Liquidity



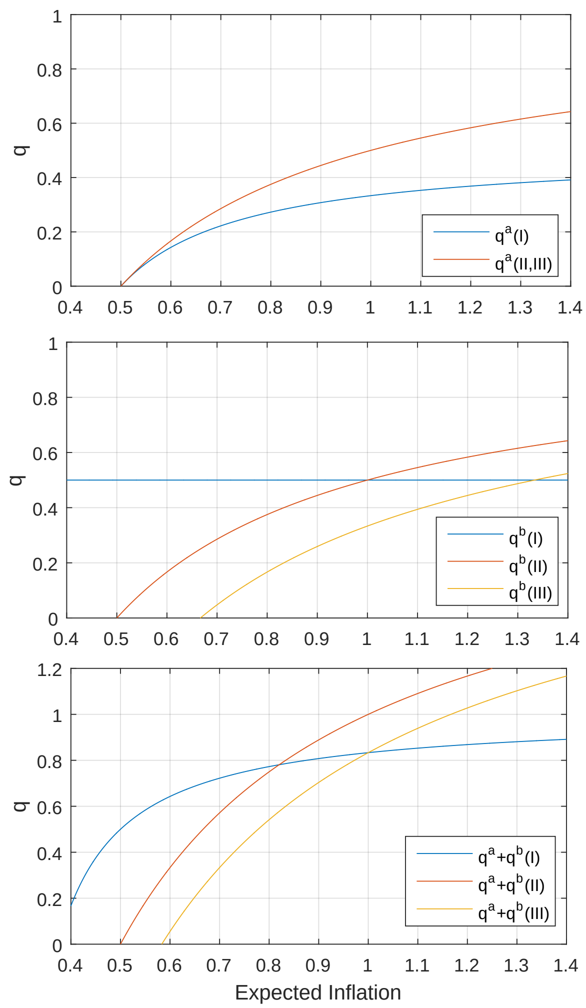
$q^i(w/o)$ and $q^i(I)$ denote liquid asset investments of Island i in cases without and with securitization, respectively. The utility function is assumed to be $u(c) = \ln c$, and the parameter values are given as $\lambda = 0.5$ and $R = 2$.

Figure 3: Effects of Inflation on ABS Markets (1)



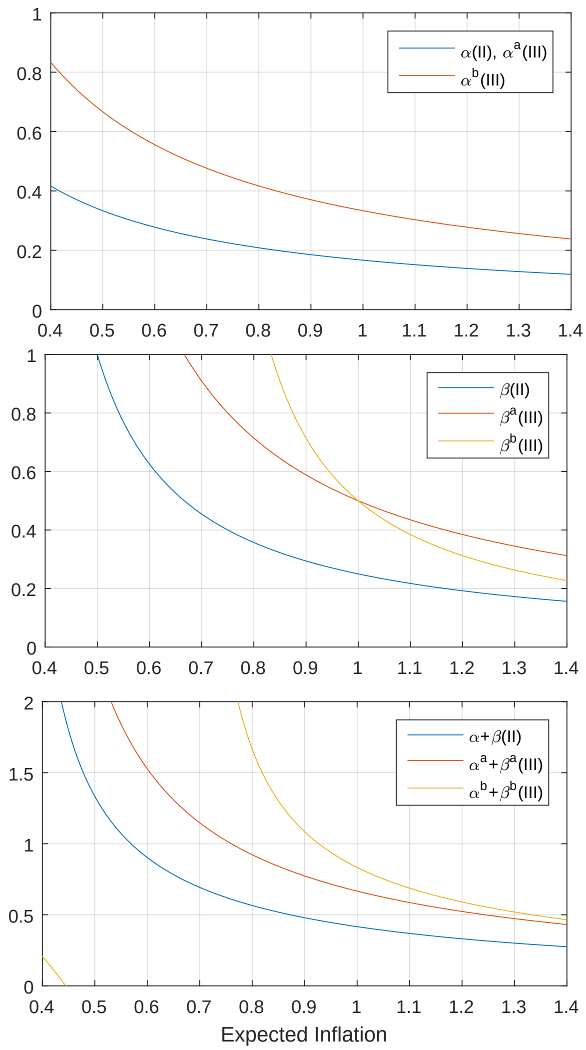
$\alpha^i(s)$ and $\beta^i(s)$ denote the ratio of capital being securitized and the ratio of fiat money being traded for ABSes of Island i in case of s. The utility function is assumed to be $u(c) = \ln c$, and the parameter values are given as $\lambda = 0.5$ and $R = 2$, $R^a = 2$ and $R^b = 1.5$.

Figure 4: Effects of Inflation on Liquid Asset Investment



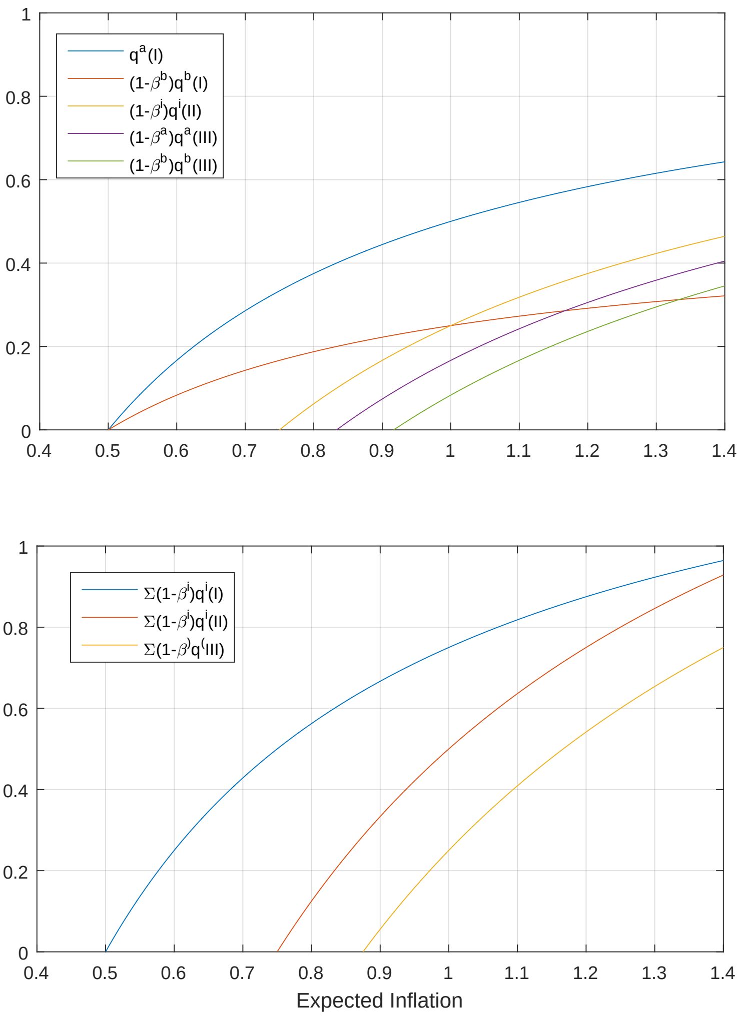
$q^i(s)$ denotes liquid asset investments of Island i in Case s. The utility function is assumed to be $u(c) = \ln c$, and the parameter values are given as $\lambda = 0.5$ and $R = 2$, $R^a = 2$ and $R^b = 1.5$.

Figure 5: Effects of Inflation on ABS Markets (2)



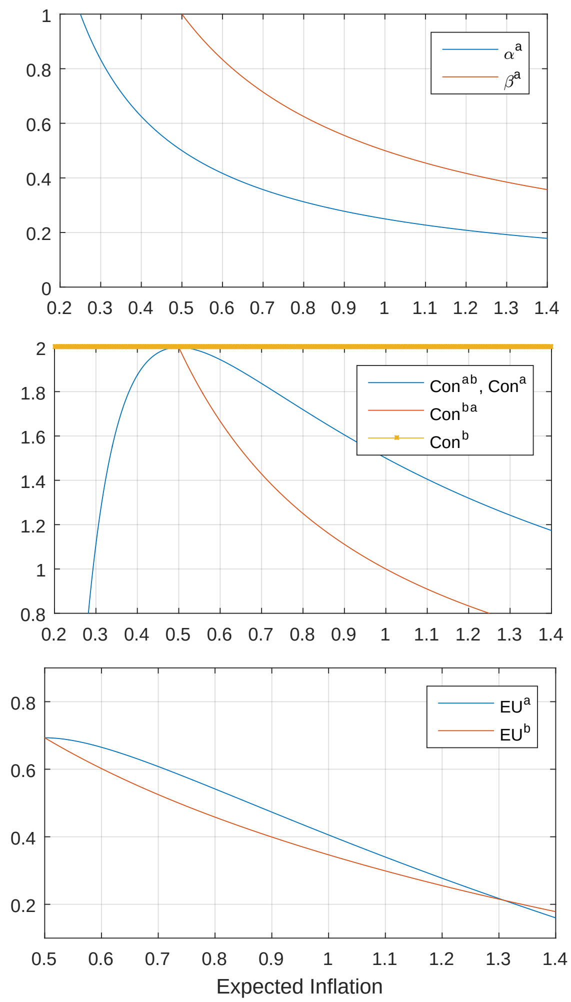
$\alpha^i(s)$ and $\beta^i(s)$ denotes the ratio of capital being securitized and the ratio of fiat money being traded for ABSes of Island i in the case of s. The utility function is assumed to be $u(c) = \ln c$, and the parameter values are given as $\lambda = 0.5$, $R^a = 3$ and $R^b = 1.5$.

Figure 6: Fiat Money Holding Ratio



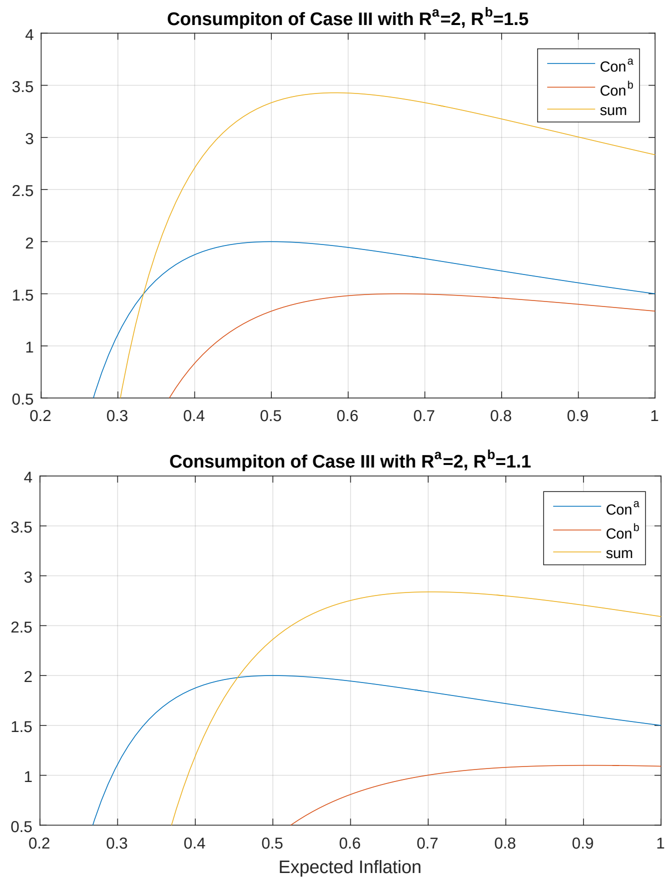
Each line denotes a fiat money holding ratio, $(1-\beta^i)q^i(s)$ in the case of s. The utility function is assumed to be $u(c) = \ln c$, and the parameter values are given as $\lambda = 0.5$ and $R = 2$, $R^a = 2$ and $R^b = 1.5$.

Figure 7: Effects of Inflation on Consumption (Case I)



α and β in the top figure denote the ratio of capital being securitized and the ratio of fiat money being traded for ABSes in case 1. The middle figure denotes the consumption level of movers, Con^{ij}, and non-movers Con^{i}. The bottom figure shows the expected utility of Island i. The utility function is assumed to be $u(c) = \ln c$, and the parameter values are given as $\lambda = 0.5$ and $R = 2$.

Figure 8: Effects of Inflation on Consumption (Case III)



con^i denotes the consumption of agents per capita born on Island i. The utility function is assumed to be $u(c) = \ln c$, and the parameter values are given as $\lambda = 0.5$, $R^a = 2$, $R^b = 1.5$ (top), and $R^b = 1.1$ (bottom).