Evolutionary effects of non compliant behavior in public procurement.

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Abstract

The main goal is to analyze the evolution of non-compliant firms in public procurement. The dynamic setting is formalized by an evolutionary adaptation process which describes whether honest or dishonest behavior prevails in society at any given time t. Firms will either behave in an honest or dishonest way depending on the payoffs and the type of firms they meet, through a word of mouth process. The *honesty-propensity assumption* is introduced. By making use of both analytical tools and numerical techniques, we show that monomorphic configurations may represent stable or unstable equilibria, while coexistence of both groups in the steady state is possible and its asymptotic stability is sensitive to parameter modifications. Moreover, a rich variety of integrated dynamic behavior can be observed due to different kinds of bifurcations, smooth or border collision. A social planner can evaluate social and/or fiscal policies to manipulate preferences for honest behavior.

Keywords: Complex Dynamics, Multiple Equilibria, Procurement, Word of Mouth, Evolutionary game, non compliant behavior.

JEL classification codes: C61, C63, H57, C73
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1 Introduction

According to Transparency International, “Public procurement, also known as public contracting, is a multi-step process of established procedures to acquire goods and services by a government entity”[1]. “It involves the full cycle from needs assessments through to the preparation of the procurement documentation, the awarding of contracts, the implementation and the final accounting of a completed contract. Public procurement is applicable to any government contract for goods, works or services, including for consultancies”[2].

Public procurement is directly linked to government policy aims. Governments are now making more effort to carry out this process in an efficient manner and with high standards of behavior in order to deliver their mandates and improve service delivery[3]. There are many public investments that involve public procurement. These include building roads, public buildings, schools, hospitals, sanitation systems and the buying of land by local and national governments. With these examples we see how public procurement plays a major part in people’s lives and represents a large portion of public budgets.

In OECD countries, public procurement is an integral part of the economy as it accounts for about 12% of GDP. While in Mexico it is about 5.1%, in the Netherlands it is 20.2%.

Many corruption risks are involved in the process of public procurement due to the close interplay between the public and private sectors leading to non-transparent behavior. The real market value of products being purchased may be covered up while complex relationships between political and bureaucratic entities create an environment subject to non-compliant behavior. It is therefore fundamental to ensure effective, high quality public procurement to safeguard the public interest and maintain good governance.

It is worth mentioning the importance of public procurement reform in maintaining transparency in government procurement, both in developed and less developed countries. There is a vast amount of money involved in procurement procedures and a substantial portion comes from the taxpayers’ money (e.g. see Hui et al (2011)). The main problem, however, has been the non-compliance in developing countries as well as European countries (De Boer and Telgen, 1998) and many authors have approached this subject in their works (De Boer and Telgen, 1998; Gelderman et al, 2006; Eyaa and Oluka, 2011).

To describe the non-compliant behavior in public procurement, we study a model which combines Becker’s economic theory (1968) with psychological and sociological theories which explain what motivates a person to adhere to the rules or not. In his model, Becker shows how criminals will rationally weigh up the utility outcome of a crime. In other words, an individual will decide to commit a crime if the utility of this action is greater than the utility gained from complying with a set of regulations. The deterrence theory emphasizes the importance of sanctions in preventing crime, although there are drawbacks to this basic deterrence model. The framework’s policy prescriptions are not always feasible and the evidence available is not explained clearly. Enforcing higher penalties can be costly and impractical and does not always lead to lower levels of non-compliance. In the same way, having less severe penalties does not necessarily mean individuals will commit more crime. Therefore, as well as considering the conventional costs and benefits linked to non-compliant conduct, our model also takes into consideration social influence and moral duty.

It is well-known that social stigma can have an important influence on a person’s

[1]Transparency International, (2009).
[2]Transparency International (2006).
[3]For more details, see OECD (2017).

behavior and this includes their desire for benefit from prosocial behavior.[4] Moral duty and compliant behavior are inextricably linked. People aim to safeguard their reputation and fear reaction from others if they are discovered to be corrupt. In order to take into account such evidence, our study considers that society may benefit from prosocial behavior and avoid social disapproval. Benabou and Tirole (2011) have analyzed this prosocial behaviour, emphasizing how one must consider "a unifying framework to analyze how private decisions as well as public policies are shaped by personal and societal preferences ("values"), material or other explicit incentives ("laws") and social sanctions or rewards ("norms")". In our model we show the important role of social norms and reputation in strengthening and undermining the presence of explicit incentives like laws.

With this aim in mind, we focus on the key role social norms play when faced with "explicit incentives", in other words "laws". If non-compliant behavior becomes more widespread, the social stigma attached is seen to be less important, in other words the person breaking the law will consider his own behavior more acceptable by society (e.g. Kim (2003) and Traxler (2010)). There is an interdependent relationship between social norms and compliant/non-compliant behavior. If there is a tendency to deviate from the social norm, this in turns weakens the norm and more individuals will carry out non-compliant behavior seeing others doing the same. The costs of deviating from the norm decrease and illegal activity becomes more common. (e.g. Akerlof, 1980).

In order to describe this mechanism, we propose a dynamic framework based on the Benabou and Tirole (2011) model and analyze how the government indexes an auction for the provision of public goods[5].

Due to the fact that there are different quality levels of public goods, problems arise when firms are dishonest about the exante quality of a public good (see Bajari and Tadelis, 2001 and Brianzoni et al. 2011). The quality of a public good is confidential information and therefore can only be verified by the State's controllers. In this way the State is able to eliminate or at least cut down on non-compliant behavior.

The model is obtained by considering how the evolutionary adaptation process determines whether honest or dishonest behavior prevails in society. Agents will either behave in an honest or dishonest way depending on the payoffs and the type of firm they meet. For example, if two firms both behave in the same way (either honest or dishonest) then it is unlikely they will then choose a different behavior as no new information is acquired. If, on the other hand, a firm comes into contact with another firm of a different type and compares the payoffs, it may realize that it can increase its expected utility by changing to the same behavior as the other firm. This is a type of word of mouth process with the meeting of two firms to compare information. Therefore, honest firms which interact with dishonest firms may change their conduct if the utility they gain from being dishonest is more substantial than the one they get from being honest (see Lamantia and Pezzino (2017)). In our model, however, we also introduce the concept of *honesty-propensity*, according to which if an honest firm meets a dishonest one, it will choose to remain honest as long as he cannot increase his expected utility by changing type.

[4]The assumption that behavior is influenced by social stigma or the desire for status has a long tradition in economics. Many have studied this phenomenon: Akerlof (1980), Moffitt (1983), Gordon (1989), Besley and Coate (1992), Bernheim (1994) and Lindbeck et al. (1999). For an analysis of prosocial behavior see e.g Benabou and Tirole (2006, 2011).

[5]Although the issue of the effect of the quality of public procurement on economic growth is very relevant, it has only recently attracted a more significant attention from scholars. See for example Brianzoni et al. (2011).

The resulting model is a one-dimensional, continuous, even piecewise-smooth, discrete time dynamical system (with one possible kink point) describing the evolution of the fraction of dishonest firms depending on the monitoring level by the State on corruption and the social stigma attached to dishonest behavior. We study the model from an analytical and numerical point of view: we determine the fixed points showing that internal equilibrium may exist in addition to standard monomorphic configurations so that also bistability can emerge (and the final state reached by the economy could depend on the initial state). A rich variety of dynamic behavior can be observed: we give some conditions for the local asymptotic stability of equilibria, while the occurrence of complex dynamics are described by using both standard techniques related to smooth one dimensional maps and also new, recently developed instruments for non differentiable maps.[6] We recall that, dealing with a piecewise smooth map, two types of bifurcations can occur: bifurcations which are well known for smooth maps (local or global such as flip or fold bifurcations or homoclinic bifurcations, see Medio and Lines (2001) for a wider description) and border collision bifurcations (as first called in the seminal papers by Nusse and Yorke 1992 and 1995) related to the contact of an invariant set with the border separating the regions of different definition of the map. In our model, both kinds of bifurcations emerge and their interplay gives rise to complex long–term evolution configurations, strictly related to parameter configurations. We underline that in the present work we have focused on the description of the evolution of non-compliant firms in public procurement mostly from an economic point of view, as we want to clarify the role of our framework to describe the phenomenon and to suggest some economic insight.[7] With this aim in mind, we have focused above all on two key variables of policy: the monitoring level put in place from fight non-compliant behavior and the "inner" honesty of a country. Our analysis shows that having a high monitoring level is a sufficient condition for the system to converge towards a society in which all firms are honest; the same result can be achieved if the "inner" honesty of the country is high. In fact they are both effective tools in reducing the incentives for dishonesty. Clearly, in cases where the monitoring level of dishonesty is low and the easiness with which laws are broken is high (low "inner" honesty), the economy converges towards a equilibrium in which all businesses are dishonest. Regarding the intermediate cases, different situations may occur depending on the variable social effects associated to honest and dishonest behavior.

The paper is organized as follows. In section 2 we construct the model; in section 3 we describe some general properties of the map (i.e. fixed points and local stability as moving the parameters of interest); in section 4 we show how complex dynamics are exhibited (numerical simulations support the study); in section 5 we present results and further development; section 6 concludes.

2 Economic setup

We consider an economy composed of three types of risk neutral players: the State, bureaucrats and firms. The population of firms has a unit mass and produces a public good. In order to supply the public good free of charge, the State has to buy a unit of

[6]For the description of possible behavior in one dimensional piecewise smooth maps see for instance Sushko et al. (2016b); see also Brianzoni et al. (2010), Radi and Gardini (2018) and Sushko et al. (2016a) as examples of possible applications of such maps in economics.

[7]We will leave a more detailed mathematical study of the bifurcation structure of the economically meaningful part of the parameter space, in particular the fold and flip border collision bifurcation curves, to further development.

it from the private firms. We assume that the State procures a unit of public good from each private firm in order to provide it free. The public good can be produced at different quality levels: low–quality public good and high–quality public good.[8] Following Bose et al. (2008) we assume that the price of high–quality public good is constant and given by $p > 0$ while firms compete over the good's quality: the higher the quality offered, the lower the profit for firms and the higher the welfare of the community.
The constant cost of production for a firm is such that if the public good's quality is high, the unit cost c^h is also high, while if the public good's quality is low, the unit cost c^l is low too, that is $c^h > c^l > 0$. Furthermore, the production of public good is assumed to be profitable, i.e. $p > c^h$.

The bureaucrats organize a reverse auction for the procurement of the public good. The provision of the good is awarded to the firm that offers the best quality good in the sealed bid. As a general rule, the firm offering the highest quality wins the auction. However, the firm can lie about the quality of the public good produced. Let us define $x \in [0, 1]$ as the fraction of firms producing low level public good who lie about the quality (dishonest firms).
The State, in order to weed out or reduce non-compliant behaviors, monitors firms: we denote with $q \in [0, 1]$ the probability of being monitored according to the control level fixed by the State and, then, of being reported.[9]

Finally the firm's propensity to engage in compliant or non-compliant behavior may also be influenced by social benefits and costs, depending, in part, on social interaction.

If an honest firm is monitored then it enjoys a prosocial benefit. As a general rule, we assume that this prosocial benefit can depend on two components: a constant component a_h and a variable component that increases as the fraction of dishonest firms increases, i.e. $b_h x$. Similarly, if a dishonest firm is monitored and, hence, detected, then the cost is given by a fixed amount and a variable one that increases with the fraction of honest firms, i.e. $a_d + b_d(1 - x)$. We can consider the fixed cost a_d as a fine due to the punishment for dishonest behavior, and the proportional cost deriving from the social stigma. In social costs and benefits there is therefore a part dependent on the diffusion of dishonest behavior in society. The more widespread the non-compliant behavior is, the more honest behavior is valued at social level, and the dishonest behavior is less stigmatized. Notice that a_h, a_d, b_h, b_d are not negative parameters, hence one or more effects can be ruled out by simply setting one or more parameters equal to zero. In our setting, consider $a_d > 0$ means that also in the absence of moral factors the dishonest firm if detected is punished with a fine.

Taking into account the previous considerations, the utility of an honest firm is given by:

$$U_h = \begin{cases} U_{h,NM} = p - c^h & \text{if not monitored,} \\ \\ U_{h,M} = p - c^h + a_h + b_h x & \text{if monitored,} \end{cases} \tag{1}$$

whereas the utility of a dishonest firm is as follows:

$$U_d = \begin{cases} U_{d,NM} = p - c^l & \text{if not monitored,} \\ \\ U_{d,M} = p - c^h - (a_d + b_d(1 - x)) & \text{if monitored.} \end{cases} \tag{2}$$

[8]For the nature of the public good, e.g. infrastructure, we assume that no kind of arbitrage is possible for the inputs purchased.

[9]Following Garoupa (2007) or Lamantia and Pezzino (2017) we assume that, if audited, a non-compliant firm is found guilty without doubt.

From such utilities, we obtain the expected utility for a dishonest and for an honest firm that are respectively given by:

$$E[U_d] = qU_{d,M} + (1-q)U_{d,NM} = p - c^l - q(\Delta c + a_d + b_d(1-x)) \tag{3}$$

and

$$E[U_h] = qU_{h,M} + (1-q)U_{h,NM} = q(a_h + b_h x) + p - c^h. \tag{4}$$

where $\Delta c = c^h - c^l > 0$.

The difference in the expected utilities between dishonest and honest firms depends on q, i.e. the monitoring level, and on the fraction of dishonest firms x (due to the presence of variable social effects) and it is given by:

$$\delta = E[U_d] - E[U_h] = \Delta c - q(\Delta c + a_d + a_h + b_d + x(b_h - b_d)). \tag{5}$$

It is interesting to analyze the relationship between the payoff differential (δ) and the others variables. In order to be more precise:

- $\frac{\partial \delta}{\partial \Delta c} > 0$ as the cost differential (Δc) increases, *ceteris paribus*, the incentive to be dishonest grows too.
- $\frac{\partial \delta}{\partial q} < 0$ as the probability of being detected and punished (q) increases, the dishonest behavior becomes less convenient compared to the honest one, thus δ decreases.
- $\frac{\partial \delta}{\partial x}$ depends on the sign of the difference $(b_h - b_d)$.
  - If $(b_h > b_d)$, i.e. the social benefit (b_h) is greater than the social cost (b_h), as the level of dishonesty increases, the incentive to be dishonest decreases;
  - If $(b_h < b_d)$, as the fraction of dishonest firm grows, the dishonest incentive behavior increases too.
- $\frac{\partial \delta}{\partial a_d} < 0$, $\frac{\partial \delta}{\partial a_h} < 0$, $\frac{\partial \delta}{\partial b_d} < 0$, $\frac{\partial \delta}{\partial b_h} < 0$. As the parameters representing the social stigma increase, the convenience of being dishonest decreases.

Consider now combinations between the fraction of dishonest firms and the monitoring levels for any fixed value of the difference between expected utilities, that is:

$$C = \{(x,q) \in [0,1] \times [0,1] : \delta = \bar{\delta}, \bar{\delta} \in \mathbb{R}\}.$$

Then the following remark trivially holds.

Remark 2.1. *The difference between utilities remains constant as long as $(x,q) \in C$ where $q = c(x, \bar{\delta})$ is such that:*

- *if $b_d > b_h$ then c is a strictly increasing function;*
- *if $b_d < b_h$ then c is a strictly decreasing function;*
- *if $b_d = b_h$ then c is a constant function.*

Expected utilities deriving from the two different types of behavior are equal when $\delta = E[U_d] - E[U_h] = 0$, which gives the monitoring level $\bar{q}$ at the equilibrium depending on the fraction of dishonest firms x

$$\bar{q} = \frac{\Delta c}{\Delta c + a_d + a_h + b_d + x(b_h - b_d)}. \tag{6}$$

Notice that $\bar{q} \in [0,1]$ iff $(a_d + a_h + b_d + x(b_h - b_d)) \geq 0$ and that simple computations show that this last inequality is always verified. In addition, $\bar{q} = 0$ in the limiting case $\Delta c = 0$, that is there is no difference between production costs, while $\bar{q} = 1$ for $(a_d + a_h + b_d + x(b_h - b_d)) = 0$ that is no social costs or benefits are considered. A simple comparative static analysis shows that:

- $\frac{\partial \bar{q}}{\partial \Delta c} > 0$ thus implying that as the differential cost (Δc) increases, the level of monitoring that equals payoffs deriving from being honest and dishonest, must also increase. This derives from the fact that, as the Δc grows, ceteris paribus, the economic benefit of being dishonest grows;
- $\frac{\partial \bar{q}}{\partial a_d} < 0$; $\frac{\partial \bar{q}}{\partial a_h} < 0$; $\frac{\partial \bar{q}}{\partial b_d} < 0$; $\frac{\partial \bar{q}}{\partial b_h} < 0$; in fact, as the social costs and benefits grow, $\delta = E[U_d] - E[U_h]$ decreases and, therefore, the level of monitoring that equals payoffs deriving from being honest and dishonest decreases too;
- the sign of $\frac{\partial \bar{q}}{\partial x}$ depends on the sign of the difference between the variable social costs, i.e. $(b_h - b_d)$:
  - if $b_h > b_d$, i.e. the social benefit (b_h) is greater than the social cost (b_h), the social parameter difference $(b_h - b_d)$ grows with the dishonesty level (x), thus increasing the convenience of being honest. Therefore, in order to maintain $\delta = 0$, the monitoring level has to decrease;
  - if $b_h < b_d$, as the fraction of dishonest firm grows, the monitoring level at which $\delta = 0$ must increase.

As far as fraction of dishonest firms is concerned, when $\delta = 0$ it is given by

$$\bar{x} = \frac{\Delta c - q(\Delta c + a_d + a_h + b_d)}{q(b_h - b_d)}. \tag{7}$$

It is interesting to analyze the relationship between $\bar{x}$ and the other relevant variables. With this aim in mind, we can write the previous equation in a different and more intuitive way:

$$\bar{x}q(b_h - b_d) + q(a_d + a_h + b_d) = \Delta c(1 - q). \tag{8}$$

The right side of the equation represents the expected benefit of dishonest behavior, while on the left side we have the expected social costs linked to the social stigma, the level of monitoring and the number of dishonest firms. A simple comparative static analysis shows that:

- $\frac{\partial \bar{x}}{\partial \Delta c}$ depends on the difference in social parameters $(b_h - b_d)$:
  - if $b_h > b_d$, the increase of Δc makes it more convenient to be dishonest and, therefore, in order to have $\delta = 0$, the expected difference in social stigma (i.e. $x(b_h - b_d)$) must increase too;

- if $b_h < b_d$, the level of dishonesty x must decrease, as Δc increases, in order to discourage dishonest behavior and maintain, in this way, $\delta = 0$;

- $\frac{\partial \bar{x}}{\partial q}$ also in this case depends on the sign of the difference $(b_h - b_d)$:
  - if $b_h > b_d$, the increase of the probability of being detected q makes it less convenient to behave dishonestly and, therefore, in order to maintain $\delta = 0$, the dishonesty level x must decrease;
  - if $b_h < b_d$, the level of dishonesty x must increase, as q increases, in order to encourage dishonest behavior and maintain, in this way, $\delta = 0$.

Notice that the sign of the difference between expected utilities given by (5) can be both positive or negative depending on the social variables and the monitoring level. Let's define:

$$q_1 = \frac{\Delta c}{\Delta c + a_d + a_h + b_h} \in (0, 1)$$

and

$$q_2 = \frac{\Delta c}{\Delta c + a_d + a_h + b_d} \in (0, 1)$$

then the following cases may occur: (i) if, at the social level, being honest is rewarded more than being dishonest is stigmatized, i.e. $b_h > b_d$ then $0 < q_1 < q_2 < 1$, (ii) if $b_h < b_d$ then $0 < q_2 < q_1 < 1$, (iii) if $b_h = b_d$ then $0 < q_1 = q_2 < 1$.

Consider also that, as previously underlined, function δ is linear w.r.t. x and, in particular, it is strictly increasing iff $b_h < b_d$ or strictly decreasing iff $b_h > b_d$; furthermore in such cases the difference between expected utilities vanishes if $x = \bar{x}$. Obviously, if $\bar{x} \leq 0$ or $\bar{x} \geq 1$ then function δ does not change sign in the interval $[0, 1]$. On the other hand, if $b_h = b_d$ then δ is constant and its sign depends on the monitoring level.

Following the previous arguments, the following cases may occur.

Remark 2.2. *Regarding the sign of $\delta(x)$ given by (5), the following cases may occur.*

a. If $b_h > b_d$ then $\delta(x)$ is strictly decreasing and:
   1. *if $0 \leq q \leq q_1$ then $\bar{x} \geq 1$ and $\delta \geq 0\ \forall x \in [0, 1]$;*
   2. *if $q_1 < q < q_2$ then $\bar{x} \in (0, 1)$ and $\delta \geq 0\ \forall x \in [0, \bar{x}]$ while $\delta < 0\ \forall x \in (\bar{x}, 1]$;*
   3. *if $q_2 \leq q \leq 1$ then $\bar{x} \leq 0$ and $\delta \leq 0\ \forall x \in [0, 1]$.*

b. If $b_h < b_d$ then $\delta(x)$ is strictly increasing and:
   1. *if $0 \leq q \leq q_2$ then $\bar{x} \leq 0$ and $\delta \geq 0\ \forall x \in [0, 1]$;*
   2. *if $q_2 < q < q_1$ then $\bar{x} \in (0, 1)$ and $\delta < 0\ \forall x \in [0, \bar{x})$ while $\delta \geq 0\ \forall x \in [\bar{x}, 1]$;*
   3. *if $q_1 \leq q \leq 1$ then $\bar{x} \geq 1$ and $\delta \leq 0\ \forall x \in [0, 1]$.*

c. If $b_h = b_d$ then $\delta(x)$ is constant and:
   1. *if $0 \leq q < q_1$ then $\delta > 0\ \forall x \in [0, 1]$;*
   2. *if $q = q_1$ then $\delta = 0\ \forall x \in [0, 1]$;*
   3. *if $q_1 < q \leq 1$ then $\delta < 0\ \forall x \in [0, 1]$.*

In our evolutionary setting, the channel through which the agents change their strategy is represented, as we said, by a word-of-mouth process.

2.1 Word-of-mouth mechanism and dynamics

Following Dawid (1999) and Banerjee and Fudenberg (2004), we describe the dynamics of non-compliant firms by considering an evolutionary game with word-of-mouth mechanism. Let us consider a discrete time setup, i.e. time $t = 0, 1, 2, ...$, and let $x_t \in [0, 1]$ (resp. $(1 - x_t)$) be the fraction of dishonest (resp. honest) firms at time t. In an evolutionary process, agents have the possibility to make a comparison between their expected payoffs and those of the rest of society. In fact, evolutionary game theory deals with entire populations of players, all programmed to engage in some strategy (in our model compliant or non-compliant with the rule). It is possible, however, that some firms may change their conduct in the presence of a different type of firm. Strategies with high payoff will spread within the society, through a word-of-mouth process in our model. Then, we assume that a firm could change his type (from dishonest to honest or viceversa) if he meets a firm of a different type and, after comparing their expected utilities, a firm may find it possible to increase their own expected utility by moving from one type to another. On the other hand, no type changes occurs if a firm meets another firm of the same type, because no new information is obtained and therefore firm decides not to change this behavior. In this context, the payoffs depend on the actions of the other players and hence on the frequencies of the strategies within the society. To be precise, in our model the probability of meeting a firm manifesting dishonest behavior depends on how many dishonest firms are operating in any given period. Therefore we assume, following Lamantia and Pezzino (2017), that an honest firm which meets a dishonest firm can change its behavior if the payoff deriving from dishonest behavior is greater than the one deriving from honest behavior. It is obviously more likely that an honest firm will become dishonest if the difference between the two utilities is high. In order to determine the probability that an honest firm meeting a dishonest one changes its behavior (from honest to dishonest), the probability is defined by a non-decreasing function depending on $\delta = [E(U_d) - E(U_h)]$, which we call $\phi(\delta)$. We introduce a new assumption in order to take into account the *honesty propensity* property. The assumptions on function ϕ are summarized in what follows.

- Let $\delta > 0$, i.e. being dishonest is more profitable than being honest, then a positive fraction of honest firms that meet a dishonest one will change type. We denote this probability with $p_{h \to d} = \phi(\delta)$. Function $\phi(\delta)$ is increasing $\forall \delta > 0$ and such that $\lim_{\delta \to +\infty} \phi(\delta) = 1$, providing that if the expected utility of a dishonest firm is very high with respect to the expected utility of an honest one, then almost all honest firms meeting a dishonest one will change their type.

- Consider now the case $\delta \leq 0$, that is the expected utility of an honest firm is greater than or equal to the expected utility of a dishonest one. Lamantia and Pezzino (2017) assume that function ϕ is such that $\phi(0) = \frac{1}{2}$ (see for instance their suggested specification from Bischi et al. (2009)), that is in the case of equal expected utilities, one half of dishonest firms meeting honest ones change their type. Differently from Lamantia and Pezzino (2017) we change this assumption by assuming that $\lim_{\delta \to 0} \phi(\delta) = \phi(0) = 0$ that is as the difference between expected utilities vanishes then all honest firms meeting dishonest ones will prefer to behave in an honest way, due to the *honesty-propensity* assumption. On the other hand if $\delta < 0$ then the expected utility of being honest is greater than the expected utility of being dishonest. Hence, due to the *honesty-propensity* assumption then all honest firms meeting dishonest ones will prefer to remain honest, thus being $\phi(\delta) = 0$.

According to the *honesty-propensity* assumption, the properties of function ϕ, representing the probability that an honest firm meeting a dishonest one changes his behavior, can be summarized in the following remark.[10]

Remark 2.3. *Function* $\phi : \mathbb{R} \to [0,1]$ *is continuous and increasing and such that* $\phi(\delta) = 0$ $\forall \delta \leq 0$ *and* $\lim_{\delta \to +\infty} \phi(\delta) = 1$.

In Figure 1 panel (a) the graph of a possible function ϕ that is continuous and piecewise smooth is depicted.

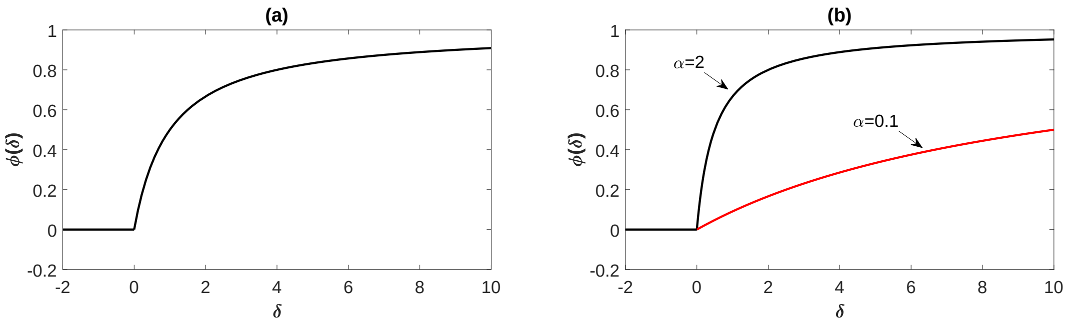

Figure 1: (a) Function ϕ verifying the assumptions in Remark 2.3. (b) Function ϕ as defined in (10) for two different α values.

Let $x_t \in [0,1]$ be the fraction of dishonest firms at time t, then the fraction of dishonest firms at time $t+1$ changes in accordance with the following equation:

$$x_{t+1} = F(x_t) = x_t + x_t(1-x_t)p_{h\to d} - x_t(1-x_t)p_{d\to h} = x_t[1 + (1-x_t)(2\phi(\delta(x_t)) - 1)] \quad (9)$$

where $p_{d\to h} = 1 - p_{h\to d}$.

The first term represents the fraction of dishonest firms at time t, then to update this fraction we have to add (subtract) the fraction of honest (dishonest) firms changing type during the time interval. An honest firm which meets a dishonest firm will change type with probability $p_{h\to d}$ if it learns that being dishonest is more convenient than being honest (second term); finally, the third term represents dishonest firms that, by meeting an honest firm, decide to become honest with probability $p_{d\to h}$ if the utility of being honest is greater than that of being dishonest.

3 Properties of the map

In the present section we study local and global dynamics exhibited by map F in (9) when simple features are produced.

Define $H(\delta) = 2\phi(\delta) - 1$. Then: (i) $H(0) = -1$, (ii) $H(\delta)$ is increasing and (iii) $H(\delta) \in [H(-\infty), H(+\infty)] = [-1, 1]$ and simple computations show that $\forall x_t \in [0,1]$ then $x_{t+1} \in [0,1]$ that is set $[0,1]$ is trapping for map F. In fact, from $H(\delta) \in [-1,1]$, it immediately follows $x_{t+1} \in [0,1]$ $\forall x_t \in [0,1]$.

Regarding the equilibria of map F the following Proposition trivially holds.

Proposition 3.1. *Points* $x_1^* = 0$ *and* $x_2^* = 1$ *are equilibria of* F *for all parameters.*

[10] Observe that in the present formulation we consider a continuous function.

The two equilibria x_1^* and x_2^* represent the boundary equilibria and correspond to the cases in which all firms are honest or dishonest (monomorphic configurations). In fact in these cases firms cannot meet other firms of different types therefore they will not change their behavior. A first interesting question is related to the local stability of these steady states belonging to the border. Since $[0, 1]$ is trapping for F and being $F(0) = 0$ and $F(1) = 1$ then no interior fixed points can exist iff $F(x_t) > x_t \forall x_t \in (0, 1)$ or $F(x_t) < x_t \forall x_t \in (0, 1)$.

Consider the second case, i.e. F is below the main diagonal $\forall x_t \in (0, 1)$. Taking into account equation (9) then it must be $\phi(\delta) < \frac{1}{2}$ for all $x_t \in (0, 1)$. A sufficient condition for this last inequality to hold is $\phi(\delta) = 0$ for all $x_t \in (0, 1)$ that is $\delta \leq 0$ for all $x_t \in (0, 1)$. In such a case x_1^* is locally stable, while x_2^* is locally unstable. From the previous considerations and taking into account Remark 2.2 then the following Proposition trivially holds.

Proposition 3.2. *Let F be given by (9). Then in cases* a.3*,* b.3*,* c.2 *and* c.3 *of Remark 2.2, the map F admits only two fixed points: x_1^* that is asymptotically stable with basin* $[0, 1)$ *and x_2^* that is unstable.*

According to the previous Proposition, if the monitoring level is high enough, i.e. greater then the maximum between q_1 and q_2, and if at the initial time there exists at least one honest firm in the economy, then in the long term the final equilibrium without corruption will be reached. In fact, in this case, a level of monitoring which makes the firm's honest payoff greater than that of the dishonest firm, induces, through the dynamic word-of-mouth mechanism, all the firms to become honest. Monitoring threshold levels are clearly influenced by social costs and benefits. In a country where these social costs and benefits are high, *ceteris paribus*, the monitoring level necessary to bring the economy to honesty will be lower.

As far as the first case is concerned, i.e. F is above the main diagonal $\forall x_t \in (0, 1)$, then it is required that $\phi(\delta) > \frac{1}{2}$ for all $x_t \in (0, 1)$ and a further analysis needs to specify function ϕ.

A second question we want to investigate is the possibility of our model to admit other fixed points belonging to the interior of $[0, 1]$, i.e. equilibrium characterized by the presence of both honest and dishonest firms (coexistence equilibrium). The interior (or inner) equilibrium is such that $\phi(\delta(x_t)) = \frac{1}{2}$ hence it requires a positive value of δ,[11] so that this equilibrium may exist only if q is not too high. In fact, according to the *honesty-propensity* property, if $\delta = 0$ then all firms will be honest and no equilibria composed by both groups can exist. To give an answer to the second question we have to better specify function $\phi(\delta)$.

3.1 Cultural norms and legal enforcement

In order to conduct a more in-depth analysis we must specify the probability function ϕ, i.e. the probability (easiness) with which an honest firm can become, on the basis of an economic convenience, dishonest. This probability is not the same among all countries but depends on the "intrinsic honesty" of each country. To take this into account, we introduce a parameter (α) that captures the greater "easiness" with which an honest firm becomes

[11] In Lamantia and Pezzino (2017), $\delta = 0$ is necessary for the interior fixed point to exist, as they assume $\phi(0) = 1/2$ implying that with no difference between expected utilities one-half of agents chooses to change group. Differently from that assumption, in the present model we consider that with no difference between expected utilities all agents choose to be honest so $\delta = 0$ does not characterize a steady state.

dishonest with the same economic advantage (δ). This parameter allows us to consider countries with a different culture of honesty. As indeed it clearly emerges from the paper by Fisman and Miguel (2007) that it is important to consider the role of both social norms and legal enforcement in non-compliant behavior. The authors' experiment considers parking violations by diplomats from different societies in NY city in the period in which there was essentially no enforcement of diplomats' parking violations. Therefore, the parking violations represent only the "culture of illegality" of a country. For example, Kuwaiti diplomats (the first on the list for number of violations) committed 250 pre-enforcement violations and only 0.15 post-enforcement violations. This means that in this country it is very easy to undertake illegal behavior and therefore a policy of enforcement is necessary to reduce dishonest behavior. They show that the propensity to break rules for private gain when enforcement is not a consideration is very different between countries reflecting the own specific country corruption level. In fact, in low corruption countries (e.g. Norway) people behave well even in situations in which there is no legal enforcement; viceversa in high corruption countries where people have a high corruption propensity especially in absence of legal enforcement.

Now we move on to the study of qualitative and quantitative dynamics of map (9) by considering the following function ϕ that is continuous, piecewise smooth and verifying the properties stated in Remark 2.3:

$$\phi(\delta) = \begin{cases} \phi_1(\delta) = 1 - \frac{1}{\alpha\delta+1} \quad \text{if } \delta \geq 0. \\ \\ \phi_2(\delta) = 0 \quad \text{if } \delta < 0. \end{cases} \tag{10}$$

where parameter $\alpha > 0$ measures the *propensity to become dishonest* characterizing the country: two sketches of function ϕ for different α values are in Figure 1 (b) where α is a parameter introduced to capture the different levels of propensity to become dishonest when this behavior is more convenient in terms of payoff. In fact, as α grows, then for any fixed positive level of difference between payoffs, the probability for an honest firm to become dishonest increases too. The inclusion of this parameter allows us to consider different countries in which honesty is less or more socially rooted. Notice that, if δ does not change sign for all $x_t \in [0, 1]$ then only ϕ_1 or ϕ_2 are involved. These cases are summarized in Remark 2.2. Observing firstly that cases a.3, b.3, c.2 and c.3 of that Remark have been discussed in the previous Proposition 3.2, so that we now focus on cases a.1, b.1 and c.1 stating conditions such that δ is not negative for all $x_t \in [0, 1]$.

Assuming one of the cases a.1, b.1 or c.1 of Remark 2.2 holds, then by substituting ϕ_1 in map (9), the following final one dimensional and differentiable map describing the evolution of the fraction of dishonest firms over time is obtained:

$$x_{t+1} = F_1(x_t) = x_t \left[1 + (1 - x_t)\frac{\alpha\delta(x_t) - 1}{\alpha\delta(x_t) + 1}\right], \quad x_t \in [0, 1]. \tag{11}$$

Trivially, map F_1 admits interior fixed points iff equation

$$\delta(x_t) = \frac{1}{\alpha}$$

admits a solution $x_3^* \in (0, 1)$. Consider Remark 2.2 and recall that the linear function $\delta(x_t)$ can be both strictly decreasing (case a.1), strictly increasing (case b.1) or constant (case c.1) so that the image set $I = \delta([0, 1])$ is given respectively by $I_a = [d_2, d_1]$ (case a.1),

$I_b = [d_1, d_2]$ (case $b.1$) and $I_c = \{d_1\}$ (case $c.1$), where:

$$d_1 = \Delta c - q(\Delta c + a_d + a_h + b_d) \tag{12}$$

while

$$d_2 = \Delta c - q(\Delta c + a_d + a_h + b_h). \tag{13}$$

Hence a unique interior fixed point exists as long as $\frac{1}{\alpha} \in I_j$, $j = a, b, c$ and the following Proposition trivially holds.

Proposition 3.3. *Let F_1 be given by (11) and consider Remark 2.2. Then the following cases may occur:*

- (i) *if condition $a.1$ holds and $\alpha \in (1/d_1, 1/d_2)$ then F_1 admits a unique interior fixed point, otherwise no interior fixed point is owned;*
- (ii) *if condition $b.1$ holds and $\alpha \in (1/d_2, 1/d_1)$ then F_1 admits a unique interior fixed point, otherwise no interior fixed point is owned;*
- (iii) *if condition $c.1$ holds and $\alpha = 1/d_1$ then F_1 admits infinitely many interior fixed point given by $(0,1)$, otherwise no interior fixed point is owned.*

The interior fixed point, when it exists and it is unique, is given by $x_3^* = \frac{d_1 - \frac{1}{\alpha}}{q(b_h - b_d)}$.

Regarding the local stability of the boundary equilibria under the assumptions $a.1$, $b.1$ and $c.1$ of Remark 2.2, we consider the derivative of F_1 in a given point $x_t \in [0,1]$ that is given by:

$$F_1'(x_t) = 1 + (1 - x_t)\frac{\alpha\delta(x_t) - 1}{\alpha\delta(x_t) + 1} + x_t\left[-\frac{\alpha\delta(x_t) - 1}{\alpha\delta(x_t) + 1} - \frac{2\alpha q(b_h - b_d)(1 - x_t)}{(\alpha\delta(x_t) + 1)^2}\right]. \tag{14}$$

Observe that in cases $a.1$, $b.1$ and $c.1$ of Remark 2.2 $\delta(x_t) \geq 0$, $\forall x_t \in [0,1]$.

Furthermore

$$F_1'(0) = 1 + \frac{\alpha\delta(0) - 1}{\alpha\delta(0) + 1}$$

while

$$F_1'(1) = 1 - \frac{\alpha\delta(1) - 1}{\alpha\delta(1) + 1}$$

where $\frac{\alpha\delta(x_t) - 1}{\alpha\delta(x_t) + 1} \in [-1, 1]$, $\forall x_t \in [0,1]$. Hence the following cases may occur:

$$\frac{\alpha\delta(0) - 1}{\alpha\delta(0) + 1} > (\leq) 0 \text{ iff } \alpha > (\leq) 1/\mathrm{d}_1$$

and, similarly,

$$\frac{\alpha\delta(1) - 1}{\alpha\delta(1) + 1} > (\leq) 0 \text{ iff } \alpha > (\leq) 1/\mathrm{d}_2$$

so that conclusions for the local stability of the boundary equilibria can be obtained. Notice also that when no interior fixed point is owned (see Proposition 3.3), then F_1 must lie above or below the main diagonal $\forall x_t \in (0,1)$ so that conclusions also on the *global* stability of the monomorphic configurations can be obtained, as summarized later.

Considering now parameter combinations such that an interior fixed point is owned (Proposition 3.3). As far as its stability is concerned, the following Proposition holds.

Proposition 3.4. *Consider cases* $a.1$, $b.1$ *and* $c.1$ *of Remark 2.2 and assume that* F_1 *admits a unique interior fixed point. Then function* F_1 *is strictly increasing in* $[0, 1]$.

Proof. F_1' has the same sign of a third degree polynomial $Ax^3 + Bx^2 + Cx + D$, whose coefficients are given by: $A = -\alpha^2 q^2 (b_h - b_d)^2$, $B = \alpha^2 q^2 (b_h - b_d)^2 + 2q(b_h - b_d)d_1 + \alpha q(b_h - b_d)$, $C = -2(\alpha d_1 + 1)\left[\alpha q(b_h - b_d) - \alpha d_1 + 1\right]$, $D = \alpha d_1(\alpha d_1 + 1)$.
Notice that $A < 0$ and, consequently, $lim_{x\to-\infty} F_1' > 0$ and $lim_{x\to+\infty} F_1' < 0$. Moreover, in cases $a.1$, $b.1$ and $c.1$ of Remark 2.2 the parameter d_1 is always non-negative, hence $D \geq 0$. In order to apply Descartes' Rule, we study the signs of B and C. Under the assumptions of Remark 2.2 ($b.1$ or $c.1$) we obtain three possible scenarios: $B, C > 0$ or $B < 0, C > 0$ or $B, C < 0$. There is one sign change between consecutive coefficients of the third degree polynomial, as a consequence there is one positive root at most. Being $F_1'(0) > 0$, $F_1'(1) > 0$ and $lim_{x\to+\infty} F_1' < 0$, this root is greater than 1, in other words $F_1'(x) > 0\ \forall x \in [0, 1]$.

By following the same steps, in the case $a.1$ of Remark 2.2 ($b_h > b_d$), we observe that the coefficient B is always positive, hence by assuming $b_h < \frac{\alpha d_1 - 1}{\alpha q} + b_d$ we guarantee $C > 0$, too. By using formulae (12) and (13), some algebra shows that this last condition is always verified. Also in this case there is one sign change between coefficients A, B, C, D and the unique (possible) positive root must belong to $(1, +\infty)$. □

The following Proposition concerning the global dynamics of F_1 can be stated.

Proposition 3.5. *Consider cases* $a.1$, $b.1$ *and* $c.1$ *of Remark 2.2 as listed below. Regarding the fixed points and global dynamics of* F_1*, the following statement holds.*

a.1. If $\alpha \in (0, 1/d_1)$ *then* 0 *is stable with basin* $[0, 1)$ *while* 1 *is unstable; at* $\alpha = 1/d_1$ x_1^* *undergoes a fold bifurcation after which the interior fixed point* x_3^* *is created; if* $\alpha \in (1/d_1, 1/d_2)$ *the interior fixed point* x_3^* *is stable with basin* $(0, 1)$ *while the boundary fixed points are unstable; at* $\alpha = 1/d_2$ *point* x_3^* *merge with* x_2^* *that is stable with basin* $(0, 1]$ *while* 0 *is unstable; if* $\alpha > 1/d_2$ *then* 1 *is stable with basin* $(0, 1]$ *while* 0 *is unstable.*

b.1. If $\alpha \in (0, 1/d_2)$ *then* 0 *is stable with basin* $[0, 1)$ *while* 1 *is unstable; at* $\alpha = 1/d_2$ x_2^* *undergoes a fold bifurcation after which the interior fixed point* x_3^* *is created; if* $\alpha \in (1/d_2, 1/d_1)$ *the interior fixed point* x_3^* *is unstable and it separates the basin of attraction of* x_1^**, given by* $[0, x_3^*)$*, from the basin of attraction of* x_2^**, given by* $(x_3^*, 1]$*; at* $\alpha = 1/d_1$ *point* x_3^* *merge with* x_1^* *that is unstable, while* 1 *is stable with basin* $(0, 1]$*; if* $\alpha > 1/d_1$ *then* 1 *is stable with basin* $(0, 1]$ *while* 0 *is unstable.*

c.1. If $\alpha \in (0, 1/d_1)$ *then* 0 *is stable with basin* $[0, 1)$ *while* 1 *is unstable; at* $\alpha = 1/d_1$ *infinitely many fixed points are owned, all the points belonging to the interval* $[0, 1]$*, and are stable; if* $\alpha > 1/d_1$ *then* 1 *is stable with basin* $(0, 1]$ *while* 0 *is unstable.*

To summarize, Proposition 3.5 deals with the case in which the monitoring level by the State q is low. As already mentioned, the monitoring threshold levels depend on the social parameters: the greater the social stigma for dishonest behavior and the reward for honest behavior, the lower the thresholds for monitoring will be, *ceteris paribus*.

In such cases, the origin is *globally* stable (except for $x_0 = 1$) as long as the propensity to become dishonest α is low enough. In fact, as we have already stressed reporting the experiment of Fisman and Miguel (2007), in countries in which the culture of honesty is high (i.e. α is low in our model), there is no need for a high level of monitoring or/and enforcement. On the other hand, if α is high enough (i.e. corruption is commonly accepted in society), and the State does not enforce a strong monitoring level, then society will be characterized by all dishonest firms.

At intermediate values of α, different situations may occur depending on the variable social effects associated to honest and dishonest behavior. In fact if $b_h > b_d$ then the coexistence equilibrium is *globally stable* so that in the long term both groups will coexist. On the other hand if $b_h < b_d$, then both boundary equilibria coexist stable (bistability is exhibited) and the final outcome will depend on the initial condition, i.e. the initial fraction of dishonest firms in the country. The case $b_h = b_d$ is a particular case.

Taking into account the previous study we observe that the open cases are given by points $a.2$ and $b.2$ of Remark 2.2. In such cases, it can be observed that $\delta(x_t)$ assumes both positive and negative values and consequently both branches ϕ_1 and ϕ_2 of map (10) are involved. Then, by simple substitutions, the evolution of the fraction of dishonest firms can be obtained and the final map is given by the following.

Consider F_1 as given by (11), let $F_2(x_t) = x_t^2$ and recall that

$$\bar{x} = \frac{\Delta c - q(\Delta c + a_d + a_h + b_d)}{q(b_h - b_d)}.$$

Then we have to distinguish between the following cases.

If $b_h > b_d$ and $q_1 < q < q_2$ (case $a.2$ of Remark 2.2) then $x_{t+1} = f_a(x_t)$ is defined by the following one-dimensional continuous and piecewise smooth map:

$$x_{t+1} = f_a(x_t) = \begin{cases} F_1(x_t) = x_t \left[1 + (1 - x_t)\left(\frac{\alpha(\Delta c - q(\Delta c + a_d + a_h + b_d + x_t(b_h - b_d))) - 1}{\alpha(\Delta c - q(\Delta c + a_d + a_h + b_d + x_t(b_h - b_d))) + 1}\right)\right] & \text{if } 0 \leq x_t \leq \bar{x}, \\ F_2(x_t) = x_t^2 \quad \text{if } \bar{x} < x_t \leq 1. \end{cases} \tag{15}$$

If $b_h < b_d$ and $q_2 < q < q_1$ (case $b.2$ of Remark 2.2) then $x_{t+1} = f_b(x_t)$ is defined by the following one-dimensional continuous and piecewise smooth map.

$$x_{t+1} = f_b(x_t) = \begin{cases} F_2(x_t) = x_t^2 \quad \text{if } 0 \leq x_t \leq \bar{x}. \\ F_1(x_t) = x_t \left[1 + (1 - x_t)\left(\frac{\alpha(\Delta c - q(\Delta c + a_d + a_h + b_d + x_t(b_h - b_d))) - 1}{\alpha(\Delta c - q(\Delta c + a_d + a_h + b_d + x_t(b_h - b_d))) + 1}\right)\right] & \text{if } \bar{x} < x_t \leq 1. \end{cases} \tag{16}$$

Notice that the kink point $(\bar{x}, F_1(\bar{x}))$ is always below the main diagonal so that it cannot be a fixed point. Regarding the internal fixed points, we can observe that $F_2(x_t)$ does not admit fixed points different from one of the two boundary equilibria so that a steady state with coexistence of honest and dishonest firms must be a solution of $F_1(x_t) = x_t$ belonging to the opportune subset of $(0, 1)$.

Following the previous considerations, a possible interior fixed point $x^* \in (0, 1)$ must solve $\delta(x^*) = 1/\alpha$ and this solution is given by:

$$x^* = \frac{d_1 - 1/\alpha}{q(b_h - b_d)} = \bar{x} - \frac{1}{\alpha q(b_h - b_d)}. \tag{17}$$

Let's observe that:

- x^* is a feasible fixed point of f_a iff $b_h > b_d$, $q_1 < q < q_2$ and $x^* \in (0, \bar{x})$,
- x^* is a feasible fixed point of f_b iff $b_h < b_d$, $q_2 < q < q_1$ and $x^* \in (\bar{x}, 1)$.

Considering firstly the case $b.2$ of Remark 2.2, i.e. $b_h < b_d$ and $q_2 < q < q_1$ so that $x^* > \bar{x}$, then we have to verify that $x^* < 1$. Notice that $x^* < 1$ iff $\frac{d_1 - 1/\alpha}{q(b_h - b_d)} < 1 \Leftrightarrow \alpha > \frac{1}{d_2}$ where $d_2 > 0$ being $q < q_1$ so that conditions for the existence of the interior fixed point $x_3^* = x^*$ are obtained.

As a consequence, if $\alpha \leq 1/d_2$ then no interior fixed points exist so that the same arguments concerning local and global stability of the boundary equilibria previously stated can be used to conclude that all initial conditions $x_0 \in [0, 1)$ will produce trajectories converging to the equilibrium without corruption.

On the other hand, in the case with $\alpha > 1/d_2$ an interior fixed point exists. In this case the following Proposition concerning the shape of f_b can be proved.

Proposition 3.6. *Let $b_h < b_d$ and $q_2 < q < q_1$ (i.e. case $b.2$ of Remark 2.2) and assume $\alpha > \frac{1}{d_2}$. Then, function $f_b(x)$ is strictly increasing in $[0, 1]$.*

Proof. Firstly, given the continuity of the map f_b and $F_2'(x) > 0 \, \forall x \in (0, \bar{x}]$, we need to prove that $F_1'(x) > 0 \, \forall x \in (\bar{x}, 1]$. To this end, remembering some properties of F_1 from Proposition 3.4: its derivative has the same sign of a third degree polynomial $Ax^3 + Bx^2 + Cx + D$, with $A = -\alpha^2 q^2 (b_h - b_d)^2 < 0$, $B = \alpha^2 q^2 (b_h - b_d)^2 + 2q(b_h - b_d)d_1 + \alpha q(b_h - b_d)$, $C = -2(\alpha d_1 + 1)\left[\alpha q(b_h - b_d) - \alpha d_1 + 1\right]$, $D = \alpha d_1(\alpha d_1 + 1)$.

Notice that for $b_h < b_d$ and $q_2 < q < q_1$, the parameter d_1 is always negative. Moreover, $\alpha d_1 + 1 < 0$ implies $D > 0$. Notice that the coefficient C is positive if and only if $\alpha q(b_h - b_d) - \alpha d_1 + 1 < 0$ which is always verified by considering (12) and (13). Hence, for the parameter values considered, there is one positive root at most. Being $lim_{x \to +\infty} F_1' < 0$ and $F_1'(1) > 0$, this root is bigger than 1 and F_1 is strictly increasing in $(\bar{x}, 1]$.

□

According to the previous results, the kink point plays no particular role so that following similar steps to that used to study the previous cases, the following result holds.

Proposition 3.7. *Considering case $b.2$ of Remark 2.2. If $\alpha < 1/d_2$ then 0 is stable with basin $[0, 1)$ while 1 is unstable; at $\alpha = 1/d_2$ a fold bifurcation occurs; if $\alpha > 1/d_2$ then the interior fixed point $x_3^* = x^*$ exists and it separates the basin of attraction of x_1^* from the basin of attraction of x_2^*.*

The previous result is shown in Figure 2. If, from a social point of view, the fact of being honest is rewarded more than the fact of being dishonest is stigmatized, i.e. $b_h > b_d$, then with the monitoring level at an intermediate value, the monomorphic equilibrium with all honest firms can be reached as long as α is not too high. Otherwise, if corruption is more *accepted* in society, i.e. greater α, then the economy will converge to one of the boundary equilibria (all honest or all dishonest) as bistability emerges. The initial condition is then crucial in determining the final outcome of the economy as is shown in the third panel of Figure 2.

Finally we consider the open case $a.2$ of Remark 2.2, i.e. $b_h > b_d$ and $q_1 < q < q_2$. The point x^* given by (17) is an interior fixed point for map f_a iff $x^* \in (0, \bar{x})$. Notice that $x^* < \bar{x}$ and that it is not negative iff $\alpha > 1/d_1 > 0$. Therefore, regarding the number of steady stats, the following result trivially holds.

Proposition 3.8. *Consider case $a.2$ of Remark 2.2. Then if $\alpha < 1/d_1$, 0 is a stable fixed point with basin $[0, 1)$ while 1 is the unstable fixed point; at $\alpha = 1/d_1$ a fold bifurcation occurs; for $\alpha > 1/d_1$ three fixed points exist: the boundary fixed points x_1^* and x_2^* and the interior fixed point $x_3^* = x^*$ given by (17).*

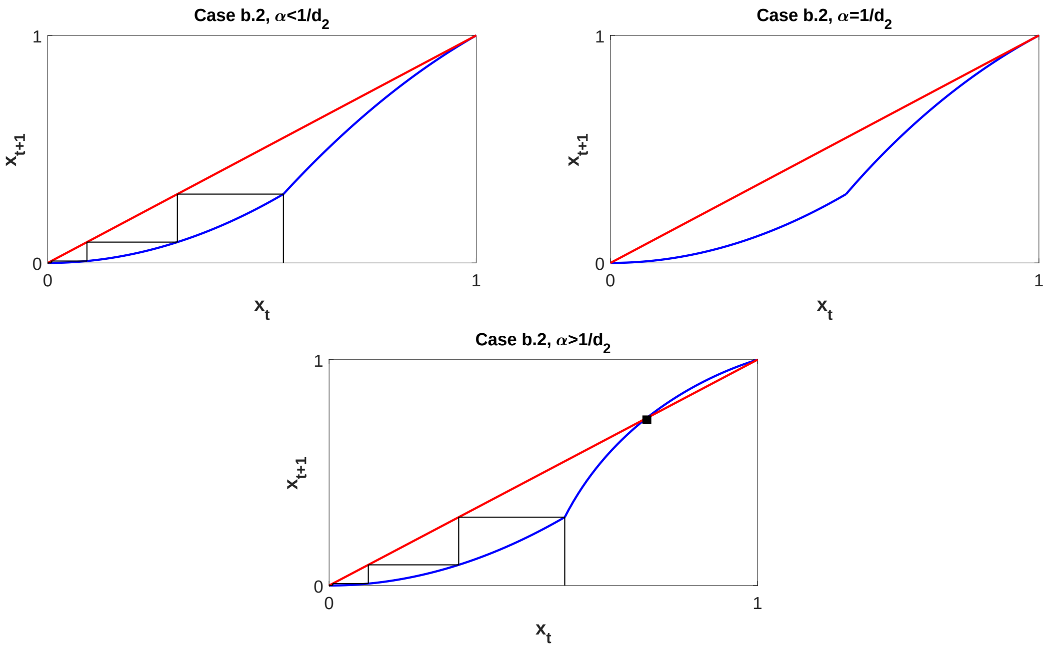

Figure 2: Qualitative scenario in the case $b.2$ of Remark 2.2.

According to the previous Proposition, as long as $\alpha \in (0, 1/d_1)$ then only two equilibria exist given by the monomorphic configuration and the situation without dishonest firms will prevail in the long term (for all initial states $x_0 \neq 1$). When $\alpha = 1/d_1$ then a fold bifurcation occurs creating a new equilibrium characterized by the coexistence of both groups. When it emerges the inner equilibrium will be reached by economies starting from any initial non-monomorphic configurations and the system monotonically converges to the interior equilibrium while no more complex features are exhibited. The interior fixed point x_3^* is created via a fold bifurcation of the origin: when it borns it is stable with basin $(0, 1)$.

According to the previous result, also in this case a high level of honesty, i.e. low α, is a sufficient condition for the economy to converge towards a situation in which all firms are honest. The data from the Fisman and Miguel experiment (2007) confirm that even in the absence of a system of monitoring and/or punishment, those who have a high culture of honesty behave honestly. Notice that while the global dynamics are well known in case $a.2$ as long as $\alpha \leq 1/d_1$, a more in-depth study is required for $\alpha > 1/d_1$.

In fact, by looking at Figure 3, it can be immediately observed that function f_a can be not monotonic, so the following questions must be investigated: (i) if complex dynamics emerge, (ii) if multistability is exhibited and (iii) if non-smooth bifurcations can occur. In order to carry out the study, both analytical tools and numerical instruments must be used. The following Section is devoted to this study.

4 Bifurcations and complex dynamics

In the previous section we studied the global dynamics of the model in the cases in which the map is upward sloping in $[0, 1]$. In such cases, the presence of the kink point does not affect the long term dynamics, qualitative dynamics are simple and when an interior fixed point is admitted then both the boundary equilibria are stable (the unstable interior fixed

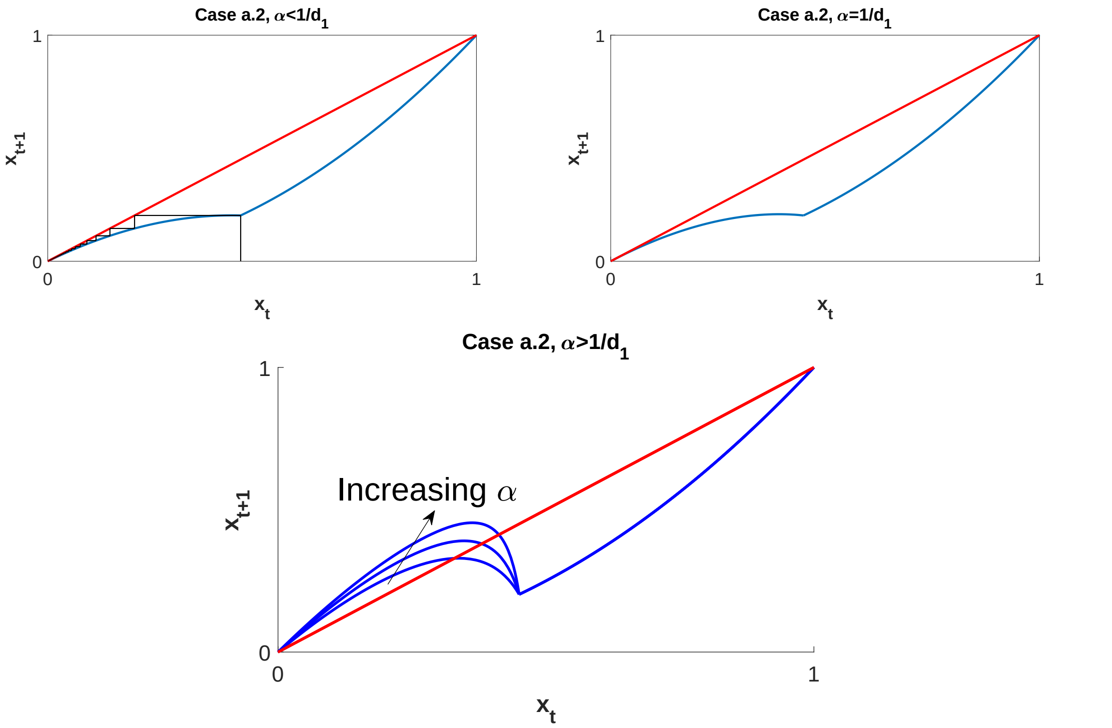

Figure 3: Possible shapes of map f_a for different α values.

point separates their basin of attraction and multistability emerges so that the economy will reach a monomorphic long term configuration depending on the initial state) or the interior fixed point is stable (so that all non-monomorphic initial configurations will converge in the long term to an equilibrium with coexistence of both groups). No more complex features are observed.

In this section we focus on the open case described in point $a.2$ of Remark 2.2 with $\alpha > 1/d_1$, i.e. f_a admits three fixed points. The assumptions follow:

Assumption 4.1. *Let* $b_h > b_d$*,* $q_1 < q < q_2$ *and* $\alpha > \frac{1}{d_1}$ *where* $q_1 = \frac{\Delta c}{\Delta c + a_d + a_h + b_h} \in (0, 1)$*,* $q_2 = \frac{\Delta c}{\Delta c + a_d + a_h + b_d} \in (0, 1)$ *and* $d_1 = \Delta c - q(\Delta c + a_d + a_h + b_d)$*.*

Recall that as long as $\alpha \in (0, 1/d_1)$ then only two equilibria exist given by the monomorphic configurations, and the situation without dishonest firms will prevail in the long term (for all initial states $x_0 \neq 1$, see the first panel of Figure 3). When $\alpha = 1/d_1$ then a fold bifurcation occurs at the origin creating a new equilibrium x_3^* characterized by the coexistence of both groups (Figure 3 second panel). When it is created the interior fixed point will be reached by economies starting from any non-monomorphic initial configuration: in this case the system monotonically converges to the equilibrium with coexistence while no more complex features are exhibited.

In order to understand what occurs $\forall \alpha > 1/d_1$, we consider that, being F_1' as given by (14) and $x_3^* = x^*$ as given by (17) then

$$F_1'(x_3^*) = -d_1[q(b_h - b_d) + d_1]\alpha^2 + 2[q(b_h - b_d) + d_1]\alpha - 1.$$

It is easy to verify that if $\alpha = 1/d_1$ then $F_1'(x_3^*) = 1$, i.e. the fold bifurcation previously described occurs. Furthermore $\frac{\partial F_1'(x_3^*)}{\partial \alpha} < 0$, $\forall \alpha > 1/d_1$ and $\lim_{\alpha \to \infty} F_1'(x_3^*) = -\infty$ so that there exists $\alpha_0 > 1/d_1$ such that $F_1'(x_3^*) = 0$ and the fixed point is super stable. In addition there exists $\alpha_1 = 2/d_1 > \alpha_0$ so that $F_1'(x_3^*) = -1$ and a flip bifurcation occurs.

The scenario previously described shows that for some parameter configurations, i.e. $\alpha \in (1/d_1, 2/d_1)$, the social stigma attached to dishonest behavior is not too low (i.e. α is not too high), if the monitoring effort by the State is at an intermediate level then long term simple dynamics converging to x^* will be produced while a flip bifurcation occurs at $\alpha = \alpha_1$. Immediately after this bifurcation, a stable 2-period cycle is created via flip bifurcation. The economy will fluctuate in the long term between two configurations characterized by different fractions of dishonest firms (for all $x_0 \in (0, 1)$) and there exists a fraction of firms changing group at each period. Hence fluctuations emerge for intermediate values attached to social stigma if the monitoring effort by the State is at an intermediate level.
Notice that for all $\alpha > \alpha_1$, F_1 is decreasing in x_3^* hence F_1 admits one maximum point $x_M \in (0, x_3^*)$. In addition, numerical simulations show that $\forall \alpha > \alpha_1$, F_1 is a unimodal function and that $f_a(x_M)$ increases w.r.t. α (see Figure 3 third panel). On the other hand, f_a admits the kink point $\bar{x} > x_3^*$ below the main diagonal so that map f_a is a piecewise bimodal map having a maximum differentiable point $(x_M, F_1(x_M))$ above the main diagonal and a minimum kink point $(\bar{x}, F_1(\bar{x}))$ below the main diagonal. The following Proposition trivially holds.

Proposition 4.2. *Considering assumption 4.1 and let $\alpha > \alpha_1 = 2/d_1$. Then map f_a admits a compact, positive invariant, absorbing the interval $J = [f_a(\bar{x}), f_a(x_M)]$.*

In fact, recalling that simple algebra shows that $f_a(J) \subseteq J$ and that any orbit of the map enters J in a finite number of iterations and stays there forever.

As long as $f_a(x_M) < \bar{x}$ the map is smooth in J since the absorbing interval J belongs to the definition region of F_1 so that smooth flip bifurcations may occur and different attracting sets may appear inside the absorbing interval J. The map has the *logistic* bifurcation structure which depends on α.

Nevertheless, as long as situations with higher α are considered, $f_a(x_M)$ crosses $\bar{x}$, (see Figure 4 (b)), then more complex dynamics may emerge due to the fact that map f_a is bimodal with one kink point in J, so that some standard (smooth) or non-standard (border collision) bifurcations can be produced. This gives rise to more complex features of the economy in the long term, together with very sensitive variations of such configurations depending on the initial state and on the parameter values of the model.

In what follows we will describe these bifurcations with the main aim of giving insight in terms of economic intuitions and policy suggestions, while we will leave a more in-depth mathematical oriented analysis (i.e. the bifurcation structure of the parameters space of the economically meaningful region) to future development. After the first period-doubling bifurcation, the cascade of smooth bifurcations can be observed. In fact as long as $f_a(x_M) < \bar{x}$ only smooth bifurcations can be observed as function f_a is differentiable and smooth in J.

As previously discussed, when F_1 is unimodal, it admits a maximum point $x_M \in (0, \bar{x})$ and $F_1(x_M)$ increases as α increases. Then the map is downward sloping in $(x_M, \bar{x})$ while it is upward sloping in $(0, x_M)$ and $(\bar{x}, 1)$. If the fixed point x_3^* belongs to its decreasing branch, then as α increases, a period-doubling smooth bifurcation occurs, the fixed point becomes unstable and a stable two period cycle is created. The flip curve $PDBC = \{(\alpha, q) \in [0, +\infty] \times [q_1, q_2] : F_1'(x_3^*) = -1\}$ in the plane (α, q) can be obtained. It is shown in Figure 4 (a). The curve $\alpha = 1/d_1$ corresponds to the fold bifurcation curve (FBC).

Our aim is to analyze non-smooth bifurcations due to the presence of the kink point, i.e. border collision bifurcations (BCB) are exhibited. We would like to emphasize here that these bifurcations are not related to an eigenvalue of the cycle crossing the threshold values ± 1, but to the collision of one of the cycle points with the border separating the region of different definitions on the map. When a kink point is a periodic point of any period, then a border collision is occurring to the cycle. This is the situation occurring in the present case.

The one dimensional bifurcation diagram is presented: the red curve is $f_a(\bar{x})$ while the black one is $f_a(x_M)$ (see Figure 4 (d)). We observe that as α increases then $f_a(x_M)$ increases and crosses the value $\bar{x}$ thus f_a is bimodal and non differentiable in J. The situation is in Figure 4 (c). Notice that when a point of a stable cycle collides with the kink point, a BCB occurs. These bifurcations and their structure in the plane (α, q) are presented in Figure 4 (e) and its enlargement in (f). We observe the corresponding periodicity regions to be organized in the so-called period adding structure (see Panchuck et al. 2013 and 2015 for a detailed discussion of this bifurcation structure). For example, between the period-2 and period-3 regions we observe a period-5 region, between period-3 and period-4 there is a period-7 region, and so on.

The sensitivity of the model to parameter configurations is a key aspect. The bifurcations diagrams presented in Figures 4 (d), (e) and (f), both one-dimensional (showing the long run behavior of the state variable as one parameter is varied) and two dimensional (showing the bifurcation structure of the model as a function of two relevant parameters), provide an example of bifurcations occurring in piecewise smooth systems. The structure of the bifurcations on the plane will be investigated further in a future more mathematical oriented work. As far as the economic insight is concerned, we underline how fluctuations in the long term can be produced. In this case, we are considering an economy in which the monitoring level is intermediate and the "inner" honesty of firms is low. Starting from a low level of dishonesty, i.e. low x_t, the disincentive effect on dishonesty $(x(b_h - b_d))$ cannot compensate for the dishonesty incentive represented by a low level of control. In addition, in this case also a small payoff differential between being dishonest and being honest (δ) is enough for the firm to become dishonest (high level of α). This will cause some firms to become dishonest, thus leading x_t to the point where economic conveniences are reversed: for high levels of x_t, the disincentive to dishonesty $(x(b_h - b_d))$ more than compensates for the incentive to dishonesty, thus decreasing the number of dishonest firms, and so on. All of this can produce the economic fluctuations described in the model.

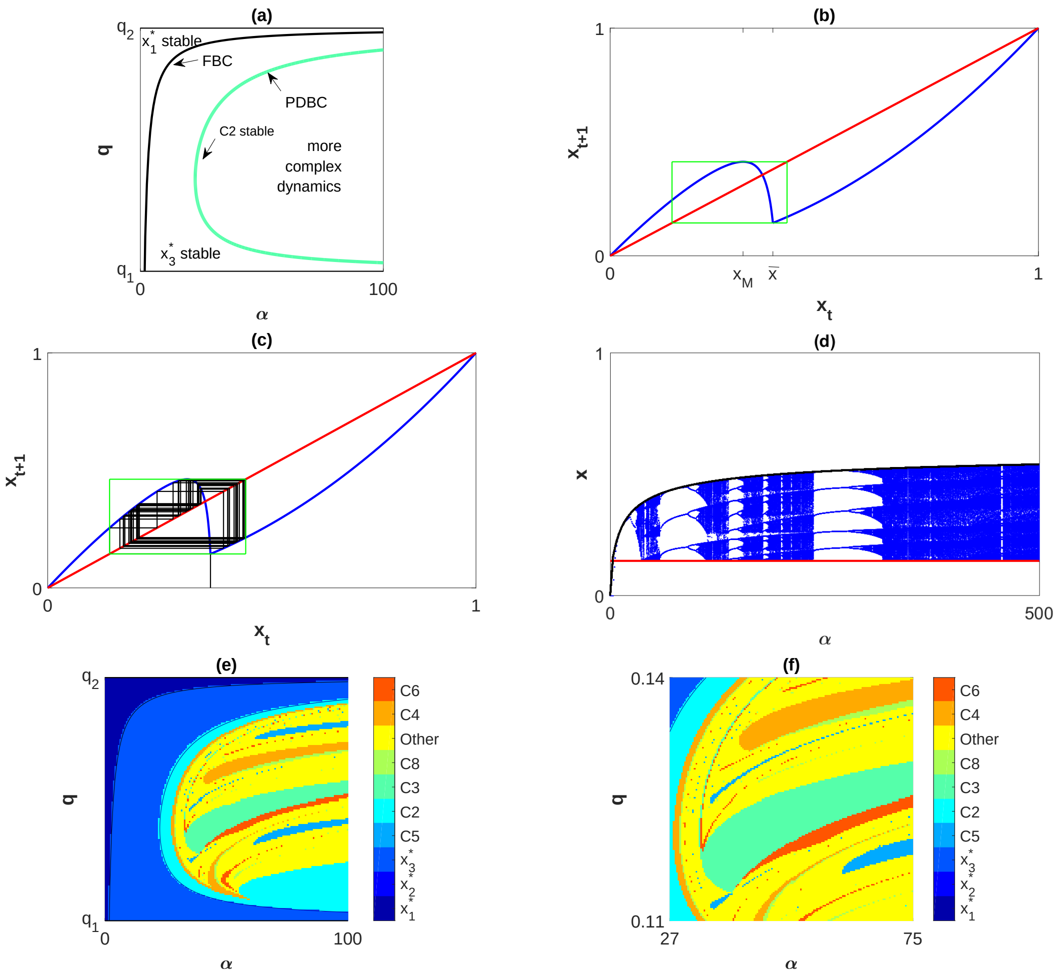

Figure 4: Common parameters: $p = 10$; $c_l = 7$, $c_h = 8.5$, $a_d = 2$, $b_h = 9$, $a_h = 3$ and $b_d = 3$. (a) Fold bifurcation curve (FBC) and flip bifurcation curve (PDBC). (b) The trapping set for $\alpha = 1/d_1 + 50$ and $q = (q_1 + q_2)/2$. (c) The trapping set for $\alpha = 1/d_1 + 100$ and $q = (q_1 + q_2)/2$. (d) One dimensional bifurcation diagram w.r.t. α. (e) two dimensional bifurcation diagram in the plane (α, q) and (f) its enlargement.

5 Results and further developments

In this section we want to summarize results through a table which gives a more intuitive representation of different cases.

	High q	**Intermediate** q	**Low** q
High α	**All Honest**	$b_h > b_d$: Fluctuations and complex dynamics $b_h < b_d$: Bistability. Monomorphic configuration depending on initial state	**All Dishonest**
Intermediate α	**All Honest**	$b_h > b_d$: Coexistence Equilibrium $b_h < b_d$: Bistability. Monomorphic configuration depending on initial state	$b_h > b_d$: Coexistence Equilibrium $b_h < b_d$: Bistability. Monomorphic configuration depending on initial state
Low α	**All Honest**	**All Honest**	**All Honest**

Table 1: Monitoring level versus *honesty-propensity*

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In the Table 5 we have collected all the cases analyzed except for the most trivial case in which $b_h = b_d$. We focused on two key variables of our model: the level of monitoring q put in place by the State in order to fight the dishonesty and the *honesty propensity* α. For both the parameters we considered a low, intermediate and high level. Our analysis shows that having a high level of monitoring is a sufficient condition for the system to converge towards a society in which all firms are honest. The same result can be achieved if the "inner honesty" of the country is high. In fact, they are both effective tools in reducing the incentives for dishonesty.

From a policy point of view, they are two very different tools. In order to implement a high level of monitoring, we need a lot of resources, so the State must decide to invest a substantial part of its budget to eliminate non-compliant behaviors. On the other hand, increasing the culture of legality and honesty in a country is a process that is not necessarily expensive but which takes time, as changing the social vision of dishonesty requires investments in long-term human and social capital. Clearly, in cases where the monitoring level of dishonesty is low and the easiness with which laws are broken is high (low "intrinsic honesty"), the economy converges towards an equilibrium in which all businesses are dishonest. Regarding the intermediate cases, different situations may occur depending on the variable social

effects associated to honest and dishonest behavior: when $b_h > b_d$ and the converse case, i.e. $b_h < b_d$. If $b_h < b_d$, i.e. the case in which, at a social level, honest behavior is rewarded less than dishonest behavior is stigmatized then both boundary equilibria have stable coexistence (bistability is exhibited). The final outcome will depend on the initial condition, i.e. the initial fraction of dishonest firms in the country. On the other hand, if $b_h > b_d$ then the coexistence equilibrium is *globally stable* so that in the long term both groups will coexist. In the case in which the monitoring level is intermediate and the "inner" honesty of firms is low, low level of dishonesty (x_t) combined with low control stimulates dishonest behavior. The dishonesty level increases to the point where economic conveniences are reversed: for high levels of x_t, the disincentive to dishonesty ($x(b_h - b_d)$) more than compensates for the incentive to dishonesty, thus making decrease the number of dishonest firms, and so on. All of this can produce the economic fluctuations described in the model.
Further development will focus on the following two questions. From a mathematical point of view, the situation in which the monomorphic equilibria are both repelling and attracting periodic orbits or bounded aperiodic dynamics are exhibited, needs further analysis. In particular, the bifurcation structure in the parameter plane must be described by using techniques recently developed to study non-smooth bimodal maps (see for instance the studies on linear piecewise smooth maps by Panchuk et al. 2013 and 2015). In a future work we want to give a qualitative description of the bifurcation structure of the parameter plane.

From an economic point of view, one of the possible extensions of the model is to endogenize the level of monitoring. This monitoring, assumed to be exogenous in our model, could be endogenous compared to, for example, public resources. This analysis requires the transition from a partial equilibrium analysis to a general equilibrium analysis in which we consider the existence of a State that finances the fight against dishonesty through fiscal revenues. Furthermore, in our model we can endogenize the monitoring level compared to the dishonesty level of the country considered.

6 Conclusion

In this paper, we have analyzed the behavior of non-compliant firms in an evolutionary, dynamic model. With this aim in mind, we have constructed a model which combines Becker's economic theory (1968) with psychological and sociological theories explaining what motivates a person to adhere or not to assigned rules (as for instance social stigma). In fact, our study considers that society may benefit from prosocial behaviour and avoid social disapproval (see Benabou and Tirole (2006, 2011)). The dynamic setting is formalized by an evolutionary adaptation process which describes whether honest or dishonest behavior prevails in society at any given time t. Agents (firms) will either behave in an honest or dishonest way depending on the payoffs and the type of firms they meet, through a word-of-mouth process in which two meeting firms compare information. Therefore, honest firms interacting with dishonest ones may change their conduct if the utility they gain from being dishonest is more substantial than the one they get from being honest (Lamantia and Pezzino (2017)) and viceversa. Compared to previous models, we introduce the *honesty-propensity assumption* according to which all honest firms meeting a dishonest one will choose to remain honest if higher expected payoffs cannot be reached. The evolutionary game herewith studied is a continuous and piecewise smooth one-dimensional map with

one kink point which highly influences the system dynamics: one inner equilibrium can be attracting while, when it is repelling, interesting dynamics may exist. The related bifurcations (both standard and border collision) depending on two parameters of economic interest (the monitoring level and the propensity to be dishonest related to social stigma) have been investigated. In particular the appearance of the equilibrium with both groups can occur only via fold bifurcations, while simple or more complex dynamics are exhibited depending on the values of the economic meaningful parameters. In particular, we have described some bifurcations leading to a complex long-run evolution of the economic model, as the standard sequence of period doubling bifurcations is presented until border collision bifurcations emerge, leading to a cycle of different periods or to a chaotic attractor.
The model has shown that a State that wants to combat dishonesty can either set up a high level of monitoring, or " build" a high level of "inner honesty" in society. In fact, as showed in Fisman and Miguel (2007) experiment, countries with a high culture of honesty do not need a monitoring and/or enforcement system.

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