The network-codetermination game

Luca Gori • Luciano Fanti

Abstract The aim of this work is to continue the strand of research dealing with the institution of codetermination by considering network externalities in consumption. Amongst the different purposes of the existing theoretical literature, so far focused only on standard (non-network) goods, Kraft (1998) showed that codetermination might emerge as the outcome of a game played by quantity-setting duopoly firms that have to choose between profit maximisation or being bargainers under codetermination in a market with a homogeneous good. This led to the existence of a prisoner's dilemma, i.e. players have an incentive to coordinate to play profit maximisation, but no one has a unilateral incentive to deviate from codetermination. The present research shows that network externalities may solve the dilemma and let codetermination become the Pareto efficient outcome of the game (deadlock).

Keywords Codetermination; Network externalities; Quantity competition

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L. Gori (corresponding author)
Department of Law, University of Pisa, Via Collegio Ricci, 10, I–56126 Pisa (PI), Italy
e-mail: luca.gori@unipi.it or dr.luca.gori@gmail.com
tel.: +39 050 22 12 847
L. Fanti
Department of Economics and Management, University of Pisa, Via Cosimo Ridolfi, 10, I–56124 Pisa (PI), Italy
e-mail: luciano.fanti@unipi.it
tel.: +39 050 22 16 369; fax: +39 050 22 10 603

1. Introduction

The aim of this work is to combine two different strands of research belonging to the Industrial Organisation literature – namely, codetermination and network externalities – by building on a tractable model describing a strategic competitive framework (duopoly) with quantity-setting firms.

On one hand, there exists an increasing attention to the institution of codetermination in several countries of Western Europe. This institution implies, broadly speaking, that employees' representatives sit on the supervisory board (or similar structures) in large companies. Codetermination is a relevant feature of the German industry but it is widespread also in other European countries: comprehensive legislation on board-level representation can be found in Austria, Denmark, Finland, France, Luxembourg, the Netherlands, Norway and Sweden (Schulten and Zagelmeyer, 1998).[1] In Germany, codetermination rules – which are relevant at least since the 1950s – have been recently extended and are high on the political agenda in other North European countries (see the discussion in Kraft, 2001). Moreover, we note that bargaining over employment is not restricted to the case of codetermination laws. Indeed, it may be also applied when unions are involved in an efficient bargaining institution whenever either employment is determined at the firm level (with centralised unions that negotiate over wages) or the length of wage contracts is larger than the length needed to employment adjustments. More in general, an active worker involvement may be considered as a crucial element of the European social model.[2] Despite this fact, there is little economic analysis on the effects of codetermination and this constitutes a gap on the side of policy recipes at the European level: "The practice of board participation and its impact are very

[1] To show the empirical relevance of codetermination, it suffices to note that, amongst the 16 countries covered by European Industrial Relations Observatory (EIRO), only the UK stands alone in having no statutory form of board-level representation or significant collectively agreed provisions. In countries such as Belgium and Italy there is no general legislation or widely applicable collective agreements providing for board-level representation; however, there exist specific provisions for board-level employee representatives in some public companies (e.g., the state railway in Belgium and a number of state holding companies in Italy).

[2] As reported by Schulten and Zagelmeyer (1998), also the European Commission (i.e. Davignon report, published on October 27th, 1970) acknowledged the relevance of the institution of co-determination together with the fundamental questions regarding the power of social partners within the company: "Globalisation of the economy and the special place of European industry raises. The type of labour needed by European companies – skilled, mobile, committed, responsible, and capable of using technical innovations… cannot be expected simply to obey the employers' instructions. Workers must be closely and permanently involved in decision-making at all levels of the company."

hard to gauge, given a general lack of research and evidence." (Schulten and Zagelmeyer, 1998). However, the number of theoretical and empirical works on the subject is growing.

On theoretical grounds, McCain (1980) represents a pioneering work that opened the route to causes for reflections about codetermination, working conditions and labour productivity. Some years later, Kraft (1998) showed – in a standard Cournot duopoly with homogenous products – that profit maximising firms have an incentive to become bargainers over employment. This in turn implies that codetermination results to be the dominant strategy in a game played by firms that must choose between profit maximisation and codetermination. However, this outcome is inefficient for firms, which would prefer, as expected by the common wisdom, to have the full power to decide over employment: the institution of codetermination emerges as the inefficient Nash equilibrium of a game played by strategic competitive firms, which remain therefore entrapped in a prisoner's dilemma. Subsequently Kraft (2001) extended his previous work by accounting for a general oligopolistic market and discussed the effects of employment bargaining from both theoretical and empirical perspectives, confirming the existence of a prisoner's dilemma for a sizable range of the union's bargaining power. Three other contributions studied the effects of R&D activities in codetermined firms. First, Granero (2006)[3] showed that codetermination could help a firm to increase market share, employment and innovativity. Second, Kraft et al. (2011)[4] built on a work mixing theoretical and empirical analyses and concluded that their results "do not support the view that co-determination slows down technological progress and reduces innovativity" (Kraft et al., 2011, p. 145) by taking the number of patents as a benchmark. Third, Fanti et al. (2018)[5] found that

[3] The author built on a quantity setting duopoly by assuming that the objective function of the firm/manager is a weighted sum of profits and the income paid to workers, where the relative proportions of board votes of shareholders and workers represent the weight of the problem.

[4] By taking Kraft (1998, 2001) as a starting point, Kraft et al. (2011) studied the effects of the German Codetermination Act of 1976 – that introduced the possibility of equal representation on the supervisory board of large companies for of employers and employees' representatives – on the innovative activity of German firms. The authors proposed a duopoly model by (exogenously) comparing profits and R&D innovative activity under codetermination and profit maximisation by assuming that R&D was not the subject of negotiations between firms and unions.

[5] The authors first revisited the codetermined duopoly of Kraft (1998) by extending with horizontal differentiation and then accounted for R&D activities as in Kraft et al. (2011) by letting the choice between profit maximisation and codetermination be endogenous to the model.

the main results of Kraft (1998) and Kraft et al. (2011) might not be robust to a more general setting including horizontal product differentiation (Singh and Vives, 1984).

On the side of the empirical evidence, many authors argue that the economic effects of codetermination were not adequately considered[6] and, in any case, the results of the existing literature on the subject are rather controversial. Amongst a few, we recall here the following contribution: Cable and FitzRoy (1980) found that codetermination positively affects labour productivity, FitzRoy and Kraft (1993) obtained no statistically significant evidence of productivity gains due to codetermination laws, Baums and Frick (1998) studied how decisions of German courts during the period 1974-1995 on co-determination issues affect the stock price developments of the firms concerned, finding no statistically significant stock market response to court verdicts. More recently, Gorton and Schmid (2004) and FitzRoy and Kraft (2005) concluded, respectively, for a negative effect of codetermination on the market value of firms and a positive labour productivity effect of near-parity codetermination, whereas Kraft (2018) considered a model to study empirically the effects of extending codetermination rights on both productivity and bargaining power. He pinpointed no productivity disadvantages of codetermined firms. More generally, increasing codetermination rights is neutral on the side of the efficiency. However, it positively affects the bargaining power of labour and modifies the distribution of rents. To sum up, notwithstanding the empirics on codetermination is still a small field and there exist results of opposite signs, it seems to prevail a negative effect of the codetermination laws on the performance of firms (productivity, market value, etc.), which seems to be coherent with the theoretical result of the Pareto-inefficiency of the codetermination at equilibrium.

On the other hand, the issue of the effects of network externalities in consumption[7] has been recently investigated in the Industrial Organization literature by an increasing number of works that

[6] As noted first by FitzRoy and Kraft (1993, p. 366) "there have been few attempts to quantify economic effects, and they all suffer from inadequate data and methodology" and then by Gorton and Schmid (2004, p. 867) "There is relatively little quantitative work on the effects of codetermination at the supervisory board level".

[7] In fact, for several products the utility derived by a consumer increases with the number of consumers, so that the total sales increase the welfare of each consumer (e.g., Katz and Shapiro, 1985).

modified several established results. For instance, in an oligopoly context, Hoernig (2012), Bhattacharjee and Pal (2014), and Chirco and Scrimitore (2013) have shown that the standard results of the managerial delegation literature may change dramatically when markets deal with network goods. Differently, Fanti and Buccella (2017, 2018) investigated whether and how the network effects may modify the common wisdom regarding the bargaining agenda (between unions and firms) and corporate social responsibility. Finally, Song and Wang (2017) studied problems of firms' collusion in a network market showing that when the strength of the network effects is large enough collusion becomes sustainable when products are close substitutes. This is in contrasts with the case of no network goods as collusion between firms will be destabilised when the degree of product substitutability is large in that case.

To show the importance of network goods in modern economies it suffices to recall, for example, products such as telephones, mobile devices and software whose markets are dramatically expanding on the market. When there exist network externalities, the utility of a single consumer from using network goods increases with the number of users. In other words, network goods are products whose demand has a positive externality. More in general, a positive network externality may exist for products that a consumer wishes to possess in part because others are using them (i.e., the so-called Bandwagon Effect), and an example may be clearly represented by products dealt with in the fashion industry. Moreover, a consumer/user's demand of a (network) good positively depends on the number of other consumers/users through other ways. This is the case of goods that are perceived as a signal of 1) the availability of after-sale services for long-lasting consumers or 2) product quality.

Empirical evidence of network effects for industries located in countries with the institution of codetermination (e.g., Germany) does also exist. For example, by focusing on the specific case of telecommunications,[8] Doganoglu and Grzybowski (2007) studied network effects in the German

[8] In the mobile telecommunication market, there exist several sources of network effects. By following Baraldi (2008), they are listed as follows. 1) If the number of subscribers is increasing, it becomes attracting for other consumers to buy a mobile phone and belong to the same network. 2) The network expansion drives the usage volume of people already

mobile telecommunication market. They estimated a system of demand functions for mobile subscribers in Germany (data on mobile subscriptions was collected from the Internet site run by the German regulator – RegTP) from January 1998 to June 2003 finding that network effects played a significant role in the diffusion of mobile services in Germany. In particular, they concluded that, as a proxy that measures the intensity of the network effect, if the previous period total installed base increased by 1%, current period sales would surge on average by 0.69% (which is considered a strong network effect).[9] Another example is represented by the work of Baraldi (2008) that accounted for 30 OECD countries from 1989 to 2006 to specify and estimate a model of consumer demand for mobile telephone calls aimed at identifying the extent of network externalities. This work showed that also for countries such as Austria and Germany the network effect is significantly large (though smaller than that found by Doganoglu and Grzybowski, 2007), thus confirming that the competition analysis under codetermination in industries producing network goods (such as the telecommunications industry) should consider the extent and intensity of network effects.

Given this empirical background, different labour market institutions may affect the consumers' expectations about the total sales of the (network) goods. Despite the possible theoretical and empirical relevance of positive consumption externalities on market outcomes, the issues related to network goods have been largely ignored in the literature on co-determined industries. This article aims at filling this gap by providing a theoretical analysis based on a strategic competitive framework with quantity-setting duopoly firms.

Amongst other things, result shows that codetermination emerges as the endogenous equilibrium market outcome of a Cournot game. In addition, the article describes a mechanism able to solve the prisoner's dilemma raised in Kraft (1998).

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using mobile telecommunication: then the usage volume of existing subscribers is expected to be increasing with the total number of mobile telephone subscribers. 3) By considering the recent approach of the social interaction theory (e.g. Schoder, 2000), another source of network externality is the need of people to buy, consume and behave like their follows. This is the case of a network effect driven by a conformist behaviour.

[9] As they note, "If there were no network effects, the penetration of mobiles at the end of the period analyzed could be at least 50% lower."

The rest of the article proceeds as follows. Section 2 builds on a game played by strategic competitive duopoly firms in a Cournot market with a homogenous good. The owner of each firm must choose to be a profit maximiser or a bargainer under codetermination in a network industry. Section 3 extends the model to the case of product differentiation. Section 4 outlines the main conclusions.

2. A quantity-setting duopoly with network externalities: codetermination versus profit maximisation

Let us assume the existence of an economy where there are two types of agents: firms and consumers. The economy is bi-sectorial, that is a competitive sector produces the numeraire good m and a duopolistic sector with firm 1 and firm 2 produces horizontally differentiated products of variety 1 and variety 2, respectively (Singh and Vives, 1984). Let $p_i \geq 0$ and $q_i \geq 0$ be firm i's price and quantity, respectively, with $i = \{1, 2\}$. Different from the traditional industrial organisation literature (that generally assumed that a demand for a good is independent of one another), we assume that there are network externalities in consumption. This implies that one person's demand also depends on the demand of other consumers. The simple mechanism of network effects considered in the present work follows the tradition initiated by Katz and Shapiro (1985), so that the surplus that a firm's client obtains increases with the number of other clients of this firm. The issue of network externalities has become relevant especially due to the tremendous growth of the internet-related activities in the market (e.g., online games, telephone and so on).

There exists a continuum of identical consumers with preferences represented by a separable utility function $V(q_1, q_2, y_1, y_2, m)$, which is linear in the numeraire good m. The representative consumer maximises $V(q_1, q_2, y_1, y_2, m) = U(q_1, q_2, y_1, y_2) + m$ subject to the budget constraint $p_1 q_1 + p_2 q_2 + m = R$, where $U(q_1, q_2, y_1, y_2)$ is a twice continuously differentiable function, q_1 and q_2

are the control variables of the problem and R is the consumer's exogenous nominal income. This income is assumed to be high enough to avoid the existence of corner solutions.

By following the spirit of several recent works dealing with network effects in a strategic competitive framework (Hoernig, 2012; Bhattacharjee and Pal, 2013; Chirco and Scrimitore, 2013; Pal, 2014, 2015; Song and Wang 2017), we assume that consumers' preferences are represented by the following utility function:

$$U(q_i,q_j,y_i,y_j)=q_i+q_j-\frac{1}{2}(q_i^2+q_j^2+2dq_iq_j)+n[q_i(y_i+dy_j)+q_j(y_j+dy_i)]-\frac{n}{2}(y_i^2+y_j^2+2dy_iy_j),(1)$$

where $i,j=\{1,2\}$ $(i\neq j)$, y_i denotes consumers' expectations about firm i's equilibrium total sales and represents a consumption externality. The strength of the network effect is denoted by $-1\leq n<1$. The model boils down to the standard non-network case if $n=0$. When $n>0$ (resp. $n<0$) there exists a positive (resp. negative) consumption externality. Though several network industries exhibit positive externalities (mobile communications, software, internet-related activities, online social networks, fashion, etc.), as a greater number of users increases the value to each consumer (there exists a positive feedback loop as long as the network becomes more valuable), one may think about real-world industries that exhibit both kinds of externalities and are quite likely to have codetermined firms. In this direction, the automobile industry (in Germany) is worth to me mentioned. Indeed, this industry might also show negative consumption externalities as the more cars are sold, the greater the traffic, the greater the difficulty in parking and so on. A negative network externality therefore implies that an increasing number of users reduces the value of the good for each consumer as is the case of traffic congestion or network congestion over limited bandwidth. Therefore, it may be interesting and instructive discussing from a theoretical perspective also the implications of negative network externalities in a strategic competitive duopoly with quantity-setting firms. The degree of product differentiation perceived by customers is denoted by $-1\leq d\leq 1$. When $d=1$ (resp. $d=-1$) products are perfect substitutes (resp. perfect complements), whereas $d>0$ (resp. $d<0$) reflects the case of imperfect substitutability (resp.

imperfect complementarity). The case $d=0$ implies that each firm behaves as if it were a monopolist for its own product. We note that the last addendum in (1) is a specific symmetric function of expectations such that for each given consumption vector (q_1,q_2) utility is highest if expectations are correct.

The utility function in (1) is a modified version of the one used by Singh and Vives (1984) allowing studying network effects also in the case of homogeneous products ($d=1$). This is because the formulation popularised by Hoernig (2012) and used by other scholars in related works is not well defined for the case of perfect substitutability (as is also pointed out by Song and Wang, 2017, Footnote 3, p. 24).

By solving the utility maximisation programme gives the following linear inverse demand of product of variety i:

$$p_i = 1 - q_i - dq_j + n(y_i + dy_j), \quad i, j = \{1,2\}, i \neq j. \tag{2}$$

From (2), it is easy to see that network externalities enter additively in the demand function. If the network externality is positive (resp. negative), an increase in the positive (resp. negative) feedback loop of the network effect causes an outward (resp. inward) shift in the demand curve that in turn implies an increase (resp. reduction) in the quantity bought by consumers for any given value of the price. This externality therefore acts as a device that increases (resp. reduces) the market size.

Given the results of the existing literature on codetermination in a strategic competitive Cournot duopoly (Kraft, 1998; Fanti et al., 2018), it would be instructive to begin the analysis of the effects of network externalities with the case of perfect substitutability ($d=1$), studying later the case of product differentiation (Section 3). This allowed us to bring to light the strict relationship between network externalities and codetermination in determining the market outcome of the game as well as to stress similarities and differences between Kraft (1998) and the present work.

By assuming homogeneous products, the demand for good i as was expressed in (2) boils down to:

$$p_i = 1 - q_i - q_j + n(y_i + y_j)\,. \tag{3}$$

The technology used by firm i to produce product of variety i employs a production function with constant returns to labour, that is $q_i = L_i$, where L_i is the labour force. Each firm faces a constant marginal (and average) cost $0 \le w < 1$ representing the wage per unit of labour set in a centralised or industry-wide bargaining, which is taken as given by each single firm. Firm i's profits are therefore usually expressed as $\Pi_i = (p_i - w)q_i$, $i = \{1, 2\}$.

By following Kraft (1998, 2001), Kraft et al. (2011) and Fanti et al. (2018), we assume that firms are subject to the rules of the institution of codetermination. This implies that firms' representatives bargain with employees' representatives over employment but not over wages on the supervisory board. The objective of firm i is to maximise its own profits Π_i with respect to q_i. By using the inverse demand Eq. (3), profits of firm i are the following:

$$\Pi_i = [1 - q_i - q_j + n(y_i + y_j) - w]q_i\,. \tag{4}$$

Differently, each firm-specific union aims at maximising its own utility $Z_i = (w_i - w^\circ)L_i$ with respect to employment L_i, where w° is the reservation (or competitive) wage. Without loss of generality, we set $w^\circ = 0$ henceforth. Knowing that technology implies that one unit of labour is transformed into one unit of output ($L_i = q_i$), the utility of trade union i can be expressed as follows:

$$Z_i = w q_i\,. \tag{5}$$

The Nash bargaining between firm i and its own union bargaining unit over employment is given by $\mathrm{N}_i = \Pi_i^{\beta} Z_i^{1-\beta}$ so that using the expressions in (4) and (5) it takes the form:

$$\mathrm{N}_i = \{1 - q_i - q_j + n(y_i + y_j) - w]q_i\}^{\beta}(w q_i)^{1-\beta}, \tag{6}$$

where the control variable is q_i and $0 < \beta \leq 1$ is the relative bargaining power of firm i. By following Kraft (1998) and the literature cited therein, the threat points have been set to zero.

The timing of the events of this two-stage game is the following. At the *codetermination stage* (stage 1), each owner must choose to be either a codetermined or profit maximisation firm. At the *bargaining market stage* (stage 2), firms either choose output in the case of profit maximisation or bargain together with unions in the case of codetermination in a network industry. This game resembles Fanti et al. (2018). As is usual from Katz and Shapiro (1985) and Hoernig (2012), we assume that consumers have rational expectations. Therefore, at the second stage of the game we impose that $q_1 = y_1$ and $q_2 = y_2$ hold in equilibrium. We proceed into the analysis according to the standard backward logic.

First, we consider that both firms are codetermined ($\beta < 1$) so that output of firm i at the second stage of the game is chosen by firms and employees' representatives by maximising Eq. (6) with respect to q_i. Therefore, the reaction function of the ith firm is given by:

$$\frac{\partial \mathrm{N}_i}{\partial q_i} = 0 \Leftrightarrow q_i(q_j, y_i, y_j) = \frac{1 - w - q_j + n(y_i + y_j)}{1 + \beta}, \tag{7}$$

From (7), an increase in the strength of the network externality shifts upward the reaction function of firm i and then causes an increase in the quantity produced by the firm (the reaction functions are negatively sloped, and goods are strategic substitutes). By using (7) together with the corresponding counterpart of firm j and knowing that $y_i = q_i$ and $y_j = q_j$, $i, j = \{1, 2\}$ ($i \neq j$), we get the equilibrium outcome of firm i, that is:

$$q_i^{B/B} = \frac{1 - w}{2(1 - n) + \beta}, \tag{8}$$

where the superscript B denotes "bargaining" under codetermination. Therefore, equilibrium profits are given by:

$$\Pi_i^{B/B} = \frac{\beta (1 - w)^2}{[2(1 - n) + \beta]^2}. \tag{9}$$

Straightforward algebra from (8) and (9) show that an increase in n causes a monotonic increase in the quantity produced by both firms and their own profits.

If both firms are profit maximisers ($\beta = 1$), equilibrium output and profit of firm i are the following:

$$q_i^{PM/PM} = \frac{1-w}{3-2n}, \tag{10}$$

and

$$\Pi_i^{PM/PM} = \frac{(1-w)^2}{(3-2n)^2}, \tag{11}$$

where the superscript PM denotes “profit maximisation”.

Let us now consider the asymmetric case in which firm 1 is codetermined and firm 2 is profit maximiser. At the bargaining market stage, firm 1 and its corresponding union bargain unit are involved in a bargaining aimed at maximising N_1 with respect to q_1, whereas firm 2 maximises Π_2 with respect to q_2. The reaction functions are given by:

$$\frac{\partial \mathrm{N}_1}{\partial q_1} = 0 \Leftrightarrow q_1(q_2, y_1, y_2) = \frac{1-w-q_2+n(y_1+y_2)}{1+\beta}, \tag{12}$$

and

$$\frac{\partial \Pi_2}{\partial q_2} = 0 \Leftrightarrow q_2(q_1, y_1, y_2) = \frac{1-w-q_1+n(y_1+y_2)}{2}. \tag{13}$$

By imposing the conditions $y_1 = q_1$ and $y_2 = q_2$, we easily get:

$$q_1^{B/PM} = \frac{1-w}{1-n+\beta(2-n)}, \tag{14}$$

and

$$q_2^{B/PM} = \frac{\beta(1-w)}{1-n+\beta(2-n)}. \tag{15}$$

Therefore, equilibrium profits of firm 1 and firm 2 are the following:

$$\Pi_1^{B/PM} = \frac{\beta(1-w)^2}{[1-n+\beta(2-n)]^2}, \tag{16}$$

and

$$\Pi_2^{B/PM} = \frac{\beta^2(1-w)^2}{[1-n+\beta(2-n)]^2}. \tag{17}$$

The equilibrium outcomes of this game are summarised in Table 1 (quantities) and Table 2 (profits) according to the strategies available to each player.[10]

Firm 1 \ Firm 2	PM	B
PM	$\frac{1}{3-2n}, \frac{1}{3-2n}$	$\frac{\beta}{1-n+\beta(2-n)}, \frac{1}{1-n+\beta(2-n)}$
B	$\frac{1}{1-n+\beta(2-n)}, \frac{\beta}{1-n+\beta(2-n)}$	$\frac{1}{2(1-n)+\beta}, \frac{1}{2(1-n)+\beta}$

Table 1. Equilibrium values of quantities under B and PM (homogeneous products).

Firm 1 \ Firm 2	PM	B
PM	$\frac{1}{(3-2n)^2}, \frac{1}{(3-2n)^2}$	$\frac{\beta^2}{[1-n+\beta(2-n)]^2}, \frac{\beta}{[1-n+\beta(2-n)]^2}$
B	$\frac{\beta}{[1-n+\beta(2-n)]^2}, \frac{\beta^2}{[1-n+\beta(2-n)]^2}$	$\frac{\beta}{[2(1-n)+\beta]^2}, \frac{\beta}{[2(1-n)+\beta]^2}$

Table 2. Payoff matrix (profits) under B and PM (homogeneous products).

Let $n_a(\beta) := \frac{1-2\beta-\sqrt{\beta}}{1-\beta}$, where $0 \le n_a(\beta) < 1$ for any $0 < \beta \le 0.25$ and $n_a(\beta) < 0$ for any $0.25 < \beta \le 1$, be a threshold value of n such that the profit differential $\Delta_a = \Pi_i^{B/PM} - \Pi_i^{PM/PM} = 0$ for any $i, j = \{1, 2\}, i \ne j$. This profit differential allows us to check whether firm i has an incentive to deviate from PM to B when the rival is playing PM. Let $n_c(\beta) := 1 - \frac{1}{2}\sqrt{\beta}$, where $1/2 \le n_c(\beta) < 1$ for any $0 < \beta \le 1$, be a threshold value of n such that the profit differential $\Delta_c = \Pi_i^{PM/PM} - \Pi_i^{B/B} = 0$ for

[10] Note that the equilibrium values of output (resp. profits) reported in the corresponding tables throughout the manuscript are net of the common term $1-w$ (resp. $(1-w)^2$).

any $i, j = \{1,2\}, i \neq j$. This profit differential allows us to check whether B is dominated by PM for each firm i. The shape of $n_a(\beta)$ and $n_c(\beta)$ is depicted in Figure 1 in the parameter space (β, n). We note that the threshold $n_b(\beta)$ such that the profit differential $\Delta_b = \Pi_i^{PM/B} - \Pi_i^{B/B} = 0$ for any $i, j = \{1,2\}, i \neq j$ is larger than one for any $0 < \beta \leq 1$ and then it is not economically meaningful in the parameter space (β, n) when the network-codetermination game is played with homogeneous products. It will become a meaningful threshold in the case of product differentiation as we will see later in this article. This profit differential allows us to check whether firm i has an incentive to deviate from B to PM when the rival is playing B. Then, Lemma 1 and Proposition 1 clarify the outcomes of the game at *the codetermination stage* (stage 1), where each owner must choose to be either a codetermined or profit maximising firm.

Lemma 1. If the strength of the network effect if sufficiently small ($n < n_c(\beta)$), then $\Pi_i^{PM/PM} > \Pi_i^{B/B}$. If the strength of the network effect if sufficiently large ($n > n_c(\beta)$), then $\Pi_i^{B/B} > \Pi_i^{PM/PM}$.

Proof. The proof follows by studying the sign of

$$\Delta_c = \frac{(1-w)^2(1-\beta)[4n^2 - 8n + 4 - \beta]}{(3-2n)^2[\beta + 2(1-n)]^2}.$$

If $n < n_c(\beta)$ (resp. $n > n_c(\beta)$) then $\Delta_c > 0$ (resp. $\Delta_c < 0$). **Q.E.D.**

Proposition 1. (1) If $-1 \leq n < n_a(\beta)$ then there exist two pure-strategy Nash equilibria given by (B,B) and (PM,PM), and PM payoff dominates B (coordination game). (2) If $n_a(\beta) < n < n_c(\beta)$ then (B,B) is the unique Pareto inefficient SPNE of the game (prisoner's dilemma). (3) If $n_c(\beta) < n < 1$ then (B,B) is the unique Pareto efficient SPNE of the game (deadlock).

Proof. Profit differentials Δ_a and Δ_b are the following:

$$\Delta_a = \frac{(1-w)^2(1-\beta)[-(1-\beta)n^2 + 2n(1-2\beta) + 4\beta - 1]}{(3-2n)^2[\beta + 2(1-n)]^2},$$

and

$$\Delta_b = \frac{-(1-w)^2\beta(1-\beta)[(1-\beta)n^2 - 2n + 1 + \beta + \beta^2]}{[1-n+\beta(2-n)]^2[\beta + 2(1-n)]^2} < 0.$$

The sign of Δ_a and Δ_c change depending on the relative size of β and n. Given Lemma 1, we have that (1) if $-1 \le n < n_a(\beta)$ then $\Delta_a < 0$, $\Delta_b < 0$ and $\Delta_c > 0$, (2) if $n_a(\beta) < n < n_c(\beta)$ then $\Delta_a > 0$, $\Delta_b < 0$ and $\Delta_c > 0$, (3) if $n_c(\beta) < n < 1$ then $\Delta_a > 0$, $\Delta_b < 0$ and $\Delta_c < 0$. **Q.E.D.**

The main result of Proposition 1 is represented by the solution of the dilemma raised in Kraft (1998). In fact, when the strength of the network effect is sufficiently large, (B,B) becomes the unique Pareto efficient Nash equilibrium of the game. More in general, the proposition shows the existence of a wide spectrum of equilibrium outcomes in a Kraft-like game played by quantity-setting duopoly firms in a network industry with negative and positive consumption externalities rather than in a standard Cournot (non-network) setting. We now discuss the mechanisms through which the network effect works in this model. We restrict the discussion to changes in n as the analysis of the mechanics of what happens when β varies has been already pointed out in Fanti et al. (2018). Let us begin the discussion with the case of a non-network industry ($n = 0$). As is clear by looking at the β-axis in Figure 1, in this case our model boils down to Kraft (1998). This means that (B,B) is the unique Pareto inefficient Nash equilibrium of the game for any $\beta > 0.25$, implying that B is the dominant strategy, and there exist two pure-strategy Nash equilibria when $\beta < 0.25$. We want to stress that in contrast with this, Kraft (1998) stated that "for values of $\beta < 0.25$ profit-maximization is the dominant strategy." (p. 199). Indeed, there do not exist dominant strategies when $\beta < 0.25$ in a market for homogeneous products (see Fanti et al., 2018). Negative values of n do not modify the qualitative outcomes of Kraft (1998). Let us now turn to the case of positive values of n. We recall that n represents the strength of the positive network externality on the consumers' side. Therefore, ceteris paribus, larger values of n cause an increase in the quantity produced by the firms in both cases of profit maximisation and codetermination by also implying an outward shift in the market demand, which in turn contributes to increase the market price.[11] Then, an increase n causes a twofold effect on profits of firm i. A positive direct effect through the augmented production. A positive indirect effect through the increase in the market price. Both these effects cause an increase in firms' profits. Therefore, it is important to understand the relative strength of these two effects under PM and B. As the codetermined firm produces more than the PM firm, the effect of an increase in n on the market demand is to let the price increase more under B rather than under PM. This implies that the outward shift in the market demand in a network codetermined industry is larger than the outward shift in the market demand in a network profit-maximising industry. This is because the network effect strengthens the codetermination effect on the side of production. Then, for any given level of the bargaining power, β, players do not have dominant strategies in a non-network industry ($n = 0$) or in an industry where the strength of the network effect is sufficiently weak ($n < n_a(\beta)$) and each player has the incentive to play the same strategy of the rival. As far as the network externality becomes stronger and the market size increases, a codetermined firm increases profits more than its profit maximising rival does. In fact, when n belongs to intermediate values ($n < n_c(\beta)$), players have an incentive to be a profit maximiser, but no one has a unilateral incentive to deviate from codetermination as the codetermined firm increases its profit dramatically in the case of asymmetric behaviour. This implies that B becomes a dominant strategy, but the result of the game is still a prisoner's dilemma (Kraft, 1998). However, larger values of n ($n > n_c(\beta)$) allow each player losing the incentive to be a profit maximiser due to a further increase in profits under B caused by a much stronger consumption externality. This is illustrated in Figure 1 and clarified in Tables 3, 4 and 5, representing payoff matrices (profits) built on by taking $w = 0$, the same level of β (0.2) and three

[11] Alternatively, for any given level of the market price consumers are willing to buy more in a network industry rather than in a standard Cournot industry. Indeed, the essence of a network externality is to increase the utility of a consumer depending on the number of users joining the network (internet, online games and so on).

different values of n (0.05, 0.35 and 0.8). This is an example of the working of the network externality as an instrument able to solve the prisoner's dilemma raised in Kraft (1998) by letting the Nash equilibrium (B,B) become Pareto efficient. Of course, if one takes smaller values of the union's bargaining power in line with the codetermination rules, i.e. values of β ranging from 0.5 to 0.6667 (Kraft, 1998; Gorton and Schmid, 2004), as shown in the region bounded in red lines in Figure 1, the network-codetermination game has only two possible outcomes where (B,B) emerges as the unique SPNE, one is a prisoner's dilemma and the other is a cooperation game. This is shown in the numerical example of Tables 6 and 7, where we have fixed $\beta = 0.5$ (employees have near-parity rights on the supervisory board) and let n be 0.5 and 0.7, respectively.

Firm 2 / Firm 1	PM	B
PM	0.118, 0.118	0.02, 0.111
B	0.111, 0.02	0.04, 0.04

Table 3. Payoff matrix (profits) under B and PM when $\beta = 0.2$ and $n = 0.05$. Coordination game: (B,B) and (PM,PM) are two pure-strategy Nash equilibria.

Firm 2 / Firm 1	PM	B
PM	0.18, 0.18	0.04, 0.2
B	0.2, 0.04	0.08, 0.08

Table 4. Payoff matrix (profits) under B and PM when $\beta = 0.2$ and $n = 0.35$. Prisoner's dilemma: (B,B) is the unique Pareto inefficient SPNE of the game.

Firm 2 / Firm 1	PM	B
PM	0.51, 0.51	0.2, 1.03
B	1.03, 0.2	0.55, 0.55

Table 5. Payoff matrix (profits) under B and PM when $\beta = 0.2$ and $n = 0.8$. Deadlock: (B,B) is the unique Pareto efficient SPNE of the game.

Firm 2 / Firm 1	PM	B
PM	0.25, 0.25	0.16, 0.32
B	0.32, 0.16	0.22, 0.22

Table 6. Payoff matrix (profits) under B and PM when $\beta = 0.5$ and $n = 0.5$. Prisoner's dilemma: (B,B) is the unique Pareto inefficient SPNE of the game.

Firm 2 / Firm 1	PM	B
PM	0.39, 0.39	0.27, 0.55
B	0.55, 0.27	0.41, 0.41

Table 7. Payoff matrix (profits) under B and PM when $\beta = 0.5$ and $n = 0.7$. Deadlock: (B,B) is the unique Pareto efficient SPNE of the game.

The result of Kraft (1998) holds only when the network effects are sufficiently small. This is because the positive effects of consumption externalities on quantities and the market size are not enough to modify the incentive of both firms to coordinate to play profit maximisation (but no one has a unilateral incentive to deviate from codetermination). When no dominated strategies do exist, there are multiple Nash equilibria in pure strategies (this holds when the bargaining power of the union is sufficiently high, and the strength of the consumption externality is sufficiently small). This implies that each firm does not have an incentive to deviate from PM when the rival plays PM (indeed, a reduction either in β or n reduces profits) and the game from a prisoner's dilemma becomes a coordination game. In order to solve the problem of equilibrium selection, we consider that a Nash equilibrium in mixed strategies can be obtained by defining probabilities x_1 and $1-x_1$ (resp. x_2 and $1-x_2$) that firm 1 (resp. firm 2) plays either B or PM. The unique mixed-strategy Nash equilibrium is the following:

$$x_1 = x_2 = x_*^{B/PM} = \frac{\Pi_i^{B/PM} - \Pi_i^{PM/PM}}{(\Pi_i^{B/PM} - \Pi_i^{PM/PM}) + (\Pi_i^{PM/B} - \Pi_i^{B/B})} = \\ = \frac{[\beta + 2(1-n)]^2[-(1-\beta)n^2 + 2n(1-2\beta) + 4\beta - 1]}{(1-n)(1-\beta)[4(1+\beta)n^2(n-3) + 3(-\beta^2 + 3\beta + 4)n + 5\beta^2 - \beta - 4]} . \quad (18)$$

From (18) it is easy to check that $x_*^{B/PM} > 0$ only when $0 \leq n < n_a(\beta)$ and it is a decreasing function of β. This probability vanishes when $n = n_a(\beta)$ and approaches 1 when $n_a(\beta) \to 1$, that is when $\beta \to 0$. This is in line with the results summarised in Proposition 1. Eq. (18) represents the mixed-strategy Nash equilibrium of the game. The rule that comes from (18) is the following: each firm will choose to play B (resp. PM) as a pure strategy if the rival plays B (resp. PM) with a probability $x > x_*^{B/PM}$ (resp. $x < x_*^{B/PM}$). The lower β, the higher the probability of playing PM as a pure strategy. By using the Pareto dominance criterion for the parameter configuration β - n represented by the areas where there is multiplicity of equilibria in pure strategies in Figure 1,[12] we may conclude that in a network industry with codetermination (PM,PM) Pareto dominates (B,B).

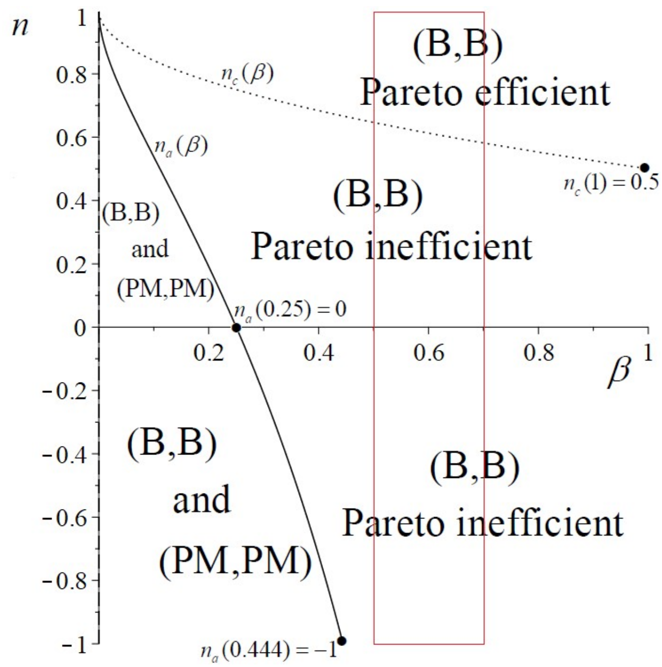


Figure 1. Codetermination and network externalities in a quantity-setting duopoly with homogeneous products. Profit differentials in (β, n) space. The solid (resp. dotted) line represents the threshold value $n_a(\beta)$ (resp. $n_c(\beta)$) related to profit differential $\Delta_a = 0$ (resp. $\Delta_c = 0$). The strength of the network externality allows solving the prisoner's dilemma of Kraft (1998), which holds only in the area of Pareto inefficiency. The area bounded by the red rectangle represents values of the union bargaining power (ranging from 0.5 to almost 0.7) that are consistent with 1) the Works Constitution Act (*Betriebsverfassungsgesetz*) issued in 1952 (with small changes since 2004) that introduced 1/3 representation of employees on supervisory boards in all industries with firms employing more than 500 workers, and 2) the Co-determination Act (*Mitbestimmungsgesetz*) issued in 1976 that introduced 1/2 representation on supervisory boards in all industries with firms employing more than 2000 workers.

The analysis above allows writing down the following results on the effects of a positive network consumption externality in a codetermined duopoly with homogeneous products.

Result 1. The strength of the network effect is sufficiently small ($n < 1/2$). The Nash equilibrium (B,B) cannot be a Pareto efficient outcome. However, an increase in n promotes the emergence of

[12] A Nash equilibrium is *Payoff-dominant* if it Pareto dominates all the other Nash equilibria in the game.

(B,B) as the unique Pareto inefficient SPNE of the game, as it reduces the parameter space (β, n) with multiplicity of equilibria in pure strategies, where PM is the payoff dominant strategy.

Result 2. The strength of the network effect is sufficiently large ($n > 1/2$). An increase in n promotes the emergence of (B,B) as the unique Pareto efficient SPNE of the game (deadlock).

3. Product differentiation

This section extends the results of Section 2 to the case of horizontal product differentiation. Therefore, the inverse market demand for product of variety i is expressed by Eq. (2). Tables 8 and 9 summarise the equilibrium values of quantity and profits in this case.

Firm 1 \ Firm 2	PM	B
PM	$\frac{1}{1+(1-n)(1+d)}, \frac{1}{1+(1-n)(1+d)}$	$\frac{\beta+(1-n)(1-d)}{(2-n)(1-n+\beta+nd^2)}$, $\frac{1+(1-n)(1-d)}{(2-n)(1-n+\beta+nd^2)}$
B	$\frac{1+(1-n)(1-d)}{(2-n)(1-n+\beta+nd^2)}$, $\frac{\beta+(1-n)(1-d)}{(2-n)(1-n+\beta+nd^2)}$	$\frac{1}{\beta+(1-n)(1+d)}, \frac{1}{\beta+(1-n)(1+d)}$

Table 8. Equilibrium values of quantities under B and PM (heterogeneous products).

Firm 1 \ Firm 2	PM	B
PM	$\frac{1}{[1+(1-n)(1+d)]^2}, \frac{1}{[1+(1-n)(1+d)]^2}$	$\frac{[\beta+(1-n)(1-d)]^2}{(2-n)^2(1-n+\beta+nd^2)^2}$, $\frac{\beta[1+(1-n)(1-d)]^2}{(2-n)^2(1-n+\beta+nd^2)^2}$
B	$\frac{\beta[1+(1-n)(1-d)]^2}{(2-n)^2(1-n+\beta+nd^2)^2}$, $\frac{[\beta+(1-n)(1-d)]^2}{(2-n)^2(1-n+\beta+nd^2)^2}$	$\frac{1}{[\beta+(1-n)(1+d)]^2}, \frac{1}{[\beta+(1-n)(1+d)]^2}$

Table 9. Payoff matrix (profits) under B and PM (heterogeneous products).

Let $n_a(\beta,d)$, $n_b(\beta,d)$ and $n_c(\beta,d):=1-\frac{\sqrt{\beta}}{1+d}$ be three threshold values of n such that $\Delta_a=\Pi_i^{B/PM}-\Pi_i^{PM/PM}=0$, $\Delta_b=\Pi_i^{PM/B}-\Pi_i^{B/B}=0$ and $\Delta_c=\Pi_i^{PM/PM}-\Pi_i^{B/B}=0$ for any $i,j=\{1,2\}, i\neq j$, respectively. Then, Lemma 2 and Propositions 2 and 3 clarify the outcomes of the network-codetermination game at *the codetermination stage* (stage 1) in the case of heterogeneous products.

Lemma 2. If the strength of the network effect if sufficiently small ($n<n_c(\beta,d)$), then $\Pi_i^{PM/PM}>\Pi_i^{B/B}$. If the strength of the network effect if sufficiently large ($n>n_c(\beta,d)$), then $\Pi_i^{B/B}>\Pi_i^{PM/PM}$. This holds on both cases of product substitutability ($0<d<1$) and product complementarity ($-1\leq d\leq 0$).

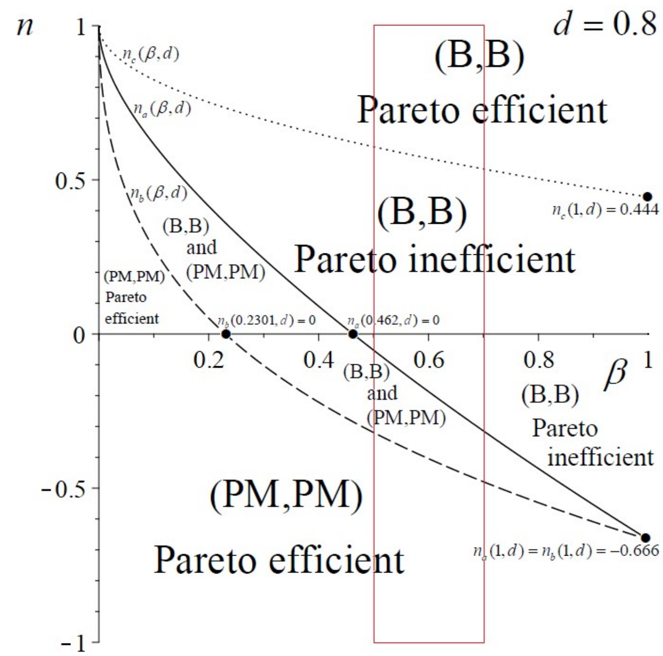
Proposition 2. [Product substitutability ($0<d<1$)]. (1) If $-1<n<n_b(\beta,d)$ then (PM,PM) is the unique Pareto efficient SPNE of the game (deadlock). (2) If $n_b(\beta,d)<n<n_a(\beta,d)$ then there exist two pure-strategy Nash equilibria given by (B,B) and (PM,PM), and PM payoff dominates B (coordination game). (3) If $n_a(\beta,d)<n<n_c(\beta,d)$ then (B,B) is the unique Pareto inefficient SPNE of the game (prisoner's dilemma). (4) If $n_c(\beta,d)<n<1$ then (B,B) is the unique Pareto efficient SPNE of the game (deadlock).

Proposition 3. [Product complementarity ($-1\leq d\leq 0$)]. (1) If $-1<n<n_c(\beta,d)$ then (PM,PM) is the unique Pareto efficient SPNE of the game (deadlock). (2) If $n_c(\beta,d)<n<n_b(\beta,d)$ then (PM,PM) is the unique Pareto inefficient SPNE of the game (prisoner's dilemma). (3) If $n_b(\beta,d)<n<n_a(\beta,d)$ then there exist two pure-strategy Nash equilibria given by (B,B) and (PM,PM), and B payoff dominates PM (coordination game). (4) If $n_a(\beta,d)<n<1$ then (B,B) is the unique Pareto efficient SPNE of the game (deadlock).

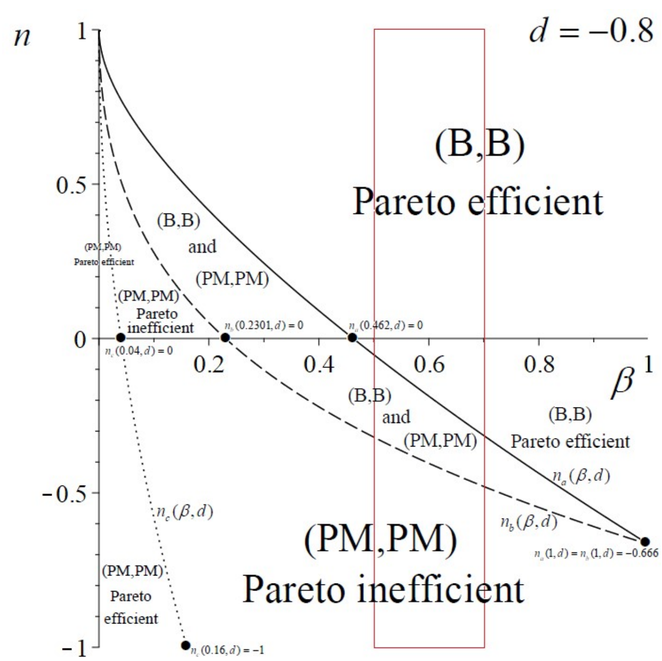
The proof of Lemma 2 and Propositions 2 and 3 follows by applying the same line of reasoning used to show Lemma 1 and Proposition 1. Specifically, Proposition 2 shows that product differentiation allows (PM,PM) to become the unique SPNE of the game, i.e. product differentiation works out against codetermination. This is in line with the results obtained by Fanti et al. (2018). However, the interaction between network externalities and product differentiation can bring to light an interesting (and at first glance counterintuitive) outcome. As an increase in both the degree of product differentiation and the strength of the network effect allows firms to increase their own profits, a reduction in d in a network industry can make the profitability of codetermined firms larger than that of profit maximising firms. In other words, product differentiation strengthens the working of the network effects as a device increasing profits under codetermination. This aims to let (B,B) being the Pareto efficient outcome of the game for a wider range of values of β and n. The following proposition shows this result, which is also illustrated in Figure 2 by contrasting Panels (a) and (c), related to product substitutability, plotted for $d=0.8$ and $d=0.5$, respectively, and Panels (b) and (d), related to product complementarity, plotted for $d=-0.8$ and $d=-0.5$, respectively.

Proposition 4. An increase in the degree of product differentiation ($d\downarrow$) shifts downward the threshold curve $n_c(\beta,d):=1-\frac{\sqrt{\beta}}{1+d}$ in the space (β,n).

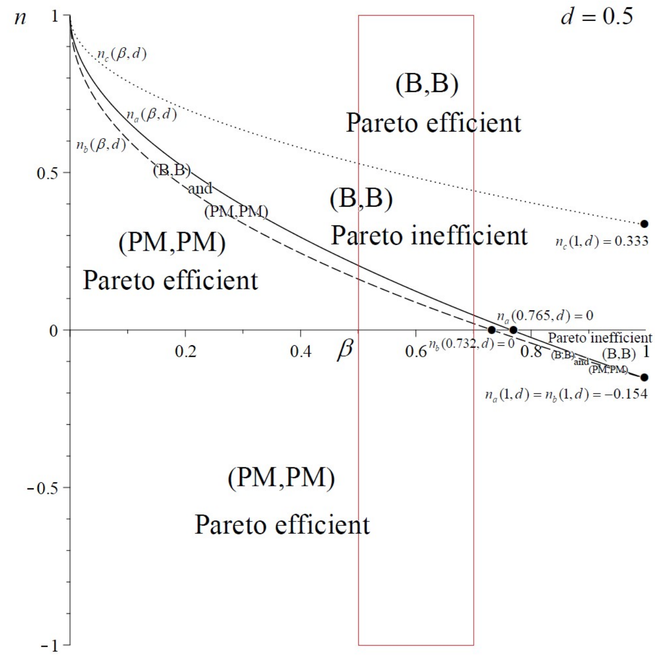
Proof. The proof follows by noting that $n_c(\beta,d)$ is a monotonic increasing function of d. **Q.E.D.**


n
d = 0.8
1
0.5
0
-0.5
-1
0.2
0.4
0.6
0.8
1
β
n_c(β,d)
n_a(β,d)
n_b(β,d)
(B,B)
Pareto efficient
(B,B)
Pareto inefficient
n_c(1,d) = 0.444
(B,B)
and
(PM,PM)
(PM,PM)
Pareto
efficient
n_b(0.2301,d) = 0
n_a(0.462,d) = 0
(B,B)
and
(PM,PM)
(B,B)
Pareto
inefficient
(PM,PM)
Pareto efficient
n_a(1,d) = n_b(1,d) = −0.666

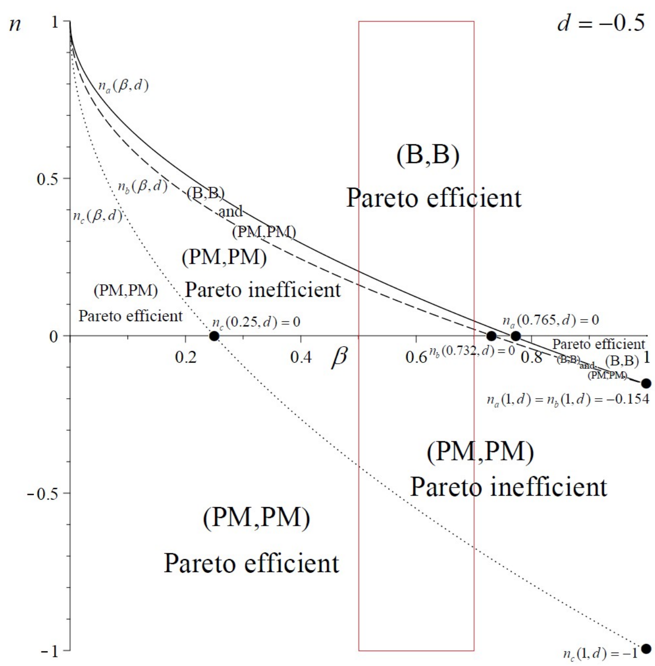
(a)


n
d = −0.8
1
0.5
0
-0.5
-1
0.2
0.4
0.6
0.8
1
β
(B,B)
Pareto efficient
(B,B)
and
(PM,PM)
(PM,PM)
Pareto efficient
(PM,PM)
Pareto
inefficient
n_c(0.04,d) = 0
n_b(0.2301,d) = 0
n_a(0.462,d) = 0
(B,B)
and
(PM,PM)
(B,B)
Pareto efficient
n_a(β,d)
n_b(β,d)
n_a(1,d) = n_b(1,d) = −0.666
n_c(β,d)
(PM,PM)
Pareto inefficient
(PM,PM)
Pareto
efficient
n_c(0.16,d) = −1

(b)



(c)



(d)

Figure 2. Codetermination and network externalities in a quantity-setting duopoly with heterogeneous products. Profit differentials in (β, n) space. The solid (resp. dashed) [resp. dotted] line represents the threshold value $n_a(\beta,d)$ (resp. $n_b(\beta,d)$) [resp. $n_c(\beta,d)$] related to $\Delta_a = 0$ (resp. $\Delta_b = 0$) [resp. $\Delta_c = 0$]. Panel (a) $d = 0.8$. Panel (b) $d = -0.8$. Panel (c) $d = 0.5$. Panel (d) $d = -0.5$. Amongst other things, the figure shows that an increase in product differentiation ($d \downarrow$) increases the area in which the prisoner's dilemma of Kraft (1998) is solved. The area bounded by the red rectangle represents values of the union bargaining power (ranging from 0.5 to almost 0.7) that are consistent with 1) the Works Constitution Act (*Betriebsverfassungsgesetz*) issued in 1952 (with small changes since 2004) that introduced 1/3 representation of employees on supervisory boards in all industries with firms employing more than 500 workers, and 2) the Co-determination Act (*Mitbestimmungsgesetz*) issued in 1976 that introduced 1/2 representation on supervisory boards in all industries with firms employing more than 2000 workers.

4. Endogenous codetermination and network externalities

The results obtained in the previous section allow to have some policy recipes (mandatory codetermination versus voluntary codetermination) depending on the values of the main parameters of the problem. However, one of the drawbacks of the proposed approach (following the original idea of Kraft, 1998) lies in the fact that the degree of codetermination (i.e., the strength with which trade unions negotiate with firms) is an exogenous parameter $(1-\beta)$. Differently, firms might decide to bargain not with any trade union, but with the one just allowing to maximise their own profits. Indeed, in actual economies there may be different types of union bargaining units that should not necessarily be appreciated by the firm as part of the bargaining process. To overcome this gap and accounting for this heterogeneity, this section speculates in this direction and extends the model previously developed with an exogenous degree of codetermination by assuming that each firm is aware of the union's attitude at the time of bargaining and chooses to bargain with a union bargaining unit under codetermination only whether the firm's bargaining power is the profit-maximising one. In doing this, *we assume that the firm has the right to choose the composition of the board of representatives (including or not workers' representatives) making production decisions*. This amounts to say that firms may choose the optimal union's bargaining effort by choosing the optimal corresponding number of workers' representative to be co-opted within the supervisory board.

Let us first assume the existence of a continuum of firm-specific unions differentiated amongst them based on their relative attitude to bargain ($0<1-\beta_i\leq 1$). The research question, which is novel at the best of our knowledge, arising in this context is the following: do firms always prefer to bargain with a trade union with a little bargaining power? The answer is not so obvious, and the aim of this section is to show that the strategic interacting effects between the degree of product differentiation and the strength of the network effect may lead a quantity-setting duopoly firm to bargain with a union-unit with a sizeable bargaining power, as this choice allows a firm to maximise its own profit.

The stages of the game change and become the following. At stage 1 (*the codetermination stage*), each owner must choose to be either a codetermined or profit-maximising firm. At stage 2 (*the union-strength stage*) the owner of each firm chooses to bargain with a union bargaining unit only whether its bargaining attitude is exactly the profit-maximising one. At stage 3 (*the bargaining market stage*), firms either choose the quantity in the output market in the case of profit maximisation or bargain it together with unions in the case of codetermination. The game follows the backward induction logic. We now briefly discuss the main features of a network-codetermination non-cooperative (three-stage) game with quantity competing firms, complete information and endogenous codetermination. Of course, equilibrium outcomes are still those reported in Tables 8 and 9 (Section 3) if both firms are profit maximising (PM) so that $\beta_1 = \beta_2 = 1$. When both firms are codetermined (B), the Nash bargaining function $\mathrm{N}_i = \Pi_i^{\beta} Z_i^{1-\beta}$ modifies to become $\mathrm{N}_i = \Pi_i^{\beta_i} Z_i^{1-\beta_i}$. This implies that firm 1 bargains with type-1 union bargaining unit with an effort or bargaining strength β_1 to choose the quantity of product of variety 1. Correspondingly, firm 2 bargains with type-2 union bargaining unit with an effort or bargaining strength β_2 to produce the quantity of product of variety 2. Then, there will be quantities and prices as a function of β_1 and β_2 that should be used to compute profits of firm i ($i, j = \{1,2\}$, $i \neq j$), that is:

$$\overline{\Pi}_i^{B/B} = \frac{\beta_i (1-w)^2 [(1-n)(1-d) + \beta_j]^2}{[(1-n)^2(1-d^2) + (1-n)(\beta_i + \beta_j) + \beta_i \beta_j]^2}. \tag{19}$$

As each firm chooses to bargain with its own union bargaining unit if and only if there exists a profit-maximising bargaining power, we get the following *reaction-bargaining-function* of firm i, that is:

$$\frac{\partial \overline{\Pi}_i^{B/B}}{\partial \beta_i} = 0 \Leftrightarrow \beta_i(\beta_j) = \frac{(1-n)[(1-n)(1-d^2) + \beta_j]}{1-n+\beta_j}. \tag{20}$$

By using the corresponding counterpart version of (20) for firm j, one can get the *optimal value of firm i's bargaining strength* (outcomes are symmetric), that is

$$\beta_i^{*(B/B)} = (1-n)\sqrt{1-d^2}, \quad i,j = \{1,2\}. \tag{21}$$

The expression in (21) gives all the couples (n,d) such that the owner maximises profits by choosing to be bargainer under codetermination and it is meaningful if and only if $\beta_i^{*(B/B)} \leq 1$. This condition implies that

$$n \geq n_\beta(d) := 1 - \frac{1}{\sqrt{1-d^2}}, \tag{22}$$

should hold, otherwise there would be no economically meaningful profit-maximising value of β_i. The expression in (22) tells us that each firm would decide to be codetermined by choosing a profit-maximising bargaining effort if and only if the network externality is strong enough, otherwise it would prefer to be a profit-maximiser. In the case of positive network externalities, the condition in (21) is meaningful for any $0 < n < 1$ and $-1 \leq d < 1$ so that (22) is always fulfilled. In the case of negative network externalities, the condition in (21) is meaningful only whether (22) holds, that is for any $-n_\beta(d) < n < 0$ and $-\sqrt{3}/2 < d < \sqrt{3}/2$.

By substituting out (21) into (19) for β_i one gets profits of firm i under optimal codetermination, that is

$$\Pi_i^{B/B} = \frac{(1-w)^2\sqrt{1-d^2}(1-d+\sqrt{1-d^2})^2}{4(1-n)(1-d^2+\sqrt{1-d^2})^2}. \tag{23}$$

When firm 1 is codetermined (B) and firm 2 is profit maximiser (PM), firm 1 bargains with type-1 union bargaining unit with an effort β_1 and firm 2 does not bargain at all ($\beta_2 = 1$). Then, by considering quantities and prices as a function of β_1 profits of firm 1 and firm 2 are the following:

$$\overline{\Pi}_1^{B/PM} = \frac{\beta_1(1-w)^2[1+(1-n)(1-d)]^2}{\{(1-n)[1+(1-n)(1-d^2)]+\beta_1(2-n)\}^2}. \tag{24}$$

and

$$\overline{\Pi}_2^{B/PM} = \frac{(1-w)^2[\beta_1+(1-n)(1-d)]^2}{\{(1-n)[1+(1-n)(1-d^2)]+\beta_1(2-n)\}^2}. \tag{25}$$

The profit-maximising bargaining power β_1 is the following:

$$\frac{\partial \overline{\Pi}_1^{B/PM}}{\partial \beta_1} = 0 \Leftrightarrow \beta_1^{*(B/PM)} = \frac{(1-n)[1+(1-n)(1-d^2)]}{2-n}. \tag{26}$$

In the case of positive network externalities, the condition in (26) implies that $\beta_1^{*(B/PM)} \le 1$ for any $0 < n < 1$ and $-1 < d < 1$. In the case of negative network externalities, the condition in (26) is meaningful only whether $-n_\beta(d) < n < 0$ and $-\sqrt{3}/2 < d < \sqrt{3}/2$. By substituting out (26) into (24) and (25) for β_1 one gets

$$\Pi_1^{B/PM} = \frac{(1-w)^2[1+(1-n)(1-d)]^2}{4(1-n)(2-n)[1+(1-n)(1-d^2)]}. \tag{27}$$

and

$$\Pi_2^{B/PM} = \frac{(1-w)^2[n+(2-n)(1-d)+(1-n)(2-d^2)]^2}{4(2-n)^2[1+(1-n)(1-d^2)]^2}. \tag{28}$$

To sum up, Table 10 summarises the equilibrium outcomes of the optimal bargaining strength in the cases of both symmetric and asymmetric behaviours and Table 11 refers to the corresponding values of firms' profits (payoff matrix).

Firm 2 / Firm 1	PM	B
PM	1,1	$1, \frac{(1-n)[1+(1-n)(1-d^2)]}{2-n}$
B	$\frac{(1-n)[1+(1-n)(1-d^2)]}{2-n}, 1$	$(1-n)\sqrt{1-d^2}, (1-n)\sqrt{1-d^2}$

Table 10. Endogenous codetermination. Equilibrium values of the bargaining strength under B and PM. The optimal firm's bargaining strength in the case of symmetric and asymmetric behaviours are meaningful if and only if $n \ge n_\beta(d)$.

Firm 2 / Firm 1	PM	B
PM	$\frac{1}{[1+(1-n)(1+d)]^2}, \frac{1}{[1+(1-n)(1+d)]^2}$	$\frac{[n+(2-n)(1-d)+(1-n)(2-d^2)]^2}{4(2-n)^2[1+(1-n)(1-d^2)]^2}$, $\frac{[1+(1-n)(1-d)]^2}{4(1-n)(2-n)[1+(1-n)(1-d^2)]}$
B	$\frac{[1+(1-n)(1-d)]^2}{4(1-n)(2-n)[1+(1-n)(1-d^2)]}$, $\frac{[n+(2-n)(1-d)+(1-n)(2-d^2)]^2}{4(2-n)^2[1+(1-n)(1-d^2)]^2}$	$\frac{\sqrt{1-d^2}(1-d+\sqrt{1-d^2})^2}{4(1-n)(1-d^2+\sqrt{1-d^2})^2}$, $\frac{\sqrt{1-d^2}(1-d+\sqrt{1-d^2})^2}{4(1-n)(1-d^2+\sqrt{1-d^2})^2}$

Table 11. Endogenous codetermination. Equilibrium values of profits under B and PM (payoff matrix).

Define $n_b^1(d)=n_c^1(d)=1-\frac{1}{\sqrt{1-d^2}}=n_\beta(d)$, and let $n_b^2(d)$ and $n_c^2(d)$ be two threshold values of the strength of the network effect such that the corresponding profit differentials $\Delta_b=\Pi_i^{PM/B}-\Pi_i^{B/B}=0$ and $\Delta_c=\Pi_i^{PM/PM}-\Pi_i^{B/B}=0$ $(i,j=\{1,2\}, i\neq j)$. The shape of $n_\beta(d)$, $n_b^2(d)$ (red line) and $n_c^2(d)$ (black line) is depicted in Figure 3 in the parameter space (n,d).[13] The red region in the figure refers to the couples (n,d) corresponding to which every firm does not find it convenient to bargain with its own union under codetermination. Figure 3 classifies the outcomes of the game at *the codetermination stage* (stage 1), where each owner must choose to be either a codetermined or profit-maximising firm under *endogenous codetermination*.

Results from Figure 3 under endogenous codetermination are in line with those obtained under exogenous codetermination. The figure shows that the larger the degree of product substitutability and the larger the network effect, the lower the optimal bargaining effort of the firm needed to maximise profits. The red area represents the unfeasible parameter space of optimal codetermination, where firms behave as profit maximisers and codetermination can be applied only through legislation. In all other cases, codetermination can emerge through voluntary agreements (irrespective of the number of employees). When products are substitutes, a voluntary

[13] Note that there exists no closed-form expression for $n_b^2(d)$, whereas the expression of $n_c^2(d)$ cannot be dealt with in a neat analytical form. However, this is not relevant for the results of the model with endogenous codetermination as Figure 3 helps clarifying the shapes of the profit differentials.

codetermination agreement is efficient when the strength of the network effect is sufficiently large. However, it is possible to have also multiple mixed Nash equilibria corresponding to which only one firm voluntarily chooses to be codetermined. In this case, no one has a dominant strategy and both equilibria are Pareto optimal. The solution of the game may emerge from the credible disclosure of a player’s will to do not play B. Then, the rival will be forced to (be the first to) play B to avoid obtaining a lower pay-off unilaterally.

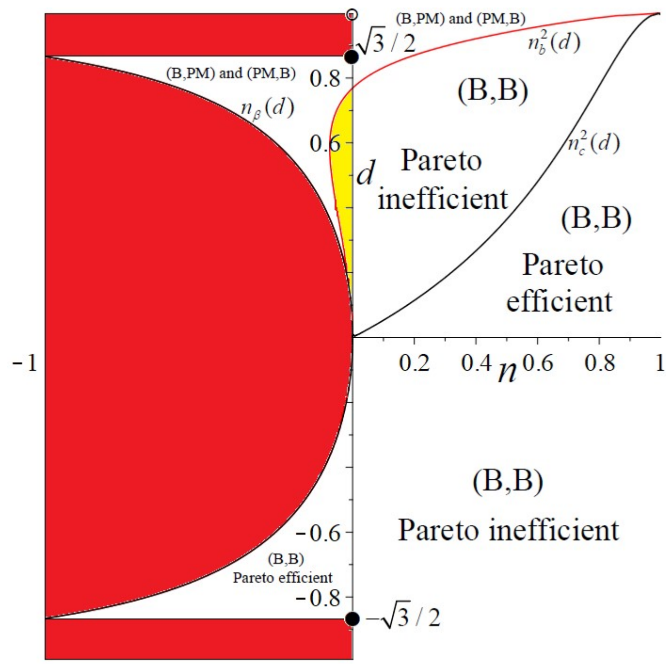


Figure 3. Endogenous codetermination and network externalities in a quantity-setting duopoly. Nash equilibrium 5outcomes in (n,d) plane. The red (resp. black) solid line represents the threshold value $n_b^2(d)$ (resp. $n_c^2(d)$) related to the profit differential $\Delta_b = 0$ (resp. $\Delta_c = 0$). The red region represents the unfeasible parameter space of optimal codetermination. Its boundary (black solid line) is given by $n_\beta(d) = n_b^1(d) = n_c^1(d) = 1 - \frac{1}{\sqrt{1-d^2}}$, which applies only for negative values of n and it is meaningful if and only if $1 - \frac{1}{\sqrt{1-d^2}} > -1$, i.e., $-\sqrt{3}/2 < d < \sqrt{3}/2$. The yellow region represents the parameter space of optimal codetermination (corresponding to product substitutability and negative consumption externalities) where (B,B) is the unique Pareto inefficient SPNE of the game.

5. Conclusions

The present article extended the strand of research dealing with the institution of codetermination by considering the presence of network externality in consumption. Codetermination is an institution that plays an important role in the protection of workers' rights and the improvement of working conditions. Amongst other North European countries, it has become relevant in the German industry since at least 1976 (The German Codetermination Act), extending codetermination rules to all industries and firms with more than 2000 employees, by also affecting the designing of German industrial policy. In modern economies, the existence of network effects in the market works in the direction of increasing the quantity produced by firms (irrespective of whether they are profit maximising or codetermined) and expanding the market size (or reducing the market size in the case of negative network externalities). This article showed that the strength of these positive effects on profits is larger under codetermination by using a strategic competitive framework à la Kraft (1998). This is because codetermination per se allows firms to produce more than standard profit maximisation and a network externality broadens this effect. Therefore, in contrast to Kraft (1998), who showed that codetermination might emerge as a Pareto inefficient market outcome (prisoner's dilemma) in a standard Cournot duopoly, this article identified a possible solution to the dilemma by letting codetermination be a Pareto efficient equilibrium of a network industry with quantity-setting duopoly firms. The result is also extended to the case of horizontal product differentiation. Finally, as codetermination enhances the consumer's surplus, we must emphasise such an institution may represent a Pareto-superior policy in comparison to profit maximisation. As consequence, in network industries codetermination might emerge as the endogenous (Pareto efficient) outcome of a strategic competitive framework (duopoly) with quantity-setting firms, thus coming not only from legislative rules.

A possible future development of this study is represented by Yang et al. (2015) about manufacturers' channel structures. In this regard, the theoretical analysis of codetermination in strategic competitive industries is still silent thus representing a promising research agenda.

Conflict of Interest The authors declare that they have no conflict of interest.

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