WEALTH INEQUALITY, FINANCIAL CRISES AND GOVERNMENT INTERVENTION IN A HETEROGENEOUS BANKING SYSTEM*

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Abstract

Does wealth inequality make financial crises more likely? To answer this question, we study a banking environment where strategic complementarities among wealth-heterogeneous depositors might trigger systemic self-fulfilling runs, and a government chooses taxes to subsidize banks and counteract them under different levels of commitment. We find that higher aggregate wealth makes systemic self-fulfilling runs less likely, but higher wealth inequality makes them more likely. Government intervention has an effect on the expectations of systemic self-fulfilling runs through a twofold effect on banks' balance sheets: a direct one through subsidies, and an indirect one due to banks anticipating the intervention and modifying maturity mismatch accordingly. A government intervention against bank illiquidity makes systemic self-fulfilling runs less likely and redistributes resources towards the poor. An intervention either time consistent or against bank insolvency makes systemic self-fulfilling runs more likely. Redistribution is larger under a time-consistent intervention than under an intervention against bank insolvency.

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1 Introduction

Does wealth inequality make financial crises more likely? If so, how can a government intervene, and how does this government intervention affect the distribution of resources in the economy? The aim of the present paper is to answer these questions in a theory of banking with wealth-heterogeneous depositors and systemic self-fulfilling runs.

The motivation for these questions comes from the debate over the role of wealth inequality in causing large financial crises. Kumhof et al. (2015) show that increasing wealth inequality preceded both the Great Depression and the Great Recession. The two most famous arguments to explain this observation (particularly regarding the latter episode) focus on the role played by government intervention. In fact, Stiglitz (2012) argues that higher wealth inequality depressed aggregate demand, forcing monetary authorities to lower interest rates too much for too long, thus fuelling a credit bubble and the following crisis. Similarly, Rajan (2010) maintains that higher wealth inequality called for some form of redistribution, and politicians promoted it by allowing households to collateralize their real-estate wealth, thus again fuelling a credit bubble and a crisis.

The present paper instead offers a different mechanism: wealth inequality exacerbates the probability of systemic self-fulfilling bank runs, which have always been considered a crucial element of financial crises.[1] The mechanism abstracts from government intervention, but goes through the self-fulfilling expectations of a systemic bank run itself: higher wealth inequality lowers the incentives to run of the rich, but increases more the incentives of the poor, thus increasing the probability of a systemic bank run overall.

To formalize our argument, our starting point is the seminal work by Diamond and Dybvig (1983). In it, banks provide insurance to their depositors against idiosyncratic shocks that force them to consume in an interim date, i.e. before their investments mature. To this end, banks engage in maturity mismatch: they issue short-term liabilities (i.e. deposits) backed by long-term assets. We modify this framework by assuming that the depositors are divided into groups that are homogeneous within themselves but heterogeneous between themselves with respect to per-capita wealth. The economy is also populated by a large number of banks. As wealth is observable,

[1] This goes back to the seminal work of Friedman and Schwartz (1963) on the US National Banking Era. Yet, self-fulfilling bank runs have been critical in several more recent financial crises, like in Ecuador in 1998, Argentina in 2001, Uruguay in 2002, Greece in 2015 and the Great Recession itself (Gorton, 2010).

the banks create separate accounts for each wealth group and offer group-specific (or equivalently wealth-specific) deposit contracts. To repay the depositors, the banks invest the deposits into a productive asset, which yields a positive return with some positive probability. This positive return represents the aggregate state of the economy, and negatively depends on the total fraction of depositors who withdraw in the interim date in the whole economy. In this way, we introduce an investment externality across wealth groups, which is similar to Morris and Shin (2000) and is in the spirit of the production externality of Romer (1990).

In such an environment, the depositors' decisions to withdraw in the interim period are subject to within-group strategic complementarities: the more a depositor expects the other depositors in her own wealth group to withdraw in the interim date, the higher her incentives to withdraw in the interim date are, too. Because of this, the economy exhibits two equilibria: one in which only the depositors who are hit by the idiosyncratic shocks withdraw in the interim date, and one in which all depositors withdraw because they expect everybody else to do the same, thus triggering a crisis in the form of a self-fulfilling "run". To characterize a unique equilibrium, we follow the "global game" literature (Carlsson and van Damme, 1993; Morris and Shin, 1998; Goldstein and Pauzner, 2005) and assume that each depositor observes a private noisy signal about the realization of the aggregate state. Based on this, a depositor forms posterior beliefs about the true aggregate state and the running behavior of the other depositors in her wealth group, and ultimately decides whether to run on her bank. This happens if the signal that she receives is lower than a certain wealth-specific threshold, which therefore is a measure of the financial fragility of each wealth group.

The presence of the investment externality has a twofold consequence on this mechanism. First, the strategic complementarities operate also between wealth groups. As such, all depositors must form posterior beliefs also about the running behavior of the other depositors in the whole economy. This represents a theoretical challenge, that we solve by adapting to this framework the concept of "Belief Constraint" (Sakovics and Steiner, 2012): it is optimal for a depositor to have "agnostic" beliefs about the running decisions of the depositors in the whole economy, i.e. assign an equal probability to every possible realization of the collective action. The second consequence of the investment externality is that the wealth-specific thresholds that trigger a run are all functions one of the others. Yet, as the volatility of the noisy signals goes to zero all thresholds cluster around a unique value, that summarizes the average incentives to run of the depositors in the whole economy. In other words, a self-fulfilling run becomes “systemic”: depositors with different wealth run together, following a common threshold strategy. This unique threshold, which is indeed a measure of systemic financial fragility, turns out to be an increasing function of the terms of the deposit contracts offered to all wealth groups, and in particular is increasing in the endogenous maturity mismatch in the banks' balance sheets. This means that this economy features financial contagion through expectation formation: high maturity mismatch in a bank's balance sheets increases the likelihood of a systemic self-fulfilling run in the whole economy. Moreover, the unique threshold turns out to be a decreasing and convex function of depositors' wealth. Thus, increasing aggregate wealth while keeping wealth inequality constant lowers systemic financial fragility. In contrast, increasing wealth inequality while keeping aggregate wealth constant increases systemic financial fragility. In fact, increasing the wealth of the rich lowers their incentives to run, but lowering the wealth of the poor increases more their incentives to run. Thus, the average incentives to run are higher than before.

The presence of systemic financial fragility justifies a government intervention against self-fulfilling runs, which are inefficient because are not based on bad fundamentals (Allen and Gale, 2004b). We assume the existence of an economy-wide government that, in order to maximize welfare, taxes wealth outside the banking system (that the depositors could consume if not taxed) and provides subsidies to affect the depositors' incentives to run. We analyze interventions under different levels of government commitment.

When a government cannot commit and can only intervene in a time-consistent manner, i.e. after a systemic self-fulfilling run has taken place and the banks have gone insolvent, the intervention is purely redistributive: richer depositors pay higher taxes and receive lower subsidies than poor ones, so that the marginal costs of the intervention as well as its marginal benefits are equal across wealth groups. This intervention has the effect of increasing systemic financial fragility in two ways. First, by anticipating that they will receive a subsidy at bank insolvency, the depositors are less afraid of the consequences of withdrawing in the interim date. Second, the banks, anticipating higher systemic financial fragility, rebalance the depositors' expected welfare by increasing the amount of insurance against the idiosyncratic shocks. In doing so, they do not fully internalize their influence on the probability of a systemic self-fulfilling run and increase the maturity mismatch in their balance sheets, thus further increasing systemic financial fragility.

When instead the government can commit to an intervention against bank insolvency, the only difference with the time-consistent case is in the allocation of the subsidies, that now takes into account that increasing them has a increasing effect on systemic financial fragility. Then, the government tries to minimize this by partially subsidizing the rich depositors, whose incentives to run are less sensitive to subsidization. Hence, the redistributive impact of an intervention with commitment against bank insolvency is lower than that of a time-consistent intervention.

We also study the commitment to intervene against bank illiquidity. Bearing in mind that illiquidity is not a possible ex-post outcome, a government intervention can nevertheless have an effect on the formation of the depositors' expectations of a run, and therefore on the equilibrium. Under this intervention, subsidies to banks lower systemic financial fragility in two ways. First, they allow banks to retain a larger amount of resources to distribute to the depositors in the final date, thus lowering their incentives to run. Second, the anticipation of lower systemic financial fragility allows the banks to reduce the maturity mismatch in their balance sheets, thus further reducing systemic financial fragility. This means that a commitment to a full liquidity assistance, that allows the banks to serve all depositors even when they all withdraw in the interim date, can rule out systemic self-fulfilling runs altogether. However, this is subject to an explicit feasibility condition, whose tightness positively depends on the depositors' relative risk aversion. In other words, when feasibility is satisfied, no depositor has incentives to run, and the government, just by announcing a commitment to intervene, resolves systemic financial fragility at zero costs. Therefore, when feasibility is satisfied, this is the optimal policy that the government can implement.

When instead a full liquidity assistance is not feasible, the government can still announce a commitment to a partial intervention against bank illiquidity and lower systemic financial fragility. The intervention consists of a set of subsidies that takes into account as before that the rich depositors' incentives to run are less sensitive to subsidization. Hence, a commitment to intervene against bank illiquidity results in an intervention in which poor depositors would receive higher subsidies than rich ones. However, as previously mentioned, bank illiquidity is not a possible ex-post equilibrium outcome, so the government announces its commitment to intervene but never implements the intervention. Moreover, the resulting redistribution would not depend on a mere welfare motivation as in a time-consistent intervention, but on the "trickle-up" effect of lowering systemic financial fragility for the whole economy.

The rest of the paper is organized as follows: in section 2, we summarize the contribution to the literature; in section 3, we lay down the environment of the model; in section 4, we study the strategic complementarities in the depositors' decisions to run, and the banking equilibrium without government intervention; in section 5, we characterize the government intervention scheme; finally, section 6 concludes.

2 Contribution to the Literature

The present paper contributes to the literature in several respects. First, by developing a theory of a heterogeneous banking system, this paper is one of the first – to the best of our knowledge – to explicitly study how systemic financial fragility is connected to wealth heterogeneity, which some new evidence suggests is a key driver of depositors' withdrawing behavior (Iyer et al., 2015). Importantly, this link is not conveyed through credit bubbles, which is a channel that, although appealing, arguably applies well to the US (Kumhof et al., 2015) but is far from general.[2] In contrast, our focus is based on financial contagion (Allen and Gale, 2000; Aghion et al., 2000; Freixas et al., 2000; Diamond and Rajan, 2005; Brusco and Castiglionesi, 2007; Allen et al., 2012) in particular through expectation formation, which has been analyzed in the past in two-group environments (Corsetti et al., 2004; Dasgupta, 2004; Goldstein, 2005; Leonello, 2018; Ahnert and Georg, 2018).[3] In this literature, the work more closely related to ours is Choi (2014). In it, the author studies an economy in which banks are heterogeneous with respect to their vulnerability to strategic risk. Then, a "stronger" bank's concern about financial fragility induces a "weaker" bank to disinvest preemptively, which in turns self-confirms the initial stronger bank's concern and leads to a systemic crises. In such a scheme, it makes sense for a government to contrast systemic risk by bolstering the strongest bank in the contagion chain. Moreover, systemic risk is lower the more heterogeneous the banking system is, because coordination issues are less severe among heterogeneous banks. In contrast, in our framework we explicitly focus on wealth as the

[2] Atkinson and Morelli (2010, 2015) and Bordo and Meissner (2012) find little evidence of a connection between inequality, household credit bubbles and financial crises, and Gu and Huang (2014) find that the relation holds only in Anglo-Saxon countries. See Van Treeck (2014) and Bazillier and Hericourt (2017) for two literature surveys on inequality, credit booms and financial crises.

[3] The existing empirical evidence on financial contagion through the expectations of self-fulfilling crises is mainly based on mutual funds (Chen et al., 2010; Schmidt et al., 2016) and finds a strong presence of strategic complementarities among heterogeneous investors.

source of heterogeneity, and show how this results in systemic financial fragility that depends on the average fragility of the whole banking system. Thus, a government intervention (in particular, the one against bank illiquidity) should focus on the trickle-up effect of subsidizing more the poor depositors, whose incentives to run are more sensitive to subsidization than rich ones'.

Second, our work contributes to the analysis of the economics of government intervention in the face of self-fulfilling uncertainty. In a recent paper, Allen et al. (2018) extend the bank-run framework of Goldstein and Pauzner (2005) by introducing a benevolent regulator who provides a bank guarantee. However, the authors study an homogenous economy, which is not suitable to analyze financial contagion in a heterogeneous banking system and the redistributive implications of government intervention. Cooper and Kempf (2016) and Mitkov (2016) instead independently develop a banking model with wealth heterogeneity, and study taxation and redistribution after a self-fulfilling run. However, they only analyze self-fulfilling runs as sunspot-driven coordination failures. In other words, in their environments systemic financial fragility is exogenous by assumption. Additionally, in their models wealth inequality has an effect on systemic financial fragility only because of the way the government intervenes, otherwise there is no financial contagion across wealth groups. In this way, they miss the direct causal link between wealth inequality and systemic financial fragility, which is at the core of our mechanism.

More generally, our work contributes to the literature on government intervention and financial crises. The main message of this literature (Schneider and Tornell, 2004; Acharya and Yorulmazer, 2007; Bianchi, 2016; Chari and Kehoe, 2016; Keister, 2016) is that a time-consistent government intervention in the aftermath of a financial crisis, while being optimal from an ex-post perspective, creates anticipation in the financial system, thus fuelling risk taking and systemic risk. In particular, the work of Farhi and Tirole (2012) argues that the anticipation of a time-consistent bailout creates strategic complementarities in the leverage decisions of the banks and in maturity mismatch. Differently from them, our direct link between wealth inequality and systemic financial fragility abstracts from government intervention. Moreover, in the present environment a government intervention against insolvency has the effect of increasing systemic financial fragility both indirectly, by affecting maturity mismatch as in their paper, and directly through the subsidies. Importantly, our two channels are present irrespective of the level of commitment of the government, as long as it intervenes only against bank insolvency.

Finally, the present paper contributes to the theoretical literature on bank runs as "global games" (Morris and Shin, 2000; Rochet and Vives, 2004; Goldstein and Pauzner, 2005) by studying the role of wealth heterogeneity and adapting the concept of Belief Constraint. Sakovics and Steiner (2012) characterize the Belief Constraint and apply it to a canonical problem of investment subsidization. They find that a regulator who wants to maximize investments should subsidize more the agents who have a relatively large influence on the investment decisions of the others, and at the same time are relatively less sensitive to them. Drozd and Serrano-Padial (2018) instead study a model of a debt-financed entrepreneur subject to enforcement externalities. Theoretically, their contribution lies in the characterization of an equilibrium in which the threshold strategies of the agents, differently from our work and from Sakovics and Steiner (2012), might cluster around more than one value.

3 A Model of a Heterogeneous Banking System

3.1 Preferences and Endowments

The economy lives for three dates, labeled $t = 0, 1, 2$, and is populated by a unitary continuum of agents, divided into G groups indexed by j, each of equal mass. The groups are heterogeneous with respect to wealth: all agents in group j have an initial endowment e^j at date 0, $\bar{e}^j$ at date 1, and nothing at date 2. At date 1, an agent i in wealth group j is hit by a private idiosyncratic shock θ^{ij}, that takes value 0 with probability $1 - \pi$ and 1 with probability π. The shock affects the point in time at which the agent wants to consume, in accordance with the welfare function:

$$U(c_1^j, c_2^j, \theta^{ij}) = \theta^{ij} u(c_1^j) + (1 - \theta^{ij}) u(c_2^j) + w(\bar{e}^j). \tag{1}$$

The agents gain utility from consumption either at date 1 or at date 2, and from the extra endowment $\bar{e}^j$ that they receive at date 1.[4] If $\theta^{ij} = 1$, the agent only wants to consume at date 1, while if $\theta^{ij} = 0$ she only wants to consume at date 2. Thus, in line with the literature, we call type-0 and type-1 agents late (or "patient") consumers and early (or "impatient") consumers,

[4]The assumption that wealth directly enters welfare in an additive-separable manner is not to ensure a positive marginal propensity to save as in Kumhof et al. (2015), but to allow the banking problem to be independent of the financing of the government intervention, as we show in section 5. Furthermore, contrary to Allen et al. (2018), no assumption rules out direct transfers to the depositors, as we will study them in section 5.1.

respectively. The law of large numbers holds, so π and $1-\pi$ are the fractions of agents in the whole economy who turn out to be early or late consumers. The utility functions $u(c)$ and $w(c)$ are twice continuously differentiable, increasing and concave. Moreover, $u(c)$ has a coefficient of relative risk aversion greater than 1, $u(0) = w(0) = 0$ and the Inada conditions hold: $\lim_{c\to 0} u'(c) = +\infty$ and $\lim_{c\to +\infty} u'(c) = 0$.[5]

3.2 Banks and Technologies

Each wealth group is served by a large number of competitive banks. The relationship between depositors and banks is mutually exclusive: a depositor can only deposit her endowments into one bank, and a bank can only collect deposits from one wealth group. This latter assumption is only for convenience and bears no effect on the relation between inequality and systemic financial fragility, which is going to be the center of the analysis of the next section.[6] At date 0, the banks collect the initial endowments e^j of the agents/depositors – which are the only liability on their balance sheets – and invest them so as to maximize their profits, subject to depositors' participation and to budget constraints. Perfect competition ensures that the banks solve the equivalent dual problem of maximizing the expected welfare of their depositors subject to budget constraints.

The banks invest the deposits in a common productive asset yielding a stochastic return A at date 2 for each unit invested at date 0. This stochastic return takes values $R(1-\ell)$ with probability p, and 0 with probability $1-p$, where ℓ is the total fraction of depositors who withdraw at date 1 in the whole economy. The probability of success of the productive asset p represents the aggregate state of the economy, and is distributed uniformly over the interval $[0,1]$, with $(1-\pi)\mathbb{E}[p]R > 1$. Moreover, the productive asset can be liquidated at date 1, i.e. before its natural maturity, and yields 1 unit of consumption for each unit liquidated. Intuitively, this productive asset represents an investment opportunity whose return in case of success depends on how much of the initial

[5] A typical utility function satisfying these assumptions is the CRRA function $u(c) = ((c+\psi)^{1-\gamma} - \psi^{1-\gamma})/(1-\gamma)$, with $\gamma > 1$. The constant ψ can be interpreted as an endowment that the agents did not deposit in the banks. It ensures that $u(0) = 0$, but it should be arbitrarily close to zero for the coefficient of relative risk aversion to be constant and equal to γ. In this way, $\lim_{c\to 0} u'(c) = \psi^{-\gamma}$, which is arbitrarily large but finite.

[6] On top of that, the assumption of wealth-specific banks is not without basis. In fact, historically bank segmentation has been an intentional choice of the regulator. For example, in Japan different financial institutions had specified services or classes of customers that they could serve (Ito, 1992). Yet, even after the market liberalization of the Eighties a substantial part of the banking industry has kept its specialization, either because of demand-driven factors such as asymmetric information giving rise to relationship banking (DeYoung, 2009), or because of supply-driven factors leading to banks' comparative advantages in serving specific customers (Paravisini et al., 2017).

investment reaches maturity in the whole economy. Put differently, a common productive asset exhibits an investment externality across wealth groups.[7]

The banks employ the productive asset to repay the depositors. To this end, the banks offer group-specific (or – equivalently – wealth-specific) standard deposit contracts, stating the uncontingent amount d^j that the depositors can withdraw at date 1 and the state-dependent amount $d_L^j(A)$ that they can withdraw at date 2, which is an equal share of the residual available resources.[8] As the realizations of the idiosyncratic shocks θ^{ij} are private information, the depositors must have the incentives to truthfully report them. This implies that the deposit contracts must satisfy the incentive compatibility constraint $d^j \leq d_L^j(A)$ in every group j. The banks commit to the deposit contracts at date 0, and pay early withdrawals by liquidating the productive asset until their resources are exhausted. When this happens, and the banks are not able to fulfill their contractual obligations, they go into insolvency. In this case, they must liquidate all the productive assets at date 1, and equally share the proceeds among all the depositors who withdraw early before closing down.[9]

We assume that the depositors cannot observe the true value of the realization of the aggregate state p, but receive at date 1 a noisy private signal $\sigma^{ij} = p + \eta^{ij}$. The term η^{ij} is an idiosyncratic noise, indistinguishable from the true value of p and drawn from a uniform distribution over the interval $[-\epsilon, +\epsilon]$, with ϵ positive but negligible. Given the received signal, each late consumer decides whether to withdraw from her bank at date 2, as the realization of her idiosyncratic shock would command, or "run on her bank" and withdraw at date 1, in accordance with the scheme to be described in the next section.

3.3 Timing and Definitions

The timing of actions is the following: at date 0, the banks collect the initial endowments, and choose the deposit contracts $\{d^j, d_L^j(A)\}$; at date 1, all depositors get to know their private types and signals, and the early consumers withdraw, while the late consumers, once observed their own

[7]In Appendix A we show that this economy is qualitatively similar to one with asset fire sales and a pecuniary externality.

[8]In order to rule out uninteresting run equilibria, the amount of early consumption d^j must be smaller than $\min\{1/\pi, R\}$. The fact that the banks have to offer a standard deposit contract here is assumed. Farhi et al. (2009) show that a standard deposit contract, with an uncontingent amount of early consumption, endogenously emerge as part of the banking equilibrium, in the presence of non-exclusive deposit contracts.

[9]The assumption of equal shares at insolvency simplifies the analysis without altering its results.

signals, decide whether to run on their banks or not; finally, at date 2, those late consumers who have not run at date 1 receive an equal share of the available resources.

We solve the model by backward induction, and characterize a perfect Bayesian equilibrium, where a representative bank in each wealth group chooses wealth-specific deposit contracts, and the late consumers decide whether to run in accordance with the threshold strategy:[10]

$$a^{ij}(\sigma) = \begin{cases} \text{wait} & \text{if } \sigma^{ij} \geq \sigma^{j*}, \\ \text{run} & \text{if } \sigma^{ij} < \sigma^{j*}. \end{cases} \tag{2}$$

The definition of equilibrium is as follows:

Definition 1 *Given the distributions of the idiosyncratic and aggregate shocks and of the private signals, a perfect Bayesian banking equilibrium is a set of deposit contracts* $\{d^j, d^j_L(A)\}$ *and depositors' threshold strategies, such that for every realization of signals and idiosyncratic shocks* $\{\sigma^{ij}, \theta^{ij}\}$*:*

- *the depositors' decisions to run maximize their expected welfare;*
- *the deposit contract maximizes the depositors' expected welfare, subject to budget constraints;*
- *the beliefs of the banks and depositors are updated according to the strategies employed and the Bayes rule.*

3.4 Banking Equilibrium with Perfect Information

As a benchmark for the results that follow, we start our analysis with the characterization of the banking equilibrium with perfect information, in which the representative banks can observe the realization of the private idiosyncratic shocks hitting the depositors. More formally, for each wealth group j a representative bank solves:

$$\max_{d^j} \ \pi u(d^j) + (1-\pi) \int_0^1 pu\left(R(1-\pi)\frac{e^j - \pi d^j}{1-\pi}\right) dp. \tag{3}$$

[10] Selecting threshold strategies comes at no loss of generality, as Goldstein and Pauzner (2005) show in a similar environment that every equilibrium strategy is a threshold strategy.

The bank knows that, with probability π, a depositor will turn out to be an early consumer and consume d^j and, with probability $1-\pi$, she will turn out to be a late consumer.[11] In this case, the total amount of available resources at date 2 depends on the realization of the aggregate state p, on the total number of late consumers in the whole economy, equal to $1-(1/G)\sum_j \pi = 1-\pi$, and on the amount of productive assets that are not liquidated to pay early consumption, $e^j - \pi d^j$. The first-order condition with respect to early consumption d^j gives the equilibrium condition:

$$u'(d^j) = (1-\pi)\mathbb{E}[p]Ru'(R(e^j - \pi d^j)). \tag{4}$$

Intuitively, this result shows that the bank provides an allocation such that the marginal rate of substitution between early and late consumption is equal to the expected return of the productive asset (equivalent to the expected marginal rate of transformation of a production technology). Moreover, as the utility function $u(c)$ is concave, the equilibrium amounts d^j and $d^j_L(R) = R(e^j - \pi d^j)$ are both increasing in the initial endowment e^j,[12] even if the ratio d^j/e^j is constant across wealth groups. In fact, by inverting marginal utility and rearranging the Euler equation:

$$d^j = \frac{u'^{-1}((1-\pi)\mathbb{E}[p]R)R}{1+\pi Ru'^{-1}((1-\pi)\mathbb{E}[p]R)}e^j. \tag{5}$$

Finally, the concavity of the utility function and the assumption that $(1-\pi)\mathbb{E}[p]R > 1$ imply that the incentive compatibility constraint is satisfied. In other words, a banking equilibrium without perfect information, i.e. in which a bank needs to ensure truth-telling, would be equivalent to the banking equilibrium with perfect information.

4 Systemic Self-fulfilling Runs

We now move to the analysis of the banking equilibrium in the presence of private signals regarding the aggregate state of the economy. To this end, we go by backward induction, and start by studying the Bayesian Nash equilibrium of the stage game in which the depositors choose their threshold

[11] In equilibrium, by the Inada conditions, both early and late consumption must be positive.

[12] To see that d^j is increasing in e^j, notice that the objective function is supermodular, as its cross derivative with respect to d^j and e^j is positive (see the definition of supermodular function in footnote 22). Also, with a simple change of variable, namely by letting $x^j = e^j - \pi d^j$, we can show that the objective function is supermodular in (x^j, e^j). This is equivalent to saying that $d^j_L(R) = R(e^j - \pi d^j)$ is increasing in e^j.

strategies according to which they run.

As in Ennis and Keister (2006), we assume that at date 1 the depositors arrive at the bank in random order, and know neither how many of them are in line nor their positions in the line. As a result, the depositors do not accept a contract contingent on either their position in line or the number of early withdrawals. Due to its commitment to pay an amount of early consumption d^j, the bank must liquidate the productive asset to pay early withdrawals until the resources are exhausted. As a consequence, if a late consumer expects only the early consumers to withdraw at date 1, she will withdraw at date 2 and receive the incentive-compatible consumption $d_L^j(R) > d^j$. However, if a late consumer expects all the other depositors to withdraw at date 1, she will rather withdraw at date 1 as well, because in that case she will be served pro-rata at date 1 instead of getting zero at date 2. This means that this economy, as any Diamond-Dybvig environments, features a “no run” equilibrium and a “run” equilibrium.[13]

As we will show, the private signals allows us to resolve the multiplicity by forcing the depositors to coordinate their actions: run under some range of signals, and not run under another. The effect of the signals is twofold: they provide private information about the aggregate state of the economy, and about the signals of the other depositors. Intuitively, obtaining a high signal increases the incentives for a late consumer to wait until date 2 and not withdraw (i.e. not “run on her bank”) at date 1, because it induces the belief that the realization of the aggregate state is good, and the signals of the other depositors are also high (under the assumption that the volatility of the signal is negligible).

More formally, a late consumer i in group j receives a private signal σ^{ij} at date 1, and takes as given the deposit contract fixed at date 0. Based on these, she creates her posterior beliefs about how many depositors withdraw at date 1 in her own group as well as in the whole economy, and the probability of the realization of the aggregate state, and decides whether to withdraw at date 1 or not. We assume the existence of two regions of extremely high and extremely low signals, where the decision of a late consumer is independent of her beliefs. In the “lower dominance region”, the signal is so low that a late consumer always runs. This happens below the threshold signal $\underline{\sigma}^j$, that

[13]For this argument to hold, we need to assume that a government cannot credibly commit to suspend deposit convertibility in the case of a run. Ennis and Keister (2009) and Keister (2016) study the time inconsistency of suspension policies in a banking model with multiple equilibria.

makes her indifferent between withdrawing or not, and is defined by:

$$u(d^j) = \underline{\sigma}^j u\left(R(e^j - \pi d^j)\right). \tag{6}$$

From here, it is easy to see that the threshold signal $\underline{\sigma}^j$ is decreasing in the initial endowment e^j and increasing in the early consumption d^j: the more a bank promises to an early consumer in group j, the larger is the set of signals below which the depositors in that group run irrespective of what the others do. In the "upper dominance region", instead, the signal is so high that a late consumer always wait until date 2 to withdraw. Following Goldstein and Pauzner (2005), we assume that this happens above a threshold $\bar{\sigma}^j$, where the investment is safe, i.e. $p = 1$, and gives the same return $R(1-\pi)$ at date 1 and date 2. In this way, a late consumer is sure to get $R(e^j - \pi d^j)$ at date 2, irrespective of the behavior of all the other late consumers, and prefers to wait.

The existence of the lower and upper dominance regions, regardless of their size, ensures the existence of an equilibrium in the intermediate region $[\underline{\sigma}^j, \bar{\sigma}^j]$, where the late consumers decide whether to run or not based on their posterior beliefs. In this region, a late consumer runs if her signal is lower than a threshold signal σ^{j*}, which is the value of the signal that makes her indifferent between running or not given her beliefs. More formally, define the utility advantage of waiting versus running as:

$$v^j(n, n^j) = \begin{cases} \sigma^{ij} u\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right) - u(d^j) & \text{if } \pi \leq n^j < \frac{e^j}{d^j}, \\ -u\left(\frac{e^j}{n^j}\right) & \text{if } \frac{e^j}{d^j} \leq n^j \leq 1, \end{cases} \tag{7}$$

where n^j and n are the total fraction of depositors withdrawing at date 1 in group j and in the whole economy, respectively. By the law of large numbers, these fractions are given by:

$$n^j = \pi + (1-\pi)\text{prob}(\sigma^{ij} \leq \sigma^{j*}), \tag{8}$$

$$n = \sum_k n^k = \pi + (1-\pi)\sum_k \text{prob}\{\sigma^{ik} \leq \sigma^{k*}\}, \tag{9}$$

i.e. the fraction of depositors withdrawing at date 1 is the sum of the π early consumers who withdraw for sure plus those among the $1-\pi$ late consumers who get a signal below the threshold signal σ^{j*}.

The expression for $v^j(n, n^j)$ highlights that, when the fraction of depositors running is between π (i.e., when there is no run) and e^j/d^j (i.e. the maximum fraction of depositors that a bank in wealth group j can serve according to the contract with the available resources), a late consumer receiving a signal σ^{ij} holds the belief that the productive asset yields a positive return with probability $\mathbb{E}[p] = \mathbb{E}[\sigma - \eta^{ij}] = \sigma^{ij}$. In that case, if she waits until date 2, she consumes either $d_L^j(R, n, n^j) = R(1-n)\frac{e^j - n^j d^j}{1-n^j}$ or $d_L^j(0, n, n^j) = 0$, and if she withdraws she consumes d^j. In contrast, when the fraction of depositors running is higher than e^j/d^j, the representative bank of wealth group j goes into insolvency: it is forced to liquidate all productive assets and equally share the proceeds among the depositors who withdraw. Hence, a late consumer gets zero if she waits, and e^j/n^j if she withdraws at date 1.

The function $v^j(n, n^j)$ exhibits both between- and within-group strategic complementarities. To see that, calculate:

$$\frac{\partial v^j}{\partial n^{\ell \neq j}} = \begin{cases} -R\sigma^{ij} u'\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right)\frac{e^j - n^j d^j}{1-n^j} & \text{if } \pi \leq n^j < \frac{e^j}{d^j}, \\ 0 & \text{if } \frac{e^j}{d^j} \leq n^j \leq 1, \end{cases} \tag{10}$$

and notice that the derivative in the first interval is always negative. As far as the within-group strategic complementarity, instead:

$$\frac{\partial v^j}{\partial n^j} = \begin{cases} R\sigma^{ij} u'\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right)\left[-\frac{e^j - n^j d^j}{1-n^j} + (1-n)\frac{e^j - d^j}{(1-n^j)^2}\right] & \text{if } \pi \leq n^j < \frac{e^j}{d^j}, \\ u'\left(\frac{e^j}{n^j}\right)\frac{e^j}{n^{j2}} > 0 & \text{if } \frac{e^j}{d^j} \leq n^j \leq 1. \end{cases} \tag{11}$$

Again, the derivative in the first interval is negative (i.e. we have one-sided strategic complementarity) as $n^j < 1$.

Given the function $v^j(n, n^j)$, we derive the threshold signal σ^{j*} as the value of the signal such that $\mathbb{E}[v^j(n, n^j)|\sigma^{j*}] = 0$, or the one solving:

$$\int_\pi^1 \int_\pi^{\frac{e^j}{d^j}} \sigma^{j*} u\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right) dn^j dn = \int_\pi^1 \left[\int_\pi^{\frac{e^j}{d^j}} u(d^j) dn^j + \int_{\frac{e^j}{d^j}}^1 u\left(\frac{e^j}{n^j}\right) dn^j\right] dn. \tag{12}$$

This gives:

$$\sigma^{j*} = \frac{(1-\pi)\left[\int_{\pi}^{\frac{e^j}{d^j}} u(d^j)dn^j + \int_{\frac{e^j}{d^j}}^{1} u\left(\frac{e^j}{n^j}\right)dn^j\right]}{\int_{\pi}^{1}\int_{\pi}^{\frac{e^j}{d^j}} u\left(R(1-n)\frac{e^j-n^jd^j}{1-n^j}\right)dn^j dn}, \tag{13}$$

for every group $j = 1, \ldots, G$. In a similar problem with a global game among heterogeneous agents, Frankel et al. (2003) show that, as the noise ϵ of the signals vanishes, there exists a unique threshold signal σ^* around which the thresholds signals σ^{j*} tend to cluster, which is the solution to the system of equation of (13) for every group j. However, finding a solution to that system is cumbersome, as the expressions for σ^{j*} are highly non-linear. Instead, we bypass the problem by applying the concept of "Belief Constraint" (Sakovics and Steiner, 2012). This leads to the following Proposition:

Proposition 1 *The set of equilibrium threshold strategies characterizing the withdrawing decisions of the depositors is unique. As the volatility of the noise ϵ goes to zero, all threshold signals σ^{j*} converge to a common limit σ^*, which is characterized by the average indifference condition:*

$$\sum_j \mathbb{E}[v^j(n,n^j)|\sigma^*] = 0, \tag{14}$$

and gives:

$$\sigma^*(\mathbf{d}) = \frac{(1-\pi)\sum_j\left[\int_{\pi}^{\frac{e^j}{d^j}} u(d^j)dn^j + \int_{\frac{e^j}{d^j}}^{1} u\left(\frac{e^j}{n^j}\right)dn^j\right]}{\sum_j\int_{\pi}^{1}\int_{\pi}^{\frac{e^j}{d^j}} u\left(R(1-n)\frac{e^j-n^jd^j}{1-n^j}\right)dn^j dn}, \tag{15}$$

where $\mathbf{d} = \{d^j\}_{j=1}^G$.

Proof. In Appendix B. ■

Intuitively, the proof of the Proposition can be summarized as follows. In principle, every group j should have its own threshold signal σ^{j*} below which a signal triggers a self-fulfilling run. This threshold signals should be characterized by the wealth-specific indifference conditions for a late consumer between withdrawing early and waiting, given her beliefs. However, the presence of between-group strategic complementarities implies that the running behavior of a late consumer in a group j influences the running behavior of the late consumers in all the other groups, too.

That would mean that we should solve for the groups-specific threshold signals σ^{j*} by solving a system of G indifference conditions in G unknowns. However, as the volatility of the noise ϵ goes to zero, all depositors tend to form the same posterior beliefs about the aggregate state. Moreover, the depositors have to form posterior beliefs about the behavior of all the other depositors, in their own group as well as in the others. The Laplacian Property (Morris and Shin, 1998) ensures that the cumulative distribution functions of the random signals σ^{ij} in all groups j are uniformly distributed over the interval $[0, 1]$. Hence, the fraction of depositors withdrawing early in group j, which is given by (8), is a random variable uniformly distributed over the interval $[\pi, 1]$, and its probability distribution function is $f(n^j) = 1/(1 - \pi)$.

To characterize the distribution of the total fraction of depositors running in the whole economy, we instead adapt to our environment the concept of "Belief Constraint" of Sakovics and Steiner (2012). The Belief Constraint shows that the Laplacian Property holds on average. Hence, the total fraction of depositors withdrawing early in the whole economy, as given by (9), is also a random variable uniformly distributed over the interval $[\pi, 1]$, as the average cumulative distribution function of the signals is uniformly distributed over the interval $[0, 1]$. In other words, given their signals all depositors tend to assign the same probability to the future realization of the aggregate state and are all agnostic about how many depositors run in their own wealth group as well as in the others. Thus, all their threshold signals σ^{j*} must cluster around a common threshold signal $\sigma^*(\mathbf{d})$, which uniquely determines the probability of a systemic self-fulfilling run occurring in the economy. The characterization of this value should come from the solution of a system of G indifference conditions in 1 unknown. Then, to perfectly identify the system and solve for the common threshold signal σ^* we average the indifference conditions across all wealth groups.

Importantly, the common threshold signal $\sigma^*(\mathbf{d})$ depends on the deposit contracts chosen by the representative banks of all wealth groups. The following Corollary sheds light on this relationship:

Corollary 1 *The threshold signal $\sigma^*(\mathbf{d})$ is an increasing function of every d^j.*

Proof. In Appendix B. ■

This result highlights the channels of financial contagion from one wealth group to the rest of the economy via expectation formation: as a representative bank in a wealth group promises a higher amount of early consumption to its depositors, the latter anticipate that the bank might not be able to serve them all, in the case of a systemic self-fulfilling run. In addition to that, also the depositors in the other wealth groups internalize the fact that a run in one wealth group might reduce the return on the productive asset, and force their own banks to also go into insolvency, too. Hence, the range of signals for which a systemic self-fulfilling run occurs increases with the early consumption offered in any wealth group.

Finally, the expression for the endogenous threshold signal $\sigma^*(\mathbf{d})$ in (15) allows us to study the connection between inequality and the probability of a systemic self-fulfilling run. The following Corollary is instrumental to this end:

Corollary 2 *The threshold signal $\sigma^*(\mathbf{d})$ is a decreasing and convex function of the initial endowment e^j.*

Proof. In Appendix B. ■

In the proof of the Corollary, we show that increasing e^j has two effects on the threshold signal $\sigma^*(\mathbf{d})$. On the one hand, a higher e^j means that at insolvency a depositor of wealth group j receives a higher liquidation value, and this increases $\sigma^*(\mathbf{d})$. On the other hand, a higher e^j increases the consumption of a late consumer who does not run at illiquidity, i.e. when the fraction of depositors running in her wealth group lies in the interval $[\pi, e^j/d^j]$, and this lowers $\sigma^*(\mathbf{d})$. This second channel dominates and its dominance increases with e^j: with a higher e^j those late consumers not running just before insolvency (i.e. when n^j approaches e^j/d^j) consume a positive amount instead of zero, and this has a large and increasing effect on their marginal utility by the Inada conditions. Hence, the threshold signal $\sigma^*(\mathbf{d})$ is decreasing and convex in e^j: ceteris paribus higher initial endowments non-linearly lower the probability of a systemic self-fulfilling run. Moreover, this mechanism is independent of the assumptions made regarding the structure of the banking system, in particular of the presence of one representative bank for each wealth group. With this result in hand, we can study how the inequality in the distribution of the initial endowments impacts the probability of a systemic self-fulfilling run. For this, assume an increase in inequality: marginally increase the endowment e^k for a wealth group k and lower for the same amount the endowment e^ℓ for another wealth group for which $e^\ell < e^k$ so that the aggregate initial endowment $\sum_j e^j$ remains constant.

The effect of this change on $\sigma^*(\mathbf{d})$ is represented by the total differential:

$$d\sigma^*(\mathbf{d}) = \frac{\partial\sigma^*(\mathbf{d})}{\partial e^k} de^k + \frac{\partial\sigma^*(\mathbf{d})}{\partial e^\ell} de^\ell = \left[\frac{\partial\sigma^*(\mathbf{d})}{\partial e^k} - \frac{\partial\sigma^*(\mathbf{d})}{\partial e^\ell}\right] de^k. \tag{16}$$

As the threshold signal is a decreasing convex function of the initial endowment e^j, the increasing effect on σ^* induced by a low e^ℓ is larger than the decreasing effect on σ^* induced by a high e^k.[14]

Proposition 2 *Higher aggregate initial endowment ceteris paribus leads to a lower probability of a systemic self-fulfilling run. Higher inequality in the distribution of the initial endowments instead leads ceteris paribus to a higher probability of a systemic self-fulfilling run.*

Proof. In the text above. ■

4.1 Banking Equilibrium

Having characterized the endogenous threshold strategy played by the late consumers at date 1, in this section we proceed by backward induction and determine the deposit contract offered by the representative bank in each wealth group at date 0. To this end, the bank solves the following problem:

$$\max_{d^j} \int_0^{\sigma^*(\mathbf{d})} u(e^j)dp + \int_{\sigma^*(\mathbf{d})}^1 \left[\pi u(d^j) + (1-\pi)pu\left(R(e^j - \pi d^j)\right)\right] dp. \tag{17}$$

Whenever the signal is between 0 and $\sigma^*(\mathbf{d})$ a systemic run happens, and all depositors receive the pro-rata return from the liquidation of the productive assets available in portfolio. When instead the signal is between $\sigma^*(\mathbf{d})$ and 1, no systemic run happens, and the depositors turn out to be early consumers with probability π and late consumers with probability $1-\pi$, as in the banking equilibrium with perfect information.

To complete the characterization of the banking equilibrium, define the welfare gain from avoiding a run in a wealth group j when a depositor i receives a signal $\sigma^{ij} = \sigma^*(\mathbf{d})$ as:

$$\Delta U^j = \pi u(d^j) + (1-\pi)\sigma^*(\mathbf{d})u(R(e^j - \pi d^j)) - u(e^j), \tag{18}$$

which is decreasing in the initial endowment e^j as the effect of a higher e^j on the threshold signal

[14] The relaxation of the assumption of wealth-specific banks would not affect this result in any way. In fact, as the initial endowments are observable, a "universal" bank serving all depositors irrespective of their wealth would still offer them wealth-specific deposit contracts.

$\sigma^*(\mathbf{d})$ is large and negative, as showed in Corollary 2. Then, the first-order condition with respect to d^j implicitly determines the equilibrium deposit contract:

$$\pi \int_{\sigma^*}^{1} \left[u'(d^j) - (1-\pi)pRu'\left(R(e^j - \pi d^j)\right)\right] dp = \frac{\partial \sigma^*(\mathbf{d})}{\partial d^j} \Delta U^j. \tag{19}$$

This Euler equation highlights that the endogeneity of the threshold signal $\sigma^*(\mathbf{d})$ forces the bank to impose a wedge between the marginal rate of substitution between early and late consumption and the expected return on the productive asset. To see that more clearly, rewrite (19) in terms of the marginal rate of substitution:

$$\begin{aligned} MRS^j \equiv \frac{u'(d^j)}{u'\left(R(e^j - \pi d^j)\right)} =& \frac{1}{\pi(1-\sigma^*(\mathbf{d}))} \frac{1}{u'(R(e^j - \pi d^j)} \frac{\partial \sigma^*(\mathbf{d})}{\partial d^j} \Delta U^j + \\ &+ (1-\pi)\mathbb{E}[p]R(1+\sigma^*(\mathbf{d})). \end{aligned} \tag{20}$$

The right-hand side of (20) is higher than the expected return on the productive asset, namely $(1-\pi)\mathbb{E}[p]R$, which is equal to the marginal rate of substitution between early and late consumption in the banking equilibrium with perfect information. In other words, the endogeneity of the threshold signal $\sigma^*(\mathbf{d})$ forces the banks to increase the marginal rate of substitution, i.e. lower the amount of early consumption offered, with respect to the banking equilibrium with perfect information. Yet, in the aggregate the banking equilibrium still represents a welfare improvement with respect to an autarkic equilibrium without banks, even if systemic self-fulfilling runs are possible. To see this, we prove the following:

Proposition 3 *In the banking equilibrium $d^j > e^j$ for all wealth groups j.*

Proof. In Appendix B. ■

The proof of this Proposition is based on showing that having $d^j = e^j$ for every wealth group j would leave some marginal benefits unexploited. Hence, for at least one wealth group k we must have that $d^k > e^k$. However, this creates a positive probability of a systemic self-fulfilling run, to which the banks in the other wealth groups $j \neq k$ react by increasing insurance against the idiosyncratic shock, i.e. by increasing d^j above e^j. This result implies that the banking equilibrium Pareto-dominates autarky for every wealth group, even if systemic self-fulfilling runs are possible. To see that, assume that in autarky an agent could not join a bank but could invest all her initial endowment e^j in the same productive asset as the banks. Then, an early consumer would liquidate all her assets and consume $c_1^j = e^j$, while a late consumer would keep them and consume either $c_2^j(R) = R(1-\pi)e^j$ or $c_2^j(0) = 0$. Since this consumption profile is feasible in the banking equilibrium but not chosen, it must be the case that it is worse than the one chosen by the banks. Hence, every agent has an incentive to deposit her whole wealth in the banking system, i.e. the economy is endogenously fully intermediated.

5 Government Intervention

Having characterized the banking equilibrium of the heterogeneous economy, in this section we study different types of government interventions, and how these affect the formation of the depositors' self-fulfilling expectations, the redistribution of resources across the economy, and the amount of risk sharing provided by the banks against idiosyncratic risk. To this end, we assume the existence of an economy-wide benevolent government, who maximizes the total expected welfare of the depositors in the economy. This government is different from a social planner, in the sense that it cannot prevent markets (in this particular case, the banking system) from operating, but it can influence its behavior through policy. We model this restriction by assuming that the banks collect the deposits and choose the deposit contracts before government intervention. The intervention is made of wealth-specific proportional taxes τ^j on the endowments $\bar{e}^j$ at date 1, and of wealth-specific lump-sum non-negative subsidies s^j. An intervention is feasible if:[15]

$$\sum_j s^j \leq \sum_j \tau^j \bar{e}^j. \tag{21}$$

In what follows, we characterize government intervention under different assumptions about the level of commitment of the government itself. We start from an economy in which the government does not commit to an intervention before the depositors decide whether to run or not and only intervenes ex post, in a time-consistent manner. We compare this to other interventions in which the government commits to choose taxes and subsidies before the depositors decide about whether to run or not. In this last case, we study government interventions against bank insolvency, against

[15] An alternative intervention in which the government purely redistribute lump-sum across wealth groups, i.e. such that $\sum_j s^j \leq 0$, would yield qualitatively similar results.

bank illiquidity, and a combined intervention against both.

5.1 Time-consistent Intervention

Assume that the government cannot commit to intervene before the depositors decide whether to run or not, and chooses taxes and subsidies in a time-consistent manner, i.e. only after she observed that a systemic run has taken place and the banks have closed down. For the sake of clarity, we repeat here the timing of actions: at date 0, the banks collect the initial endowments, and choose the deposit contracts $\{d^j, d_L^j(A)\}$; at date 1, all depositors get to know their private types and signals, and the early consumers withdraw, while the late consumers, once observed the signals, decide whether to run on their banks or not; if a run takes place, the government chooses the intervention $\{\tau^j, s^j\}$; finally, at date 2, those late consumers who have not withdrawn at date 1 receive an equal share of the available resources.

As before, we solve for the equilibrium by backward induction, starting from the government intervention. The government problem reads:

$$\max_{\tau^j, s^j} \sum_j \left[u(e^j + s^j) + w((1-\tau^j)\bar{e}^j) \right], \tag{22}$$

subject to the budget constraint (21). As a systemic run has already taken place, the government maximizes the total ex-post welfare of the economy. All banks have completely liquidated the productive asset, equally shared the proceeds from liquidation among their depositors, and closed down. Then, the government taxes the extra endowment $\bar{e}^j$ and distributes the subsidy s^j directly to the depositors, so their total consumption is $e^j + s^j$. It is straightforward to see that the equilibrium intervention satisfies:

$$w'((1-\tau^j)\bar{e}^j) = w'((1-\tau^\ell)\bar{e}^\ell), \tag{23}$$

$$u'(e^j + s^j) = u'(e^\ell + s^\ell), \tag{24}$$

for any two groups j and ℓ. The intervention equalizes across wealth groups its marginal costs and its marginal benefits. By the concavity of the utility function, the intervention without commitment optimally redistributes resources from rich wealth groups to poor wealth groups: the higher the taxable endowment $\bar{e}^j$ of a depositor of a wealth group j is, the higher the tax τ^j that she pays; the lower the initial endowment e^j of a depositor of a wealth group j is, the higher the subsidy s^j that she receives.

Having characterized the intervention, we analyze how it affects the incentives of a late consumer to join a run. To this end, as in the previous section, we study the advantage of waiting versus running, which is given by:

$$v^j(n, n^j, s^j) = \begin{cases} \sigma^{ij} u\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right) - u(d^j) & \text{if } \pi \leq n^j < \frac{e^j}{d^j}, \\ -u\left(\frac{e^j}{n^j} + s^j\right) & \text{if } \frac{e^j}{d^j} \leq n^j \leq 1. \end{cases} \tag{25}$$

From this expression it is clear that the intervention only affects the incentives of the depositors to run through the subsidy that they receive after insolvency, i.e. when $n^j \geq e^j/d^j$. Taxes instead do not enter $v^j(n, n^j, s^j)$ (not even when $n^j \geq e^j/d^j$) as the depositors pay them irrespective of whether they run or not. Applying the Belief Constraint, we derive the threshold signal below which a late consumer runs as a function of the vectors of deposit contracts $\mathbf{d} = \{d^j\}$ and subsidies $\mathbf{s} = \{s^j\}$ offered to all groups j as:

$$\sigma^*(\mathbf{d}, \mathbf{s}) = \frac{(1-\pi)\sum_j \left[\int_\pi^{\frac{e^j}{d^j}} u(d^j) dn^j + \int_{\frac{e^j}{d^j}}^1 u\left(\frac{e^j}{n^j} + s^j\right) dn^j\right]}{\sum_j \int_\pi^1 \int_\pi^{\frac{e^j}{d^j}} u\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right) dn^j dn}. \tag{26}$$

The threshold signal $\sigma^*(\mathbf{d}, \mathbf{s})$ turns out to be increasing in any subsidy s^j. In fact, higher subsidies increase the incentives of a late consumer to run, as she internalizes that the ex-post intervention increases her consumption when her bank is insolvent. This result highlights the crucial effect that the lack of commitment makes in the case of government intervention: the anticipation of an ex-post intervention, while redistributing in a time-consistent fashion across wealth groups, has the unintended consequence of increasing the probability of a systemic self-fulfilling run ex ante.

5.2 Commitment against Bank Insolvency

The fact that a government intervention without commitment leads to a higher probability of a systemic run raises the question of whether this is a exclusively a consequence of the level of commitment. In what follows, we characterize the equilibrium intervention against bank insolvency in the case when the government can instead commit to a set of taxes and subsidies before the depositors decide whether to run or not. This means that, with respect to the intervention without commitment, the timing of actions at date 1 changes as follows: first, the government chooses the intervention $\{\tau^j, s^j\}$; then, all depositors get to know their private types and signals, and the early consumers withdraw, while the late consumers, once observed the signals, decide whether to run on their banks or not.

Again, we solve for the equilibrium by backward induction, starting now from the depositors' decision of whether to run or not, as represented by the advantage of waiting versus running:

$$v^j(n, n^j, s^j) = \begin{cases} \sigma^{ij} u\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right) - u(d^j) & \text{if } \pi \leq n^j < \frac{e^j}{d^j}, \\ -u\left(\frac{e^j + s^j}{n^j}\right) & \text{if } \frac{e^j}{d^j} \leq n^j \leq 1. \end{cases} \tag{27}$$

Differently from the previous case, the government intervening against insolvency transfers the subsidies s^j directly to the banks when the number of depositors running is higher than e^j/d^j. In other words, at insolvency the total amount of available resources that the banks can share between their depositors is equal to $e^j + s^j$. Nevertheless, the effect of the subsidies on the advantage of waiting versus running is the same as for the case without commitment: the subsidies s^j increase the threshold signal $\sigma^*(\mathbf{d}, \mathbf{s})$, and such an intervention like that cannot make the banking system run proof. Going backward, at the beginning of date 1 the government chooses taxes and subsidies to solve:

$$\max_{\{\tau^j, s^j\}_{j=1,\ldots,G}} \sum_j \left[\int_0^{\sigma^*(\mathbf{d},\mathbf{s})} \left[u(e^j + s^j) + w((1-\tau^j)\bar{e}^j) \right] dp + \right.$$
$$\left. + \int_{\sigma^*(\mathbf{d},\mathbf{s})}^1 \left[\pi u(d^j) + (1-\pi) p u(R(e^j - \pi d^j)) + w(\bar{e}^j) \right] dp \right], \tag{28}$$

subject to $\sigma^*(\mathbf{d}, \mathbf{s})$ in (26) and to the budget constraint (21). Differently from the previous case, here the government maximizes the total ex-ante welfare of the economy. In other words, it also takes into account the effect that the subsidy has on the endogenous threshold signal $\sigma(\mathbf{d}, \mathbf{s})$ when choosing the intervention. The equilibrium taxes and subsidies satisfy the equilibrium conditions:

$$w'((1-\tau^j)\bar{e}^j) = w'((1-\tau^\ell)\bar{e}^\ell), \tag{29}$$

$$\sigma^*(\mathbf{d}, \mathbf{s})u'(e^j + s^j) - \frac{\partial \sigma^*(\mathbf{d}, \mathbf{s})}{\partial s^j} \sum_k \Delta U^k = \sigma^*(\mathbf{d}, \mathbf{s})u'(e^\ell + s^\ell) - \frac{\partial \sigma^*(\mathbf{d}, \mathbf{s})}{\partial s^\ell} \sum_k \Delta U^k. \tag{30}$$

As in the case without commitment, the government equalizes the marginal cost of financing the intervention across wealth groups by choosing a set of taxes that is increasing in the taxable endowment $\bar{e}^j$. However, the equilibrium allocation of the subsidies follow a different path. The subsidy that a wealth group j receives is higher the higher the statistic:

$$\Psi^j = \sigma^*(\mathbf{d}, \mathbf{s})u'(e^j + s^j) - \frac{\partial \sigma^*(\mathbf{d}, \mathbf{s})}{\partial s^j} \sum_k \Delta U^k. \tag{31}$$

In this way, the government takes into account the redistributive effect that a subsidy must have when a run takes place with probability $\sigma^*(\mathbf{d}, \mathbf{s})$ (the first term of (31)). Additionally, the government also takes into account the increasing effect that a subsidy to a wealth group j has on the threshold signal $\sigma^*(\mathbf{d}, \mathbf{s})$, which has a negative effect on the aggregate welfare gains from avoiding a run in the whole economy (the second term of (31)).

The expression for Ψ^j allows us to study the redistributive effect of the intervention with commitment against bank insolvency through the subsidy. To this end, calculate:

$$\frac{\partial \Psi^j}{\partial e^j} = \frac{\partial \sigma^*(\mathbf{d}, \mathbf{s})}{\partial e^j} u'(e^j + s^j) + \sigma^*(\mathbf{d}, \mathbf{s})u''(e^j + s^j) - \frac{\partial^2 \sigma^*(\mathbf{d}, \mathbf{s})}{\partial s^j \partial e^j} \sum_k \Delta U^k - \frac{\partial \sigma^*(\mathbf{d}, \mathbf{s})}{\partial s^j} \sum_k \frac{\partial \Delta U^k}{\partial e^j}. \tag{32}$$

Remember that the threshold signal $\sigma^*(\mathbf{d}, \mathbf{s})$ is decreasing in the initial endowments e^j, as we proved in Corollary 2. Moreover, the redistributive effect that a subsidy must have ex post implies that the government finds optimal to provide higher subsidies to the poor wealth groups. All in all, this means that the first two terms of (32) command higher subsidies to poor wealth groups. In contrast, the marginal effect of a subsidy s^j on the threshold signal $\sigma^*(\mathbf{d}, \mathbf{s})$, which we already argued is positive, is decreasing in the initial endowments e^j:

$$\begin{aligned}\frac{\partial^2\sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j \partial e^j} =& \frac{1-\pi}{\sum_j \int_\pi^1 \int_\pi^{\frac{e^j}{d^j}} u\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right) dn^j dn} \times \\ & \times \left[-\frac{1}{e^j} u'\left(\frac{e^j+s^j}{e^j} d^j\right) + \int_{\frac{e^j}{d^j}}^1 u''\left(\frac{e^j+s^j}{n^j}\right) \frac{1}{n^{j2}} dn^j + \right. \\ & \left. - \frac{\partial \sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j} \left[\int_\pi^1 \int_\pi^{\frac{e^j}{d^j}} u'\left(R(1-n)\frac{e^j+s^j-n^j d^j}{1-n^j}\right) \frac{R(1-n)}{1-n^j} dn^j dn \right] \right]. \end{aligned} \qquad (33)$$

Intuitively, as the initial endowment increases, the maximum fraction of depositors that a bank can serve in a wealth group without breaching the deposit contract also increases. Moreover, an increasing initial endowment increases the consumption that the depositors enjoys at bank insolvency, and this lowers the marginal effect of a subsidy by the concavity of the utility function. Finally, increasing the initial endowment also has the effect of increasing the consumption of a late consumer who does not run. All in all, these three channels make the marginal effect of a subsidy s^j on the threshold signal $\sigma^*(\mathbf{d},\mathbf{s})$ decreasing in e^j. Thus, the third term of (31) commands higher subsidies to rich wealth groups. Finally, since the welfare gain from avoiding a run ΔU^j is decreasing in all initial endowments e^j-s, and the threshold signal $\sigma^*(\mathbf{d},\mathbf{s})$ is increasing in the subsidy s^j, also the fourth term is positive.

To sum up, a government who commits to an intervention against bank insolvency finds optimal to minimize its distortive effect on the incentives to run by partially subsidizing the rich wealth groups, whose incentives to run are less sensitive to the subsidization. Hence, when committing to intervene against bank insolvency a government is willing to lower the redistributive impact of its intervention with respect to a time-consistent intervention.

5.3 Commitment against Bank Illiquidity

Assume now that the government can commit to choose an intervention only against bank illiquidity. Put differently, it commits to a set of taxes and subsidies for the different wealth groups before the depositors decide whether to run or not, and only as long as their banks are illiquid but solvent. This means that the intervention takes place only as long as the fraction of depositors running in wealth group j is lower than e^j/d^j. The budget constraint of the representative bank of wealth group j at date 1 reads:

$$X^j + s^j = n^j d^j, \tag{34}$$

where X^j is the amount of productive assets that needs to be liquidated to pay early consumption. Thus, the amount of productive assets that gets to maturity is equal to $e^j - X^j$, and affects the amount of consumption that a late consumer gets if she does not withdraw at date 1. Moreover, the subsidy affects the maximum fraction of depositors that can be served before the bank goes into insolvency, i.e. $(e^j + s^j)/d^j$. Thus, the advantage of waiting versus running in the presence of a subsidy now reads:

$$v^j(n, n^j, s^j) = \begin{cases} \sigma u\left(R(1-n)\frac{e^j+s^j-n^j d^j}{1-n^j}\right) - u(d^j) & \text{if } \pi \le n^j < \frac{e^j+s^j}{d^j}, \\ -u\left(\frac{e^j}{n^j}\right) & \text{if } \frac{e^j+s^j}{d^j} \le n^j \le 1. \end{cases} \tag{35}$$

As before, by the Belief Constraint the endogenous threshold signal $\sigma^*(\mathbf{d}, \mathbf{s})$ is given by the average indifference condition between running or not:

$$\sigma^*(\mathbf{d}, \mathbf{s}) = \frac{(1-\pi)\sum_j \left[\int_\pi^{\frac{e^j+s^j}{d^j}} u(d^j)dn^j + \int_{\frac{e^j+s^j}{d^j}}^1 u\left(\frac{e^j}{n^j}\right)dn^j\right]}{\sum_j \int_\pi^1 \int_\pi^{\frac{e^j+s^j}{d^j}} u\left(R(1-n)\frac{e^j+s^j-n^j d^j}{1-n^j}\right)dn^j dn}. \tag{36}$$

From here, we can calculate the effect of a marginal increase of a subsidy s^j on the common threshold signal $\sigma^*(\mathbf{d}, \mathbf{s})$:

$$\frac{\partial \sigma^*(\mathbf{d}, \mathbf{s})}{\partial s^j} = (1-\pi)\frac{\frac{u(d^j) - u\left(\frac{e^j}{e^j+s^j}d^j\right)}{d^j} - \sigma^*(\mathbf{d}, \mathbf{s})\int_\pi^1 \int_\pi^{\frac{e^j+s^j}{d^j}} u'\left(R(1-n)\frac{e^j+s^j-n^j d^j}{1-n^j}\right)\frac{R(1-n)}{1-n^j}dn^j dn}{\sum_j \int_\pi^1 \int_\pi^{\frac{e^j+s^j}{d^j}} u\left(R(1-n)\frac{e^j+s^j-n^j d^j}{1-n^j}\right)dn^j dn}. \tag{37}$$

Intuitively, a subsidy has two effects. On the one hand, they increase the fraction of depositors that the banks can serve before insolvency, thus increasing the incentives to run (the first part of the numerator of (37)). On the other hand, positive subsidies allow the banks to liquidate a lower amount of productive assets, that stay until maturity and finance higher late consumption at date 2, thus lowering the depositors' incentives to run. By the Inada conditions the second effect dominates: providing a subsidy to a late consumer who is not running just before insolvency would allow her to consume a positive amount instead of zero, and that would have a large effect on her marginal utility. Hence, the threshold signal $\sigma^*(\mathbf{d}, \mathbf{s})$ is decreasing in the subsidies s^j.

The previous result clarifies that it is possible for a government to commit to a scheme of subsidies so that the economy becomes run proof. Such an intervention should allow banks in all wealth groups to serve all depositors at date 1, even in the case of a run. In formulae, this means the maximum fraction of depositors that can be served before a bank goes into insolvency $(e^j + s^j)/d^j$ must be equal to 1, or $s^j = d^j - e^j$ for all groups j. If this full liquidity assistance is feasible, every depositor internalizes that there are sufficient resources to pay early withdrawals in the case of a run, so no one runs and the banks can implement the equilibrium with perfect information. More formally, assume that the government fully taxes all wealth groups, i.e. $\tau^j = 1$ for all groups j. Then, the following can be proved:

Proposition 4 *If the condition:*

$$\sum_j \bar{e}^j \geq \left[\frac{(1-\pi)Ru'^{-1}((1-\pi)\mathbb{E}[p]R) - 1}{1 + \pi Ru'^{-1}((1-\pi)\mathbb{E}[p]R)}\right] \sum_j e^j \tag{38}$$

is satisfied, then the equilibrium government intervention with commitment against bank illiquidity is $\tau^j = s^j = 0$ *for all wealth groups* j*, and the banking equilibrium is equivalent to the equilibrium with perfect information.*

Proof. In Appendix B. ■

The Proposition states the feasibility condition to rule out a systemic self-fulfilling run: the total taxable resources (the left-hand side of (38)) must be larger than or equal to the total liabilities of the banking system, as represented by $\sum_j e^j$, times a constant. If this condition is satisfied, there is no need for the government to intervene, and the mere announcement of a commitment to intervene is sufficient to implement the equilibrium with perfect information at zero costs. In other words, under the feasibility condition the announcement of a commitment to intervene at bank illiquidity is the optimal policy.

The tightness of the feasibility condition crucially depends on the constant:

$$\Xi = \left[\frac{(1-\pi)Ru'^{-1}((1-\pi)\mathbb{E}[p]R) - 1}{1 + \pi Ru'^{-1}((1-\pi)\mathbb{E}[p]R)}\right]. \tag{39}$$

As we show in the proof of the Proposition, this expression is equal to $(d^j - e^j)/e^j$, which is a measure of bank relative maturity mismatch in the equilibrium with perfect information. Moreover, it is homogeneous in the whole economy, as it depends on the common probability of the idiosyncratic shock and on technology. Finally, Ξ is a function of the depositors' relative risk aversion, as the following Corollary shows:

Corollary 3 *There exists an upper bound $\bar{R}$ such that if $R \leq \bar{R}$ then Ξ is smaller than or equal to 1. Assume $\pi > 1/2$. Under CRRA utility, $\bar{R} = 2/(1-2\pi)$ and Ξ is increasing in the coefficient of relative risk aversion.*

Proof. In Appendix B. ■

We use this result to further analyze the feasibility of the optimal policy of Proposition 4. If the endowments are constant across time in every wealth group, i.e. $\bar{e}^j = e^j$, the fact that the constant Ξ is less than or equal to 1 implies that feasibility is satisfied. Otherwise, a weaker condition that satisfies feasibility is that $\sum_j \bar{e}^j \geq \sum_j e^j$, i.e. the aggregate endowment is weakly increasing between date 0 and date 1. The second part of the Corollary instead highlights that the more risk averse the depositors are, the higher the amount of risk sharing that they require against the idiosyncratic shocks is, hence the higher the amount of maturity mismatch in which the banks must engage. In turns, this tightens the feasibility condition of the optimal policy.

The characterization of the feasibility condition for a full intervention against bank illiquidity calls for a further issue: how does the government intervene when it is committed against bank illiquidity, but feasibility is not satisfied? Clearly, only a partial intervention is possible, that leaves behind some systemic financial fragility. Then, the question becomes how a government should allocate taxes and subsidies to maximize the expected welfare of the whole economy:

$$\max_{\{\tau^j, s^j\}_{j=1,\ldots,G}} \sum_j \left[\int_0^{\sigma^*(\mathbf{d},\mathbf{s})} u(e^j)dp + \int_{\sigma^*(\mathbf{d},\mathbf{s})}^1 \left[\pi u(d^j) + (1-\pi) p u(R(e^j - \pi d^j)) \right] dp + w(\bar{e}^j) \right], \tag{40}$$

subject to the definition of $\sigma^*(\mathbf{d},\mathbf{s})$ in (36), to the budget constraint in (21), and to $s^j \in [0, d^j - e^j]$ and $0 \leq \tau^j \leq 1$ for all groups j.

Proposition 5 *The optimal partial intervention with commitment against bank illiquidity fully* *taxes all wealth groups j, and subsidizes them according to the statistics:*

$$\Psi^j = -\frac{\partial \sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j} \sum_k \Delta U_B^k. \tag{41}$$

There exists a unique threshold group $\hat{j}$ such that all wealth groups with $\Psi^{(j)} > \Psi^{(\hat{j})}$ are fully subsidized (i.e. $s^j = d^j - e^j$), all wealth groups with $\Psi^{(j)} < \Psi^{(\hat{j})}$ receives zero (i.e. $s^j = 0$) and all wealth groups with $\Psi^{(j)} = \Psi^{(\hat{j})}$ receives $s^j \in (0,1)$.

Proof. In Appendix B. ■

Intuitively, as the marginal benefit of imposing taxes lies in the relaxation of the government budget constraint, which is joint for the whole economy, the equilibrium tax scheme should tax all wealth groups in the same way. Moreover, the economy exhibits only two possible allocations ex post: run and no-run. This means that a partial government intervention against bank illiquidity, like a fully feasible one, is announced, committed to, but never implemented. That is the reason why it prescribes a 100-percent tax rate on all groups j.

As the subsidies are paid only when the banks are illiquid but solvent, their allocation only maximizes the impact that the subsidies have on the depositors' expectations and therefore on the probability of a systemic self-fulfilling run. The government achieves this by calculating the statistic Ψ^j for each group j. This depends on the initial endowment in the following way:

$$\frac{\partial \Psi^j}{\partial e^j} = -\frac{\partial^2 \sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j \partial e^j} \sum_k \Delta U^k - \frac{\partial \sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j} \sum_k \frac{\partial \Delta U^k}{\partial e^j}. \tag{42}$$

This expression is negative. To see that, notice that the threshold signal $\sigma^*(\mathbf{d},\mathbf{s})$ is decreasing in the subsidy s^j, and the welfare gains from avoiding a run ΔU^k in any group k is decreasing in the initial endowment e^j of a group j. Moreover, from (37):[16]

$$\begin{aligned}\frac{\partial^2 \sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j \partial e^j} =& \frac{(1-\pi)}{\sum_j \int_\pi^1 \int_\pi^{\frac{e^j+s^j}{d^j}} u\left(R(1-n)\frac{e^j+s^j-n^j d^j}{1-n^j}\right) dn^j dn} \times \\ & \times \left[-u'\left(\frac{e^j}{e^j+s^j} d^j\right) \frac{s^j}{(e^j+s^j)^2} + \right.\end{aligned}$$

[16]Notice that the utility function $u(c)$ has a kink at $c=0$, hence $u'(0)$ is undefined.

$$
\begin{aligned}
&-\frac{\partial \sigma^*(\mathbf{d},\mathbf{s})}{\partial e^j}\left[\int_\pi^1\int_\pi^{\frac{e^j+s^j}{d^j}} u'\left(R(1-n)\frac{e^j+s^j-n^jd^j}{1-n^j}\right)\frac{R(1-n)}{1-n^j}dn^jdn\right]+\\
&-\sigma^*(\mathbf{d},\mathbf{s})\int_\pi^1\int_\pi^{\frac{e^j+s^j}{d^j}} u''\left(R(1-n)\frac{e^j+s^j-n^jd^j}{1-n^j}\right)\left(\frac{R(1-n)}{1-n^j}\right)^2 dn^jdn+\\
&-\frac{\partial \sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j}\left[\int_\pi^1\int_\pi^{\frac{e^j+s^j}{d^j}} u'\left(R(1-n)\frac{e^j+s^j-n^jd^j}{1-n^j}\right)\frac{R(1-n)}{1-n^j}dn^jdn\right]\Bigg]. \qquad (43)
\end{aligned}
$$

Again, the threshold signal $\sigma^*(\mathbf{d},\mathbf{s})$ is decreasing both in the initial endowment e^j and in the subsidy s^j. Then, as the utility function is concave, (43) must be positive: the marginal utility of consumption of a late consumer who waits until date 2 and consumes just before insolvency (i.e. as n^j approaches $(e^j+s^j)/d^j$) tends to be large by the Inada conditions. In other words, the marginal effect of a subsidy to group j on the probability of a systemic self-fulfilling run is negative and increasing with the initial endowment.

This has a crucial consequence for the allocation of subsidies in this partial intervention against bank illiquidity. The government finds optimal to put relatively higher in its ranking the poorer wealth groups, because they are "more systemic" (remember that $\sigma^*(\mathbf{d},\mathbf{s})$ is decreasing in s^j): a subsidy to them has a larger effect on the probability of a systemic self-fulfilling run than a subsidy to rich wealth groups. Finally, the third part of Proposition 5 suggests a practical rule to allocate subsidies: rank wealth groups from the most to the least systemic according to (41), and start fully subsidizing them from top to bottom, until the government budget constraint clears. This means that the wealth groups at the bottom of the ranking, which incidentally are also the rich ones, not only would pay full taxes, but might receive no subsidy. Put differently, the equilibrium partial intervention would highly reduce the initial inequality, but purely from efficiency considerations. Additionally, as we already mentioned, bank illiquidity is not an ex-post outcome of this economy, hence such a redistribution never takes place. Finally, this intervention (indeed, the mere announcement of a commitment to it) has the "trickle-up" effect of lowering the probability of a systemic self-fulfilling run in the whole economy, and this improves expected welfare also for the rich wealth groups that are promised a 100-percent tax and no subsidy.

5.4 Commitment against Bank Illiquidity and Insolvency

In the last two sections, we implicitly assumed that the government could distinguish between bank illiquidity and insolvency, and act against one or the other accordingly. However, in the real world this distinction is less straightforward.[17] This argument calls for a comparison of the government interventions against bank illiquidity and bank insolvency, and in particular for the analysis of which of the two effects of the subsidies (positive or negative) on the probability of a systemic self-fulfilling run dominates once a combined intervention is put into place. To this end, here we characterize the equilibrium intervention when the government commits to intervene against both bank illiquidity and insolvency with the same set of taxes and subsidies. As before, we start from the analysis of the depositors' advantage of waiting versus running and derive the threshold signal:

$$\sigma^*(\mathbf{d},\mathbf{s}) = \frac{(1-\pi)\sum_j \left[\int_{\pi}^{\frac{e^j+s^j}{d^j}} u(d^j)dn^j + \int_{\frac{e^j+s^j}{d^j}}^{1} u\left(\frac{e^j+s^j}{n^j}\right)dn^j\right]}{\sum_j \int_{\pi}^{1}\int_{\pi}^{\frac{e^j+s^j}{d^j}} u\left(R(1-n)\frac{e^j+s^j-n^jd^j}{1-n^j}\right)dn^j dn}. \tag{44}$$

Notice that here a subsidy has the simultaneous effects of altering the maximum fraction of depositors that can be served before the bank goes into insolvency, the amount of productive assets that needs to be liquidated to pay early consumption, and the amount that they depositors receive at insolvency. The effect of the subsidy s^j on the threshold signal is summarized by:

$$\begin{aligned}\frac{\partial\sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j} =& \frac{(1-\pi)}{\sum_j \int_{\pi}^{1}\int_{\pi}^{\frac{e^j+s^j}{d^j}} u\left(R(1-n)\frac{e^j+s^j-n^jd^j}{1-n^j}\right)dn^j dn}\times \\ &\times\left[\frac{u(d^j)-u\left(\frac{e^j}{e^j+s^j}d^j\right)}{d^j} + \int_{\frac{e^j+s^j}{d^j}}^{1} u'\left(\frac{e^j+s^j}{n^j}\right)\frac{1}{n^j}dn^j + \right. \\ &\left. -\sigma^*(\mathbf{d},\mathbf{s})\int_{\pi}^{1}\int_{\pi}^{\frac{e^j+s^j}{d^j}} u'\left(R(1-n)\frac{e^j+s^j-n^jd^j}{1-n^j}\right)\frac{R(1-n)}{1-n^j}dn^j dn\right]\end{aligned} \tag{45}$$

[17] This argument has been recognized in the past academic literature (Goodhart, 1999) and is shared by several policymakers. For example, the former member of the Fed Board of Governors Jeremy Stein (2013) recognizes that "the line between illiquidity and insolvency is far blurrier in real life than it is sometimes assumed to be in theory". Similarly, the other former member of the Fed Board Daniel Tarullo (2014) argues that "particularly in periods of stress, when the value of important asset classes may be quite volatile and very difficult to determine, the central bank cannot always easily disentangle illiquidity and insolvency risks".

As a mix of the two previous cases, the subsidy has the effect of increasing the fraction of depositors that the banks can serve before insolvency and the amount of consumption that the depositors enjoy after insolvency, and of reducing the amount of productive assets that the banks need to liquidate to pay early consumption. By the Inada conditions, this second effect dominates: letting some late consumers consume a positive amount instead of zero just before insolvency has a large effect on their marginal utility. Hence, the threshold signal $\sigma^*(\mathbf{d}, \mathbf{s})$ is decreasing in the subsidies s^j. This means that the same result of the government intervention against bank illiquidity holds: if feasible, a commitment to a full liquidity assistance completely rules out a systemic self-fulfilling run at zero costs for the economy.

Then, the question becomes how a government should allocate taxes and subsidies to maximize the expected welfare of the whole economy when only a partial intervention is feasible:

$$\max_{\{\tau^j, s^j\}_{j=1,\dots,G}} \sum_j \left[\int_0^{\sigma^*(\mathbf{d},\mathbf{s})} \left[u(e^j + s^j) + w((1-\tau^j)\bar{e}^j) \right] dp + \right. \\ \left. + \int_{\sigma^*(\mathbf{d},\mathbf{s})}^1 \left[\pi u(d^j) + (1-\pi) p u(R(e^j - \pi d^j)) + w(\bar{e}^j) \right] dp \right], \tag{46}$$

subject to the definition of $\sigma^*(\mathbf{d}, \mathbf{s})$ in (44) and to the budget constraint in (21). The first-order conditions of the problem yield:

$$w'((1-\tau^j)\bar{e}^j) = w'((1-\tau^\ell)\bar{e}^\ell), \tag{47}$$

$$\sigma^*(\mathbf{d}, \mathbf{s}) u'(e^j + s^j) - \frac{\partial \sigma^*(\mathbf{d}, \mathbf{s})}{\partial s^j} \sum_k \Delta U^k = \sigma^*(\mathbf{d}, \mathbf{s}) u'(e^\ell + s^\ell) - \frac{\partial \sigma^*(\mathbf{d}, \mathbf{s})}{\partial s^\ell} \sum_k \Delta U^k. \tag{48}$$

As for the case of intervention with commitment against bank insolvency, the government chooses a set of taxes that is increasing in the taxable endowment $\bar{e}^j$ so as to equalize the marginal cost of financing the intervention across wealth groups. The subsidies are instead allocated according to the statistic:

$$\Psi^j = \sigma^*(\mathbf{d}, \mathbf{s}) u'(e^j + s^j) - \frac{\partial \sigma^*(\mathbf{d}, \mathbf{s})}{\partial s^j} \sum_k \Delta U^k, \tag{49}$$

from which it is clear that the government takes into account the effect of the subsidy on the marginal utility of consumption when a run takes place together with the marginal effect on the threshold signal $\sigma^*(\mathbf{d}, \mathbf{s})$. However, the redistributive effect of the intervention with commitment against bank illiquidity and insolvency is different from the case of intervention against bank insolvency only. To this end, calculate:

$$\frac{\partial \Psi^j}{\partial e^j} = \frac{\partial \sigma^*(\mathbf{d},\mathbf{s})}{\partial e^j} u'(e^j+s^j) + \sigma^*(\mathbf{d},\mathbf{s}) u''(e^j+s^j) - \frac{\partial^2 \sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j \partial e^j} \sum_k \Delta U^k - \frac{\partial \sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j} \sum_k \frac{\partial \Delta U^k}{\partial e^j}. \quad (50)$$

The first two terms of (50) are both negative: the threshold signal is decreasing in the initial endowment, and the utility function is concave. As far as the third term is concerned, calculate instead:

$$\begin{aligned}
\frac{\partial^2 \sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j \partial e^j} =& \frac{(1-\pi)}{\sum_j \int_\pi^1 \int_\pi^{\frac{e^j+s^j}{d^j}} u\left(R(1-n)\frac{e^j+s^j-n^j d^j}{1-n^j}\right) dn^j dn} \times \\
& \times \left[-u'\left(\frac{e^j}{e^j+s^j} d^j\right) \frac{s^j}{(e^j+s^j)^2} - u'(d^j)\frac{1}{e^j+s^j} + \int_{\frac{e^j+s^j}{d^j}}^1 u''\left(\frac{e^j+s^j}{n^j}\right) \frac{1}{n^{j2}} dn^j + \right. \\
& - \frac{\partial \sigma^*(\mathbf{d},\mathbf{s})}{\partial e^j} \left[\int_\pi^1 \int_\pi^{\frac{e^j+s^j}{d^j}} u'\left(R(1-n)\frac{e^j+s^j-n^j d^j}{1-n^j}\right) \frac{R(1-n)}{1-n^j} dn^j dn \right] + \\
& - \sigma^*(\mathbf{d},\mathbf{s}) \left[\int_\pi^1 \int_\pi^{\frac{e^j+s^j}{d^j}} u''\left(R(1-n)\frac{e^j+s^j-n^j d^j}{1-n^j}\right) \left(\frac{R(1-n)}{1-n^j}\right)^2 dn^j dn \right] + \\
& \left. - \frac{\partial \sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j} \left[\int_\pi^1 \int_\pi^{\frac{e^j+s^j}{d^j}} u'\left(R(1-n)\frac{e^j+s^j-n^j d^j}{1-n^j}\right) \frac{R(1-n)}{1-n^j} dn^j dn \right] \right]. \quad (51)
\end{aligned}$$

The first three terms of (51) are all negative, but the last three terms are positive as the utility function is concave and the threshold signal $\sigma^*(\mathbf{d},\mathbf{s})$ is decreasing in both the initial endowment e^j and the subsidy s^j. Yet, again by the Inada conditions the last three dominates. Hence, the marginal effect of a subsidy on the threshold signal is increasing in the initial endowment e^j. Finally, the fourth term of (50) is positive, as both the threshold signal and the welfare gains from avoiding a run are decreasing in the initial endowment e^j of group j. To sum up, the equilibrium government intervention against bank illiquidity and insolvency commands a strong redistribution from rich to poor wealth groups: redistributing resources towards the poor is helpful from a welfare perspective, because it allows the government to offer insurance ex post against the realization of a systemic self-fulfilling run, and because it lowers the probability of a systemic self-fulfilling run.

5.5 Banking Equilibrium with Government Intervention

To close the characterization equilibrium, we conclude this section by studying how the banks at date 0 react to the anticipation of a government intervention at date 1. We already argued that under the feasibility condition of Proposition 4 a commitment to intervene at bank illiquidity is sufficient to rule out systemic self-fulfilling runs. In fact, all depositors internalize in their expectations that the government has sufficient resources to provide full liquidity assistance to the whole economy. Therefore, no depositor runs, the government does not intervene, and the banks can offer a deposit contract equivalent to the one in the banking equilibrium with perfect information, in which early consumption is higher than in the banking equilibrium with systemic self-fulfilling runs of section 4.1. Put differently, a feasible government commitment to intervene at illiquidity allow banks to provide better risk sharing against the idiosyncratic shocks.

This result significantly changes for different levels of commitment and when the feasibility condition is not satisfied. To see that, we solve for the banking equilibrium with government intervention. Formally, a bank in wealth group j maximizes the expected welfare of their depositors subject to the different expressions for the threshold signal $\sigma^*(\mathbf{d}, \mathbf{s})$ in (26),(36) and (44), that depend on the level of commitment and type of government intervention. The following Proposition summarizes our results:

Proposition 6 *For any wealth groups j and k, in the banking equilibrium with a time-consistent government intervention d^j is non-decreasing in s^k. In the banking equilibrium with government intervention with commitment:*

- *against bank insolvency d^j is non-decreasing in s^k;*
- *against bank illiquidity d^j is non-increasing in s^k;*
- *against bank illiquidity and insolvency d^j is non-increasing in s^k.*

Proof. In Appendix B. ■

The proof of the Proposition is based on showing that, depending on the type of intervention, the amount of early consumption offered by the banks and the subsidy that they receive and/or other groups receive are either strategic complements or strategic substitutes. Intuitively, because of perfect competition banks offer insurance to their depositors against both idiosyncratic risk (via early consumption) and the risk of a systemic self-fulfilling run. When the government intervenes in a time-consistent manner or against bank insolvency, the intervention, although ex-post efficient, increases the ex-ante probability of a systemic self-fulfilling run, independently of which wealth group is subsidized. In turn, all banks anticipate this effect and counterbalance it by increasing early consumption, to provide more insurance against idiosyncratic risk. In a symmetric way, when the government commits to a partial intervention against bank illiquidity, the mere announcement of lower subsidies increases the probability of a systemic self-fulfilling run, and all banks anticipate this by increasing early consumption. Finally, when the government commits to a joint partial intervention against bank illiquidity and insolvency we proved that it is the first effect that dominates. Hence, the banks anticipate this by lowering early consumption.

To sum up, the main takeaway of our analysis of government intervention in the presence of systemic financial fragility is twofold. First, a government intervention has a direct effect on systemic financial fragility through the depositors' incentives to run. Second, in anticipation of government intervention the banks change the deposit contract that they offer to their depositors. In this way, government intervention also has an indirect effect that amplifies systemic financial fragility. In fact, the anticipation of an intervention either time consistent or against insolvency creates incentives for the banks to increase early consumption. This has the consequence of further increasing the maturity mismatch in banks' balance sheets, thus indirectly enhancing the increasing effect of the intervention on the probability of a systemic self-fulfilling run. In contrast, the anticipation of an intervention against bank illiquidity makes the banks lower the maturity mismatch in their balance sheets, and this indirectly enhances the decreasing effect of the intervention on the probability of a self-fulfilling run.

Corollary 4 *A time-consistent government intervention increases systemic financial fragility. A government intervention with commitment:*

- *against bank insolvency increases systemic financial fragility;*
- *against bank illiquidity lowers systemic financial fragility;*
- *against bank illiquidity and insolvency lowers systemic financial fragility.*

Proof. In the text above. ■

6 Concluding Remarks

The present paper proposes a novel mechanism explaining how wealth inequality could exacerbate financial fragility via the self-fulfilling expectations of a systemic run: higher inequality lowers the incentives to run of the rich, but increases more the incentives to run of the poor, thus increasing systemic financial fragility overall. Our analysis highlights two distortions arising in such an economy: the coordination failure among depositors that brings about systemic financial fragility, and banks not fully internalizing their systemic contribution when choosing the level of maturity mismatch in their balance sheets. Do these inefficiencies justify a direct government intervention against wealth inequality, for example through taxation? As far as the coordination failure is concerned, our main argument shows that lowering wealth inequality reduces systemic financial fragility. However, financial fragility arises in wealth-homogeneous economies, too. Moreover, a government commitment to a full liquidity assistance to the banks, if feasible, is extremely effective at ruling out self-fulfilling runs, independently of the level and of the distribution of wealth inequality in the economy. In other words, taxing wealth is neither necessary nor sufficient to eliminate the coordination failure leading to systemic financial fragility. Similarly, banks' failure to recognize their systemic contribution is not a consequence of wealth inequality per se but of the segmentation of the banking market across wealth groups. That would naturally rationalize some government intervention, e.g. bank-specific limits to maturity mismatch in the spirit of Hellman et al. (2000), without the need to directly tax wealth.

The assumption of segmented banking markets across wealth groups, which as already argued does not affect any of the results regarding the link between wealth inequality and systemic financial fragility, gives rise to some further considerations. On the one hand, a "universal" bank could better account for the strategic complementarities across wealth-heterogeneous depositors. As a consequence, while still keeping separate balance sheets for each wealth group, it would choose lower maturity mismatch in each of them, thus lowering systemic financial fragility.[18] On the other hand, a universal bank can be an unstable business model. In other words, segmented banking markets, like the ones assumed here, could be easily endogenized. To this end, assume that there exists a bank-formation stage in which the wealth groups decide whether or not to create a coalition and

[18] This argument rationalizes a negative relation between bank concentration and systemic financial fragility, which is supported by an extensive empirical evidence (Beck et al., 2006, 2007).

form a universal bank at a cost. Further assume that there is no enforcement mechanism that forces the wealth groups to stay in the coalition. Under these assumptions, a wealth group that forms its own bank would benefit from lower systemic financial fragility but not incur in any coalition cost. Hence, the coalition would not be stable.[19]

Finally, in our analysis the government can only observe whether a run happens or not, without distinguishing whether the depositors run because of their self-fulfilling expectations. This is important, because self-fulfilling runs due to mere coordination failures require government intervention, while "fundamental" runs due to real shocks might not induce any failure of the fundamental theorems of welfare economics (especially when the liquidation of the productive asset is technologically efficient) and should not be counteracted (Allen et al., 2018). Nevertheless, the ability to distinguish between self-fulfilling and fundamental runs would require the government to hold some privileged information about the realization of the aggregate state of the economy, which would make the characterization of the banking equilibrium with government intervention rather pointless. Instead it would be more interesting to analyze an economy where the government is part of the "global game", i.e. it decides whether and how to intervene based on the reception of a noisy signal. In this way, the government would resemble the large investor (the "Soros") analyzed in the context of currency attacks by Corsetti et al. (2004). We leave a formal analysis of these issues for future work.

[19]This result is reminiscent of the free riding problem that arises in the climate-agreement literature (Carraro, 1997).

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Appendices

A Pecuniary Externalities

In this section, we show an alternative way to model the strategic complementarities in a banking model with heterogeneous depositors. In particular, we introduce in the model a pecuniary externality in the spirit of Allen and Gale (2004a), and show that it brings about results that are qualitatively similar to the investment externality that we model in the main text.

To this end, extend the environment of the main text in the following directions. The economy is populated also by a continuum of measure 1 of agents with endowment $e = \{0, 1, 0\}$ and utility:

$$u(c_1, c_2) = c_2. \tag{52}$$

These agents do not have access to any storage technology, but they can access a secondary market for the productive assets, where they can buy those that the banks sell in order to finance the depositors' withdrawals. Then, the clearing condition in this secondary market reads:

$$1 = \mathcal{P} \sum_k n^k d^k, \tag{53}$$

where the cash-in-the-market price of the productive asset:

$$\mathcal{P} = \frac{1}{\sum_k n^k d^k} \tag{54}$$

equalizes their supply, coming from the banks liquidating them in order to finance early withdrawals, to their demand, coming from the risk-neutral buyers. With this in hand, further define X^j as the amount of productive assets that the bank in a wealth group j has to liquidate. Then, the bank budget constraint at date 1 reads:

$$\mathcal{P} X^j = n^j d^j. \tag{55}$$

Finally, assume that, if the bank goes insolvent, it cannot access the secondary market, and has to liquidate the productive asset by using a costly liquidation technology with recovery rate $r < 1$.

Under these assumptions, the advantage of waiting versus running for a late consumer in wealth group j is given by:

$$v^j(n, n^j) = \begin{cases} \sigma u\left(R\frac{e^j - (\sum_k n^k d^k) n^j d^j}{1 - n^j}\right) - u(d^j) & \text{if } \pi \leq n^j < \frac{e^j}{d^j \sum_k n^k d^k} \\ -u\left(\frac{r e^j}{n^j}\right) & \text{if } \frac{e^j}{d^j \sum_k n^k d^k} \leq n^j < 1 \end{cases} \tag{56}$$

It is easy to see the sign of the cross-group strategic complementarities: $v^j(n, n^j)$ is decreasing in $n^{k \neq j}$ in the first interval, and equal to 0 in the second. The within-group strategic complementarities, however, are more complex. Clearly, $v^j(n, n^j)$ is increasing in n^j in the second interval. In the first interval, instead:

$$\frac{\partial v^j(n,n^j)}{\partial n^j} = -\sigma R u'(d_L^j(R,n,n^j)) \frac{n^j d^{j2} + \left(\sum_{k\neq j} n^k d^k\right) d^j - e^j}{(1-n^j)^2}. \tag{57}$$

This derivative is negative whenever:

$$d^j \geq \frac{-\sum_{k\neq j} n^k d^k + \sqrt{\left(\sum_{k\neq j} n^k d^k\right)^2 + 4e^j n^j}}{2n^j} \tag{58}$$

Under this assumption, the economy exhibits one-sided strategic complementarities as in the text. Thus, the assumption of a investment externality allows us to convey the same message of a pecuniary externality, regarding how strategic complementarities arise in this economy, but in a more parsimonious and elegant way.

B Proofs

Proof of Proposition 1. We start by proving the first part of the Proposition. The utility advantage of waiting versus running is:

$$v^j(p,n,n^j) = \begin{cases} \sigma u\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right) - u(d^j) & \text{if } \pi \leq n^j < \frac{e^j}{d^j} \\ -u\left(\frac{e^j}{n^j}\right) & \text{if } \frac{e^j}{d^j} \leq n^j < 1, \end{cases} \tag{59}$$

where n^j and n are the aggregate actions, i.e. the total fraction of depositors who are withdrawing at date 1 in group j and in the whole economy, respectively. These are given by:

$$n^j = \pi + (1-\pi)\text{prob}(\sigma \leq \sigma^{j*}), \tag{60}$$

$$n = \sum_k m^k n^k = \pi + (1-\pi)\sum_k m^k \text{prob}\{\sigma \leq \sigma^{k*}\}. \tag{61}$$

Define $\Delta^j = (\sigma^{j*} - \sigma^*)$ as the difference between the threshold signal σ^{j*} of group j and the threshold signal σ^* of a generic group (which will turn out to be the unique equilibrium threshold).

Given this definition, we can rescale the aggregate actions as:

$$\tilde{n}^j = \pi + (1-\pi)(1 - F(\sigma^{j*} - p)) = \pi + (1-\pi)(1 - F(\Delta^j - \zeta)) \equiv \tilde{n}^j(\zeta, \Delta^j), \tag{62}$$

$$\tilde{n} = \sum_k n^k = \pi + (1-\pi)\sum_k (1 - F(\sigma^{k*} - p)) \equiv \tilde{n}(\zeta, \Delta), \tag{63}$$

where Δ is the vector of Δ^j-s. Moreover, define $\vartheta(\tilde{n}, \Delta)$ as the inverse of $\tilde{n}(\zeta, \Delta)$ with respect to ζ. Finally, define:

$$H^j(\sigma^*, \Delta) = \mathbb{E}[v^j(\sigma^* + \vartheta(\tilde{n}, \Delta), \tilde{n}(\zeta, \Delta), \tilde{n}^j(\zeta, \Delta^j))]. \tag{64}$$

We follow Frankel et al. (2003) and prove by contradiction that the solution to the system of indifference conditions:

$$H^j(\sigma^*, \Delta) = 0, \tag{65}$$

for all $j = 1, ..., G$ is unique. Assume there exist two distinct solutions, namely (σ^*, Δ^*) and $(\sigma^{*\prime}, \Delta^{*\prime})$. We distinguish two cases: $\Delta^* = \Delta^{*\prime}$ and $\Delta^* \neq \Delta^{*\prime}$. Suppose first that $\Delta^* = \Delta^{*\prime}$, then it must be that $\sigma^* \neq \sigma^{*\prime}$ and without loss of generality, $\sigma^* < \sigma^{*\prime}$. Since $H^j(\sigma^*, \Delta)$ is increasing in σ^*, this implies that $H(\sigma^*, \Delta^*) < H(\sigma^{*\prime}, \Delta^{*\prime})$. However, given that both (σ^*, Δ^*) and $(\sigma^{*\prime}, \Delta^{*\prime})$ are solutions to the system, we should have that $H(\sigma^*, \Delta^*) = H(\sigma^{*\prime}, \Delta^{*\prime}) = 0$, and that is a contradiction.

Now suppose that $\Delta^* \neq \Delta^{*\prime}$ and $\sigma^* \leq \sigma^{*\prime}$. Choose $h \in \arg\max_j(\Delta^{j*\prime} - \Delta^{j*})$ and let $D = \max_j(\Delta^{j*\prime} - \Delta^{j*}) \geq 0$. Observe that $\Delta^{h*\prime} - \Delta^{j*\prime} \geq \Delta^{h*} - \Delta^{j*}$, for all $j = 1, ..., G$, with strict inequality for at least one j. Define $\tilde{\sigma} = \sigma^{*\prime} + D > \sigma^{*\prime} \geq \sigma^*$, hence:

$$H^h(\tilde{\sigma}, \Delta^*) \geq H^h(\sigma^*, \Delta^*) = 0.$$

In order to prove the contradiction, we have to show that:

$$H^h(\tilde{\sigma}, \Delta) \geq H^h(\sigma^{*\prime}, \Delta^{*\prime}) = 0.$$

To this end, rewrite:

$$H^h(\tilde{\sigma}, \Delta^*) = \int_0^1 \int_0^1 v^h(p, n, n^h) f(n^h) dn^h f(n) dn =$$

$$= \int_{-\epsilon}^{\epsilon} v^h(\tilde{\sigma}^h - \eta^h, \tilde{n}(\Delta^{h*} - \eta^h, \Delta^*), n^h(\Delta^{h*} - \eta^h, \Delta^{h*}))f(\eta^h)d\eta^h, \tag{66}$$

where $\tilde{\sigma}^h = \tilde{\sigma} + \Delta^{h*}$, and:

$$H^h(\sigma^{*\prime}, \Delta^{*\prime}) = \int_{-\epsilon}^{\epsilon} v^h(\sigma^{h*\prime} - \eta^h, \tilde{n}(\Delta^{h*\prime} - \eta^h, \Delta^{*\prime}), n^h(\Delta^{h*\prime} - \eta^h, \Delta^{h*\prime}))f(\eta^h)d\eta^h, \tag{67}$$

where $\sigma^{h*\prime} = \sigma^{*\prime} + \Delta^{h*\prime}$. It is easy to see that $\sigma^{h*\prime} = \tilde{\sigma}^h$, as $\tilde{\sigma}^h = \tilde{\sigma} + \Delta^{h*} = \sigma^{*\prime} + D + \Delta^{h*} = \sigma^{*\prime} + \Delta^{h*\prime} - \Delta^{h*} + \Delta^{h*} = \sigma^{*\prime} + \Delta^{h*\prime} = \sigma^{h*\prime}$. Moreover:

$$\tilde{n}(\Delta^{h*\prime} - \eta^h, \Delta^{*\prime}) \geq \tilde{n}(\Delta^{h*} - \eta^h, \Delta^*), \tag{68}$$

for all η^h, as:

$$\sum_j (1 - F(\Delta^{j*\prime} - \Delta^{h*\prime} + \eta^h)) \geq \sum_j (1 - F(\Delta^{j*} - \Delta^{h*} + \eta^h)) \tag{69}$$

holds due to the observation above. Similarly:

$$F(\Delta^{j*\prime} - \Delta^{h*\prime} + \eta^h) \leq F(\Delta^{j*} - \Delta^{h*} + \eta^h) \tag{70}$$

for all η^h. Hence, $H^j(\tilde{\sigma}, \Delta) \geq H^h(\sigma^{*\prime}, \Delta^{*\prime})$ because $H^j(\sigma, \Delta)$ is decreasing in $\tilde{n}(\zeta, \Delta)$ and $\tilde{n}^j(\zeta, \Delta)$. This gives a contradiction, and concludes the proof of the first part of the Proposition.

As far as the second part of the Proposition is concerned, we start by showing that, when ϵ is small, the system of indifference conditions $H^j(\sigma^*, \Delta)(\epsilon) = 0$ is well approximated by $H^j(\sigma^*, \Delta)(0) = 0$. Notice that, as $\epsilon \to 0$, we have that $\zeta = 0$ and $\vartheta(\tilde{n}, \Delta) = 0$. Hence:

$$H^j(\sigma^*, \Delta)(\epsilon) = \int_0^1 \int_\pi^{\frac{e^j}{d^j}} \left[(\sigma^* + \vartheta(\tilde{n}, \Delta)) u \left(\frac{R(1 - \tilde{n}(\zeta, \Delta))(e^j - \tilde{n}^j(\zeta, \Delta) d^j)}{1 - \tilde{n}^j(\zeta, \Delta)} \right) - u(d^j) \right] \times$$
$$\times f(\tilde{n}^j) d\tilde{n}^j(\zeta, \Delta) f(\tilde{n}) d\tilde{n}(\zeta, \Delta) - \int_0^1 \int_{\frac{e^j}{d^j}}^1 u\left(\frac{e^j}{d^j}\right) f(\tilde{n}^j) d\tilde{n}^j(\zeta, \Delta) f(\tilde{n}) d\tilde{n}(\zeta, \Delta), \tag{71}$$

$$H^j(\sigma^*, \Delta)(0) = \int_0^1 \int_\pi^{\frac{e^j}{d^j}} \left[\sigma^* u \left(\frac{R(1 - \tilde{n}(0, \Delta))(e^j - \tilde{n}^j(0, \Delta) d^j)}{1 - \tilde{n}^j(0, \Delta)} \right) u(d^j) \right] \times$$
$$\times f(\tilde{n}^j) d\tilde{n}^j(0, \Delta) f(\tilde{n}) d\tilde{n}(0, \Delta) - \int_0^1 \int_{\frac{e^j}{d^j}}^1 u\left(\frac{e^j}{d^j}\right) f(\tilde{n}^j) d\tilde{n}^j(0, \Delta) f(\tilde{n}) d\tilde{n}(0, \Delta). \tag{72}$$

The intervals of integration of the two functions are the same. Moreover, the integrands are both

Lipschitz continuous in σ^*. Hence, there exists a constant C_1 such that:

$$|H^j(\sigma^*, \Delta)(\epsilon) - H^j(\sigma^*, \Delta)(0)| \leq C_1\epsilon. \tag{73}$$

In other words, as ϵ goes to zero, the two systems of equations coincide. To see that also the solutions of the two systems of equations coincide, let σ^* and Δ^* be the solution of the system of indifference conditions $H^j(\sigma^*, \Delta)(0) = 0$. Given any neighbourhood N of (σ^*, Δ^*), the function $H^j(\sigma^*, \Delta)(0)$ is uniformly bounded from 0 by some ι on $S \backslash N$. Choosing $\bar{\epsilon}$ such that $|H^j(\sigma^*, \Delta)(\epsilon) - H^j(\sigma^*, \Delta)(0)| \leq \iota$ for all $\epsilon < \bar{\epsilon}$, the system of equations $H^j(\sigma^*, \Delta)(\epsilon) = 0$ has no solution outside of N.

Finally, to characterize the unique threshold signal $\sigma^*(\mathbf{d})$ in (15), we take the average:

$$\begin{aligned}\frac{1}{G}\sum_j H^j(\sigma^*, \Delta)(0) =&\frac{1}{G}\sum_j \left[\int_0^1 \int_\pi^{\frac{e^j}{d^j}} \left[\sigma^* u\left(\frac{R(1-\tilde{n}(0,\Delta))(e^j - \tilde{n}^j(0,\Delta)d^j)}{1-\tilde{n}^j(0,\Delta)}\right) - u(d^j)\right] \times \right.\\ &\times f(\tilde{n}^j(0,\Delta))d\tilde{n}^j(0,\Delta)f(\tilde{n}(0,\Delta))d\tilde{n}(0,\Delta) + \\ &\left. - \int_0^1 \int_{\frac{e^j}{d^j}}^1 u\left(\frac{e^j}{d^j}\right) f(\tilde{n}^j(0,\Delta))d\tilde{n}^j(0,\Delta)f(\tilde{n}(0,\Delta))d\tilde{n}(0,\Delta)\right].\end{aligned} \tag{74}$$

By the Laplacian Property, $\tilde{n}^j(0,\Delta) \sim U[\pi, 1]$, hence the probability distribution $f(\tilde{n}^j(0,\Delta)) = 1/(1-\pi)$ is independent of Δ. In a similar way, by the Belief Constraint (Sakovics and Steiner, 2012), the Laplacian Property holds on average, meaning that also $\tilde{n}(0,\Delta) \sim U[\pi, 1]$, therefore the probability distribution $f(\tilde{n}(0,\Delta)) = 1/(1-\pi)$ is independent of Δ. Thus, the average indifference condition takes the form:

$$\sum_j \int_\pi^1 \int_\pi^{\frac{e^j}{d^j}} \sigma^* u\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right) dn^j dn = \sum_j \int_\pi^1 \int_\pi^{\frac{e^j}{d^j}} u(d^j)dn^j + \int_{\frac{e^j}{d^j}}^1 u\left(\frac{e^j}{n^j}\right) dn^j dn. \tag{75}$$

Rearranging this expression, we get threshold signal $\sigma^*(\mathbf{d})$ in (15). This ends the proof. ■

Proof of Corollary 1. We study the sign of:

$$\frac{\partial\sigma^*(\mathbf{d})}{\partial d^j} = \frac{1}{\sum_j \int_\pi^1 \int_\pi^{\frac{e^j}{d^j}} u\left(R(1-n)\frac{e^j - n^j d^j}{1-n^j}\right) dn^j dn} \times$$

$$\times \left[(1-\pi)u'(d^j)\left(\frac{e^j}{d^j}-\pi\right) + \sigma^*(\mathbf{d})\int_{\pi}^{1}\int_{\pi}^{\frac{e^j}{d^j}} u'(d_L^j(R,n,n^j))\frac{R(1-n)n^j}{1-n^j}dn^j dn \right] \tag{76}$$

This is clearly positive, as the utility function $u(c)$ is increasing in c and e^j/d^j is larger than π. This ends the proof. ■

Proof of Corollary 2. To prove that the threshold signal $\sigma^*(\mathbf{d})$ is decreasing in e^j, calculate:

$$\frac{\partial\sigma^*(\mathbf{d})}{\partial e^j} = \frac{1-\pi}{DEN_{\sigma^*}} \left[\int_{\frac{e^j}{d^j}}^{1} u'\left(\frac{e^j}{n^j}\right)\frac{1}{n^j}dn^j + \right. \tag{77}$$

$$\left. - \sigma^*(\mathbf{d})\int_{\pi}^{1}\int_{\pi}^{\frac{e^j}{d^j}} u'\left(R(1-n)\frac{e^j-n^jd^j}{1-n^j}\right)\frac{R(1-n)}{1-n^j}dn^j dn \right], \tag{78}$$

where DEN_{σ^*} is the denominator of $\sigma^*(\mathbf{d})$. By the Inada conditions:

$$\lim_{n^j\to\frac{e^j}{d^j}} u'\left(R(1-n)\frac{e^j-n^jd^j}{1-n^j}\right) = \lim_{c\to 0} u'(c) = +\infty. \tag{79}$$

Hence, the derivative is negative.[20] For the second part of the proof regarding the convexity, calculate instead:

$$\frac{\partial^2\sigma^*(\mathbf{d})}{\partial e^{j2}} = \frac{1-\pi}{DEN_{\sigma^*}} \left[-\frac{u'(d^j)}{e^j} + \int_{\frac{e^j}{d^j}}^{1} u''\left(\frac{e^j}{n^j}\right)\frac{1}{n^{j2}}dn^j + \right. \tag{80}$$

$$- \frac{\partial\sigma^*(\mathbf{d})}{\partial e^j}\left[\int_{\pi}^{1}\int_{\pi}^{\frac{e^j}{d^j}} u'\left(R(1-n)\frac{e^j-n^jd^j}{1-n^j}\right)\frac{R(1-n)}{1-n^j}dn^j dn\right] + \tag{81}$$

$$- \sigma^*(\mathbf{d})\left[\int_{\pi}^{1}\int_{\pi}^{\frac{e^j}{d^j}} u''\left(R(1-n)\frac{e^j-n^jd^j}{1-n^j}\right)\left[\frac{R(1-n)}{1-n^j}\right]^2 dn^j dn\right] + \tag{82}$$

$$\left. - \frac{\partial\sigma^*(\mathbf{d})}{\partial e^j}\left[\int_{\pi}^{1}\int_{\pi}^{\frac{e^j}{d^j}} u'\left(R(1-n)\frac{e^j-n^jd^j}{1-n^j}\right)\frac{R(1-n)}{1-n^j}dn^j dn\right]\right]. \tag{83}$$

As $\sigma^*(\mathbf{d})$ is decreasing in e^j and the utility function is concave, this expression must be positive by the Inada conditions. Hence $\sigma^*(\mathbf{d})$ is a convex function of e^j. This ends the proof. ■

Proof of Proposition 3. The proof of the Proposition is divided in two parts. First, we want to prove that We prove that $d^j = e^j$ for all groups j cannot be an equilibrium. We show this by

[20] Notice that $u(c)$ has a kink at $c = 0$, so it is not differentiable at that point. The derivative tends to infinite, without ever getting there.

contradiction. Assume that in equilibrium $d^j = e^j$ for all $j = 1, \ldots, G$. In this case, no self-fulfilling run can happen in any group, because the representative banks are able to serve all depositors, even in the case of a run. Hence, each group j is left only with runs happening in their lower dominance regions, below the thresholds signal $\underline{\sigma}^j$. Then, the first-order conditions of the banking problems at $d^j = e^j$ read:

$$FOC^j = \pi(1-\underline{\sigma}^j)\left[u'(e^j) - \frac{1-\pi}{2}(1+\underline{\sigma}^j)Ru'(R(1-\pi)e^j)\right] + \\ -\frac{u'(e^j) - \pi\underline{\sigma}^j Ru'(R(1-\pi)e^j)}{u(R(1-\pi)e^j)}(1-\pi)\left[\underline{\sigma}^j u(R(1-\pi)e^j) - u(e^j)\right], \tag{84}$$

for every group j. By definition of the threshold of the lower dominance region, the last term of (84) is equal to zero. Moreover, as the coefficient of relative risk aversion is larger than 1, we have that:[21]

$$\frac{u'(e^j)}{u'(R(1-\pi)e^j)} \geq \frac{R(1-\pi)e^j}{e^j}. \tag{85}$$

Hence:

$$FOC^j \geq \pi(1-\underline{\sigma}^j)R(1-\pi)u'(R(1-\pi)e^j)\left[1 - \frac{1+\underline{\sigma}^j}{2}\right] \tag{86}$$

which is strictly positive, as $\underline{\sigma}^j$ is smaller than 1. As the first-order condition is positive, this cannot be an equilibrium. In other words, there must be at least one group k for which $d^k > e^k$.

In the second part of the proof, we want to show that $d^k > e^k$ for one group k implies that $d^j > e^j$ also for the other groups $j \neq k$. To this end, we show that the bank objective function of the group j is supermodular in $(d^j - e^j)$ and $(d^k - e^k)$. The only place in the bank objective function of the group j where $(d^j - e^j)$ and $(d^k - e^k)$ interact is in the threshold signal $\sigma^*(\mathbf{d})$. Hence, proving supermodularity boils down to proving that the cross derivative of $\sigma^*(\mathbf{d})$ with respect to $(d^j - e^j)$ and $(d^k - e^k)$ is positive. To this end, apply the following change of variable: define $m^j = d^j = -e^j$ for all groups j, so that $d^j = m^j + e^j$. Then, the sign of the cross derivative of $\sigma^*(\mathbf{d})$ with respect to $(d^j - e^j)$ and $(d^k - e^k)$ is clearly equal to the sign of the cross derivative with respect to d^j and d^k. From (76), it is easy to argue that the cross derivative is positive, as d^k enters negatively in the

[21] The assumption about the coefficient of relative risk aversion is crucial for this result to hold. To see this, rewrite $-\frac{u''(c)c}{u'(c)} > 1$ as $-\frac{u''(c)}{u'(c)} > \frac{1}{c}$. This, in turn, means that $-(log[u'(c)])' > (log[c])'$. Integrate between z_1 and $z_2 > z_1$ so as to obtain $log[u'(z_1)] - log[u'(z_2)] > log[z_2] - log[z_1]$. Once taken the exponent, the last expression gives $\frac{u'(z_1)}{u'(z_2)} > \frac{z_2}{z_1}$. If $z_1 > z_2$, the inequality is reversed.

denominator of $\partial\sigma^*(\mathbf{d})/\partial d^j$ and $\sigma^*(\mathbf{d})$ (which is in the numerator) is increasing in d^k by Corollary 1. To sum up, this means that the bank objective function exhibits strategic complementarities in $(d^j - e^j)$ and $(d^k - e^k)$. As $(d^k - e^k) > 0$ for at least one group k, that must mean that $(d^j - e^j) > 0$ also for the other groups $j \neq k$. This ends the proof. ■

Proof of Proposition 4. As argued in the main text, if a subsidy scheme $s^j = d^j - e^j$ for all groups j is feasible, no run is possible and the banks in all groups can implement the equilibrium with perfect information of section 3.4. Rearranging the Euler equation (4), we derive the amount of early consumption d^j with perfect information as:

$$d^j = \frac{Ru'^{-1}((1-\pi)\mathbb{E}[p]R)}{1 + \pi Ru'^{-1}((1-\pi)\mathbb{E}[p]R)} e^j. \tag{87}$$

By the government budget constraint (21), the subsidy scheme that makes the economy run proof under $\tau^j = 1$ for all groups j is feasible if:

$$\sum_j \bar{e}^j \geq \sum_j \left[d^j - e^j\right] = \left[\frac{(1-\pi)Ru'^{-1}((1-\pi)\mathbb{E}[p]R) - 1}{1 + \pi Ru'^{-1}((1-\pi)\mathbb{E}[p]R)}\right] \sum_j e^j. \tag{88}$$

■

Proof of Corollary 3. For the first part of the Corollary, it is sufficient to prove that $\Xi \leq 1$ is equivalent to:

$$u'^{-1}((1-\pi)\mathbb{E}[p]R) \leq \frac{2}{(1-2\pi)R}. \tag{89}$$

As the utility function $u(c)$ is increasing and concave, its first derivative is decreasing and $u'^{-1}(c)$ is increasing. Therefore, the left-hand side of (89) is increasing in R, and the right-hand side is decreasing. In other words, there exists an upper bound $\bar{R}$ for R that satisfies the inequality. Under CRRA utility, $u'^{-1}((1-\pi)\mathbb{E}[p]R) = [(1-\pi)\mathbb{E}[p]R]^{-1/\gamma}$, where $\gamma > 1$ is the coefficient of relative risk aversion. As $(1-\pi)\mathbb{E}[p]R > 1$ by assumption, the left-hand side of (89) is strictly smaller than 1. Hence, for the inequality to hold it is sufficient to have that the right-hand side is larger than or equal to 1, meaning that $R \leq 2/(1-2\pi)$, which is positive as $\pi > 1/2$. To show that Ξ is increasing in the coefficient of relative risk aversion under CRRA, calculate:

$$\frac{\partial \Xi(\gamma)}{\partial \gamma} = \frac{[(1-\pi)\mathbb{E}[p]R]^{-\frac{1}{\gamma}} ln((1-\pi)\mathbb{E}[p]R)R}{\gamma^2 \left[1 + \pi R[(1-\pi)\mathbb{E}[p]R]^{-\frac{1}{\gamma}}\right]}[(1-\pi) - \pi\Xi(\gamma)]. \tag{90}$$

Under $\pi > 1/2$ and $R < \bar{R}$, $\Xi(\gamma) \leq 1$ and $1/\pi - 1 > 1$, hence this expression is larger than or equal to zero. This ends the proof. ■

Proof of Proposition 5. To prove that the optimal partial intervention fully taxes all groups, i.e. $\tau^j = 1$ for all j-s, attach the Lagrange multipliers $\bar{e}^j\zeta^j$ to $\tau^j \geq 0$ and $\bar{e}^j\eta^j$ to $\tau^j \leq 1$. The first-order condition with respect to τ^j reads:

$$\bar{e}^j \left[\xi + \zeta^j - \eta^j\right] = 0, \tag{91}$$

where ξ is the multiplier on the government budget constraint (21). As ξ is the same across all groups, it must be that:

$$\eta^j - \zeta^j = \eta^\ell - \zeta^\ell \tag{92}$$

for any two groups j and ℓ, meaning that taxes must be the same across groups. If $\tau^j = 0$ for all j-s, then also $s^j = 0$ for all groups j. Clearly, this cannot be an equilibrium, because the threshold signal $\sigma^*(\mathbf{d}, \mathbf{s})$ is decreasing in the subsidies s^j, so the economy would be better off by using the proceeds from taxation to positively subsidize at least one group. Similarly, if $\tau^j \in (0, 1)$ for all groups j, then $\eta^j = \zeta^j = 0$ for all groups j by complementary slackness, meaning that $\xi = 0$, which cannot be an equilibrium. Hence, the only possible case left is $\tau^j = 1$ for all groups j.

For the second part of the Proposition, attach the Lagrange multipliers λ^j and χ^j to the upper and lower bounds of s^j. The first-order condition with respect to s^j then reads:

$$-\frac{\partial \sigma^*(\mathbf{d}, \mathbf{s})}{\partial s^j} \sum_k \Delta U_B^k - \lambda^j + \chi^j = \xi, \tag{93}$$

for all $j = 1, \ldots, G$, where ΔU_B^k is defined as in (18). Then, the government bailout scheme satisfies the equilibrium condition:

$$-\frac{\partial \sigma^*(\mathbf{d}, \mathbf{s})}{\partial s^j} \sum_k \Delta U^k - \lambda^j + \chi^j = -\frac{\partial \sigma^*(\mathbf{d}, \mathbf{s})}{\partial s^\ell} \sum_k \Delta U^k - \lambda^\ell + \chi^\ell, \tag{94}$$

for any two groups j and ℓ. Calculate Ψ^j according to (41), which is obviously positive for every group j, because we proved that $\sigma^*(\mathbf{d},\mathbf{s})$ is a decreasing function of the subsidy s^j and $u(c)$ is increasing. Then, rank the groups by decreasing Ψ^j. For the condition (94) to hold, it must be the case that:

$$-\lambda^{(1)} + \chi^{(1)} < -\lambda^{(2)} + \chi^{(2)} < \cdots < -\lambda^{(G)} + \chi^{(G)}, \tag{95}$$

where (j) indicates the j-th group in the ranking. Assume that $-\lambda^{(1)} + \chi^{(1)} > 0$. For this to be true, it must be that $\lambda^{(1)} = 0$ and $\chi^{(1)} > 0$, meaning that the group with the highest Ψ^j gets the lowest possible subsidy, or $s^{(1)} = 0$. But if $-\lambda^{(1)} + \chi^{(1)} > 0$, also $-\lambda^j + \chi^j > 0$ for all groups j. This means that all groups get the lowest possible subsidy, or $s^j = 0$ for all groups j, which cannot be an equilibrium for the same argument that we make above. Hence, we must have that $-\lambda^{(1)} + \chi^{(1)} \leq 0$. On the contrary, assume that $-\lambda^{(G)} + \chi^{(G)} < 0$. Then $\chi^{(G)} = 0$ and $\lambda^{(G)} > 0$, implying that $s^{(G)} = d^{(G)} - e^{(G)} > 0$. However, if $-\lambda^{(G)} + \chi^{(G)} < 0$, also $-\lambda^j + \chi^j < 0$ for all groups j, and $s^j = d^{(j)} - e^{(j)}$ for all groups j. This is not possible, as we ruled out the possibility of complete subsidization. Thus, the only possible equilibrium features $-\lambda^{(G)} + \chi^{(G)} \geq 0$: some groups are fully subsidized and some others get zero. This implies that there exists a unique threshold group $\hat{j}$ for which there is indifference. This ends the proof. ■

Proof of Proposition 6. When the government intervenes in a time-consistent manner, the banking problem in group j reads:

$$\max_{d^j} \int_0^{\sigma^*(\mathbf{d},\mathbf{s})} \Big[u(e^j + s^j) + w((1-\tau^j)\bar{e}^j)\Big] dp + \\ + \int_{\sigma^*(\mathbf{d},\mathbf{s})}^1 \Big[\pi u(d^j) + (1-\pi) p u(R(e^j - \pi d^j)) + w(\bar{e}^j)\Big] dp, \tag{96}$$

subject to the expression for $\sigma^*(\mathbf{d},\mathbf{s})$ in (26). To prove that d^j is non-decreasing in s^k for any k, we need to prove that d^j and s^k are strategic complements. In turn, that means that we need to prove that the bank's objective function is supermodular in d^j and s^k, i.e. that its cross-derivative with respect to d^j and s^k is positive. Clearly, the only place in the bank objective function where d^j and s^j interact is in the threshold signal $\sigma^*(\mathbf{d},\mathbf{s})$. Hence, proving supermodularity boils down to prove that the cross derivatives of $\sigma^*(\mathbf{d},\mathbf{s})$ with respect to d^j and s^j are positive:[22]

$$\frac{\partial\sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j} = \frac{(1-\pi)\int_{\frac{e^j}{d^j}}^{1} u'\left(\frac{e^j}{n^j}+s^j\right)\frac{1}{n^j}dn^j}{\sum_j \int_\pi^1 \int_\pi^{\frac{e^j}{d^j}} u\left(R(1-n)\frac{e^j-n^jd^j}{1-n^j}\right)dn^j dn} > 0, \tag{97}$$

$$\begin{aligned}\frac{\partial^2\sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j\partial d^j} =& \frac{(1-\pi)}{\sum_j \int_\pi^1 \int_\pi^{\frac{e^j}{d^j}} u\left(R(1-n)\frac{e^j-n^jd^j}{1-n^j}\right)dn^j dn}\left[\frac{1}{d^j}u'\left(d^j+s^j\right)+\right.\\ &\left.+\frac{\partial\sigma^*}{\partial s^j}\left[\int_\pi^1\int_\pi^{\frac{e^j}{d^j}} u'(d_L(R,n,n^j))\frac{R(1-n)n^j}{1-n^j}dn^j dn\right]\right] > 0.\end{aligned} \tag{98}$$

In a similar way, we prove that the objective function of a bank in group j is supermodular in s^j and d^k for any $k \neq j$. In fact:

$$\begin{aligned}\frac{\partial^2\sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j\partial d^k} =& \frac{(1-\pi)\int_{\frac{e^j}{d^j}}^{1} u'\left(\frac{e^j}{n^j}+s^j\right)\frac{1}{n^j}dn^j}{\left[\sum_j \int_\pi^1 \int_\pi^{\frac{e^j}{d^j}} u\left(R(1-n)\frac{e^j-n^jd^j}{1-n^j}\right)dn^j dn\right]^2}\times\\ &\times\left[\int_\pi^1\int_\pi^{\frac{e^j}{d^j}} u'\left(R(1-n)\frac{e^j-n^jd^j}{1-n^j}\right)\frac{R(1-n)n^j}{1-n^j}dn^j dn\right]\end{aligned} \tag{99}$$

is clearly positive.

When the government commits to intervene against bank insolvency, as we proved in the text the effect of the subsidy on the threshold signal $\sigma^*(\mathbf{d},\mathbf{s})$ is the same as in the time-consistent case. Hence, d^j is non-decreasing in s^k for any group j and k as before.

When the government commits to intervene against bank illiquidity and a full liquidity assistance is not feasible, the banking problem in group j reads:

$$\max_{d^j} \int_0^{\sigma^*(\mathbf{s})} u(e^j)dp + \int_{\sigma^*(\mathbf{s})}^1 \left[\pi u(d^j) + (1-\pi)pu(R(e^j-\pi d^j))\right]dp + w(\bar{e}^j), \tag{100}$$

subject to the expression for $\sigma^*(\mathbf{d},\mathbf{s})$ in (36). To prove that d^j is non-increasing in s^k for any k, we need to prove that d^j and s^k are strategic substitutes, which means that the cross-derivative of the

[22]Let X be an open sublattice of $\mathbb{R}^m$. A twice-continuously differential function $F: X \to \mathbb{R}$ is supermodular on X if and only if for all $\mathbf{x} \in X$ we have that $\partial^2 F/\partial x_i \partial x_j \geq 0$ for any $i, j = 1, \ldots, m$ and $i \neq j$ (Topkis, 1998).

bank objective function with respect to d^j and s^k is negative. Again, the only place in the bank objective function where d^j and s^k interact is in the threshold signal $\sigma^*(\mathbf{d}, \mathbf{s})$. Hence, in order to prove supermodularity we just need to prove that the cross derivative of $\sigma^*(\mathbf{d}, \mathbf{s})$ with respect to d^j and s^k is negative. Differentiating (37) with respect to d^j, we obtain:

$$
\begin{aligned}
\frac{\partial^2 \sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j \partial d^j} = & \frac{(1-\pi)}{\sum_j \int_\pi^1 \int_\pi^{\frac{e^j+s^j}{d^j}} u\left(R(1-n)\frac{e^j+s^j-n^j d^j}{1-n^j}\right) dn^j dn} \times \\
& \times \left[-\frac{u(d^j) - u\left(\frac{e^j}{e^j+s^j} d^j\right)}{d^{j2}} + \frac{u'(d^j) - u'\left(\frac{e^j}{e^j+s^j} d^j\right)\frac{e^j}{e^j+s^j}}{d^j} + \right. \\
& - \frac{\partial \sigma^*(\mathbf{d},\mathbf{s})}{\partial d^j}\left[\int_\pi^1 \int_\pi^{\frac{e^j+s^j}{d^j}} u'(d_L(R,n,n^j))\frac{R(1-n)}{1-n^j} dn^j dn\right] + \\
& + \sigma^* \int_\pi^1 \int_\pi^{\frac{e^j+s^j}{d^j}} u''(d_L(R,n,n^j))\frac{R^2(1-n)^2 n^j}{(1-n^j)^2} dn^j dn + \\
& \left. + \frac{\partial \sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j}\left[\int_\pi^1 \int_\pi^{\frac{e^j+s^j}{d^j}} u'(d_L(R,n,n^j))\frac{R(1-n)n^j}{1-n^j} dn^j dn\right]\right]. \qquad (101)
\end{aligned}
$$

This expression is negative: the first term in the square brackets is positive and enters negatively, the second term is negative as the coefficient of relative risk aversion is larger than 1,[23] the threshold signal $\sigma^*(\mathbf{d}, \mathbf{s})$ is increasing in d^j and decreasing in s^j, and the utility function is concave. To prove that d^j is non-increasing in s^k for any $k \neq j$, calculate instead:

$$
\begin{aligned}
\frac{\partial^2 \sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j \partial d^k} = & \frac{(1-\pi)}{\sum_j \int_\pi^1 \int_\pi^{\frac{e^j+s^j}{d^j}} u\left(R(1-n)\frac{e^j+s^j-n^j d^j}{1-n^j}\right) dn^j dn} \times \\
& \times \left[-\frac{\partial \sigma^*}{\partial d^k}\left[\int_\pi^1 \int_\pi^{\frac{e^j+s^j}{d^j}} u'(d_L(R,n,n^j))\frac{R(1-n)}{1-n^j} dn^j dn\right] + \right. \\
& \left. + \frac{\partial \sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j}\left[\int_\pi^1 \int_\pi^{\frac{e^k+s^k}{d^k}} u'(d_L(R,n,n^k))\frac{R(1-n)n^k}{1-n^k} dn^k dn\right]\right]. \qquad (102)
\end{aligned}
$$

For the same reasons as above, this expression is negative.

Finally, when the government commits to intervene against bank illiquidity and insolvency and

[23] See footnote 21.

a full liquidity assistance is not feasible, the banking problem in group j reads as in (96) subject to the expression for $\sigma^*(\mathbf{d},\mathbf{s})$ in (44). By taking the derivative of (45) with respect to d^j, we obtain:

$$\begin{aligned}\frac{\partial^2\sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j\partial d^j} = & \frac{(1-\pi)}{\sum_j \int_\pi^1 \int_\pi^{\frac{e^j+s^j}{d^j}} u\left(R(1-n)\frac{e^j+s^j-n^jd^j}{1-n^j}\right)dn^j dn} \times \\ & \times \left[-\frac{u(d^j)-u\left(\frac{e^j}{e^j+s^j}d^j\right)}{d^{j2}} + \frac{u'(d^j)-u'\left(\frac{e^j}{e^j+s^j}d^j\right)\frac{e^j}{e^j+s^j}}{d^j} + \frac{u'(d^j)}{d^j}\right. \\ & -\frac{\partial\sigma^*(\mathbf{d},\mathbf{s})}{\partial d^j}\left[\int_\pi^1\int_\pi^{\frac{e^j+s^j}{d^j}} u'(d_L(R,n,n^j))\frac{R(1-n)}{1-n^j}dn^j dn\right] + \\ & +\sigma^*(\mathbf{d},\mathbf{s})\int_\pi^1\int_\pi^{\frac{e^j+s^j}{d^j}} u''(d_L(R,n,n^j))\frac{R^2(1-n)^2n^j}{(1-n^j)^2}dn^j dn + \\ & \left. + \frac{\partial\sigma^*(\mathbf{d},\mathbf{s})}{\partial s^j}\left[\int_\pi^1\int_\pi^{\frac{e^j+s^j}{d^j}} u'(d_L(R,n,n^j))\frac{R(1-n)n^j}{1-n^j}dn^j dn\right]\right]. \end{aligned} \tag{103}$$

As in (101) the first two and the last three terms in the numerator of (103) are negative, but the third is positive. By the Inada conditions, the last three terms are large and dominate, hence (103) is negative. Similarly, notice from (45) that the cross-derivative of the threshold signal $\sigma^*(\mathbf{d},\mathbf{s})$ with respect to s^j and d^k is equivalent to (102), hence is negative. This ends the proof. ■