International Transmission of the Business Cycle and Environmental Policy*

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Abstract

This paper presents a baseline dynamic general-equilibrium model of environmental policy for a two-country economy and studies the international transmission of several asymmetric shocks considering three different economy-wide greenhouse gases (GHG) emission regulations: (i) national cap-and-trade, (ii) carbon tax, and (iii) international cap-and-trade system allowing for cross-border allocation of emission permits. We find that international spillovers of shocks originated in one country are strongly influenced by the environmental regime put in place. We show that, while a national cap-and-trade system reduces the international spillovers by dampening the response of the country hit by shocks, the cross-border reaction to supply-side shocks is found to be magnified under an international cap-and-trade system and demand shocks are more intensively transmitted under a carbon tax. The pattern of trade and the underlying monetary regime influence the cross-border transmission channels interacting with the environmental policy adopted.

Keywords: Open Economy Macroeconomics, GHG Emission Control, Macroeconomic Dynamics.

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1 Introduction

This paper presents a baseline general-equilibrium theoretical model with two-interdependent economies to highlight the international aspects of environmental policies. In particular, the paper addresses the following fundamental questions. What is the role of different environmental policy regimes in shaping the transmission of shocks in open economies? What is the dynamic behavior of an economy where countries are tied by international trade and by a common environmental policy regime? How does the pattern of trade interact with the underlying environmental policy? What happens if countries share the same currency?

The impact of unilateral mitigation policies and the strategic interactions between different countries committed to regulate emissions are topics largely debated among environmental economists. Computable General Equilibrium (CGE) models and Integrated Assessment Models (IAMs) are at present the main tools used to estimate costs and benefits of different policies in climate change research. Nevertheless, only recently, another class of environmental models have been emerging in macroeconomics in which a growing attention is given to the role of uncertainty and of the business cycle in influencing the performance of environmental regulation.[1]

Methodologically this strand of literature on the interaction between climate actions and the business cycle is based on dynamic stochastic general equilibrium (DSGE) models and involves the systematic application of intertemporal optimization methods and of the rational expectation hypothesis that determine the behavior of consumption, investment and factor supply for different states of the economy.[2] As proposed by Kahn et al. (2015) we use the acronym E-DSGE to refer to dynamic stochastic general equilibrium models with environmental regulation.

For a long time environmental aspects have been neglected by the so-called "New Consensus Macroeconomics", as remarked by Arestis and González-Martínez (2015).[3] Relevant examples of E-DSGE models include Chang et al. (2009), Angelopoulos et al. (2013), Heutel (2012), Fischer and Springborn (2011), Bosetti and Maffezzoli (2014), Annicchiarico and Di Dio (2015) and Dissou and Karnizova (2016). However, the international dimension of climate actions has so far been neglected in the context of E-DSGE models, therefore the study of the interaction among environmental policy, international trade and economic uncertainty has still remained unexplored. As far as we know, the only exception in this direction is the contribution by

[1] For an accurate and comprehensive empirical analysis of the cyclical relationship between output and carbon dioxide emissions, see Doda (2014); for an interesting investigation on the behavior of emissions at business cycle frequency in response to different technology shocks, see Khan et al. (2016).

[2] Dynamic general equilibrium models are also fruitfully used for the study of energy and climate policies in deterministic analyses abstracting from the business cycle. See Conte et al. (2010), Annicchiarico et al. (2016, 2017), and Bartocci and Pisani (2013). This last paper is the only one exploring the international dimension of energy policies analysing the effects of both unilateral and simultaneous interventions throughout the EU.

[3] However, the role of uncertainty in shaping the performance of different environmental regulations has been widely addressed in the literature. Following the seminal paper by Weitzman (1974), several contributions study the performance of alternative environmental policies, accounting for uncertainty. See, e.g., Quirion (2005) and Jotzo and Pezzey (2007). On the relationship between economic fluctuations and environmental policy, see e.g. Kelly (2005).

Ganelli and Tervala (2011) who explore the international transmission of a unilateral implementation of a more stringent mitigation policy in the context of a New Keynesian - E-DSGE model of a global economy, however, they neither consider the international transmission of shocks commonly studied in the business cycle literature, nor the role played by the underlying environmental regime in shaping fluctuations and cross-border spillovers.[4]

With this paper we aim at filling this gap of this strand of literature, enriching the methodology based upon choice-theoretic stochastic models, by embodying New Keynesian aspects, such as nominal rigidities, imperfect competition and forward-looking price-setting, consistently with Annicchiarico and Di Dio (2015, 2017), and developing the analysis in an open economy model with two interdependent countries, Home and Foreign. With this model in hand, we are able to explore the international transmission of a battery shocks commonly considered in the business cycle literature, and to study the role played by different environmental regimes in shaping the dynamic response of the economy. In particular, we consider three policies for constraining emissions: a national cap-and-trade, a carbon tax and an international cap-and trade, where emission permits are traded between countries. We explore the dynamic response of the economy to five shocks hitting Home, namely (i) a positive technology shock on total factor productivity (TFP), (ii) a shock increasing emission abatement costs, (iii) a positive shock on the quality of capital, (iv) a positive investment-specific technology shock, and (v) a positive shock on the risk-free interest rate set by the monetary authorities. The first two shocks directly affect the supply side of the economy (supply shocks), while the last two directly change aggregate demand (demand shocks). The shock on the quality of capital, instead, is a hybrid shock, altering directly and simultaneously both the supply and the demand schedules of the economy. To further shed light on the influence exerted by environmental policies on the international transmission channels of shocks, we also look at the spillover effects under different assumptions regarding the pattern of trade and the underlying monetary regime.

Our main results can be summarized as follows. First, the international transmission of shocks from one economy to another may be affected by the underlying environmental regime, but the magnitude of the spillover effects crucially depends on the source of uncertainty. In particular, in response to supply-side shocks we observe major differences across regimes, especially when we look at the main macroeconomic aggregates, while in response to demand-side shocks the differences are smaller. We argue that this result is due to the role played by mon-

[4] Yet the international dimension of climate policies has been the object of several studies in the field of environmental economics. For an overview on the relationships linking trade, economic growth and environment, see Copeland and Taylor (2013). For a survey of studies focussing specifically on environmental policy analysis in open economy, see e.g. Rauscher (2005). A substantial body of literature, mostly related to CGE models, tackle problems relative to carbon leakage, strategic behaviors (e.g. Burniaux and Martins 2012 and Babiker 2005), and the loss of competitiveness (see Carbone and Rivers, 2017). For a general overview on the relationship between environmental regulation and competitiveness, see e.g. Dechezleprêtre and Sato (2017). Furthermore the effects of climate policies in open economy have been extensively analysed and estimated, mainly by means of different simulations scenarios, in the context of integrated assessment models (IAMs). Thank to their regional or global structure these models are well suited for a study of the overall costs of different policy instruments. For an overview on global scale IAMs, see Weyant (2017).

etary policy in reaction to each type-shock. On the other hand, as expected, environmental variables display plainly different dynamics across regimes, no matter the nature of the shock, and in some circumstances emissions may turn out to be countercyclical.

Second, contrary to what expected, the adoption of an international cap-and-trade regime does not necessarily exacerbate the international spillover of shocks. In particular, we observe that major effects are observed for shocks on TFP and abatement technology, while for investment specific shocks and monetary policy shocks the spillover effects are found to be larger under a carbon tax.

Third, when we solve the model assuming a trade pattern such that Home and Foreign goods are imperfect complements, rather than substitutes, we show how the two economies tend to move in the same direction in all the circumstances. In particular, in response to a TFP shock hitting Home under imperfect complementarity Foreign output increases under all the regimes, but under an international cap-and-trade the outflow of emission permits toward the more productive economy reduces the positive spillover effects from the international trade channel. When instead we assume a higher degree of openness, the correlation between domestic and foreign output does not necessarily increase. Again the underlying environmental regime and the source of uncertainty shape the size and the sign of the cross-border effects.

Finally, we show how the role played by environmental policies in shaping the international transmission channels of asymmetric shocks changes when the economies share the same currency. In particular, we show how in response to a positive TFP shock hitting the domestic economy, the correlation between Home and Foreign output turns out to be positive, since in dealing with the asymmetric shock monetary policy is now less accommodative for Home, but it becomes expansionary for Foreign. However, contrary to what observed in the benchmark case, the intensity of the relationship between the two economies is found to be stronger under a carbon tax, but weaker under an international cap-and-trade regime, where the possibility of importing emission permits from abroad for Home producers reduces the positive spillover effects on Foreign output.

The remainder of the paper is organized as follows. Section 2 describes the two-country model and introduces the various sources of uncertainty giving rise to different dynamic adjustments of the economy. Section 3 summarizes the parametrization used to numerically solve the model. Section 4 presents the dynamic response of macroeconomic and environmental variables to various types of shocks, under different environmental policy regimes, accounting for the role of international trade in the propagation of disturbances between countries. Section 5 summarizes the main results and concludes.

2 The model

We model an artificial economy with two countries, Home and Foreign, open to international trade and capital flows. Home and Foreign are modeled symmetrically, therefore the following description holds for both economies. Foreign variables are denoted by a superscript asterisk. Each country manufactures one type of tradable goods produced in a number of horizontally differentiated varieties, by using labor and physical capital as factor inputs. The goods sector is characterized by monopolistic competition and price stickiness in the form of quadratic adjustment costs of the Rotemberg (1982) type, while labor and capital are immobile between countries. On the demand side, the economy is populated by households deriving utility from consumption and disutility from labor. For convenience, we assume the existence of a perfectly competitive final good sector that produces a final good by combining domestic and foreign varieties of intermediate goods. Households supply labor and capital to domestic producers and hold two financial assets, namely domestic and foreign bonds. The economy features pollutant emissions, which are a by-product of output, and a negative environmental externality on production. Finally, we have a central bank making decisions on monetary policy and a government that sets the environmental policy.

2.1 Households

The typical infinitely lived household derives utility from consumption, C_t, and disutility from hours worked, L_t. The lifetime utility U is of the type:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\varphi_C}}{1-\varphi_C} - \xi_L \frac{L_t^{1+\varphi_L}}{1+\varphi_L} \right), \tag{1}$$

where E is the rational expectations operator, $\beta \in (0,1)$ is the discount factor, φ_C is the coefficient of relative risk aversion, ξ_L is a scale parameter measuring the relative disutility of labor, and φ_L is the inverse of the Frisch elasticity of labor supply. Households own the stock of physical capital, K_t, and provides it to firms in a perfectly competitive rental market. The accumulated capital stock K_t is subject to a quality shock determining the level of effective capital for use in production. In addition, we introduce an investment-specific technology shock affecting the extent to which investment spending increases the capital stock available for production in the following period. Therefore, the stock of capital held by households evolves according to the following law of motion:

$$K_{t+1} = e^{u_{I,t}} I_t + (1-\delta) e^{u_{K,t}} K_t, \tag{2}$$

where I_t denotes investments, K_t is physical capital carried over from period $t-1$ and $\delta \in (0,1)$ is the depreciation rate of capital, while u_I and u_K are two exogenous processes capturing, respectively, the marginal efficiency of investment and capital-quality shocks.[5] The capital-quality shock is meant to capture any exogenous variation in the value of installed capital able

[5] On the relevance of investment shocks in driving business cycle fluctuations, see Justiniano et al. (2010) and Furlanetto and Seneca (2014).

to trigger sudden variations in its market value and changes in investment expenditure.[6] The two processes are such that

$$u_{I,t} = \rho_I u_{I,t-1} + \varepsilon_{I,t}, \tag{3}$$

$$u_{K,t} = \rho_K u_{K,t-1} + \varepsilon_{K,t}, \tag{4}$$

where $0 < \rho_I, \rho_K < 1$, $\epsilon_I \sim i.i.d.\ N(0, \sigma_I^2)$ and $\epsilon_K \sim i.i.d.\ N(0, \sigma_K^2)$. Investment decisions are subject to convex capital adjustment costs of the type $\Gamma_K(I_t, K_t) \equiv \frac{\gamma_I}{2}(\frac{I_t}{K_t} - \delta)^2 K_t$, $\gamma_I > 0$. It should be noted how both shocks bring about a change in effective capital. However, while the investment-technology specific shock indirectly affects effective capital through a change in investment flows and so in aggregate demand, the capital-quality shock directly affects the capital in use for production and indirectly influences future investments by changing their expected return.

We further assume that domestic residents have access to a one-period risk free bond, B_t, sold at a price R_t^{-1} and paying one unit of currency in the following period, and to a risk-free asset traded between the two countries, F_t^*, denominated in Foreign currency, sold at a price $(R_t^*)^{-1}$ and paying one unit of foreign currency in the following period. Households receive lump-sum transfers Tr_t from the government, dividends D_t from the ownership of domestic intermediate good-producing firms, and payments for factors they supply to these firms: a nominal capital rental rate $R_{K,t}$ and a nominal wage W_t.

Denoting the consumption price index by P_t, the period-by-period budget constraint reads as:

$$\begin{aligned} P_t C_t + P_t I_t + R_t^{-1} B_t + (R_t^*)^{-1} S_t F_t^* = W_t L_t + R_{K,t} K_t + \\ + B_{t-1} + S_t F_{t-1}^* - P_t \Gamma_K(I_t, K_t) + P_t Tr_t + P_t D_t, \end{aligned} \tag{5}$$

where S_t is the nominal exchange rate expressed as the price of Foreign currency in units of Home currency. The typical household will choose the sequences $\{C_t,\ K_{t+1},\ I_t,\ L_t,\ B_t,\ F_t^*\}_{t=0}^{\infty}$ so as to maximize (1), subject to (2) and (5).

Rewriting the budget constraint in real terms, from the households' utility maximization problem, we obtain the following set of first-order conditions:

$$C_t^{-\varphi_c} = \lambda_t, \tag{6}$$

[6] This type of shock is introduced in DSGE models to mimic a recession originating from an adverse shock on the asset price. As we will see this shock is able to generate co-movement of consumption, investment, hours and output. See e.g. Gertler and Kiyotaki (2010).

$$q_t = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[r_{k,t+1} + \gamma_I \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{1}{K_{t+1}} - \frac{\gamma_I}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 \right] \right\} + \tag{7}$$
$$+\beta(1-\delta) E_t \left\{ e^{u_{K,t+1}} \frac{q_{t+1}\lambda_{t+1}}{\lambda_t} \right\},$$

$$e^{u_{I,t}} q_t - 1 = \gamma_I \left(\frac{I_t}{K_t} - \delta \right), \tag{8}$$

$$\lambda_t w_t = \xi_L L_t^{\varphi_L}, \tag{9}$$

$$\frac{1}{R_t} = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\Pi_{t+1}} \right\}, \tag{10}$$

$$\frac{1}{R_t^*} = \beta E_t \left\{ \frac{\lambda_{t+1} S_{t+1}}{\lambda_t \Pi_{t+1} S_t} \right\}, \tag{11}$$

where λ_t denotes the Lagrange multiplier associated to the flow budget constraint (5) expressed in real terms and measures the marginal utility of consumption according to condition (6), $r_{k,t} = \frac{R_{k,t}}{P_t}$, $w_t = \frac{W_t}{P_t}$, q_t is the Tobin's q and $\Pi_t = \frac{P_t}{P_{t-1}}$ measures inflation in the final-good sector. Equations (7) and (8) refer to the optimality conditions with respect to capital and investments, (9) describes labor supply, whereas (10) and (11) are the two first-order conditions with respect to domestic and foreign assets, reflecting the optimal choice between current and future consumption, given the return on the two risk-free assets, expected inflation and the expected depreciation of the domestic currency.

2.2 Production

2.2.1 Production of Domestic Intermediate Goods

The intermediate good producing sector is dominated by a continuum of monopolistically competitive polluting firms indexed by $j \in (0,1)$. Each firm charges the same price at home and abroad and faces a demand function that varies inversely with its output price $P_{j,t}^D$ and directly with aggregate demand Y_t^D for domestic production, that is $Y_{j,t}^D = \left(\frac{P_{j,t}^D}{P_t^D} \right)^{-\sigma} Y_t^D$, where $\sigma > 1$ and P_t^D is an aggregate price index defined below.

The producer of the variety j hires capital and labor in perfectly competitive factor markets to produce an intermediate goods $Y_{j,t}^D$ according to a Cobb-Douglas technology, modified to incorporate a capital-quality shock and the damage from pollution, measured in terms of intermediate output's reduction:

$$Y_{j,t}^D = \Lambda_t A_t \left(u_{K,t} K_{j,t} \right)^{\alpha} L_{j,t}^{1-\alpha}, \tag{12}$$

where $0 < \alpha < 1$ is the elasticity of output with respect to capital, A denotes total factor productivity, and Λ_t is a term capturing the negative externality of pollution on production. In particular, referring to Golosov et al. (2014), we adopt the following simplified specification for the damage function Λ_t:

$$\Lambda_t = \exp[-\chi(Z_t - \overline{Z})], \tag{13}$$

where Z_t is the global stock of carbon dioxide in period t, $\overline{Z}$ is the pre-industrial atmospheric CO_2 concentration, and χ is a positive scale parameter measuring the intensity of the negative externality on production.[7] We assume that productivity A_t is subject to shocks, that is $A_t = Ae^{u_{A_t}}$, where A denotes the steady-state productivity level, while u_{A_t} is assumed to evolve according to the following process:

$$u_{A_t} = \rho_A u_{A_{t-1}} + \varepsilon_{A,t}, \tag{14}$$

where $0 < \rho_A < 1$ and $\varepsilon_{A,t} \sim i.i.d.\ N(0, \sigma_A^2)$.

Emissions for firm are a by-product of output:

$$E_{j,t} = (1 - \mu_{j,t})\epsilon(Y_{j,t}^D)^{1-\gamma}, \tag{15}$$

where the parameter γ determines the elasticity of emissions with respect to output, ϵ is a parameter that we use to scale the emission function and $0 < \mu_t < 1$ is the abatement effort.

Firms are subject to environmental regulation and can choose to purchase emission permits on the market at the price $P_{E,t}$ (or to pay a tax in the case of price regulation), or to incur in abatement costs $AC_{j,t}$ to reduce emissions. Abatement costs, in turn, depend on firm's output and on abatement effort:

$$AC_{j,t} = e^{u_{AC,t}}\theta_1 \mu_{j,t}^{\theta_2} Y_{j,t}^D, \tag{16}$$

where $\theta_1 > 0$ and $\theta_2 > 1$ are technological parameters, while $u_{AC,t}$ is a zero mean shock process:

$$u_{AC,t} = \rho_{AC} u_{AC,t-1} + \epsilon_{AC,t}, \tag{17}$$

with $0 < \rho_{AC} < 1$ and $\epsilon_{AC} \sim i.i.d.\ N(0, \sigma_{AC}^2)$.

Let $p_{E,t} = \frac{P_{E,t}}{P_t}$ and $p_t^D = \frac{P_t^D}{P_t}$, by imposing symmetry across producers, from the solution of firm j's static cost minimization problem, we have the following optimality conditions:

$$r_{K,t} = \alpha \Psi_t \frac{Y_t^D}{K_t}, \tag{18}$$

$$w_t = (1 - \alpha)\Psi_t \frac{Y_t^D}{L_t}, \tag{19}$$

$$p_{E,t}(Y_t^D)^{(1-\gamma)} = \theta_2 \theta_1 e^{u_{AC,t}} \mu_t^{\theta_2 - 1} Y_t^D p_t^D, \tag{20}$$

[7] A similar specification is adopted by Annicchiarico et al. (2017).

where equations (18) and (19) are demands for capital and labor, equation (20) is the optimal abatement choice and Ψ_t is the marginal cost component related to the use of extra units of capital and labor needed to produce an additional unit of output. It can be easily shown that the marginal cost component Ψ_t is common to all firms and is equal to $\Psi_t = \frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}} \frac{1}{\Lambda_t A} w_t^{1-\alpha} r_{K,t}^\alpha$.

Consider now the optimal price setting problem of the typical firm j. Acting in a non-competitive setting, firms can choose their price, but they face quadratic adjustment costs *à la* Rotemberg: $\frac{\gamma_p}{2}\left(\frac{P_{j,t}^D}{P_{j,t-1}^D}-1\right)^2 P_t^D Y_t^D$, where the coefficient $\gamma_p > 0$ measures the degree of price rigidity. Formally, the firm sets the price $P_{j,t}^D$ by maximizing the present discounted value of profits subject to demand constraint $Y_{j,t}^D = \left(\frac{P_{j,t}^D}{P_t^D}\right)^{-\sigma} Y_t^D$. At the optimum we have:

$$\left(1-\theta_1\mu_t^{\theta_2}\right)(1-\sigma)+\sigma MC_t + \\ -\gamma_p\left(\Pi_t^D-1\right)\Pi_t^D+\beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\gamma_p\left(\Pi_{t+1}^D-1\right)\left(\Pi_{t+1}^D\right)^2\frac{Y_{t+1}^D}{Y_t^D}\frac{1}{\Pi_{t+1}}\right\}=0, \tag{21}$$

where we have imposed symmetry across producers and defined $\Pi_t^D = \frac{P_t^D}{P_{t-1}^D}$. The above equation is the New Keynesian Phillips curve, relating current inflation Π_t^D to the expected future rate of inflation Π_{t+1}^D and to the current (real) marginal cost, $MC_t = \frac{1}{p_t^D}\left[p_{E,t}(1-\gamma)(1-\mu_t)\epsilon\left(Y_t^D\right)^{-\gamma}+\Psi_t\right]$, which, in turn, depends on the available technology and the underlying environmental regime. Notice that in the deterministic steady state and with no trend inflation (i.e. $\Pi = \Pi^D = 1$), the so-called New Keynesian Phillips curve (21) collapses to $MC_t = \frac{\sigma-1}{\sigma}\left(1-\theta_1\mu_t^{\theta_2}\right)$,[8] or equivalently, by defining the price markup, say MU_t, as the reciprocal of the MC_t, to

$$MU_t = \frac{\sigma}{\sigma-1}\frac{1}{1-\theta_1\mu_t^{\theta_2}}. \tag{22}$$

Clearly, in the absence of any environmental policy regime, the steady-state price markup will only depend on the elasticity of substitution between intermediate goods σ. In this case, instead, the price markup is shown to be increasing in the abatement effort μ_t. Market power gives firms the possibility of transferring the burden of emission abatement to households.

2.2.2 Production of the Domestic Output Index

Each domestic producer supplies goods to the Home and to the Foreign markets. Let $Y_{j,t}^H$ and $X_{j,t}$ denote, respectively, the domestic and the foreign demand for the generic domestic variety j, then $Y_{j,t}^D = Y_{j,t}^H + X_{j,t}$. For simplicity we assume the presence of a perfectly competitive aggregator that combines domestically produced varieties into a composite Home-produced good Y_t^D, according to a CES function $Y_t^D = \left(\int_0^1 \left(Y_{j,t}^D\right)^{\frac{\sigma-1}{\sigma}} dj\right)^{\frac{\sigma}{\sigma-1}}$. Cost minimization delivers the demand schedule $Y_{j,t}^D = \left(\frac{P_{j,t}^D}{P_t^D}\right)^{-\sigma} Y_t^D$ for each variety. From the zero-profit condition, we

[8]This condition simply equates marginal cost, MC, to marginal revenues, $\frac{\sigma-1}{\sigma}\left(1-\theta_1\mu_t^{\theta_2}\right)$.

obtain the production price index, $P_t^D = \left(\int_0^1 (P_{j,t}^D)^{(1-\sigma)} dj\right)^{\frac{1}{1-\sigma}}$, at which the aggregator sells units of each sectoral output index. Clearly, this output index is allocated in both markets, therefore $Y_t^D = Y_t^H + X_t$, where X_t represents exports of Home to Foreign.

By symmetry, we assume the existence of a perfectly competitive aggregator in the Foreign economy that combines differentiated intermediate goods into a single good to be used for local production of the final good and for exportation.

2.2.3 Production of the Final Good

Competitive firms in the final sector combine a share Y_t^H of the good index Y_t^D produced in the intermediate domestic sector with a share M_t of foreign intermediate production in order to produce the final good Y_t according to the following production function:

$$Y_t = [\kappa^{\frac{1}{\rho}} (Y_t^H)^{\frac{\rho-1}{\rho}} + (1-\kappa)^{\frac{1}{\rho}} (M_t)^{\frac{\rho-1}{\rho}}]^{\frac{\rho}{\rho-1}}, \tag{23}$$

where κ represents the share of intermediate domestic good used in the production of final good and $\rho > 0$ is the elasticity of substitution between domestic and foreign intermediate goods. Clearly, $1-\kappa$ represents the degree of openness of the economy.

Final good producing firms sustain the following cost for inputs: $P_t^D Y_t^H + P_t^{D^*} S_t M_t$, where $P_t^{D^*}$ represents the price index of Foreign production expressed in Foreign currency. Taking as given the price of the domestic intermediate good, P_t^D, and the price of the imported intermediate good, $S_t P_t^{D^*}$, firms minimize their cost function choosing the optimal quantities of domestic and imported goods:

$$Y_t^H = \kappa \left(\frac{P_t^D}{P_t}\right)^{-\rho} Y_t, \tag{24}$$

$$M_t = (1-\kappa) \left(\frac{S_t P_t^{D^*}}{P_t}\right)^{-\rho} Y_t. \tag{25}$$

From the zero-profit condition we derive the consumer price index:

$$P_t = [\kappa (P_t^D)^{(1-\rho)} + (1-\kappa)(S_t P_t^{D^*})^{(1-\rho)}]^{1/(1-\rho)}. \tag{26}$$

2.3 Public Sector

2.3.1 Environmental Policy

In what follows we assume that in the two countries governments can choose among three possible environmental policies:

- National cap-and-trade: each country chooses independently a credible emission reduction target and sets the total level of national emissions tolerated ($E_t = \bar{E}$; $E_t^* = \bar{E}^*$). A regulated firm must hold one permit for each unit of pollution it emits. Permits are sold by the government and traded on a secondary market. We rule out the possibility of grandfathering.

- International cap-and-trade: Home and Foreign pursue a common environmental policy. They set the level of cumulative emissions that can be released ($E_t + E_t^* = \bar{E} + \bar{E}^*$). A central authority sells permits to firms in both countries. In this case permits are traded not only within each country, but also between countries. Also in this case we rule out the possibility of grandfathering.
- Carbon tax: each country imposes a tax rate per unit of emission (i.e. p_E is constant and can then be interpreted as a carbon tax).

For simplicity we abstract from the existence of a public debt and assume that the fiscal authority runs a balanced budget at all times. In particular, we assume that the revenues from environmental policy are distributed to households as lump-sum transfers, that is

$$p_{E,t}E_t = Tr_t, \tag{27}$$

where the term $p_{E,t}E_t$ may refer to the revenues from a carbon tax policy or from the government sale of emission permits.

2.3.2 Monetary Policy

The monetary authority manages the short-term nominal interest rate R_t in accordance to the following simple interest-rate rule:

$$\frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi}\right)^{\iota_\Pi} e^{u_{R,t}}, \tag{28}$$

where R and Π denote the deterministic steady-state of the nominal interest rate and of the inflation rate, ι_π is a policy parameter and $u_{R,t}$ is an exogenous process capturing the possibility of monetary policy shocks, that is:

$$u_{R,t} = \rho_R u_{R,t-1} + \epsilon_{R,t}, \tag{29}$$

with $0 < \rho_R < 1$ and $\epsilon_R \sim i.i.d.\ N(0, \sigma_R^2)$.

2.4 Trade Block, Current Account, Real Exchange Rate, and PTT

In a two-country setting imports of Home are translated into a exports of Foreign, therefore

$$X_t^* = M_t = (1-\kappa)\left(\frac{S_t P_t^{D^*}}{P_t}\right)^{-\rho} Y_t. \tag{30}$$

Likewise, exports of Home are translated into imports of Foreign

$$X_t = M_t^* = (1-\kappa)\left(\frac{P_t^D}{S_t P_t^*}\right)^{-\rho} Y_t^*. \tag{31}$$

The accumulation of Foreign assets for Home is determined by the current account relationship:

$$S_t F_t^* = R_t^* \left(S_t F_{t-1}^* + P_t^D X_t - S_t P_t^{D^*} M_t\right). \tag{32}$$

In the initial steady state F^* is set at zero, thus implying $P_t^D X_t = S_t P_t^{D^*} M_t$.

The assumption of perfect capital mobility between Home and Foreign implies that the nominal exchange rate is determined in the Foreign exchange market as a result of the monetary policy conduct in the two countries.[9] On the other hand, the real exchange rate, defined as $\frac{S_t P_t^*}{P_t}$ (i.e. the ratio between the Foreign price level and the Home price level, where the Foreign price level is converted into domestic currency), not only is influenced by the time path of the nominal exchange rate, but it also reflects the response of the consumption prices indexes to shocks and policy changes.

Finally, in order to capture the total emissions embodied in trade we compute the pollution terms of trade (PPT) index (Antweiler, 1996), defined as:

$$PTT_t = \frac{X_t}{M_t}\frac{E_t/Y_t^D}{E_t^*/Y_t^{D*}}. \tag{33}$$

This indicator, constructed as the ratio between the emission content of exports and the emission content of imports, captures a sort of trade-related environmental balance. Clearly, in this stylized model the PTT index has only two components. The first component refers to the relative volume of exports and imports, while the second component reflects the influence of different emission intensities between countries. A value of the index larger than 1 implies that exports have greater pollution content than imports, whereas a value between 0 and 1 implies that imports are more polluting.

2.5 Resource Constraint and Stock of Pollution

The resource constraint of the economy can be derived by plugging the government budget constraint, along with the definition of profit of the intermediate sector and the expression for

[9]It can be easily shown that by log-linearizing the two Euler equations (10) and (11) one obtains the familiar uncovered interest parity condition relating the rate of depreciation of Home currency to the nominal interest rate differential, which, in turn, depends on the inflation rates via the interest rate rules adopted by Home and Foreign monetary authorities.

the current account position, into the household budget constraint:

$$P_t^D\left(Y_t^D - AC_t\right) = P_tC_t + P_te^{u_{I,t}}I_t + P_t^DX_t - S_tP_t^{D^*}M_t + \frac{\gamma_I}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 P_tI_t + \frac{\gamma_p}{2}(\Pi_t^D - 1)^2P_t^DY_t^D, \tag{34}$$

where the term $Y_t^D - AC_t$ measures net domestic output, that is output net of the abatement costs. The stock of pollution Z_t evolves according a natural decay factor $\eta \in (0,1)$, and on the basis of current period Home emissions E_t, current period Foreign emissions E_t^*, and non-industrial emissions E_t^{NI}:

$$Z_t = \eta Z_{t-1} + E_t + E_t^* + E_t^{NI}. \tag{35}$$

3 Parametrization

The model is calibrated for the world economy and time is measured in quarters. However, the parametrization is only illustrative and designed for our positive analysis. Standard parameters, related to the New Keynesian formalization of the model, follow the existing literature. See, e.g., Galí (2015). The discount factor β is set at a value consistent with a real interest rate of 4% per year, that is $\beta = 0.99$. The inverse of the Frisch elasticity of labor supply ϕ_L is equal to 1. By assuming that the time spent working at the steady state is 0.3, we obtain an implied value for ξ_L, the scale parameter related to the disutility of labor, of 3.8826. The depreciation rate of capital δ is set at 0.025 and the capital share α at 1/3. The degree of price rigidities, the parameter γ_p, is consistent with a Calvo pricing setting with a probability that price will stay unchanged of 0.75 (i.e. average price duration of three quarters), namely $\gamma_p = 58.25$. The inverse of the intertemporal elasticity of substitution φ_C is equal to 1.2, the parameter for capital adjustment costs γ_I is set at 1.5. Regarding the goods market, we set the elasticity of substitution among intermediate good varieties σ equal to 6 and the intratemporal elasticity of substitution between domestic and foreign intermediate goods ρ equal to 1.5, implying that domestic and foreign varieties are imperfect substitute. In line with the average values of the import/GDP ratio observed for the world economy in period 2010-2015 according to World Bank data, we assume a propensity to import of 0.3, that implies a share of domestic intermediate goods used in the final sector κ equal to 0.7. The steady-state target inflation is equal to zero ($\Pi = 1$), while the relative price of intermediate goods and the real exchange rate, p^D and S^R, are both normalized to 1. Turning to parameter related to monetary policy, we set the interest rate response to inflation, ι_Π, at 1.5.

With regards to the environmental part of the model, we refer to previous environmental DSGE models and Integrated Assessment Models for climate change, in order to obtain plausible values for environmental parameters. We set the elasticity parameter of emissions to output γ at 0.304 as in Heutel (2012), the pollution decay factor η at 0.9979, following Reilly and Richards (1993), and the parameter of the abatement cost function θ_2 at 2.8 as in Nordhaus

(2008), while θ_1 is normalized to 1. To obtain the steady state level for emissions, we refer to the policy runs of the RICE-2010 model, in detail to the simulation results for year 2015. We take the level of global carbon emissions, and the level of global industrial emissions, both measured in gigatons of carbon (GtC) per year, then we assume that Home and Foreign contribute in equal way to output and emissions in the region. Through these data we are able to recover the level of global non-industrial emissions, emissions for domestic and foreign country, and the steady state level of output in the intermediate sector. Finally, by looking at the RICE model, we know that abatement costs, measured as fraction of output, are equal to 0.00013. This calibration strategy delivers implicit values for the pollution stock in model units, emission intensity and the scale parameter ϵ. Regarding the negative externality on production, we calibrate Λ on the basis of the total damage for year 2015, measured as fraction of output, that amounts to 0.0030. Estimating that the pre-industrial atmospheric CO_2 concentration ($\overline{Z}$) represents 3/4 of the total pollution stock, we obtain a value for the intensity of negative externality on output χ and for the total factor productivity A.

With regards to stochastic processes we assume a high degree of autocorrelation for the exogenous shocks by setting ρ_A, ρ_{AC}, ρ_I and ρ_K at 0.85, while ρ_R is set at 0.5. Table 1 lists all the parameters of the model.

4 International Transmission of Shocks and Environmental Policies

In this Section we analyze the international transmission of several shocks under alternative environmental regimes, namely a national cap-and-trade, a carbon tax and an international cap-and-trade. With regards to national environmental policies, we assume that countries are subject to the same type of regime. We analyze the effects of five temporary shocks hitting only Home: (i) a positive productivity shock increasing the TFP, (ii) a shock increasing emission abatement costs, (iii) a positive shock on the quality of capital, (iv) a positive investment-specific technology shock, and (v) a positive shock on the risk-free interest rate set by the monetary authorities.

In what follows we focus our attention on a selection of macroeconomic and environmental variables. Results are reported as percentage deviations from the initial steady state over 20 quarters, with the exception of emission intensities which are reported in percentage point deviations and the trade balance which is reported in percentage points. The PTT index is reported at levels.[10]

[10]The model is solved with Dynare. For details, see http://www.dynare.org/ and Adjemian et al. (2011).

4.1 TFP Shock

Figures 1 and 2 show the economy's response to a one percent increase of productivity. Continuous lines refer to the dynamic responses of the economy under a national cap-and-trade scheme, dotted lines refer to the case of a carbon tax regime, while dashed lines report the response under an international cap-and-trade policy. Overall, we observe no significant differences in the path of the Home main macroeconomic variables under different environmental policies. As expected, a slight larger effect on net output is observed under a carbon tax. In response to this positive shock, due to an intertemporal transfer of wealth, domestic consumption and investment increase.[11] The shock gives rise to a depreciation of the domestic currency and deteriorates the Home terms of trade.

On impact the effects on the trade balance are negligible, although we can observe a lower deterioration under national cap-and-trade compared with the other two environmental regimes. Starting from the fifth period we observe an improvement of the trade balance, no matter the kind of environmental policy implemented. A typical J-curve effect arises: in the first periods after the shock the price effect dominates, imports are costlier than exports and this deteriorates the trade balance. At later stages quantities adjust: the volume of export starts to rise because of the increase in Foreign demand for the domestic goods that are relatively low-priced. At the same time domestic consumers reduce their demand for more expensive Foreign goods. In the first periods we observe also a deterioration in the foreign asset position of Home, followed by a steady increase.[12]

The improvement in the Foreign terms of trade increases Foreign consumption, that remains above the steady state levels along all the time horizon. Foreign investment and net output, after an initial increase, start to decrease. Foreign net output follows the fall in Home import demand, and remains under the steady state level along all the simulation period. The international cap-and-trade environmental regime slightly reduces the positive initial response of Foreign variables, but tends to magnify the subsequent decline. This is particularly evident for investments.

Consider now the response of the environmental variables of Figure 2. Under a national cap-and-trade policy emissions are constant, while abatement costs expand sharply. In order to comply with the emissions cap and afford higher production at the same time, firms increase their abatement effort and this, along with higher production, drives the observed increase in abatement costs and in the price of emission permits. Abatement costs and permit price

[11] It can be shown that in response to a positive technology shock labor shows a countercyclical dynamics, as usual in New Keynesian models. Nominal rigidities do not allow an immediate adjustment of prices and this has a negative impact on the labor market. This result is also consistent with empirical studies that point out how a positive technology shock leads to a temporary decline in employment: firms take advantage of the productivity's increase by reducing labor demand. See e.g. Galí (1999).

[12] The response of trade balance and of net external asset position of Home crucially depends on the elasticity of substitution ρ between domestic and foreign goods. It can be shown that in the case of imperfect complementarity (i.e. $0 < \rho < 1$), in fact, Home trade balances never improve during the adjustment process, while we observe a stronger depreciation of the domestic currency.

increase also in Foreign, as a consequence of the initial increase in production.

Turning to the case of an international cap-and-trade regime, the first evidence is that emissions now rise in Home. Price of permits and abatement costs increase, but to a lesser extent than under the case of a national cap-and-trade. The price of pollution permits is now determined in the international market. Due to the pressure on the permits price to be ascribed to Home higher production, Foreign finds it convenient to increase abatement and reduce emissions. This causes a sharp increase in the abatement costs of Foreign. Therefore, more resources are devoted to the abatement, and this explains the dampened reaction of Foreign investment, consumption and output under this environmental regime.

Under a carbon tax, in response to a positive shock on productivity, Home producers increase emissions sharply. We observe a negligible initial reaction of the abatement costs both in Home and Foreign. These costs slightly rise in Home, because the increase of production overcomes the decrease of abatement effort. Both abatement effort and abatement costs fall in Foreign, while emissions rise following the dynamics of output.

Consider now the emission intensity and the PTT. The emission intensity indicator provides us with a measure for relative emissions, that is particularly important when we deal with business cycle fluctuations. Emission intensity decreases in Home in response to the positive TFP shock under all the three regimes. Intuitively, the fall is greater under the national cap-and-trade because the level of Home pollutant emissions is pegged. The highest emission intensity is observed when a carbon tax is in place.

Differently from what expected the policy ranking in terms of emission intensity is not just reversed in Foreign. First of all, we notice that, also in Foreign, at least initially, emission intensity decreases in all the three policy scenarios. The tax is confirmed as the policy that gives rise to the highest emission intensity, but it is the international cap-and-trade the regime that grants the lower emission intensity. A shock on productivity is able to generate positive environmental spillovers when countries are tied by a common environmental regime, by pushing, through a higher permit price, the country that is not hit by the shock to invest in abatement and make its production cleaner. The national cap-and-trade, instead, proves to be the only regime where, after the first periods emission intensity increases above the steady state level. In this case firms choose to reduce production keeping emissions at the same level, without any incentive to invest in abatement. In this case the transmission of the shock not only leads to a recession in Foreign, but also worsens the environmental impact of production activity. Looking at the PTT we observe that the index decreases by more under a national cap-and-trade and stays persistently below its baseline level all along the adjustment path. This result suggests that the national cap-and-trade is the only policy able to improve the trade balance and reduce the emission content of exports at the same time.

4.2 Abatement Costs Shock

Figures 3 and 4 display the economy's response to a one percent specific abatement costs shock that hits Home. In front of this type of shock the underlying environmental policy influences the dynamic behavior of the economy considerably.

In the presence of a national cap-and-trade we observe a decrease of both investment and consumption, crowded out by the higher abatement costs. Home firms reduce production, while Foreign goods become relatively cheaper. The appreciation of the domestic currency gives rise to an initial improvement of the trade balance, followed by a deterioration. As a consequence, we observe an improvement also in the external asset position of Home. Abatement is now more expensive, so Home firms are induced to reduce their abatement effort. Nevertheless, the combined decrease in abatement effort and output is not sufficient to offset the increase in abatement costs and this brings about a rise in the price of emission permits. On the other hand, also Foreign output decreases on impact, due to the initial fall in Home import demand, but immediately starts to recover, sustained by the increase in Foreign investments. In Foreign abatement effort, abatement costs and permits price start to slightly increase after the first period, given the increase in production.

Under a regime of international cap-and-trade Home consumption, investments and output decline by less than in the case of a national cap-and-trade. Abatement effort decreases and this raises the price of emission permits, but to a lesser extent than under a national environmental policy, so that the increase in domestic marginal costs is now lower. Output decreases less than under a national cap-and-trade, while emissions increase, since it is more convenient to purchase emissions permits from Foreign rather than to abate. The real exchange rate and the trade balance stay almost invariant: we just observe a slight improvement of the trade balance, while the external asset position deteriorates as consequence of the purchase of emission permits from abroad. In Foreign the shortage of emission permits and the increase in the international price of permits, due to the pressure from Home, induce firms to reduce output and to increase their abatement effort. Foreign abatement costs go up, crowding out consumption and investment. In the presence of an international cap-and-trade regime a negative shock for Home is then able to trigger a recession in Foreign.

Under a carbon tax, differently from the other two cases, Home consumption, investment and output slightly increase. The presence of a tax on emissions provides a constant-price alternative to abatement. The reduction of abatement effort is significant and brings about a reduction of the abatement costs, while emissions increase. As a result the shock is fully absorbed and the effects on the main macroeconomic variables are negligible. The emission tax dampens the macroeconomic effects of abatement costs shock in Home and the resulting transmission of the shock in Foreign is weak. Emission intensity in Home increases particularly under the tax and the international cap-and-trade regimes. In Foreign emission intensity decreases under the international cap-and-trade and slightly increases under the other two regimes, in line with the reaction of abatement costs and permits price. Finally, the PTT is found to be higher than its baseline level in all the cases, in particular under the international cap-and-trade because of the relatively lower emission intensity in Foreign.

4.3 Quality of Capital shock

We now focus the attention on the economy's response to a one percent positive shock on the quality of capital. See Figures 5 and 6. The positive shock boosts the capital value, increasing, at the same time, the effective quantity of capital available for production.

This improvement in the economy production capacity results in higher output. Since the shock is temporary, households find it optimal to increase investment immediately in response to the shock, given the higher marginal product of capital, while consumption follows a hump-shaped dynamics. In general, we observe a positive co-movement of the main real variables: consumption, investment and output. The real exchange rate slightly decreases in the first period, then depreciates. The trade balance deteriorates on impact, but we can observe an improvement already from the second period. On the other hand, in Foreign the value of capital is relatively lower and firms decide to invest less. Foreign output decreases, while consumption increases thank to the improved terms of trade. Considering the dynamic implications of the underlying environmental policy, we notice that both the international cap-and-trade and the carbon tax magnify the response of Home output. Under these regimes Home firms incur in lower abatement costs and the expansionary effects are more pronounced than those observed in a national cap-and-trade regime under which more resources are devoted to abatement. Conversely, in Foreign output decreases by more under an international cap-and-trade and under a carbon tax.

Under a national cap-and-trade we observe, as usual, a cyclical response of the environmental variables: abatement costs and permits price increase in Home and decrease in Foreign.

In the presence of an international cap-and-trade the reallocation of permits from Foreign to Home dampens the increase in the price of permits and abatement costs in Home, compared to the previous scenario. Home producers take advantage of the lower price of emissions on the market by buying permits and increasing emissions. Foreign producers find it convenient to replace permits for abatement, that is why we observe an increase in abatement costs.

Under a tax policy the shock does not affect particularly firms' abatement choices and translates entirely into higher emissions in Home and lower emissions in Foreign.

As already observed in the case of TFP shock, intensity target in Home goes down despite the environmental policy implemented. The national cap-and-trade also in this case turns out to be the policy that ensures the lowest emission intensity. The main differences compared to the TFP shock arise if we look at the Foreign emission intensity. Emission intensity now increases mainly under the national cap-and-trade policy, slightly under the tax policy, and decreases under the international cap-and-trade. This result is mainly due to the effects that different environmental policies have on output through the abatement costs channel. Following the shock, the PTT decreases below 1 as a result of the lower emission intensity and of the deterioration of the trade balance at the earlier stages of the adjustment process, then it starts to rise persistently under all the regimes. The size of the response differs across regimes, in line with the behavior of the relative emission patterns.

4.4 Investment-Specific Technology Shock

Figures 7 and 8 display the behavior of the economy in response to a positive investment-specific technology shock of one per cent. The reaction of the main macroeconomic variables is consistent across regimes both qualitatively and in size. Minor differences can be seen only for Foreign output. Following the shock, domestic net output increases, although the size of the response is negligible. As expected, given the aggregate resource constraint, Home consumption declines to accommodate the larger investment demand.

The real exchange rate initially appreciates, the trade balance deteriorates on impact and so Home external asset position worsens. Net Foreign output initially improves, sustained by Home demand, and then decreases. At the same time this shock brings about a negative effect on the main components of the Foreign aggregate demand, as we observe a decline of both investment and consumption.

Environmental variables behave as in the case of a technology shock, with the only exception of PTT, that in this case is below its baseline level in all scenarios. This result shows how an investment-specific technology shock, differently from the technology shock, can trigger a reduction of the relative emissions content of Home exports, no matter the environmental regime implemented. Clearly, this result reflects the relative change in the volume of exports and imports, that gives rise to the deterioration of the trade balance, rather than the relative change in emission intensity.

4.5 Monetary Policy Shock

In Figures 9 and 10 we consider the responses to a monetary policy shock. In detail, we assume an increase of 0.50% in the innovation $\epsilon_{R,t}$. The main macroeconomic variables show the same patterns across regimes. The rise in the interest rate reduces investment and consumption, triggering a fall of output. As a consequence of the increase in the nominal interest rate in Home the real exchange rate appreciates which, in turn, gives rise to a short-lived improvement in the trade balance, since the positive price effect on imports dominates the negative volume effect on net exports which materializes only at later stages. Consistently, the external asset position first improves and then worsens. The domestic demand channel depresses Foreign net output, following the decline of Home imports. On impact we observe a negative reaction of Foreign consumption and investment. In the following periods the expenditure switching effect prevails and these variables recover quickly, following the movement of the trade balance.

Turning to the response of environmental related variables of Figure 10, under a national cap-and-trade regime the tightening of monetary policy generates a decline of abatement costs and of permits price in both countries. Under an international cap-and-trade regime we observe a reallocation of permits in favor of Foreign, where the drop of output is lower and emissions increase, along with a sharp fall in abatement costs. Under the tax regime emissions diminish in both countries. Home producers increase abatement effort, but abatement costs shrink, carried down by the contraction of output. Foreign producers slightly raise abatement effort and abatement costs.

As a result of the shock, Home emission intensity goes up. The size of the reaction is different across regimes and in this case, given the recessionary nature of the shock, the national cap-and-trade gives rise to the highest emission intensity, while the carbon tax to the lowest. In Foreign emission intensity goes up as well. Also in this case under the tax policy we observe the lowest emission intensity. Differently, the international cap-and-trade results the policy that yields the higher emission intensity, in line with the sharp decrease in Foreign abatement that we observe under this regime in response to the shock.

4.6 Pattern of Trade and Monetary Regime

In this Section we explore the role played by the pattern of trade and by monetary policy in the transmission of the business cycle across different environmental policy regimes. In particular, we solve the model under three different assumptions in turn: (i) domestic and foreign bundles of goods are imperfect complements, rather than imperfect substitutes, (ii) higher degree of openness to international trade, (iii) currency union. To address these points in a parsimonious way we look at the standard deviations for Home and Foreign output and at the correlation between output, emissions and emission permit prices. All the statistics are computed using stochastic simulations considering each shock in turn. In this way we are able to measure the magnitude of international spillovers of the shocks.[13]

We start by considering the benchmark case, where the model is solved under the baseline calibration of Table 1. Results are reported in Table 2, where σ_{Y^D} and $\sigma_{Y^{D*}}$ denote the standard deviations of Home and Foreign output, while $\rho(\cdot,\cdot)$ is the coefficient of correlation between variables. We notice what follows.

First, the underlying environmental regime does not alter the sign of the relationship between output of the two countries. In particular, we observe, consistently with our previous results, that in response to a technology and capital quality shock Home and Foreign output are negatively correlated, while in all the other cases the relationship is positive.

Second, the size of the correlation and the size of the relative standard deviation of Foreign output are magnified under an international cap-and-trade regime in response to TFP shocks and shocks on the abatement cost function. In the former case, in fact, positive (negative)

[13] Given the optimal decision rules, for each shock we draw 200 realizations of size 10,000, dropping the first 100 observations from each realization. We set the standard deviations of all shocks to 0.001.

shocks in technology push Home firms to produce more (less) and then to pollute at a greater (minor) extent, by buying (selling) emission permits from abroad, while in the latter case variations in the abatement technology alter the cost of pollution control for Home firms that will react by buying or selling emission permits in the international market. In the case of productivity shocks the sign of the spillover is clearly negative, while in the case of shocks on abatement technology the sign is positive. This also explains the countercyclical behavior of permit prices for Foreign in response to these shocks originated from abroad.

Finally, in response to demand shocks, such as investment-specific technology shocks and monetary policy shocks, the international spillover effects are found to be slightly larger under a carbon tax and lower under an international-cap-and-trade regime. Further, in this regime we observe that Foreign emissions are countercyclical. In response to a positive monetary policy shock hitting Home, in fact, Foreign output declines because of Home export demand contraction, the excess of emission permits in the international market pushes firms to pollute more in Foreign. In response to a positive investment-specific technology shock Foreign output increases along with Home output, but there will be an outflow of emission permits from Foreign to Home.

Table 3 reports the results assuming that foreign and domestic bundles of goods are imperfect complements rather than imperfect substitutes, in particular we set the elasticity of substitution ρ in equation (23) at 0.5. As expected now domestic and foreign outputs move in the same direction in response to all the shocks. The relationship, when already positive in the baseline model, becomes stronger under the hypothesis of imperfect complementarity and the sign changes from negative to positive for technology and capital quality shocks. However, we now notice that the correlation between Home and Foreign output is always higher under a carbon tax, with the exception of the shock on abatement technology. More interestingly in response to a technology shock under the international cap-and-trade the relationship is now positive, as expected, but the relative standard deviation of Foreign output is lower than under the other regimes. This is because the drain of emission permits from Foreign to Home mitigates the positive spillovers on Foreign production due to complementarity.

Table 4 presents the results under the assumption that the share of imported varieties, κ, in the final good production function is equal to 0.5 instead of 0.3. Overall, we observe that with a higher degree of openness the relative standard deviation of Foreign output is higher that in the benchmark case. However, the degree of correlation of output found in response to an investment-specific technology shock is sharply lower than in the benchmark case. Intuitively, the depreciation of the domestic currency in this case sharply mitigates the expansionary effects on Foreign output in response to a positive investment-specific shock hitting Home. Moreover, we observe that under a national cap-and-trade policy the appreciation of the domestic currency tends to reduce the negative spillover on Foreign production following a detrimental shock on Home abatement technology and the correlation between output of the two countries reduces sharply.

Finally, Table 5 presents the results under the assumption that Home and Foreign share the same currency, therefore the economies are subject to the same monetary policy which now responds to an average of the two CPI inflation rates. We notice what follows. First, in response to the TFP shock the correlation between Home and Foreign output turns out to be positive, since now monetary policy is less accommodative for Home, but it becomes expansionary for Foreign. However, the intensity of the relationship is found to be stronger under a carbon tax, but weaker under an international cap-and-trade regime, where the possibility of importing emission permits from abroad diminishes the positive spillover effects on Foreign output. Second, an improvement in the quality of capital in Home gives rise to a negative spillover effect, which is mitigated under a common monetary regime. As in the case of the TFP shock, in fact, monetary policy is now less accommodative for Home, but it is expansionary for Foreign, so partially offsetting the negative spillover effects. Finally, consider what happens in response to a shock on abatement costs in Home. While the major spillover effects are still observed under an international cap-and-trade system, now it is under a national cap-and-trade regime, rather than in a carbon tax system, that the correlation is higher, while the relative standard deviation is higher under a carbon tax.

5 Conclusions

Climate change and global warming are among the greatest pressing current policy issues. A clear understanding of the economic aspects of the policy undertaken is needed, that is why environmental issues have been recently raising the hurdles also for DSGE modeling. In this respect, the paper presents a stylized but rigorous framework to study the international dimension of climate actions in a two-country fully interdependent economy with uncertainty. With this tool in hand, we are able to convey the main ideas about the role played by various environmental regimes in shaping the propagation of shocks between countries.

Our results show how the international transmission mechanism of uncertainty is influenced by the policy tool chosen to stabilize greenhouse gas concentrations in the atmosphere. Unexpected shocks hitting a country may generate spillover effects, whose sign and intensity depend not only on the nature of uncertainty, but also on the underlying environmental regime. Supply shocks are likely to bring into play more cross-border pressure when countries adopt an international cap-and-trade system, while demand shocks produce more intense effects abroad under a carbon tax. The degree of openness, the trade pattern and the underlying monetary policy regime are shown to play a non-trivial role in this interplay between economic and policy variables.

The model studied in this paper leaves out a number of features that have been identified as potentially important for understanding the economic implications of climate actions in open economy. First, the model does not allow for international mobility of labor and capital. Clearly, this poses a limit to the re-allocation of production activity resulting from asymmetric and persistent shocks. Second, the importance of the pattern of trade in determining the propagation mechanism is only touched upon in this paper and deserves further and deeper investigation. Third, in this paper the economy is composed by two identical economies. Similar investigations should be carried out allowing for a certain degree of asymmetry in technology and size between countries. Finally, a further step to advance this analysis should regard a thorough analysis of the interaction between stabilization policies and economy-wide emission regulations in open economy. We leave these issues for future research.

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Figure 1: Dynamic Response to a 1% TFP Shock - Macroeconomic Variables

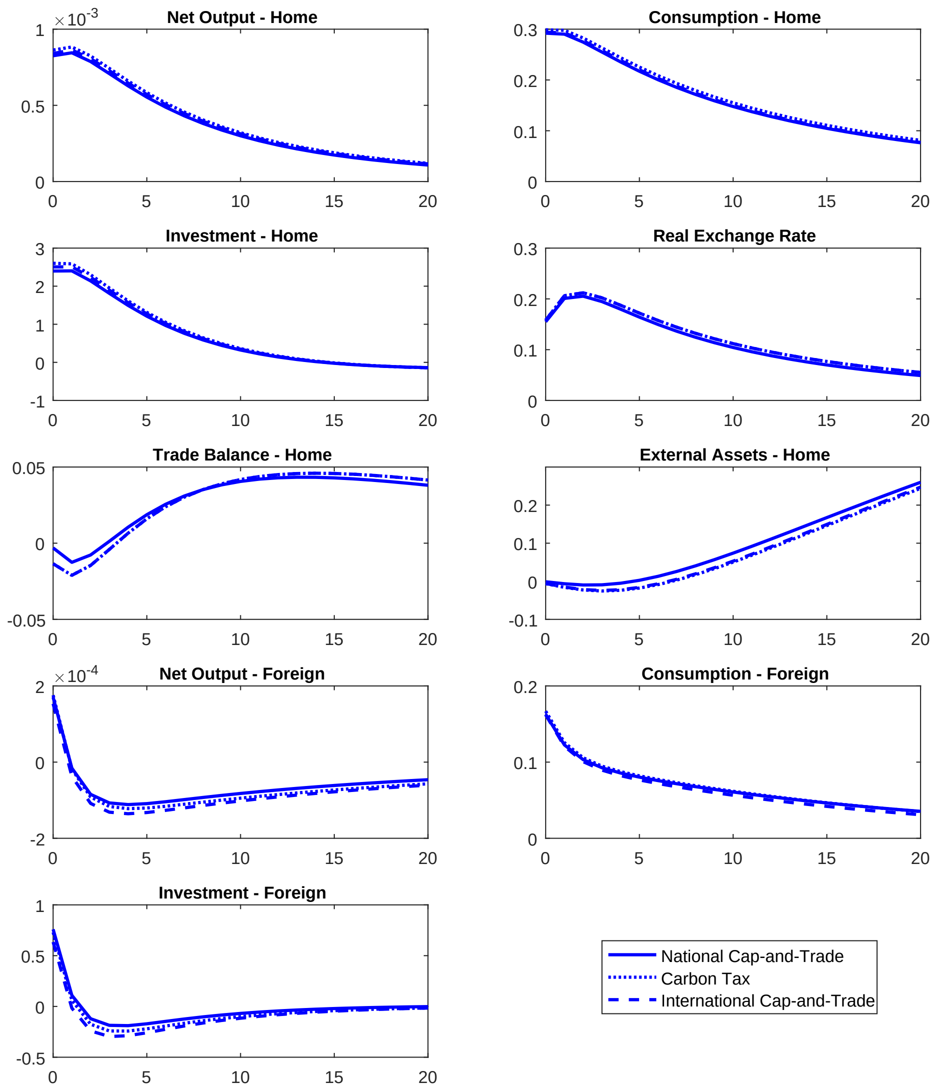

Figure 2: Dynamic Response to a 1% TFP Shock - Environmental Variables

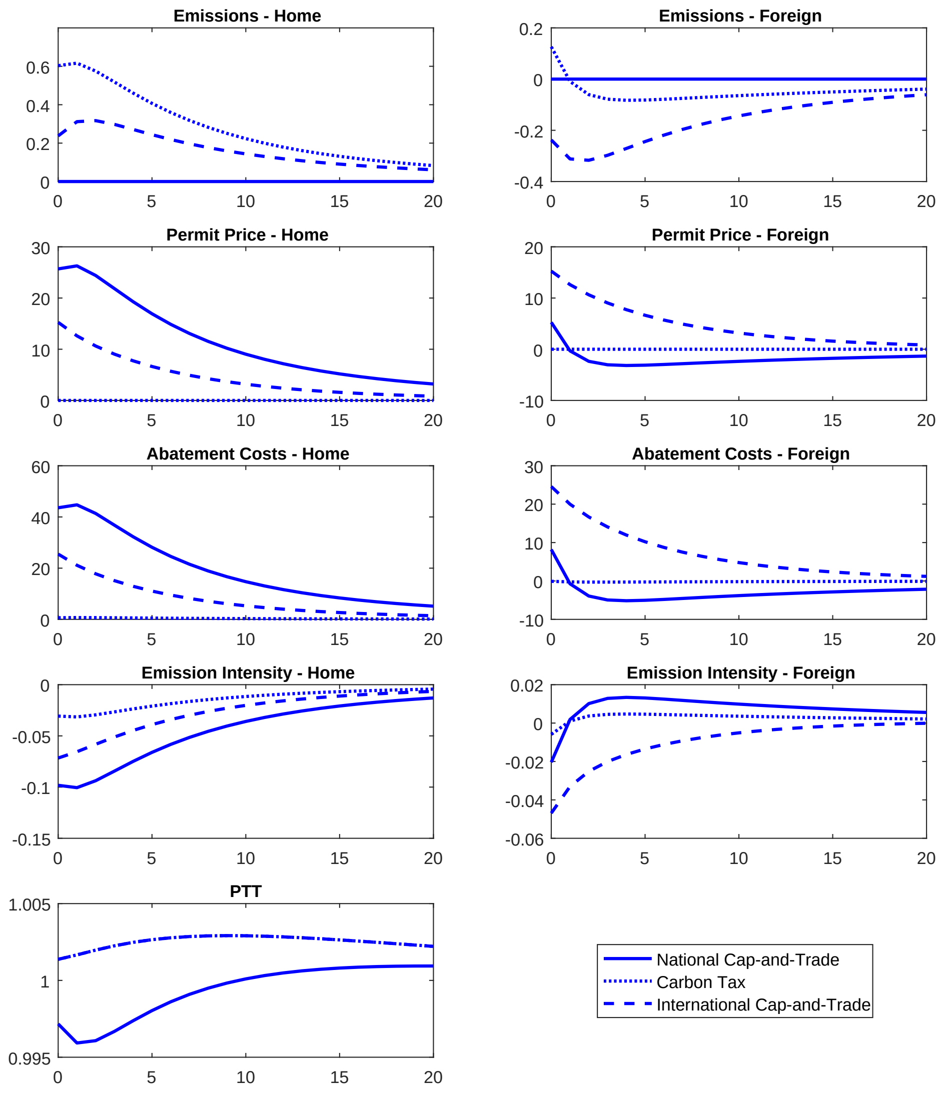

Figure 3: Dynamic Response to a 1% Abatement Costs Shock - Macroeconomic Variables

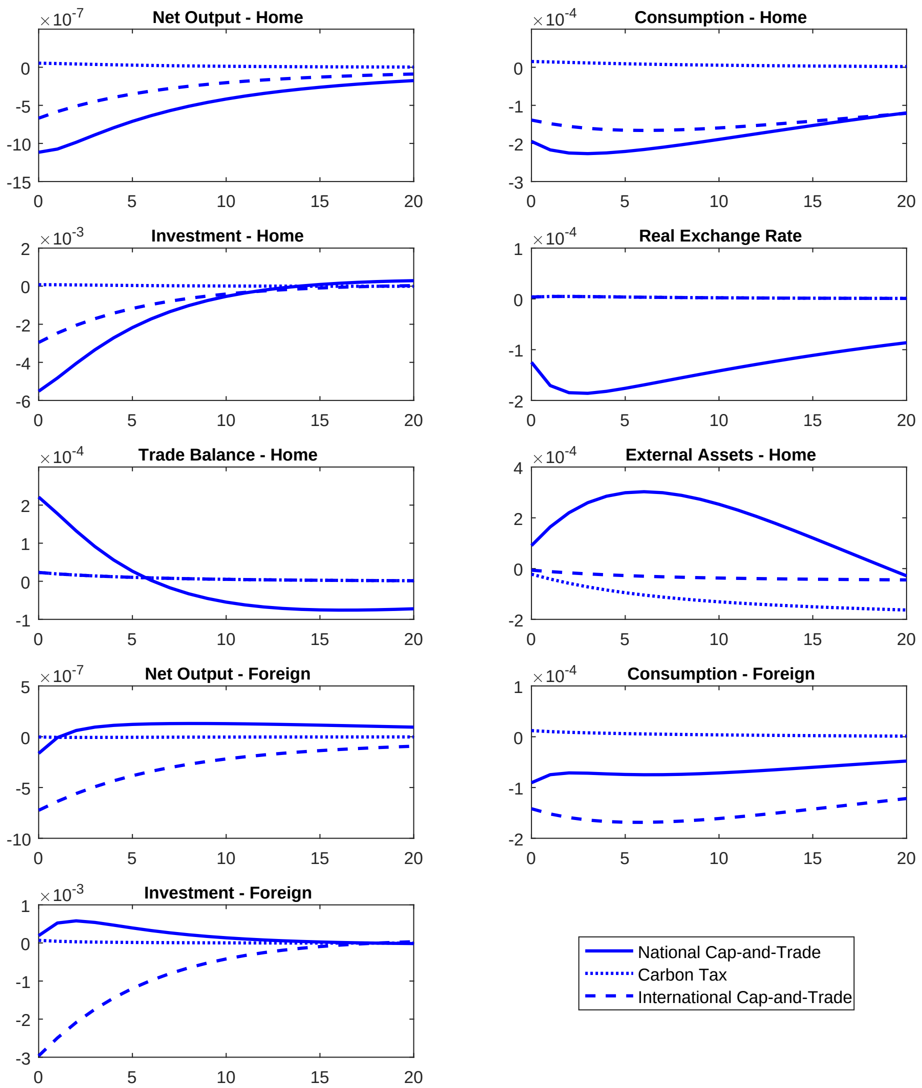

Figure 4: Dynamic Response to a 1% Abatement Costs Shock - Environmental Variables

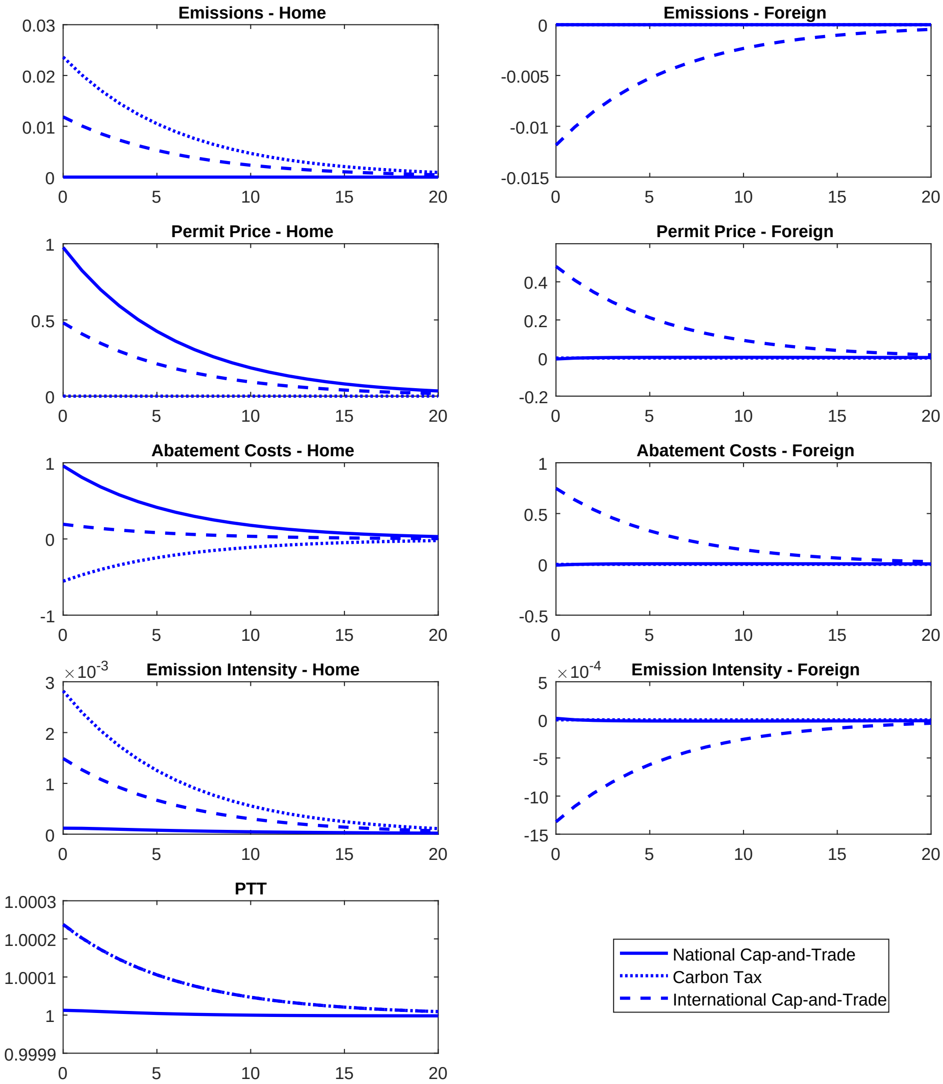

Figure 5: Dynamic Response to a 1% Capital-Quality Shock - Macroeconomic Variables

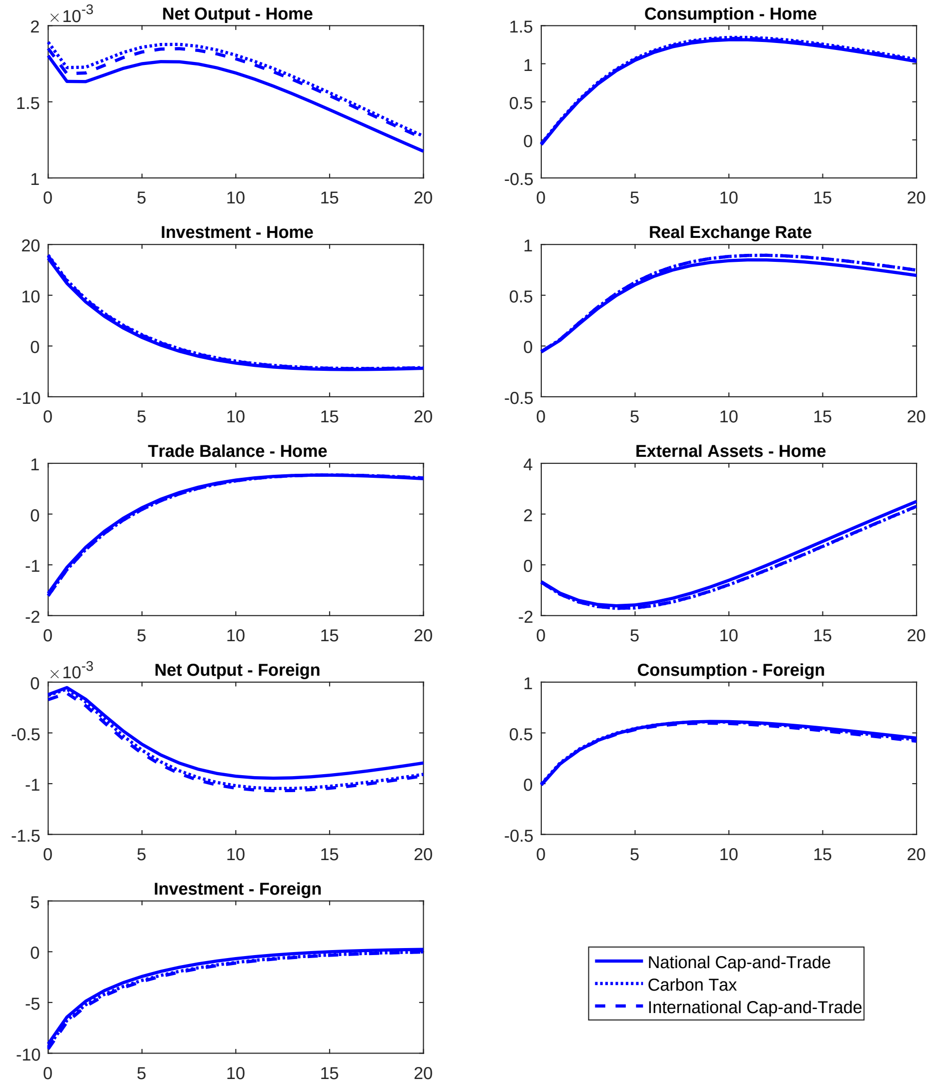

Figure 6: Dynamic Response to a 1% Capital-Quality Shock - Environmental Variables

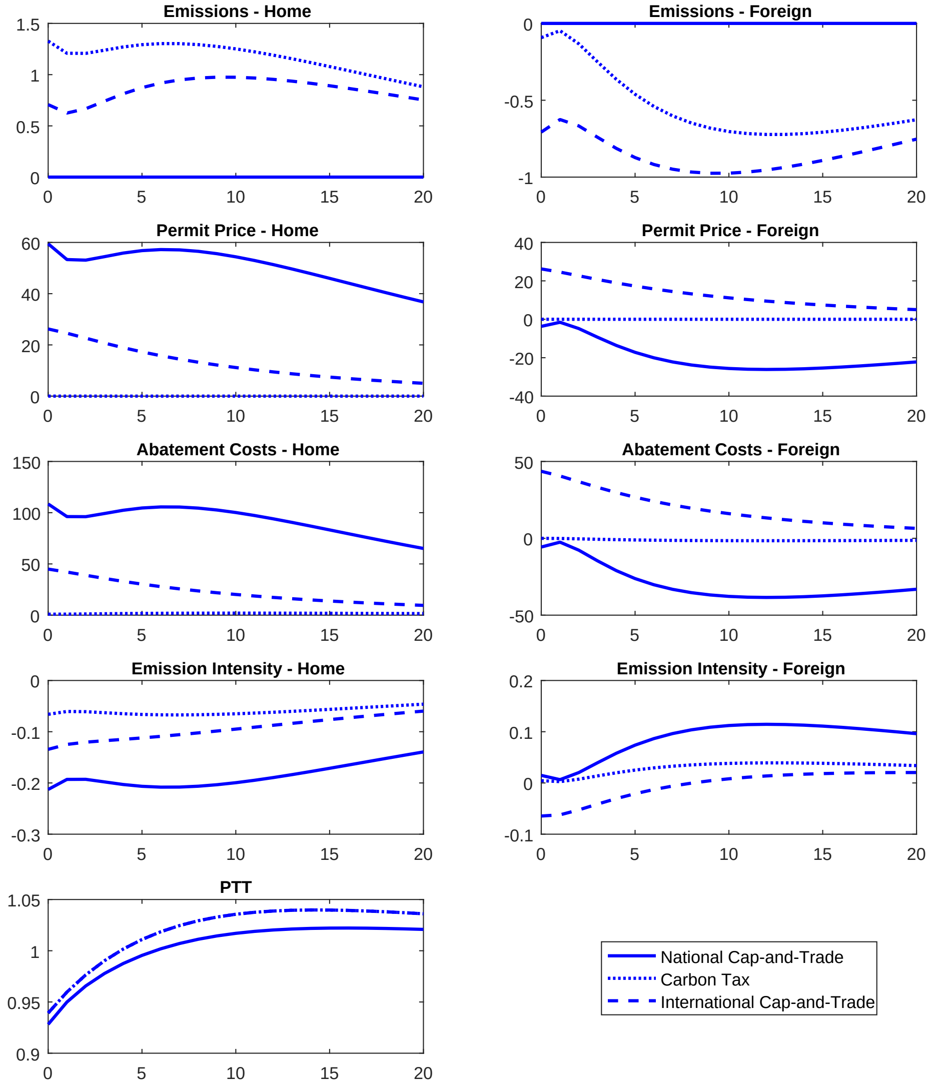

Figure 7: Dynamic Response to a 1% Investment-Specific Technology - Macroeconomic Variables

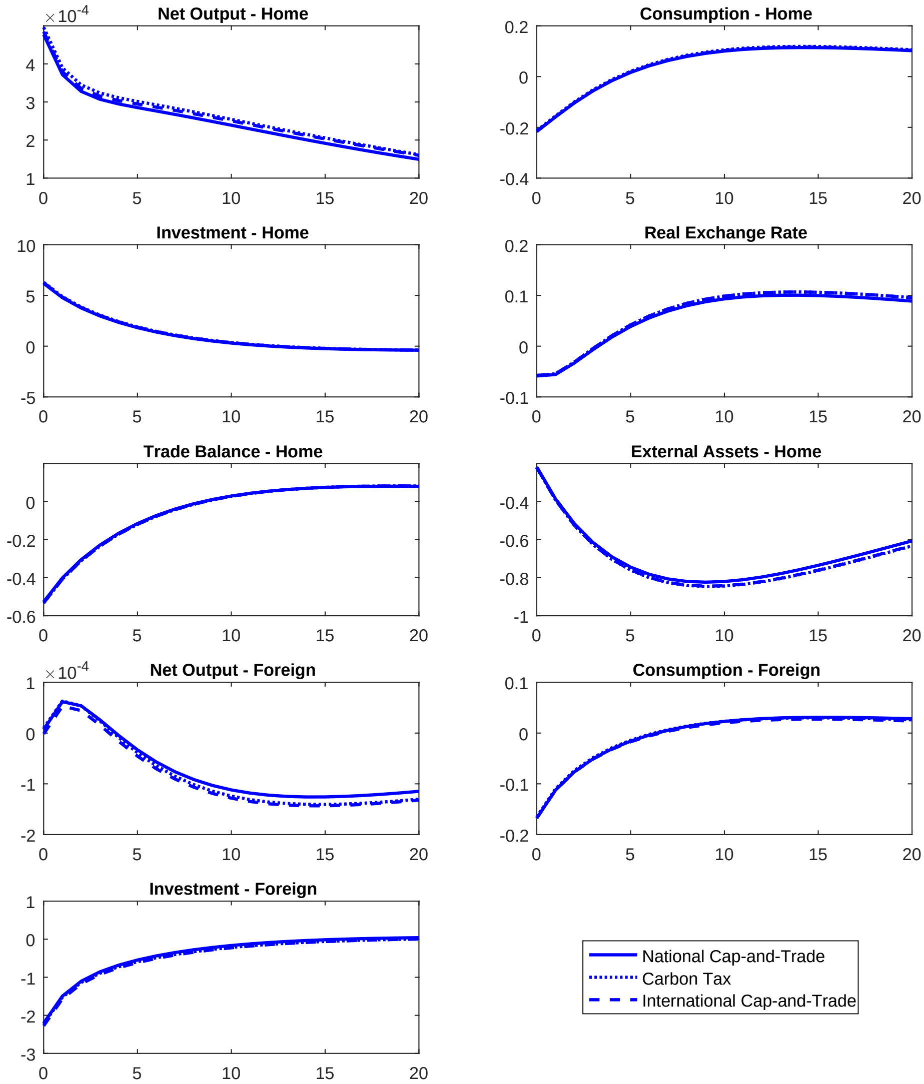

Figure 8: Dynamic Response to a 1% Investment-Specific Technology Shock - Environmental Variables

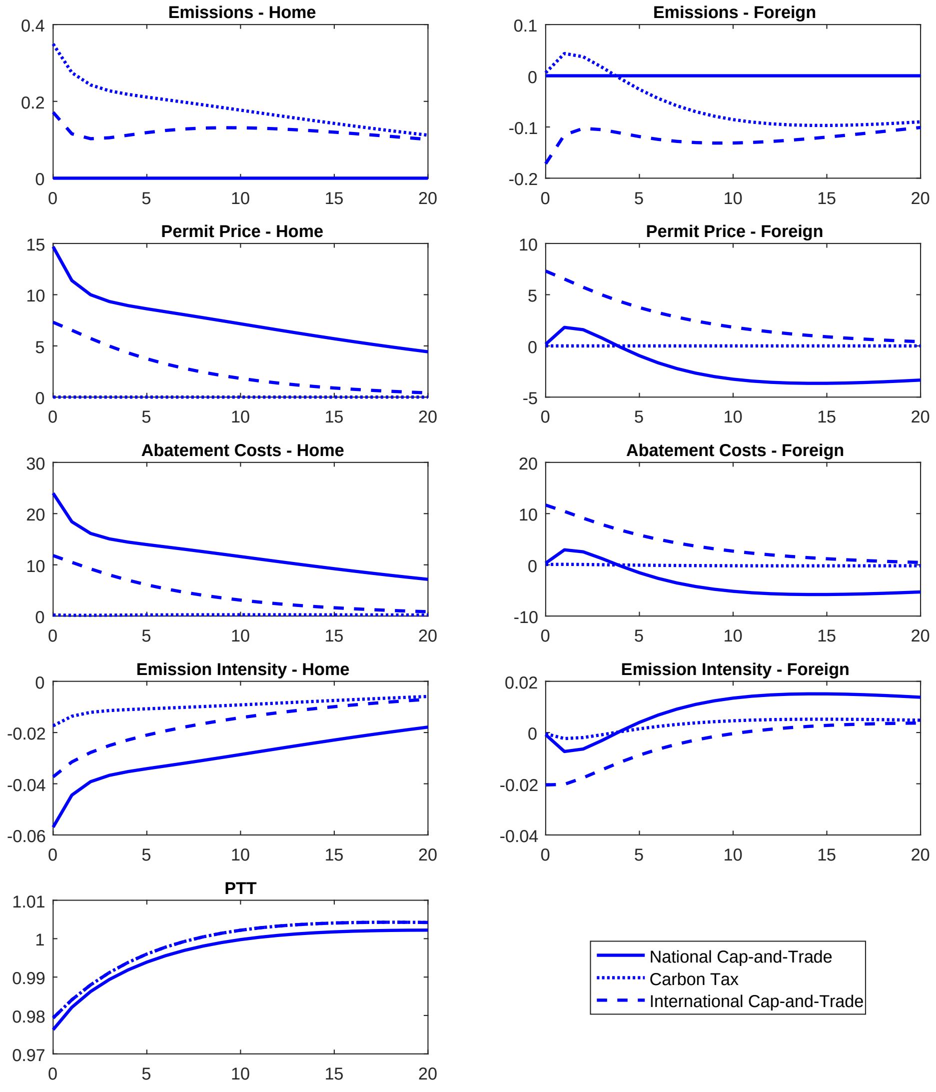

Figure 9: Dynamic Response to a 0.5% Monetary Policy - Macroeconomic Variables

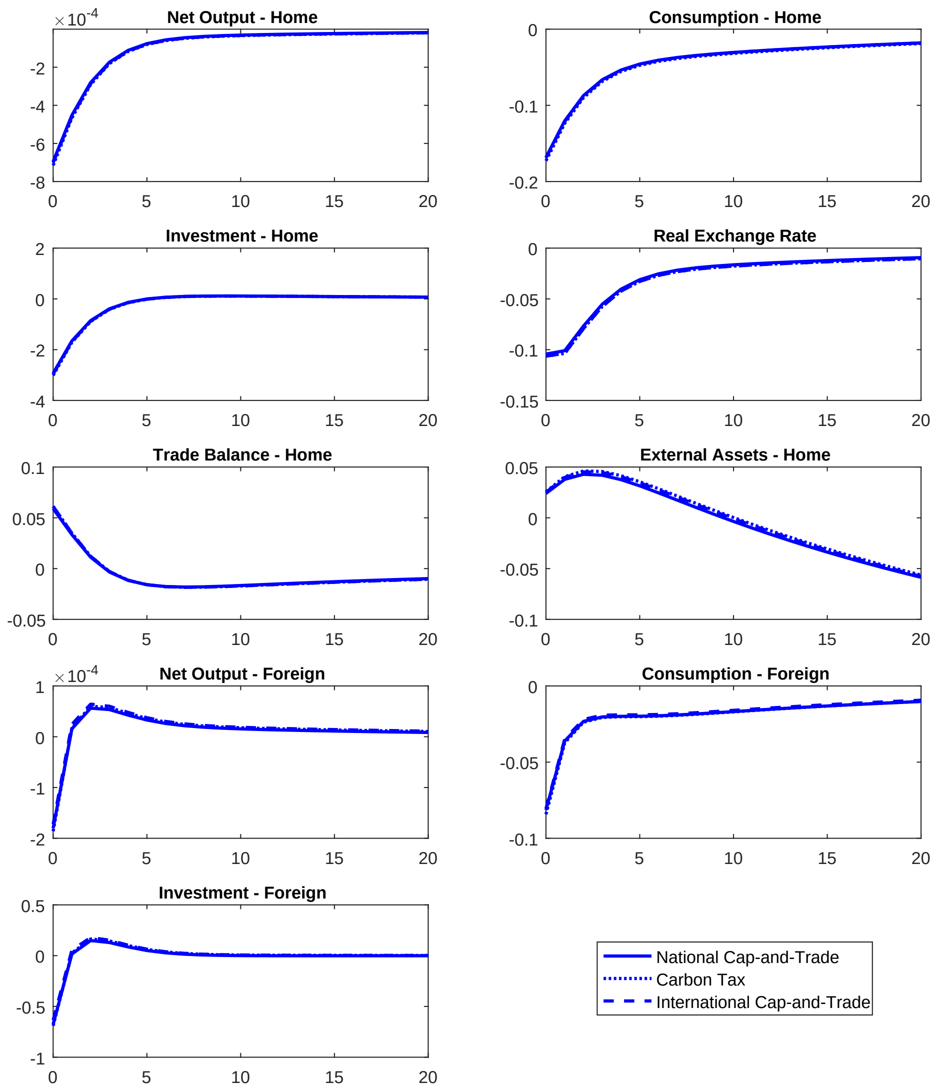

Figure 10: Dynamic Response to a 0.5% Monetary Policy - Environmental Variables

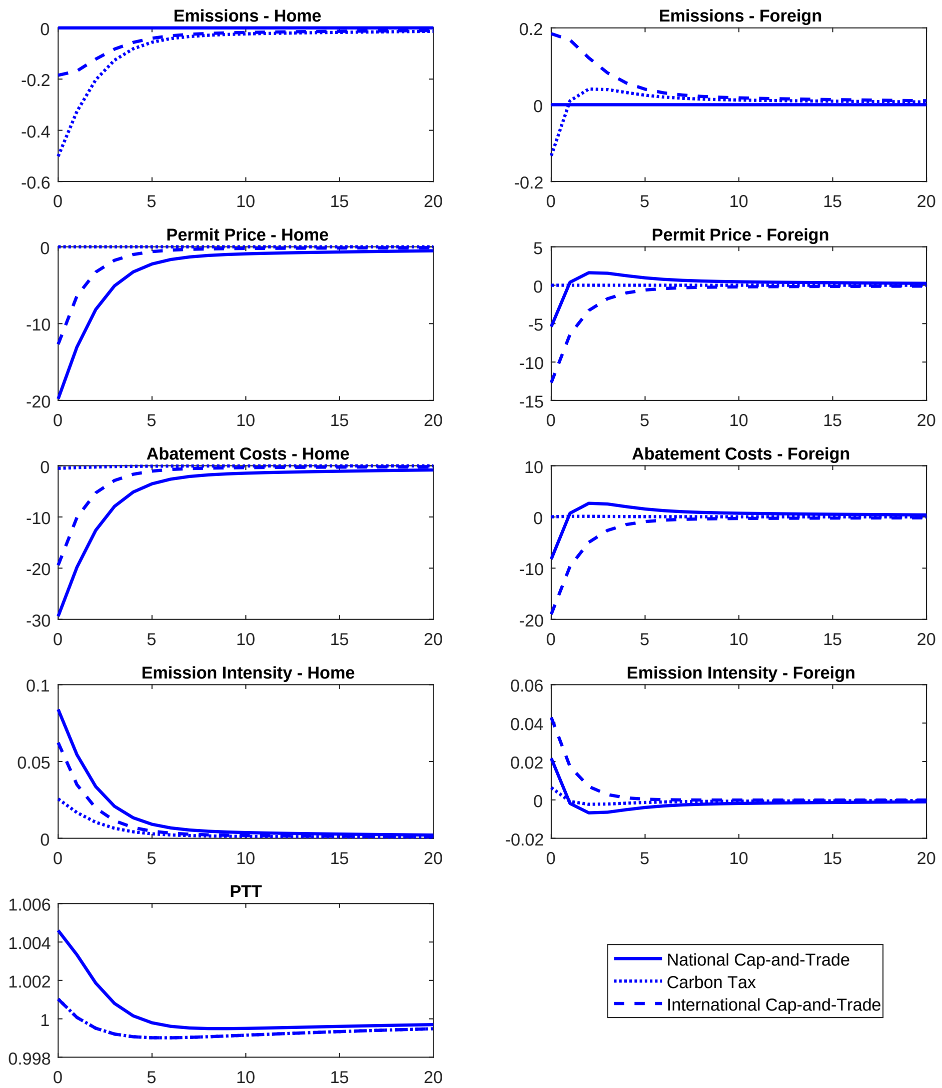

Table 1: Parametrization

Parameter	Value	Description
α	1/3	technology parameter
β	0.99	quarterly discount factor
$1-\gamma$	1-0.304	elasticity of emissions to output
γ_I	1.5	parameter for capital adjustment costs
γ_p	58.25	degree of price rigidities
δ	0.025	quarterly capital depreciation rate
ϵ	0.3829	emissions scale parameter
$1-\eta$	1-0.9979	pollution decay rate
θ_1	1	abatement cost function parameter
θ_2	2.8	abatement cost function parameter
ι_Π	1.5	Interest rate rule: inflation coefficient
κ	0.70	share of domestic goods used in the final sector
ξ_l	3.8826	disutility of labor parameter
ρ	1.5	elasticity of substitution between Home and Foreign goods
ρ_A	0.85	persistence of productivity shock
ρ_{AC}	0.85	persistence of abatement costs shock
ρ_I	0.85	persistence of investment-specific costs shock
ρ_K	0.85	persistence of quality of capital shock
ρ_R	0.5	persistence of monetary policy shock
σ	6	elasticity of substitution between good varieties
ϕ_c	1.2	coefficient of relative risk aversion
ϕ_l	1	inverse of the Frisch elasticity of labor supply
χ	2.3069e-06	intensity of negative externality on output
A	13.2581	total factor productivity - TFP

Table 2: International Transmission of Shocks - Benchmark Case (%)

	σ_{Y^D}	$\sigma_{Y^{D*}}/\sigma_{Y^D}$	$\rho(Y^D, Y^{D^*})$	$\rho(Y^D, E)$	$\rho(Y^D, p_E)$	$\rho(Y^{D^*}, E^*)$	$\rho(Y^{D^*}, p_E^*)$
National Cap-and-Trade							
technology	4.4698	17.5281	-15.7062	-	99.9999	-	99.9487
abatement cost	0.0049	16.6494	8.5068	-	-98.1710	-	99.9657
capital quality	8.0150	36.8010	-20.4032	-	99.9960	-	99.9981
investment	1.8229	35.8081	58.1057	-	99.9979	-	99.9978
monetary	5.1182	19.8823	47.2146	-	99.9999	-	99.9737
Carbon Tax							
technology	4.6263	17.9565	-16.0961	99.9999	-	99.9525	-
abatement cost	0.0001	68.2089	52.4136	-93.5913	-	99.9993	-
capital quality	8.3573	37.9101	-21.2625	99.9961	-	99.9984	-
investment	1.8920	36.5273	57.0341	99.9980	-	99.9982	-
monetary	5.2391	20.2264	46.0408	99.9999	-	99.9743	-
International Cap-and-Trade							
technology	4.5420	18.5051	-26.0515	98.5852	98.2835	41.8657	-7.7925
abatement cost	0.0028	98.9322	99.9886	-99.6154	-99.5803	99.4947	-99.4546
capital quality	8.1988	38.5498	-26.7971	95.1739	92.3889	55.0705	12.1091
investment	1.8477	35.7540	52.6556	94.2224	96.8806	-21.1382	72.0813
monetary	5.1657	19.7858	40.4458	98.1371	98.6253	-22.1215	55.0020

Table 3: International Transmission of Shocks - Imperfect Complementarity between Home and Foreign Goods (%)

	σ_{Y^D}	$\sigma_{Y^{D*}}/\sigma_{Y^D}$	$\rho(Y^D, Y^{D^*})$	$\rho(Y^D, E)$	$\rho(Y^D, p_E)$	$\rho(Y^{D^*}, E^*)$	$\rho(Y^{D^*}, p_E^*)$
National Cap							
TFP	4.1894	29.0435	80.6631	-	99.9986	-	99.9860
abatement cost	0.0050	36.5994	73.1053	-	-99.7010	-	99.9982
capital quality	9.7259	73.2355	53.5569	-	99.9953	-	99.9857
investment	2.9309	67.6913	41.7895	-	99.9920	-	99.9880
monetary	4.6998	23.8904	85.1984	-	99.9995	-	99.9798
Carbon Tax							
TFP	4.1362	22.2635	87.3517	99.9996	-	99.9807	-
abatement cost	0.0001	72.9584	35.4246	-94.0978	-	99.9975	-
capital quality	9.1583	55.8165	64.0898	99.9973	-	99.9826	-
investment	2.8021	55.0592	52.1787	99.9965	-	99.9883	-
monetary	4.6736	25.7241	83.9646	99.9998	-	99.9861	-
International cap-and-trade							
TFP	4.0324	20.2955	85.0030	99.2313	99.5869	-77.8343	89.4345
abatement cost	0.0028	97.7083	99.9959	-99.5759	-99.5454	99.6464	-99.6185
capital quality	8.8538	53.7232	61.5502	85.9514	95.2922	-12.6401	82.5500
investment	2.7202	53.4408	49.0932	86.3305	93.8224	1.5846	76.2064
monetary	4.5940	24.8802	81.9593	98.4375	99.3063	-70.5920	88.1277

Table 4: International Transmission of Shocks - High Degree of Openness (%)

	σ_{Y^D}	$\sigma_{Y^{D*}}/\sigma_{Y^D}$	$\rho(Y^D, Y^{D^*})$	$\rho(Y^D, E)$	$\rho(Y^D, p_E)$	$\rho(Y^{D^*}, E^*)$	$\rho(Y^{D^*}, p_E^*)$
National Cap-and-Trade							
TFP	3.8562	32.2853	-14.8037	-	99.9993	-	99.9388
abatement cost	0.0041	37.8203	1.9651	-	-95.9448	-	99.9549
capital quality	6.8709	84.1418	-49.4541	-	99.9965	-	99.9930
investment	1.3307	101.4315	24.5426	-	99.9938	-	99.9941
monetary	4.2778	37.6667	47.6346	-	99.9993	-	99.9658
Carbon Tax							
TFP	3.9607	33.6309	-15.1338	99.9993	-	99.9436	-
abatement cost	0.0000	144.8294	88.0188	-95.9931	-	99.9993	-
capital quality	7.1135	87.9037	-50.6317	99.9962	-	99.9937	-
investment	1.3764	104.2643	22.6499	99.9934	-	99.9944	-
monetary	4.3586	38.4496	46.8201	99.9993	-	99.9668	-
International Cap-and-Trade							
TFP	3.8920	34.1124	-20.7600	95.4718	94.1150	48.9230	13.5243
abatement cost	0.0028	100.3715	99.9997	-99.5724	-99.5374	99.5505	-99.5146
capital quality	7.0299	88.3807	-53.4440	89.1788	57.6949	85.9023	38.2003
investment	1.3436	104.5706	19.0656	61.5003	75.9728	65.6806	78.3160
monetary	4.2961	38.0520	43.8805	92.5100	95.9654	-6.4727	67.3767

Table 5: International Transmission of Shocks - Currency Union

	σ_{Y^D}	$\sigma_{Y^{D*}}/\sigma_{Y^D}$	$\rho(Y^D, Y^{D^*})$	$\rho(Y^D, E)$	$\rho(Y^D, p_E)$	$\rho(Y^{D^*}, E^*)$	$\rho(Y^{D^*}, p_E^*)$
National Cap-and-Trade							
TFP	4.1605	26.2899	21.8787	-	99.9999	-	99.9780
abatement cost	0.0046	23.1478	36.9830	-	-97.5325	-	99.9831
capital quality	7.6850	32.9326	-3.3964	-	99.9965	-	99.9926
investment	2.0913	28.7928	57.6502	-	99.9990	-	99.9937
monetary	6.2192	100.000	100.000	-	100.000	-	100.000
Carbon Tax							
TFP	4.3021	26.6966	21.8988	99.9999	-	99.9791	-
abatement cost	0.0001	50.9192	21.0023	-93.3259	-	99.9976	-
capital quality	8.0055	33.8475	-4.2973	99.9966	-	99.9937	-
investment	2.1641	29.2502	58.3058	99.9990	-	99.9947	-
monetary	6.3856	100.000	100.000	100.000	-	100.000	-
International Cap-and-Trade							
TFP	4.2166	26.5543	14.5400	96.4686	96.9465	12.0337	38.3584
abatement cost	0.0029	98.2293	99.9819	-99.6291	-99.5942	99.4744	-99.4328
capital quality	7.8429	34.1993	-10.8382	95.3987	94.2954	40.1415	22.8763
investment	2.1156	28.3384	53.1636	96.5047	97.8930	-29.1144	69.3384
monetary	6.2192	100.000	100.000	-	100.000	-	100.000

Appendix

Equilibrium Conditions

Let define $B_t^* = \frac{S_t B_t^*}{P_t}$, $S_t^R = \frac{S_t P_t^*}{P_t}$ and $\frac{S_t}{S_{t-1}} = 1 + s_t$, the following equations describe the decentralized competitive equilibrium of the model. Since we assume that the structure of the Foreign economy is isomorphic to that of the Home, we present only the equations for the Home economy and common equations.

$$C_t^{-\varphi_C} = \lambda_t, \tag{A-1}$$

$$\begin{aligned} q_t \;=\; & \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[r_{K,t+1} + \gamma_I \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{1}{K_{t+1}} - \frac{\gamma_I}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 \right] \right\} + \\ & + \beta(1-\delta) E_t \left\{ e^{u_{K,t+1}} \frac{q_{t+1} \lambda_{t+1}}{\lambda_t} \right\}, \end{aligned} \tag{A-2}$$

$$-\xi_L L_t^{\varphi_L} + \lambda_t w_t = 0, \tag{A-3}$$

$$\gamma_I \left(\frac{I_t}{K_t} - \delta \right) = e^{u_{I,t}} q_t - 1, \tag{A-4}$$

$$\frac{1}{R_t} = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\Pi_{t+1}} \right\}, \tag{A-5}$$

$$K_{t+1} = e^{u_{I,t}} I_t + (1-\delta) e^{u_{K,t}} K_t, \tag{A-6}$$

$$r_{K,t} = \alpha \Psi_t \frac{Y_t^D}{K_t}, \tag{A-7}$$

$$w_t = (1-\alpha) \Psi_t \frac{Y_t^D}{L_t}, \tag{A-8}$$

$$p_{E,t} (Y_t^D)^{(1-\gamma)} - \theta_2 \theta_1 e^{u_{AC,t}} \mu_t^{\theta_2 - 1} Y_t^D p_t^D = 0, \tag{A-9}$$

$$Y_t^D = \Lambda_t A_t \left(e^{u_{K,t}} K_t \right)^\alpha L_t^{1-\alpha}, \tag{A-10}$$

$$\begin{aligned} & \left(1 - \theta_1 \mu_t^{\theta_2}\right)(1-\sigma) + \sigma MC_t + \\ & -\gamma_p \left(\Pi_t^D - 1\right) \Pi_t^D + \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \gamma_p \left(\Pi_{t+1}^D - 1\right) \left(\Pi_{t+1}^D\right)^2 \frac{Y_{t+1}^D}{Y_t^D} \frac{1}{\Pi_{t+1}} \right\} = 0, \end{aligned} \tag{A-11}$$

$$MC_t = \frac{p_{E,t}}{p_t^D} (1-\gamma)(1-\mu_t) \epsilon (Y_t^D)^{-\gamma} + \Psi_t \frac{1}{p_t^D}, \tag{A-12}$$

$$Y_t = [\kappa^{\frac{1}{\rho}} (Y_t^H)^{\frac{\rho-1}{\rho}} + (1-\kappa)^{\frac{1}{\rho}} (M_t)^{\frac{\rho-1}{\rho}}]^{\frac{\rho}{\rho-1}}, \tag{A-13}$$

$$Y_t^H = \kappa Y_t \left(\frac{1}{p_t^D} \right)^\rho, \tag{A-14}$$

$$Y_t^D = Y_t^H + X_t, \tag{A-15}$$

$$M_t = (1-\kappa)\left(\frac{1}{p_t^{D*}}\frac{1}{S_t^R}\right)^{\rho} Y_t, \tag{A-16}$$

$$\Pi_t = \frac{p_{t-1}^D}{p_t^D}\Pi_t^D, \tag{A-17}$$

$$Tr_t = p_{E,t}E_t, \tag{A-18}$$

$$X_t = (1-\kappa)\left(\frac{S_t^R}{p_t^D}\right)^{\rho} Y_t^*, \tag{A-19}$$

$$p_t^D Y_t^D = C_t + e^{u_{I,t}} I_t + p_t^D AC_t + p_t^D X_t - S_t^R p_t^{D*} M_t + \frac{\gamma_I}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 I_t + \frac{\gamma_p}{2}(\Pi_t^D - 1)^2 p_t^D Y_t^D, \tag{A-20}$$

$$\frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi}\right)^{\iota_\Pi} e^{u_{R,t}}, \tag{A-21}$$

$$E_t = (1-\mu_t)\epsilon(Y_t^D)^{(1-\gamma)}, \tag{A-22}$$

$$AC_t = e^{u_{AC,t}}\theta_1 \mu_t^{\theta_2} Y_t^D, \tag{A-23}$$

$$u_{I_t} = \rho_I u_{I_{t-1}} + \varepsilon_{I,t}, \tag{A-24}$$

$$u_{K,t} = \rho_K u_{K,t-1} + \epsilon_{K,t}, \tag{A-25}$$

$$u_{A_t} = \rho_A u_{A_{t-1}} + \varepsilon_{A,t}, \tag{A-26}$$

$$u_{AC,t} = \rho_{AC} u_{AC,t-1} + \epsilon_{AC,t}, \tag{A-27}$$

$$u_{R,t} = \rho_R u_{R,t-1} + \epsilon_{R,t}. \tag{A-28}$$

Common equations determine the time path of the depreciation rate of the domestic currency s_t, the net external asset position f_t^*, the real exchange rate S_t^R, the stock of pollution Z_t in the atmosphere and the related damage Λ_t:

$$\frac{1}{R_t^*} = \beta E_t\left\{\frac{\lambda_{t+1}(1+s_{t+1})}{\Pi_{t+1}\lambda_t}\right\}, \tag{A-29}$$

$$f_t^* = \frac{R_{t-1}^*(1+s_t)}{\Pi_t} f_{t-1}^* - S_t^R p_t^{D*} M_t + p_t^D X_t, \tag{A-30}$$

$$S_t^R = S_{t-1}^R (1+s_t)\frac{\Pi_t^*}{\Pi_t}, \tag{A-31}$$

$$Z_t = \eta Z_{t-1} + E_t + E_t^* + E_t^{NI}, \tag{A-32}$$

$$\Lambda_t = \exp[-\chi(Z_t - \overline{Z})]. \tag{A-33}$$

The overall economy is then described by 24 variables related to Home, $\{C_t, E_t, I_t, K_t, L_t, M_t, MC_t, p_t^D, p_{E,t}, q_t, R_t, r_{K,t}, Tr_t, w_t, X_t, Y_t, Y_t^D, Y_t^H, AC_t, \lambda_t, \mu_t, \Pi_t, \Pi_t^D, \Psi_t\}$, 24 variables related to Foreign $\{C_t^*, E_t^*, I_t^*, K_t^*, L_t^*, M_t^*, MC_t^*, p_t^{D^*}, p_{E,t}^*, q_t^*, R_t^*, r_{K,t}^*, Tr_t^*, w_t^*, X_t^*, Y_t^*, Y_t^{D*},$

Y_t^{H*}, AC_t^*, λ_t^*, μ_t^*, Π_t^*, Π_t^{D*}, $\Psi_t^*\}$, and 4 common variables, $\{f_t^*$, s_t, S_t^R, $Z_t, \Lambda_t\}$.

Note that under a national cap-and-trade regime, $E_t = \bar{E}$ and $E_t^* = \bar{E}^*$ with $\bar{E} = \bar{E}^*$; under a carbon tax $p_{E,t} = p_E$ and $p_{E,t}^* = p_E^*$, with $p_E = p_E^*$; under international cap-and-trade regime, $E_t + E_t^* = \bar{E} + \bar{E}^*$ and $p_{E,t} = p_{E,t}^*$.