IDEA: a DSGE model of the euro area*

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Abstract

We develop a three-country open economy model, New Keynesian framework, of a home region, the euro area and the world economy. The model incorporates a banking sector and is designed for conducting quantitative analysis of non-standard policy measures as the ones implemented during the last years by the ECB. Banks collect deposits from domestic household and raise capital to finance loans to domestic households and firms. In order to borrow from local (regional) banks, households use domestic real estate whereas firms use both domestic real estate and physical capital as collateral. Entrepreneurs finance their investment in physical capital by borrowing from domestic banks. We provide a full description of trade balance and real exchange rate dynamics. We simulate the model to conduct quantitative policy analysis of macroeconomic interdependence across regions belonging to the euro area and between euro area regions and the world economy.

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1 The basic setup

The model is an open economy, New Keynesian framework. It represents the world economy, composed of three regions: Home, REA (Home+REA=EA), and RW. The size of the world economy is normalized to 1. Home, REA, and RW have sizes equal to n, n^*, and $(1 - n - n^*)$, respectively, with $n, n^* > 0$, and $n + n^* < 1$. For each region, the size refers to the overall households' population and to the number of firms operating in each sector (intermediate tradable, intermediate nontradable, final nontradable consumption, final nontradable investment). Home and REA share the currency and the monetary authority. The latter sets the nominal interest rate according to a standard Taylor rule, and reacts to EA-wide inflation and real GDP growth. A similar rule is followed by the monetary authority in the RW. Crucial features are those determining the financial structure. Fig.1 provides a bird's eye view of the main financial relationships in the Home region (similar relations hold in the REA). In each EA regions there are (i) two types of households – i.e., savers and borrowers, (ii) the banking sector (wholesale and retail branches), and (iii) non-financial entrepreneurs. Savers invest in deposits with domestic banks, internationally traded bonds, domestic corporate bonds, domestic real estate. Borrowers obtain loans from the domestic banking sector after pledging their real estate as collateral. The banking sector collects deposits from domestic savers, issues equities to savers, and lends to domestic borrowers and entrepreneurs.

The productive structure of the model is reported in Fig.2. The non-financial entrepreneurs choose the optimal amounts of the (end-of-period) stock of physical capital and investment. They rent capital to domestic wholesale firms. Entrepreneurs finance their investment in physical capital by borrowing from domestic banks (their loans are collateralized by the owned real estate) and by issuing uncollateralized long-term "corporate" bonds to savers in the domestic corporate bond market. All households consume and supply labor services to domestic (non-financial) firms. Savers hold domestic firms operating in the final and intermediate sectors.

Real estate is in fixed aggregate supply in each region (Fig.3). It is exchanged among entrepreneurs, savers, and borrowers, under perfect competition. It is a durable non-tradable good that provides utility (housing services) to households and that entrepreneurs rent as input to domestic wholesale firms.

The remaining features of the model are rather standard and in line with New Keynesian open economy models.

Households consume a final good, which is a composite of intermediate non-tradable and tradable goods. The latter are domestically produced or imported. All households supply differentiated labor services to domestic firms and act as wage setters in monopolistically competitive labor markets, as they charge a wage mark-up over their marginal rate of substitution between consumption and leisure.

On the production side, there are firms that, under perfect competition, produce two final goods (consumption and investment goods) and firms that, under monopolistic competition, produce intermediate (internationally) tradable and nontradable goods.

The two final goods are sold domestically and are produced combining all available intermediate goods using a constant-elasticity-of-substitution (CES) production function. The two resulting bundles can have different composition. Intermediate tradable and nontradable goods are produced combining capital and labor, supplied by the domestic households. Capital and labor are assumed to be mobile across the two intermediate sectors.

Given the assumption of differentiated intermediate goods, firms have market power, are price-setter and restrict output to create excess profits. Intermediate tradable goods can be sold domestically and abroad. It is assumed that markets for tradable goods are segmented, so that firms can set a different price in each of the three regions.

In line with other dynamic general equilibrium models of the EA (see, among the others, Warne et al. 2008 and Gomes et al. 2010, we include adjustment costs on real and nominal variables, ensuring that consumption, production, and prices react in a gradual way to a given shock. On the real side, habits and quadratic costs prolong the adjustment of consumption and investment, respectively. On the nominal side, quadratic costs make wages and prices sticky.[1]

In what follows, we report the main new equations for the Home country. Similar equations hold in the REA and in the RW (if not so, we report the differences).[2]

1.1 Firms

We initially show the final good sectors (private consumption good, investment good, public consumption good). Thereafter, the intermediate good sectors (intermediate nontradable goods, and intermediate tradable goods).

1.1.1 Final private consumption good

There is a continuum of symmetric Home firms producing final nontradable consumption goods under perfect competition. Each firm producing the consumption good is indexed by $x \in (0, n]$, where the parameter $0 < n < 1$ measures the size of Home region. Firms in the REA and in the RW are indexed by $x^* \in (n, n + n^*]$ and $x^{**} \in (n + n^*, 1]$, respectively. The CES production technology used by the

[1]See Rotemberg (1982).

[2]In what follows we report the main equations of the model largely following Pesenti (2008).

generic firm x is

$$A_t(x) \equiv \left(\begin{array}{c} a_T^{\frac{1}{\phi_A}} \left(\begin{array}{c} a_H^{\frac{1}{\rho_A}} Q_{HA,t}(x)^{\frac{\rho_A - 1}{\rho_A}} + a_G^{\frac{1}{\rho_A}} Q_{GA,t}(x)^{\frac{\rho_A - 1}{\rho_A}} \\ + (1 - a_H - a_G)^{\frac{1}{\rho_A}} Q_{FA,t}(x)^{\frac{\rho_A - 1}{\rho_A}} \end{array} \right)^{\frac{\rho_A}{\rho_A - 1} \frac{\phi_A - 1}{\phi_A}} \\ + (1 - a_T)^{\frac{1}{\phi_A}} Q_{NA,t}(x)^{\frac{\phi_A - 1}{\phi_A}} \end{array} \right)^{\frac{\phi_A}{\phi_A - 1}}, \tag{1}$$

where Q_{HA}, Q_{GA}, Q_{FA}, and Q_{NA} are bundles of, respectively, intermediate tradables produced in Home, REA, RW, and intermediate nontradables produced in the Home country. The parameter $\rho_A > 0$ is the elasticity of substitution among tradables, and $\phi_A > 0$ is the elasticity of substitution between tradable and nontradable goods. The parameter a_H ($a_H > 0$) is the weight of the Home tradable, the parameter a_G ($a_G > 0$, $1 - a_H - a_G < 1$) the weight of tradables imported from the REA, and the parameter a_T ($0 < a_T < 1$) the weight of the overall tradable goods.

1.1.2 Final investment good

The production of investment good is similar. There are symmetric Home firms under perfect competition indexed by $y \in (0, n]$. Firms in the REA and in the RW are indexed by $y^* \in (n, n + n^*]$ and $y^{**} \in (n + n^*, 1]$. Output of the generic Home firm y is

$$E_t(y) \equiv \left(\begin{array}{c} v_T^{\frac{1}{\phi_E}} \left(\begin{array}{c} v_H^{\frac{1}{\rho_E}} Q_{HE,t}(y)^{\frac{\rho_E - 1}{\rho_E}} + v_G^{\frac{1}{\rho_E}} Q_{GE,t}(y)^{\frac{\rho_E - 1}{\rho_E}} \\ + (1 - v_H - v_G)^{\frac{1}{\rho_E}} Q_{FE,t}(y)^{\frac{\rho_E - 1}{\rho_E}} \end{array} \right)^{\frac{\rho_E}{\rho_E - 1} \frac{\phi_E - 1}{\phi_E}} \\ + (1 - v_T)^{\frac{1}{\phi_E}} Q_{NE,t}(y)^{\frac{\phi_E - 1}{\phi_E}} \end{array} \right)^{\frac{\phi_E}{\phi_E - 1}}, \tag{2}$$

where Q_{HE}, Q_{GE}, Q_{FE}, and Q_{NE} are bundles of respectively intermediate tradables produced in Home, REA, RW, and intermediate nontradables produced in the Home country. The parameter $\rho_E > 0$ is the elasticity of substitution among tradables, and $\phi_E > 0$ is the elasticity of substitution between tradable and nontradable goods. The parameter v_H ($v_H > 0$) is the weight of the Home tradables, the parameter v_G ($v_G > 0$, $1 - v_H - v_G < 1$) the weight of tradables imported from the REA, and the parameter v_T ($0 < v_T, 1 - v_T < 1$) the weight of the overall tradable goods.

1.1.3 Final public good

It is assumed that the public consumption good is fully biased towards the intermediate nontradable goods

$$Q_{NA,t}\left(x\right) \equiv \left[\left(\frac{1}{n}\right)^{\theta_N} \int_0^n Q_t\left(i,x\right)^{\frac{\theta_N-1}{\theta_N}} di\right]^{\frac{\theta_N}{\theta_N-1}}, \tag{3}$$

where $\theta_N > 1$ is the elasticity of substitution among brands in the nontradable sector.

FOCs: input demands

Bundles used to produce the final consumption goods are CES indexes of differentiated intermediate goods, each produced by a single firm under conditions of monopolistic competition (see section xxx),

$$Q_{HA,t}\left(x\right) \equiv \left[\left(\frac{1}{n}\right)^{\theta_T} \int_0^n Q_t\left(h,x\right)^{\frac{\theta_T-1}{\theta_T}} dh\right]^{\frac{\theta_T}{\theta_T-1}}, \tag{4}$$

$$Q_{GA,t}\left(x\right) \equiv \left[\left(\frac{1}{n^*}\right)^{\theta_T} \int_n^{n+n^*} Q_t\left(g,x\right)^{\frac{\theta_T-1}{\theta_T}} dg\right]^{\frac{\theta_T}{\theta_T-1}}, \tag{5}$$

$$Q_{FA,t}\left(x\right) \equiv \left[\left(\frac{1}{1-n-n^*}\right)^{\theta_T} \int_{n+n^*}^1 Q_t\left(f,x\right)^{\frac{\theta_T-1}{\theta_T}} df\right]^{\frac{\theta_T}{\theta_T-1}}, \tag{6}$$

$$Q_{NA,t}\left(x\right) \equiv \left[\left(\frac{1}{n}\right)^{\theta_N} \int_0^n Q_t\left(i,x\right)^{\frac{\theta_N-1}{\theta_N}} di\right]^{\frac{\theta_N}{\theta_N-1}}, \tag{7}$$

where firms in the Home intermediate tradable and nontradable sectors are respectively indexed by $h \in (0,n]$ and $n \in (0,n]$, firms in the REA by $g \in (n, n+n^*]$, and firms in the RW by $f \in (n+n^*, 1]$. Parameters θ_T, $\theta_N > 1$ are respectively the elasticity of substitution among brands in the tradable and nontradable sector. The prices of the intermediate nontradable goods are denoted $p(i)$. Each firm x takes these prices as given when minimizing production costs of the final good. The resulting demand for intermediate nontradable input i is

$$Q_{A,t}\left(i,x\right) = \left(\frac{1}{n}\right)\left(\frac{P_t\left(i\right)}{P_{N,t}}\right)^{-\theta_N} Q_{NA,t}\left(x\right), \tag{8}$$

where $P_{N,t}$ is the cost-minimizing price of one basket of local nontradable intermediates,

$$P_{N,t} = \left[\int_0^n P_t(i)^{1-\theta_N} \, di \right]^{\frac{1}{1-\theta_N}} . \tag{9}$$

Firms y producing the final investment goods have similar demand curves. Aggregating over x and y, it can be shown that total demand for intermediate nontradable good i is

$$\int_0^n Q_{A,t}(i,x) \, dx + \int_0^n Q_{E,t}(i,y) \, dy + \int_0^n C_{N,t}^g(i,x) \, dx =$$
$$\left(\frac{P_t(i)}{P_{N,t}} \right)^{-\theta_N} \left(Q_{NA,t} + Q_{NE,t} + C_{N,t}^g \right) , \tag{10}$$

where C_N^g is public sector consumption.

Home demands for (intermediate) domestic and imported tradable goods – $Q_A(h,x)$, $Q_A(f,x)$, $Q_A(g,x)$ – and the cost-minimizing prices of the corresponding baskets – P_H, P_F, and P_G – can be derived in a similar way.

1.1.4 Intermediate goods

We report the production function and the implied first-order conditions. Finally, we show the labor bundle.

Production function The supply of each Home intermediate nontradable good i is denoted by $N^S(i)$:

$$N_t^S(i) = \left((1-\alpha_N)^{\frac{1}{\xi_N}} L_{N,t}(i)^{\frac{\xi_N - 1}{\xi_N}} + \alpha_N^{\frac{1}{\xi_N}} K_{N,t}(i)^{\frac{\xi_N - 1}{\xi_N}} \right)^{\frac{\xi_N}{\xi_N - 1}} . \tag{11}$$

Firm i uses labor $L_{N,t}(i)$ and capital $K_{N,t}(i)$ supplied by domestic households. The parameter $\xi_N > 0$ measures the elasticity of substitution. The parameter $0 < \alpha_N < 1$ is the weight of capital. Firms producing intermediate goods take the prices of labor and capital inputs as given when minimizing their costs.

FOCs: inputs demand Denoting W_t the nominal wage index and R_t^K the nominal rental price of capital, cost minimization implies that

$$L_{N,t}(i) = (1-\alpha_N) \left(\frac{W_t}{MC_{N,t}(i)} \right)^{-\xi_N} N_t^S(i) , \tag{12}$$

and

$$K_{N,t}(i) = \alpha_N \left(\frac{R_t^K}{MC_{N,t}(i)} \right)^{-\xi_N} N_t^S(i), \tag{13}$$

where $MC_{N,t}(n)$ is the nominal marginal cost:

$$MC_{N,t}(i) = \left((1-\alpha_N) W_t^{1-\xi_N} + \alpha_N \left(R_t^K\right)^{1-\xi_N} \right)^{\frac{1}{1-\xi_N}}. \tag{14}$$

The productions of each Home tradable good, $T^S(h)$, is similarly characterized.

FOCs: supply of nontradable intermediates Consider now profit maximization in the Home intermediate nontradable sector. Each firm i sets the price $P_{N,t}(i)$ by maximizing the present discounted value of profits

$$E_t \sum_{\tau=t}^{\infty} \beta^t \Lambda_{t,\tau} \left[\begin{array}{c} \frac{P_{N,\tau}(i)}{P\tau} Q_{N,\tau}(i) + \frac{S_\tau^* P_{N,\tau}^*(h)}{P_\tau} Q_{N,\tau}^*(i) + \frac{S_\tau^{**} P_{N,\tau}^{**}(i)}{P\tau} Q_{N,\tau}^{**}(i) \\ -\frac{MC_{N,\tau}(i)}{P\tau} \left(Q_{N,\tau}(i) + Q_{N,\tau}^*(i) + Q_{N,\tau}^{**}(i) \right) \end{array} \right], \tag{15}$$

subject to the demand constraint

$$Q_{N,\tau}(i,x) = \left(\frac{1}{n}\right) \left(\frac{P_{N,\tau}(i)}{P_{N,\tau}} \right)^{-\theta_N} Q_{NA,\tau}(x), \tag{16}$$

and the quadratic adjustment costs,

$$AC_{N,\tau}^p(i) \equiv \frac{\kappa_N^p}{2} \left(\frac{P_{N,\tau}(i)/P_{N,\tau-1}(i)}{\pi_{N,\tau-1}^{ind_N} \bar{\pi}^{1-ind_N}} - 1 \right)^2 \frac{P_{N,\tau}}{P_\tau} Q_{N,\tau}, \tag{17}$$

which is paid in unit of sectorial product $Q_{N,t}$, where $\kappa_N^p \geq 0$ is a parameter that measures the degree of price stickiness, $\pi_{N,t-1}$ is the previous-period gross inflation rate of nontradable goods $(\pi_{N,t} \equiv P_{N,t}/P_{N,t-1})$, $\bar{\pi}$ is the long-run (consumer-price) inflation target set by the monetary authority, and $0 \leq ind_N \leq 1$ is a parameter that measures indexation to previous-period inflation. The FOC with respect to $P_{N,t}(i)$, expressed in terms of domestic consumption units???, is

$$0 = (1-\theta_N) \frac{P_{N,t}(i)^{-\theta_N}}{P_{N,t}{}^{-\theta_N}} - \theta_N \frac{P_{N,t}(i)^{-\theta_N - 1}}{P_{N,t}{}^{-\theta_N}} MC_{N,t}(i) - A_t(i), \tag{18}$$

where $MC_{N,t}(i)$ is the nominal marginal cost of nontradable goods and $A_t(i)$ contains terms related to the presence of price adjustment costs

$$A_t(i) \equiv \kappa_N^p \frac{P_{N,t}/P_{N,t-1}(i)}{\pi_{N,t-1}^{ind_N}\bar{\pi}^{1-ind_N}} \left(\frac{P_{N,t}(i)/P_{N,t-1}(i)}{\pi_{N,t-1}^{ind_N}\bar{\pi}^{1-ind_N}} - 1 \right)$$
$$-\beta\kappa_N^p \frac{\lambda_{t+1}}{\lambda_t} \frac{P_{N,t+1}(i)\,P_{N,t+1}/P_{N,t}(i)^2}{\pi_{N,t}^{ind_N}\bar{\pi}^{1-ind_N}} \left(\frac{P_{N,t+1}(i)/P_{N,t}(i)}{\pi_{N,t}^{ind_N}\bar{\pi}^{1-ind_N}} - 1 \right) \frac{Q_{N,t+1}}{Q_{N,t}}. \qquad (19)$$

The above equations clarify the link between imperfect competition and nominal rigidities. When the elasticity of substitution θ_N is very large and, thus, the competition in the sector is high, prices closely follow marginal costs, even though adjustment costs are large. To the contrary, it may be optimal to maintain stable prices and accommodate changes in demand through supply adjustments when the average markup over marginal costs is relatively high. Under the hypothesis of a symmetric equilibrium: $MC_{N,t}(i) = MC_{N,t}$ and $P_{N,t}(i) = P_{N,t}$. The FOC can be simplified as follows

$$p_{N,t} = \frac{\theta_N}{\theta_N - 1} mc_{N,t} - \frac{A_t}{\theta_N - 1}, \qquad (20)$$

where

$$A_t \equiv \kappa_N^p \frac{\pi_{N,t-1}}{\pi_{N,t-1}^{ind_N}\bar{\pi}^{1-ind_N}} \left(\frac{\pi_{N,t}}{\pi_{N,t-1}^{ind_N}\bar{\pi}^{1-ind_N}} - 1 \right) p_{N,t}$$
$$-\beta\kappa_N^p \frac{\lambda_{t+1}}{\lambda_t} \frac{\pi_{N,t+1}^2}{\pi_{N,t}^{ind_N}\bar{\pi}^{1-ind_N}\pi_{t+1}} \left(\frac{\pi_{N,t+1}}{\pi_{N,t}^{ind_N}\bar{\pi}^{1-ind_N}} - 1 \right) \frac{q_{N,t+1}}{q_{N,t}} gr_{t+1} p_{N,t}. \qquad (21)$$

If prices were flexible, optimal pricing would collapse to the standard pricing rule of constant markup over marginal costs (expressed in units of domestic consumption)

$$p_{N,t} = \frac{\theta_N}{\theta_N - 1} mc_{N,t}. \qquad (22)$$

FOCs: supply of tradable intermediates Firms operating in the intermediate tradable sector solve a similar problem. We assume that there is market segmentation. Hence the firm producing the brand h chooses the prices $P_t(h)$ in the Home market, the price $P_t^*(h)$ in the REA, and the price $P_t^{**}(h)$ in the RW to maximize the expected flow of profits (in terms of domestic consumption units),

$$E_t \sum_{\tau=t}^{\infty} \beta^t \Lambda_{t,\tau} \left[\begin{array}{c} \frac{P_t(h)}{P_t} Q_t(h) + \frac{S_t^* P_t^*(h)}{P_t} Q_t^*(h) + \frac{S_t^{**} P_t^{**}(h)}{P_t} Q_t^{**}(h) \\ -\frac{MC_{H,t}(h)}{P_t} \left(Q_t(h) + Q_t^*(h) + Q_t^{**}(h) \right) \end{array} \right], \qquad (23)$$

subject to quadratic price adjustment costs similar to those considered for non-tradables and *standard* demand constraints. The term E_t denotes the expectation operator conditional on the information set at time t, $\Lambda_{t,\tau}$ is the appropriate discount rate, and $MC_{H,t}(h)$ is the nominal marginal cost.

$$Q_t(h) = \left(\frac{\bar{P}_{H,\tau}(j) + \eta P_{N,t}}{P_{H,\tau}} \right)^{-\theta_H} Q_t, \tag{24}$$

Similar

The first order conditions with respect to $\bar{P}_t(h)$ is:

$$\begin{aligned} 0 &= \left(\frac{\bar{P}_{H,\tau}(h) + \eta P_{N,t}}{P_{H,\tau}} \right)^{-\theta_H} \\ -\theta_H \frac{(\bar{P}^*_{H,\tau}(h) + \eta P_{N,t})^{-\theta_H - 1}}{(P^*_{H,\tau})^{-\theta_H}} & \left(\bar{P}^*_{H,\tau}(h) - MC_{\tau,t}(h) \right) - A_t(h), \end{aligned} \tag{25}$$

with respect to $\bar{P}^*_t(h)$ (a similar equation holds for $\bar{P}^{**}_t(h)$):

$$\begin{aligned} 0 &= \left(\frac{\bar{P}^*_{H,\tau}(h) + \eta P_{N,t}}{P^*_{H,\tau}} \right)^{-\theta^*_H} \\ -\theta^*_H \frac{(\bar{P}^*_{H,\tau}(h) + \eta P_{N,t})^{-\theta^*_H - 1}}{(P^*_{H,\tau})^{-\theta^*_H}} & \left(\bar{P}^*_{H,\tau}(h) - \frac{MC_{\tau,t}(h)}{S^*_\tau} \right) - A^*_t(h), \end{aligned} \tag{26}$$

where θ_T is the elasticity of substitution of intermediate tradable goods, while $A_t(h)$ and $A^*_t(h)$ (a similar equation holds for $A^{**}_t(h)$) involve terms related to the presence of price adjustment costs:

$$\begin{aligned} A_t(h) &\equiv \kappa^p_H \frac{P_{H,t}/P_{H,t-1}(h)}{\pi^{\alpha_H}_{H,t-1} \bar{\pi}^{1-\alpha_H}} \left(\frac{P_{H,t}(h)/P_{H,t-1}(h)}{\pi^{\alpha_H}_{H,t-1} \bar{\pi}^{1-\alpha_H}} - 1 \right) \\ -\beta \kappa^p_H \frac{\lambda_{t+1}}{\lambda_t} & \frac{P_{H,t+1}(h) \, P_{H,t+1}/P_{H,t}(h)^2}{\pi^{\alpha_H}_{H,t} \bar{\pi}^{1-\alpha_H} \pi_{t+1}} \left(\frac{P_{H,t+1}(h)/P_{H,t}(h)}{\pi^{\alpha_H}_{H,t} \bar{\pi}^{1-\alpha_H}} - 1 \right) \frac{Q_{H,t+1}}{Q_{H,t}}. \end{aligned} \tag{27}$$

$$A_t^*(h) \equiv \kappa_H^{p\,*} \frac{\bar{P}_{H,t}^*/\bar{P}_{H,t-1}^*(h)}{(\pi_{H,t-1}^*)^{\alpha_H^*}(\bar{\pi}^*)^{1-\alpha_H^*}} \left(\frac{\bar{P}_{H,t}^*(h)/\bar{P}_{H,t-1}^*(h)}{(\pi_{H,t-1}^*)^{\alpha_H^*}(\bar{\pi}^*)^{1-\alpha_H^*}} - 1 \right)$$

$$-\beta\kappa_H^{p\,*} \frac{\lambda_{t+1}}{\lambda_t} \frac{\bar{P}_{H,t+1}^*(h)\,\bar{P}_{H,t+1}^*/(\bar{P}_{H,t}^*(h))^2}{(\pi_{H,t}^*)^{\alpha_H^*}(\bar{\pi}^*)^{1-\alpha_H^*}\pi_{t+1}} \left(\frac{\bar{P}_{H,t+1}^*(h)/\bar{P}_{H,t}^*(h)}{(\pi_{H,t}^*)^{\alpha_H^*}(\bar{\pi}^*)^{1-\alpha_H^*}} - 1 \right) \frac{Q_{t+1}^*}{Q_t^*} \frac{S_{t+1}^*}{S_t^*}, \tag{28}$$

where $\kappa_H^p, \kappa_H^{p\,*}, \kappa_H^{p\,**} > 0$ respectively measure the degree of nominal rigidity in the Home country, in the REA, and in the RW.

Under the hypothesis of a symmetric equilibrium: $\bar{P}_{H,t}(h) = \bar{P}_{H,t}$ and $MC_{H,t}(h) = MC_{H,t}$, the same hold for $\bar{P}_{H,t}^*(h)$, $\bar{P}_{H,t}^{**}(h)$, $MC_{H,t}^*(h)$ and $MC_{H,t}^{**}(h)$.

1.2 Labor bundle

In the case of the generic firm i operating in the intermediate nontradable sector, the labor input $L_N(i)$ is a CES combination of differentiated labor inputs supplied by domestic agents and defined over a continuum of mass equal to the country size ($j \in [0, n]$):

$$L_{N,t}(i) \equiv \left(\frac{1}{n}\right)^{\frac{1}{\psi}} \left[\int_0^n L_t(i,j)^{\frac{\psi-1}{\psi}} dj\right]^{\frac{\psi}{\psi-1}}, \tag{29}$$

where $L(i,j)$ is the demand of the labor input of type j by the producer of good i and $\psi > 1$ is the elasticity of substitution among labor inputs. Cost minimization implies that

$$L_t(i,j) = \left(\frac{1}{n}\right) \left(\frac{W_t(j)}{W_t}\right)^{-\psi} L_{N,t}(j), \tag{30}$$

where $W(j)$ is the nominal wage of labor input j and the wage index W is

$$W_t = \left[\left(\frac{1}{n}\right) \int_0^n W_t(j)^{1-\psi} dj\right]^{\frac{1}{1-\psi}}. \tag{31}$$

Similar equations hold for firms producing intermediate tradable goods. Each household is the monopolistic supplier of a labor input j and sets the nominal wage facing a downward-sloping demand obtained by aggregating demand across domestic firms.

Households

In the Home country there is a countinuum of households of mass $j \in [0, n]$. Each household j maximizes its lifetime expected utility subject to the budget constraint. The lifetime utility, in consumption C and labor L, is

$$E_t \sum_{\tau=t}^{\infty} \beta_t \left(\log \left(C_t(j) - b_c \frac{C_{t-1}}{gr_t} \right) - \frac{\kappa}{1+\zeta} L_t\,(j)^{1+\zeta} \right), \tag{32}$$

where $0 < \beta < 1$ is the discount factor, $b_c \in (0,1)$ is the external habit parameter, $\zeta > 0$ is the reciprocal of the Frisch elasticity of labor supply, where

$$L_t\,(j) = \left(\frac{w_t(j)}{w_t} \right)^{-\sigma_L} L_t. \tag{33}$$

The budget constraint expressed in real terms (units of consumpion) is

$$\begin{aligned} B_t(j) + S_t B_t^*(j) \leq (1+i_{t-1}) \frac{B_{t-1}(j)}{\pi_t gr_t} + (1+i_{t-1}^*) \left[1 - \Gamma_{B,t-1}\right] \frac{\Delta S_t B_{t-1}^*(j)}{\pi_t gr_t} \\ +(1-\tau_{k,t}) r_t^k \frac{K_{t-1}(j)}{gr_t} + (1-\tau_{w,t}) w_t(j) L_t(j) \\ -\frac{\psi}{2} \left(\frac{w_t(j)/w_{t-1}(j)}{\pi_{W,t}^{\alpha} \pi_t^{1-\alpha}} - 1 \right)^2 w_t L_t + \Pi_t^{prof} - (1+\tau_{c,t}) C_t(j) - p_{I,t} I_t(j), \end{aligned} \tag{34}$$

where B_t is the end-of-period t position in a riskless nominal bond denominated in the Home currency, B_t^* is the end-of period position in a riskless nominal bond denominated in the REA currency (we'll introduce the monetary union assumption in section xxx). The two bonds pay the gross nominal interest rates $(1 + i_t)$ and $(1 + i_t^*)$ at the beginning of period $t + 1$. The interest rates are known at time t (consistent with the riskless bond assumption). The term S_t is the nominal exchange rate, defined as number of Home currency units per unit of REA currency. The term $\Gamma_{B,t-1}$ is the REA bond adjustment cost. The sources of the household income are physical capital K_t, which is rent to domestic intermediate firms at the net rate R_t^k, labor L_t, which is supplied to domestic firms and earns the nominal wage W_t, and Π_t^{prof}, which represents profits from ownership of domestic firms (the profits are rebated in a lump-sum way to households). The variable I_t reprents investment in physical capital, $\Gamma_{W,t}$ is tha djustment cost of labor,$\tau_{k,t}$, $\tau_{w,t}$ and $\tau_{c,t}$ represent taxes on capital income, labor income, consumption expenditure, respectively. $\breve{\pi}$ is the inflation target of government, which corresponds to the convergence value of the steady state $\pi_{W,t}$. ψ

The stationarized capital accumulation law is

$$K_t(j) \leq (1-\delta)\frac{K_{t-1}(j)}{gr_t} + \left[1 - \frac{\psi}{2}\left(\frac{I_t(j)}{I_{t-1}(j)} - 1\right)^2 I_t(j)\right]. \tag{35}$$

The wage adjustment is sluggish because of quadratic costs paid in terms of the total wage bill,

$$AC_t^W = \frac{\kappa_W}{2}\left(\frac{w_t(j)}{w_{t-1}(j)} - 1\right)^2 w_t L_t, \tag{36}$$

where the parameter $\kappa_W > 0$ measures the degree of nominal wage rigidity and L_t is the total amount of labor in the Home economy.

$\Gamma_{B,t}$ represents the spread between the rate paid in the Home country and the offshore rate received by domestic investors

$$1 - \Gamma_{B,t} = \left(1 - \phi_{B1}\frac{exp(\phi_{B2}[S_t b_t^*/GDP_t - b_{FDES}^*]) - 1}{exp(\phi_{B2}[S_t b_t^*/GDP_t - b_{FDES}^*]) + 1} - Z_{B,t}\right)\frac{\beta_{t-1,t}^*}{\beta_{t-1,t}} \tag{37}$$

$0\geq$ ϕ_{B1} ≤ 1 ϕ_{B2} ≥ 0. $\Gamma_{B,t}$ guarantees that international net asset positions follow a stationary process and the economies converge asymptotically to a well-defined steady state. This transaction cost is a function of the average net asset position of the whole economy. b_{FDES} is the "desired" net asset position in the country, expressed as a ratio of GDP, intended as the degree of international exposure that financial intermediates consider appropriate for the country, based on their assessment of the global economic outlook.

FOCs Households' desidered consumption today decreases when expected consumption decreases (c_{t+1}^{-1}), when the expected gross real interest rate ($R_t^{\sigma}\pi_{t+1}^{-1}$) increases and when the expected exchange rate increases ($\triangle S_{t+1}$), according to the Euler equation:

Household maximizes utility, subject to the budget constraint xxx, with respect to consumption $C_t(j)$,...The implied FOCs are

$$\begin{aligned}
\lambda_t(1+\tau_{c,t}) &= C_t^{-1}(j), && (38)\\
\lambda_t &= \beta E_t\left[r_t \pi_{t+1}^{-1}\lambda_{t+1}\right], && (39)\\
\lambda_t &= \beta E_t\left[r_t^* \frac{S_{t+1}}{S_t}\pi_{t+1}^{-1}\lambda_{t+1}\right], && (40)\\
\lambda_t &= \beta E_t\left[r_t^{**} \frac{S_{t+1}^{RW}}{S_t^{RW}}\pi_{t+1}^{-1}\lambda_{t+1}\right], && (41)
\end{aligned}$$

with respect to government bonds $B_t(j)$

$$\lambda_t(1+\tau_{c,t}) = \beta E_t \frac{\lambda_{t+1}}{gr_{t+1}}(1+\tau_{c,t+1})(1+i_t)\pi_{t+1}^{-1}, \tag{42}$$

with respect to foreign government bonds $B_t^*(j)$

$$\lambda_t(1+\tau_{c,t}) = \beta E_t \frac{\lambda_{t+1}}{gr_{t+1}}(1+i_t^*)(1-\Gamma_{B,t})\frac{\Delta S_{t+1}}{\pi_{t+1}}, \tag{43}$$

with respect to the end-of-period capital $K_t(j)$

$$Q_t = \beta E_t \left[\lambda_{t+1}(1-\tau_{k,t+1})\frac{r_{t+1}^K}{gr_{t+1}} + Q_{t+1}\frac{(1-\delta)}{gr_{t+1}}\right], \tag{44}$$

with respect to investment $I_t(j)$

$$\begin{aligned}\lambda_t p_{i,t} = Q_t &\left[1 - \frac{\psi}{2}\left(\frac{I_t(j)}{I_{t-1}(j)}gr_t - 1\right)^2 - \psi\left(\frac{I_t(j)}{I_{t-1}(j)}gr_t^2 - gr_t\right)\frac{I_t(j)}{I_{t-1}(j)}\right] \\ &+\beta E_t Q_{t+1}\psi\left[\left(\frac{I_{t+1}(j)}{I_t(j)} - \frac{1}{gr_{t+1}}\right)gr_{t+1}^2\frac{I_{t+1}^2(j)}{I_t^2(j)}\right],\end{aligned} \tag{45}$$

with respect to real wages $w_t(j)$

$$\begin{aligned}\kappa\sigma_L \frac{w_t(j)^{-\sigma_L(1+\zeta)-1}}{w_t^{-\sigma_L(1+\zeta)}}L_t^{\zeta} + (1-\sigma_L)\frac{w_t(j)^{-\sigma_L}}{w_t^{-\sigma_L}} &= \\ \lambda_t\psi\frac{\psi}{2}\left(\frac{w_t(j)/w_{t-1}(j)}{\pi_{W,t-1}^{\alpha}\breve{\pi}^{1-\alpha}} - 1\right)\frac{w_t/w_{t-1}(j)}{\pi_{W,t-1}^{\alpha}\breve{\pi}^{1-\alpha}}& \\ -\beta\lambda_{t+1}\psi\left(\frac{w_{t+1}(j)/w_t(j)}{\pi_{W,t}^{\alpha}\breve{\pi}^{1-\alpha}} - 1\right)\frac{w_{t+1}w_{t+1}(j)/w_t(j)^2L_{t+1}}{\pi_{W,t}^{\alpha}\breve{\pi}^{1-\alpha}L_t}&,\end{aligned} \tag{46}$$

that in symmetric equilibrium becomes

$$w_t = \sigma_L\kappa L_t^{\varepsilon}DD^{-1} \tag{47}$$

where

$$DD = \lambda_t(\sigma_L - 1)(1 - \tau_{w,t}) + \psi\lambda_t \left(\frac{\pi^{\alpha}_{W,t}}{\pi^{\alpha}_{W,t-1}\breve{\pi}^{1-\alpha}} - 1 \right) \frac{\pi_{W,t}}{\pi^{\alpha}_{W,t-1}\breve{\pi}^{1-\alpha}}$$
$$-\psi\beta\lambda_{t+1} \left(\frac{\pi^{\alpha}_{W,t+1}}{\pi^{\alpha}_{W,t}\overline{\pi}^{1-\alpha}} - 1 \right) \frac{\pi^{2}_{W,t+1}L_{t+1}}{\pi^{\alpha}_{W,t}\breve{\pi}^{1-\alpha}L_t}^{-1} \tag{48}$$

1.3 Monetary policy

Monetary policy follows a standard interest rate rule. For the EA,

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho R} \left(\frac{\pi_{EA,t}}{\bar{\pi}_{EA}}\right)^{(1-\rho_R)\rho\pi} \left(\frac{GDP_{EA,t}}{GDP_{EA,t-1}}\right)^{(1-\rho_R)\rho_{GDF}} \tag{49}$$

where R_t is the gross monetary policy rate. The parameter ρ_R $(0 < \rho_R < 1)$ captures inertia in interest rate setting, while the parameter $\bar{R}$ represents the steady-state gross nominal policy rate. The parameters ρ_π and ρ_{GDP} are respectively the weights of EA consumer price index (CPI) inflation rate ($\pi_{EA,t}$) (taken as a deviation from its long-run constant target $\bar{\pi}_{EA}$), and GDP ($GDP_{EA,t}$).[3]

2 Advanced setup

2.1 Impatient Households

Impatient households discount the future more heavily than the patient ones. They choose consumption C_t, housing h_t, labor L_t to maximize

$$E_t \sum_{\tau=t}^{\infty} \beta^{'}_t \left(\log\left(C^{'}_t(j) - b_c \frac{C^{'}_{t-1}}{gr_t} \right) - \frac{\kappa}{1+\zeta} L^{'}_t(j)^{1+\zeta} + \gamma_t \log(h^{'}_t) \right), \tag{50}$$

where $\beta^{'}$is the discount factor, $0 < \beta^{'} < 1$, $\beta^{'} > \beta$, $b_c \in (0,1)$ is the external habit parameter, $\zeta > 0$ is the reciprocal of the Frisch elasticity of labor supply, where

$$L^{'}_t(j) = \left(\frac{w_t(j)}{w_t}\right)^{-\sigma_L} L^{'}_t. \tag{51}$$

[3]The CPI inflation rate is a geometric average of Home and REA CPI inflation rates (respectively Π_t and Π^*_t) with weights equal to the correspondent country GDP (as a share of the EA GDP). The EA GDP, $GDP_{EA,t}$, is the sum of Home and REA GDPs.

The budget constraint is

$$B_t^{'}(j) \leq (1+i_{t-1})\frac{B_{t-1}^{'}(j)}{\pi_t g r_t} + (1-\tau_{w,t})w_t(j)L_t^{'}(j)$$
$$-\frac{\psi}{2}\left(\frac{w_t(j)/w_{t-1}(j)}{\pi_{W,t}^{\alpha}\pi^{1-\alpha}} - 1\right)^2 w_t L_t^{'} + \Pi_t^{prof} - (1+\tau_{c,t})C_t^{'}(j) - q_t\left[h_t^{'} - (1-\delta)h_{t-1}^{'}\right]. \tag{52}$$

and the borrowing constraint

$$B_t^{'} \leq mE_t\left(\frac{q_{t+1}h_t^{'}\pi_{t+1}}{r_t}\right). \tag{53}$$

Like for entrepreneurs, this guarantees an equilibrium in which impatient households will hit the borrowing constraint. Here, the subscript under γ_t allows for random disturbances to the marginal utility of housing, and, given that it directly affects housing demand, offers a parsimonious way to assess the macro effects of an exogenous disturbance on house prices.

The borrowing constraint is consistent with standard lending criteria used in the mortgage market, which limit the amount lent to a fraction of the value of the asset. One can interpret the case m 0 as the limit situation when housing is not collateralizable at all, so that households are excluded from financial markets.

Like for the entrepreneurs, the equations for consumption and housing choice (shown in Appendix A) hold with the addition of the multiplier associated with the borrowing restriction.

2.2 Enterpreneurs

There exists a continuum of entrepreneurs e having mass $0 < \lambda_E < 1$ in the Home population. The generic entrepreneur e maximizes the intertemporal utility function

$$E_0 \sum_{t=0}^{\infty} \beta_E^t Z_{PR,t} \frac{\left(C_{e,t}\left(e\right) - bbC_{e,t-1}\right)^{1-\sigma}}{(1-\sigma)}, \tag{54}$$

where E_0 denotes the expectation conditional on information set at date 0, $C_{e,t}$ is consumption of (non-durable) goods, $0 < \beta_E < 1$ is the discount factor, $1/\sigma$ is the elasticity of intertemporal substitution ($\sigma > 0$). The parameter bb ($0 < bb < 1$) represents external habit formation in consumption. $Z_{PR,t}$ is a stationary consumption preference shock (common to all EA entrepreneurs and households).

Entrepreneurs borrow from domestic banks and issue corporate bonds that are sold to domestic savers and, when the CSPP is implemented, to the central bank of the monetary union.

The entrepreneur e obtains one-period (short-term) loans from banks subject to a collateral constraint à la Kiyotaki and Moore (1997),

$$- Loan_{e,t}(e) \leq m_e E_t \left\{ \frac{Q_{t+1} h_{e,t}(e)}{R_t^{LOANS,entr}} \right\}, \tag{55}$$

where $Loan_{e,t} < 0$ is the bank loan, $0 \leq m_e \leq 1$ is the loan-to-value ratio, Q_{t+1} is the real estate price, $h_{e,t}$ is the real estate (a durable good), and $R_t^{LOANS,entr}$ is the gross interest rate on loans.[4]

The entrepreneur also issues long-term corporate bonds B_{CORP}, modelled as a perpetuity paying an exponentially decaying coupon $\kappa_{CORP} \in (0, 1]$.[5] The budget constraint is

$$\begin{aligned} & P_{CORP,t} B_{CORP,t}(e) - R_{CORP,t} P_{CORP,t} \frac{B_{CORP,t-1}(e)}{\pi_t gr_t} \\ & + Loan_{e,t}(e) - Loan_{e,t-1}(e) R_{t-1}^{LOANS,entr} + R_t^K (1 - \tau_K) \frac{K_{t-1}(e)}{gr_t} - P_{I,t} I_t(e) \\ = \; & \Pi_t^K(e) + R_t^h \frac{h_{e,t-1}(e)}{gr_t} - P_{c,t}(1 + \tau_C) C_t(e) - Q_t \left(h_{e,t}(e) - \frac{h_{e,t-1}(e)}{gr_t} \right) - AC_{CORP,t}(e), \end{aligned} \tag{56}$$

where $R_{CORP,t}$ is the gross yield to maturity on corporate bonds,

$$R_{CORP,t} = \frac{1}{P_{CORP,t}} + \kappa_{CORP}, \tag{57}$$

and $P_{CORP,t}$ is the price of the corporate bond.[6].

R_t^h is the (net) return from renting real estate to domestic firms on a period-by-period basis,[7] $P_{c,t}$ is the consumption deflator, and Finally, $AC_{CORP,t}$ is an adjustment cost paid by the entrepreneurs when issuing corporate bonds.[8]

[4]As in Iacoviello (2005), it is assumed that entrepreneurs are more impatient than savers, i.e., their discount factor is relatively low. This guarantees that the borrowing constraint holds with equality in our deterministic simulations.

[5]See Woodford (2001).

[6]See the Technical Appendix of Chen et al. (2012) for details.

[7]We assume that entrepreneurs receive a housing endowment in every period (as saving and borrowing household do), but do not receive any utility service from it.

[8]We assume a standard quadratic form for the adjustment cost, that is,

$$AC_{CORP,t}(e) \equiv \frac{\phi_{b_E^C}}{2} \left(P_{CORP,t} B_{CORP,t}(e) - \bar{P}_{CORP} \bar{B}_{CORP} \right)^2, \text{ with } \phi_{b_E^C} > 0, \tag{58}$$

Each entrepreneur optimally chooses the end-of-period capital K_t and investment I_t subject to the law of capital accumulation, the adjustment costs on investment, and taking all prices as given. The law of motion of capital accumulation for the generic entrepreneur e is

$$K_t(e) = (1-\delta) K_{t-1}(e) + \left(1 - AC_t^I(e)\right) Z_{I,t} I_t(e), \tag{59}$$

where $0 < \delta < 1$ is the depreciation rate and $Z_{I,t}$ is a stationary investment-specific shock (common to all entrepreneurs in the EA). The adjustment cost on investment AC_t^I is

$$AC_t^I(e) \equiv \frac{\phi_I}{2}\left(\frac{I_t(e)}{I_{t-1}(e)} - 1\right)^2, \tag{60}$$

where $\phi_I > 0$ is a parameter. Investment is a final non-tradable good, composed of intermediate tradable (domestic and imported) goods. Entrepreneurs buy it in the corresponding market at the price $P_{I,t}$.[9] Entrepreneurs rent existing physical capital stock K_{t-1} in a perfectly competitive market at the nominal rate R_t^K to domestic firms producing intermediate goods.

Enterpreneurs produce an intermediate good Y_t using real estate h_t and labor L_t as inputs according to

$$Y_t = A h_{t-1}^{v} L_t^{1-v}, \tag{61}$$

where A is the technology parameter. Following Bernanke et al. (1999), we assume that output cannot be transformed immediately into consumption c_t. Retailers purchase the intermediate good from entrepreneurs at the wholesale price $P_{w,t}$ and transform it into a composite final good, whose price index is P_t. With this notation, $1/p_{w,t} = P_t/P_{w,t}$ denotes the markup of final over intermediate goods.

As in Kiyotaki and Moore 1997, we assume a limit on the obligations of the entrepreneurs. Suppose that, if borrowers repudiate their debt obligations, the lenders can repossess the borrowers' assets by paying a proportional transaction cost $(1-m)E_t(q_{t+1}h_t)$. In this case the maximum amount B_t that a creditor can borrow is bound by $mE_t(Q_{t+1}h_t/R_t)$. In real terms

$$b_t \leq mE_t\left(\frac{q_{t+1}h_t\pi_{t+1}}{R_t}\right). \tag{62}$$

where $\bar{P}_{CORP}\bar{B}_{CORP}$ is the steady-state value of the corporate bond in the symmetric steady state (we solve the model for a symmetric equilibrium, which implies a representative agent for each type of households, firms, and entrepreneurs).

[9]Because of the adjustment costs on investment, a "Tobin's Q" holds.

To make matters interesting, we want a steady state in which the entrepreneurial return to savings is greater than the interest rate, which implies a binding borrowing constraint. They maximize

$$E_0 \sum_{t=0}^{\infty} \gamma^t \log C_t, \tag{63}$$

where $\gamma < \beta$, because we assume that entrepreneurs discount the future more heavily than households. The reason is that we want that entrepreneurs will not postpone consumption and accumulate wealth so that the borrowing constraint becomes nonbinding. They are subject to the technology constraint, the borrowing constraint, and the flow of funds

$$\begin{aligned} P_{L,t}B_{L,t} - (1+\kappa P_{L,t})B_{L,t-1} + B_t + P_{w,t}Ah_{t-1}^{\upsilon}L_t^{1-\upsilon} = \\ B_{t-1}R_{t-1} - P_{c,t}C_t + W_tL_t + Q_t(h_t - h_{t-1}), \end{aligned} \tag{64}$$

The first-order conditions for an optimum are the consumption Euler equation, real estate demand, labor demand and long term bonds demand, in real terms

$$\frac{1}{c_t} = E_t\left(\frac{\gamma r_t}{\pi_{t+1}c_{t+1}gr_{t+1}}\right) + \lambda_t^{BC} r_t, \tag{65}$$

$$\frac{q_t}{c_t} = E_t\left(\frac{\gamma}{c_{t+1}}\left(\frac{\upsilon p_{w,t+1}y_{t+1}gr_{t+1}}{h_t} + \frac{q_{t+1}}{gr_{t+1}}\right) + \lambda_t^{BC} m q_{t+1}\pi_{t+1}\right), \tag{66}$$

$$w_t = -\frac{(1-\upsilon)p_{w,t}^{\upsilon}y_t}{L_t}. \tag{67}$$

$$\frac{p_{L,t}}{c_t} = E_t\left(\frac{\gamma(1+\kappa p_{L,t+1})}{c_{t+1}}\right). \tag{68}$$

The Lagrange multiplier on the borrowing constraint λ_t^{BC}, equals the increase in lifetime utility that would stem from borrowing R_t dollars, consuming or investing the proceeds, and reducing consumption by an appropriate amount the following period.

Without uncertainty, the assumption $\gamma < \beta$ guarantees that entrepreneurs are constrained in and around the steady state. In fact, the steady-state consumption Euler equation for the household implies, with zero inflation, that $R = 1/\beta$, the household time preference rate. Combining this result with the steady-state entrepreneurial Euler equation for consumption yields: $\lambda^{BC} = (\beta - \gamma)/c > 0$. Therefore, the borrowing constraint will hold with equality

$$B_t = mE_t\left(\frac{Q_{t+1}h_t\pi_{t+1}}{R_t}\right), \tag{69}$$

With uncertainty, the concavity of the objective function implies that, in some states of the world, entrepreneurs might "self-insure" by borrowing less than their credit limit so as to buffer their consumption against adverse shocks. Specifically, entrepreneurs might not hit the borrowing limit after a sufficiently long run of positive shocks. In this case, the model would become asymmetric around its stationary state. In bad times, entrepreneurs would be constrained; in good times, they might be unconstrained. In such a case, a linear approximation around the deterministic steady state might give misleading results. We take as given that uncertainty is "small enough" relative to degree of impatience so as to rule out this possibility.

2.3 Banks

There is a banking sector in both Home and REA economies, whose size is the same as that of the region.

Each banking sector is composed by intertwined wholesale and retail branches.

The wholesale branch acts under perfect competition. It maximizes profits by taking all interest rates as given and subject to a bank capital requirement. It (optimally) issues deposits and equities (i.e., bank capital) to domestic savers, buys domestic long-term sovereign bonds and makes resources available to the domestic bank retail sector. The latter makes loans to domestic households (borrowers) and, crucially, to domestic entrepreneurs.

The retail branch acts under monopolistic competition. It maximizes profits by optimally setting the interest rate on loans, taking as given (i) the interest rate paid on resources it gets from the wholesale banking sector and (ii) the demand for loans by entrepreneurs and households. It also faces adjustment costs when setting the interest rate.

In what follows we initially describe the main equations of the wholesale sector and, subsequently, those of the retail sector.

2.3.1 Banks - Wholesale sector

The optimal behavior of the wholesale banking sector is dictated by the combination of balance sheet (loans, long-term sovereign bonds, deposits), capital requirement, and no-arbitrage conditions among different asset and liabilities returns, implied by price-taking profit maximization problem (wholesale branch).

The balance sheet constraint is

$$LOANS_t^{entr} + LOANS_t^{bor} + P_{m,t} B_t^{long,bank} = D_t^{bank,d} + equity_t \tag{70}$$

where $LOANS^{entr}$ and $LOANS^{bor}$ are respectively loans to entrepreneurs and households, $B^{long,bank}$ the domestic long-term sovereign bonds (P_m is its price), $D^{bank,d}$ the deposits, *equity* is the (nominal) amount of resources obtained by issuing equities and exploitable for financing loans. We assume that the following relation holds,

$$equity_t = \left(K_t^{bank,d}\right)^{\alpha} \tag{71}$$

where $K^{bank,d}$ is the volume of issued equities and $0 < \alpha \leq 1$ is a parameter. The relation represents a technology function, and captures in a parsimonious way the process that allows to convert in loans the resources obtained by issuing equities.

The profits are equal to

$$\begin{aligned} & R_{wh,t}^{LOANS} LOANS_t^{wh} + R_t^{long} P_{m,t} B_t^{long,bank} - R_t^{DEP} D_t^{bank,d} - V_t K_t^{bank,d} \\ & - \frac{\phi_{LOAN}}{2} \left(LOANS_t - \overline{LOANS}\right)^2 - \frac{\phi_D}{2} \left(D_t^{bank,d} - \overline{D^{bank,d}}\right)^2 \\ & - \frac{\phi_{BK}}{2} \left(V_t K_t^{bank,d} - \kappa LOANS_t\right)^2, \end{aligned} \tag{72}$$

where R_{wh}^{LOANS} is (gross) interest rates on loans $LOANS_t^{wh}$ to the retail banking sector, R^{long} on sovereign bonds, R^{DEP} on deposits, V is the price of one equity. The branch pays quadratic adjustment costs on loans, deposits and on the deviations from the capital requirement $\kappa LOANS$, where $0 \leq k \leq 1$ is a parameter (ϕ_{LOAN}, ϕ_D, $\phi_{BK} > 0$ are parameters as well, $\overline{LOANS}$ and $\overline{D^{bank,d}}$ are the steady-state values of loans and deposits, respectively).

The wholesale sector maximizes profits on a period-by-period basis (static problem) with respect to sovereign bonds, deposit and equities, taking all prices and the balance sheet constraint as given. To save on space, we do not report the implied static no-arbitrage conditions.

2.3.2 Banks - Retail sector

There are two retail branches, one lends to entrepreneurs, the other to households (borrowers), in two different markets. Each branch acts under monpolistic competition. It sets the interest rate on loans to maximize profits, taking as given (i) the interest rate that pays to borrow from the wholesale branch, (ii) the entrepreneurs' and borrowers' demand for loans, and (iii) subject to quadratic adjustment costs on the loans' interest rate (this allows us to get a gradual adjustment of retail interest rates to a given shock).

The resulting first-order conditions imply that the interest rates on loans to entrepreneurs and households, $R^{LOANS,entr}_{retail,t}$ and $R^{LOANS,bor}_{retail,t}$ respectively, are given by a (time-varying) markup on the interest rate paid to the wholesale sector,

$$R^{LOANS,entr}_{retail,t} = mkp^{entr}_t R^{LOANS}_{wh,t} \tag{73}$$

$$R^{LOANS,bor}_{retail,t} = mkp^{bor}_t R^{LOANS}_{wh,t} \tag{74}$$

The implied profits are rebated to the savers, according to the owned amount of bank capital (equities).

FOCs First order conditions (FOC). The representative bank maximizes lifetime utility (x) subject to its budget constraint (xx) and the cost from deviating from the capital requirement (xxx) (given excess bank capital definition xxxx) with respect to dividends, deposit supply, loans supply and interbank position. Variables are expressed in "real"terms by dividing them by the consumption price deflator PC

The implied FOCs, with respect to deposits D_t

$$\begin{aligned} R^L_t = R^{DEP}_t + \phi_{LOAN}\left(L_t - \overline{L}\right) - \phi_{BK}(BK^{REQ}_t + E(BK^{REQ}_t))R^{LONG}_t BK^{GOV}_{BANK,t} \\ -(BK^{REQ}_t + E(BK^{REQ}_t))L_t \end{aligned} \tag{75}$$

with respect to Government capital $B^{GOV}_{BANK,t}$

$$\begin{aligned} R^{LOAN}_t = R^{LONG}_t + \phi_{LOAN} R^{LONG}_t \left(L_t - \overline{L}\right) \\ +\phi_{BK}\left(1 - (BK^{REQ}_t + E(BK^{REQ}_t)\right)(R^{LONG}_t)^2 BK^{GOV}_{BANK,t} \\ -\left(BK^{REQ}_t + E(BK^{REQ}_t)\right)L_t, \end{aligned} \tag{76}$$

with respect to $TLTRO_t$

$$\begin{aligned} R^{LOAN}_t = R^{TLTRO}_t + \phi_{LOAN}\left(L_t - \overline{L}\right) \\ -\phi_{BK}(BK^{REQ}_t + E(BK^{REQ}_t)R^{LONG}_t BK^{GOV}_{BANK,t} \\ -\left(BK^{REQ}_t + E(BK^{REQ}_t)\right)L_t + \phi_{TLTRO}(TLTRO_t - 0), \end{aligned} \tag{77}$$

with respect to long term sovereign bond B^L_t

$$
\begin{aligned}
P_{m,t}R_t^{LOAN} = {} & P_{m,t}R_t^{LONG} + R_t^{TLTRO} + \phi_{LOAN}P_{m,t}\left(L_t - \overline{L}\right) \\
& -\phi_{BK}(BK_t^{REQ} + E(BK_t^{REQ})P_{m,t}R_t^{LONG}BK_{BANK,t}^{GOV} \\
& - \left(BK_t^{REQ} + E(BK_t^{REQ})\right)L_t.
\end{aligned} \tag{78}
$$

2.4 Public capital and firms' decisions

The production function of the generic firm i in the Home intermediate tradable sector is

$$
Y_{T,t}(i) = K_{T,t}^{P}(i)^{\alpha_{1T}} L_{T,t}^{U}(i)^{\alpha_{2T}} L_{T,t}^{R}(i)^{\alpha_{3T}} \left(K_{G,t-1}\right)^{1-\alpha_{1T}-\alpha_{2T}-\alpha_{3T}},
$$

where $K_{T,t}^{P}(i)$ is private capital, which is supplied by the domestic capital producers, $L_{T,t}^{R}(i)$ and $L_{T,t}^{U}(i)$ represent labor supplied by, respectively, domestic restricted and unrestricted households, $K_{G,t-1}$ is public capital, accumulated by the domestic public sector. The parameters $0 < \alpha_{1T} < 1$ $(i = 1, 2, 3)$, $\alpha_{1T}+\alpha_{2T}+\alpha_{3T} < 1$, are the weights on private capital, unrestricted households' labor, and restricted households labor, respectively.

The firm optimally chooses demand for private capital and labor taking prices and the amount of public capital as given. Thus, firms do not demand public capital and there is no price or tariff paid for its use.

A similar production function holds for the generic firm i producing the intermediate non-tradable good:

$$
Y_{N,t}(i) = K_{N,t}^{P}(i)^{\alpha_{1N}} L_{N,t}^{U}(i)^{\alpha_{2N}} L_{N,t}^{R}(i)^{\alpha_{3N}} \left(K_{G,t-1}\right)^{1-\alpha_{1N}-\alpha_{2N}-\alpha_{3N}}.
$$

For public capital projects, we follow Leeper et al. (2010) and in some simulations assume "time-to-build" Kydland and Prescott 1982: there is a delay between the authorization of a government spending plan and the completion of an investment project. The possibility of several periods of time-to-build in public capital implies that the government initiates investment projects that take N periods until they become productive and augment the public capital stock. Thus, the public capital is accumulated by the public sector according to

$$
K_{G,t-1} = (1-\delta_G)\,K_{G,t-2} + A_{I_G,t-1-N},
$$

where $0 < \delta_G < 1$ is the depreciation rate, and $A_{I_G,t-1-N}$, with $N \geq 1$, is authorized government investment in period $t-1-N$. The time-to-build lags capture the idea that it takes time before a public investment is finished and, hence, can be effectively included in the public capital stock and affect the supply side of the economy. A "classic" example is the government that authorizes funding at time (quarter) $t-8$ for a highway that takes two years to build ($N=8$). Then the highway cannot be considered as a part of the stock of public capital until quarter t ($K_{G,t-1}$ is used to produce goods in period t).

To capture the idea that spending outlays typically occur over time, we introduce the sequence $\{b_0, b_1, b_2, ..., b_{N-1}\}$ of the spending rates from the date the funding is authorized (date $t-8$) to the period before project completion (date $N-1$). For example, the highway may not be usable for two years but government investment increases during this time as construction of the highway takes place. Therefore, government investment actually implemented at time t is then given by

$$I_{G,t} = \sum_{n=0}^{N-1} b_n A_{I_G,t-n}, \tag{79}$$

$$\sum_{n=0}^{N-1} b_n = 1. \tag{80}$$

where the rate at which the construction takes place is parameterized by the b's. In the case of a one-period time-to-build technology (as assumed for private investment), public investment outlaid in period t becomes productive in period $t+1$, i.e. $N=1$ and $I_{G,t} = A_{I_G,t}$.

2.5 Fiscal sector

Fiscal policy is set at the regional level. The government budget constraint is

$$B^S_{G,t} - B^S_{G,t-1}R_{t-1} + P_{L,t}B^L_{G,t} - \sum_{s=1}^{\infty} \kappa^{s-1} B^L_{G,t-s} \leq P_{N,t}C_{G,t} + P_{I_G,t}I_{G,t} + TR_t - T_t, \tag{81}$$

where $B^S_{G,t}$, $B^L_{G,t}$ are short-term and long-term nominal sovereign bonds, respectively ($B^S_{G,t}, B^L_{G,t} > 0$ is public debt). The short-term bond is a one-period nominal bond issued in the domestic bond market that pays the (gross) monetary policy interest rate R_t. The implied gross yield to maturity at time t on the long-term bond is defined as

$$R^L_t = \frac{1}{P^L_t} + \kappa. \tag{82}$$

The variable $C_{G,t}$ represents government purchases of goods and services, $Tr_t > 0$ (< 0) are lump-sum transfers (lump-sum taxes) to households. Consistent with the empirical evidence, $C_{G,t}$ is fully biased towards the intermediate non-tradable good. Therefore, it is multiplied by the corresponding price index $P_{N,t}$.[10] Given

[10] See Mueller and Corsetti (2007).

that the public investment has its own composition, it is pre-multiplied by the public investment price deflator $P_{I_G,t}$. The investment in public capital $I_{G,t}$ is assumed, in line with empirical evidence, to have a composition that is more biased towards domestic goods. Thus, we assume that it has the same composition as the private consumption good. The same tax rates apply to every domestic household and capital producer (the latter pays the tax rate $0 \leq \tau_t^k \leq 1$ on return R_t^k on capital K_{t-1}). Total government revenues T_t from distortionary taxation are given by the identity

$$\begin{aligned} T_t \equiv & \int_0^{n\lambda_R} \tau_t^\ell W_t(j') L_t(j') \, dj' \\ & + \int_{n\lambda_R}^{n} \tau_t^\ell W_t(j) L_t(j) \, dj \\ & + \int_0^{n} \tau_t^k R_t^k K_{t-1}(e) \, de \\ & + \int_0^{n\lambda_R} \tau_t^c P_t C_t(j') \, dj' \\ & + \int_{n\lambda_R}^{n} \tau_t^c P_t C_t(j) \, dj. \end{aligned} \tag{83}$$

The government follows a fiscal rule defined on lump-sum transfers to bring the short-term public debt as a percentage of domestic GDP, $b_G^s > 0$, in line with its long-run (steady-state) target $\bar{b}_G^s$.[11] The rule is

$$\frac{TR_t}{TR_{t-1}} = \left(\frac{b_{G,t}^s}{\bar{b}_G^s} \right)^{-\phi_1}, \tag{85}$$

where the parameter $-\phi_1$ is lower than zero ($\phi_1 > 0$), calling for a reduction (increase) in lump-sum transfer whenever the current-period short-term public debt (as a ratio to GDP) is above (below) the target. We choose lump-sum transfers to stabilize public finance as they are non-distortionary and, thus, allow a "clean" evaluation of the macroeconomic effects of public investment.

For long-term public debt, it is assumed for simplicity that its rate of change is the same as that of the short-term public debt, so that the maturity composition of the overall public debt does not change.

[11] The definition of nominal GDP is

$$GDP_t = P_t C_t + P_t^I I_t + P_t^{I_G} I_{G,t} + P_{N,t} C_{G,t} + P_t^{EXP} EXP_t - P_t^{IMP} IMP_t, \tag{84}$$

where P_t, P_t^I, $P_t^{I_G}$, $P_{N,t}$, P_t^{EXP}, P_t^{IMP} are prices of private consumption, private investment, public investment, public consumption (given the assumption of fully biased composition towards intermediate non-tradable goods), exports, and imports, respectively.

Fiscal items other than (i) public deficit, (ii) public debt, (iii) lump-sum transfers, and (iv) public investment are kept at their corresponding initial (steady-state) levels when simulating the model.

3 Calibration

Tbw

4 Simulations

We now consider a number of simulations to illustrate the response of the model.[12] The first application shows how the model react to a negative monetary policy shock. In the second simulation we impose a positive consumption preference shock.

4.1 Monetary policy shock

We simulate a 0.25 (annualized) percentage point reduction in the EA monetary policy rate. Figure 4 reports the responses of the interest rates. The policy rate, after the initial drop, gradually returns to the baseline, as dictated by the Taylor rule, (eq. 49). Consistent with the lower policy rate, houseolds increase their bank deposits (Fig.5) and banks decrease interest rates on deposits, but to a lower extent than that of the policy rate, such that deposits are still more appealing than government bonds for households. Bank interest rates on loand decrease, because bank loans augment in both regions (Fig.5), consistent with the increase in the value of collateralizable real estate (Fig.7). Overall capital-to-loan ratio has a slightly negative value because banks find more profitable to finance loans by raising deposits than using their capital. Lower interest rates and higher loans favor an increase in consumption and physical capital (Fig.6). As demand is higher, imports increase. Home exports benefit from the increase in REA aggregate demand. REA exports increase to a lower extent than Home exports do, because they are more oriented towards the RW, whose aggregate demand is not affected by the interest rate shock (exchange rate depreciation is mild with respect to the RW currency). Overall, after four periods GDP increases up to 0.2% and inflation to 0.1%. The effects are similar in both countries. For what concerns the real estate market (Fig.7), the positive pattern of investments and labor market favor a persistent increase in real estate demand by entrepreneurs and constrained households. Given that in each region the overall (economy-wide) stock of real estate is constant, the higher demand for real estate from the borrowers and entrepreneurs is satisfied by the lower demand by savers (the latter sell the real estate to the former). Consistent with the increase in demand, the price of real estate increases.

[12] Model in the complete version (see Section2). It includes banks, savers, impatient households, entrepreneurs. It does not incluse public capital.

Given the higher aggregate demand, in both tradables and non tradables sectors (Fig.8), real wages increase.

4.2 Preference shock

Figures 6-10 show the effects of a temporary preference shock applied to households in Home and REA (persistence set to 0.90). The shock drives up output in both regions by around 0.5 percent on impact. GDPs get to their peaks by more than 1 percent after about 10 periods. Inflation increases by 0.2 percent on impact and then slowly increases to 0.4 percent in both countries. The short-term interest rate slowly increases (Fig.9) following the pattern of inflation. Households increase their consumption and reduce their bank deposits. Interest rates on deposits decrease on impact in both regions, in particular in Home region, then they slowly increase. As investment and real estate demand decrease, loans demand decreases too, and interest rates on loans decrease. Bank capital-to-loan ratios increase, because banks have to finance their debts using their capital as loans demand decreases (Fig.10). Qualitatively, the shock has rather similar effects on both Home and rest of the euro area regions (Fig. 11). There is a positive impact on imports to satisfy the higher domestic demand. REA exports decrease due to the negative effect of the appreciation of the exchange rate, HOME exports slightly increase because the large effect of REA demand. In both countries borrowers sell their real estate holdings, (Fig. 12), entrepreneurs are not directly affected by the shock, so on impact they slightly decrease their holdings of reals estate, then they gradually increase it again. As a consequence the price of real estate decreases on impact, then slowly increases. Labour demand augments and the production of tradables and nontradables follows the same path. Real wages decrease as effect of the higher inflation (Fig. 13).

4.3 Loan-to-value ratios shock

Figure 14 shows the implications of an exogenous rise in the loan-to-value (LTV) ratios of restricted households and entrepreneurs in REA. This shock represents a change in lending standards applied by banks to their customers, measured as the ratio of the loan given the value of the collateral available (real estate value). The LTV ratios of Home and REA increase by 10% in the first period and then gradually go back to their steady-state level (0.6 for entrepreneurs and 0.45 for restricted households). The shock mainly reflects on an increase in the demand of loans by borrowers and entrepreneurs, as it is encoded in the collateral constraint. Short-term interest rate slightly increases on impact, then it gradually decreases. Demand of loans increase by 30 percent on impact (Fig. 15), then decreases, and interest rates on loans to entrepreneurs and borrowers increase too by about 1 percent. The situation is symmetric in both countries. GDP increases in both countries by about 0.5 percent, following the path of consumption and investment (Fig. 16). Imports increase, driven by domestic demand. Export increase less, expecially in REA, because of the unfavorable exchange rate with RW region. Inflation decreases to the through of -0.1 percent, after three periods it gradually returns to the baseline level in both countries. Real estate demand increases and on impact their prices grow by more than 1 percent, then gradually decreases (Fig.17). Borrowers and entrepreneurs buy real estates from savers in both countries, using bank loans. Savers invest the revenues in bank deposits, that increase by about 10 percent on impact. Banks increase the demand for deposits to finance the higher amounts of loans and as a consequence interest rates on deposits increase. The labor market improves on impact and then goes back to the baseline level, also wages increase (Fig. 16). Figure 17) shows that the demand of real estate by entrepreneurs and borrowers increases, driving up prices.

4.4 Government spending shock

We simulate an increase in public spending by 1 percent of GDP in home and REA in the first period (persistence 0.9).

Figure 19) shows the response of the short-term interest rate. It is slightly positive and reaches a peak of 0.2 percent after eight periods, then gradually returns to the base level. Demand of loans (especially by borrowers. who buy more real estate, Fig.20)) increases in both countries and interest rates on loans increase too. Bank deposits gradually decrease. Banks raise interest rates on deposit to make them more appealing. Bank capital-to-loan ratio decreases, because banks have to finance their debts using loans. GDP increases on impact by 1 percent, then gradually goes back to the baseline level, following the path of government spending. Inflation increases by more than 0.2 percent at its peak. Consumption is stable at the baseline llevel (Fig. 21), while investment increases. Net trade is negative in both countries because of the appreciation of the exchange rate. Real estate demand gradually increases in both countries, by 1.5 percent from borrowers and by 0.5 percent from entrepreneurs (Fig. 22). Savers demand has a negative path because they sell real estates. The peak in sell is after 10 periods, in correspondence of the lowest real estate prices (-0.3p.p.). Labor demand increases by 2 percent on impact, then slowly decrease to the baseline level (Fig.23). The effect is stronger in the sector of nontradable goods (increase by 2p.p. in Home abd 3 p.p. in REA) than in the tradable one (increase by 1p.p. in both countries). As a consequence real wages increase and reach about 0.1 p.p. after 5 periods.

Appendix: List of Equations basic model

- Household
  - Budget constraint expressed in real terms

$$B_t(j) + S_t B_t^*(j) \leq (1+i_{t-1})\frac{B_{t-1}(j)}{\pi_t gr_t} + (1+i_{t-1}^*)\left[1-\Gamma_{B,t-1}\right]\frac{\Delta S_t B_{t-1}^*(j)}{\pi_t gr_t}$$
$$+(1-\tau_{k,t})r_t^k\frac{K_{t-1}(j)}{gr_t} + (1-\tau_{w,t})w_t(j)L_t(j)$$
$$-\frac{\psi}{2}\left(\frac{w_t(j)/w_{t-1}(j)}{\pi_{W,t}^{\alpha}\pi_t^{1-\alpha}} - 1\right)^2 w_t L_t + \Pi_t^{prof} - (1+\tau_{c,t})C_t(j) - p_{I,t}I_t(j), \tag{A.1}$$

  - FOC wrt consumption

$$\lambda_t\left(1+\tau_t^c\right) = C_t^{-1} \tag{A.2}$$

  - FOC wrt domestic bond

$$\lambda_t = \beta E_t\left[(1+i_t)\pi_{t+1}^{-1}\lambda_{t+1}\right] \tag{A.3}$$

  - FOC wrt foreign bond

$$\lambda_t = \beta E_t\left[(1+i_t^*)\left[1-\Gamma_{B,t}\right]\frac{S_{t+1}}{S_t}\pi_{t+1}^{-1}\lambda_{t+1}\right] \tag{A.4}$$

  - FOC wrt physical capital

$$Q_t = \beta E_t\left[\lambda_{t+1}(1-\tau_{k,t+1})\frac{r_{t+1}^K}{gr_{t+1}} + Q_{t+1}\frac{(1-\delta)}{gr_{t+1}}\right], \tag{A.5}$$

  - FOC wrt investment

$$\lambda_t p_{I,t} = Q_t\left[1-\frac{\psi}{2}\left(\frac{I_t}{I_{t+1}}gr_t - gr\right)^2 - \psi\left(\frac{I_t}{I_{t+1}}gr_t^2 - \frac{gr}{gr_t}\right)\frac{I_t}{I_{t+1}}\right] +$$
$$\beta E_t Q_{t+1}\psi\left[\left(\frac{I_t}{I_{t+1}} - \frac{gr}{gr_{t+1}}\right)gr_{t+1}^2\frac{I_t^2}{I_{t+1}^2}\right], \tag{A.6}$$

  - Law of capital accumulation

$$K_t(j) = (1-\delta)K_{t-1}(j)gr_t^{-1} + \left[1-\frac{\psi}{2}\left(\frac{I_t}{I_{t+1}}gr_t - gr\right)^2 I_t\right]. \tag{A.7}$$

- Firms (final sector)
  - Final private consumption good production function

$$A_t(x) \equiv \left(a_T^{\frac{1}{\phi_A}} \left(\begin{array}{c} a_H^{\frac{1}{\rho_A}} Q_{HA,t}(x)^{\frac{\rho_A-1}{\rho_A}} + a_G^{\frac{1}{\rho_A}} Q_{GA,t}(x)^{\frac{\rho_A-1}{\rho_A}} \\ +(1-a_H-a_G)^{\frac{1}{\rho_A}} Q_{FA,t}(x)^{\frac{\rho_A-1}{\rho_A}} \end{array} \right)^{\frac{\rho_A}{\rho_A-1}\frac{\phi_A-1}{\phi_A}} \begin{array}{c} \\ \\ +(1-a_T)^{\frac{1}{\phi_A}} Q_{NA,t}(x)^{\frac{\phi_A-1}{\phi_A}} \end{array} \right)^{\frac{\phi_A}{\phi_A-1}}, \tag{A.8}$$

  - Final investment good production function

$$E_t(y) \equiv \left(\upsilon_T^{\frac{1}{\phi_E}} \left(\begin{array}{c} \upsilon_H^{\frac{1}{\rho_E}} Q_{HE,t}(y)^{\frac{\rho_E-1}{\rho_E}} + \upsilon_G^{\frac{1}{\rho_E}} Q_{GE,t}(y)^{\frac{\rho_E-1}{\rho_E}} \\ +(1-\upsilon_H-\upsilon_G)^{\frac{1}{\rho_E}} Q_{FE,t}(y)^{\frac{\rho_E-1}{\rho_E}} \end{array} \right)^{\frac{\rho_E}{\rho_E-1}\frac{\phi_E-1}{\phi_E}} \begin{array}{c} \\ \\ +(1-\upsilon_T)^{\frac{1}{\phi_E}} Q_{NE,t}(y)^{\frac{\phi_E-1}{\phi_E}} \end{array} \right)^{\frac{\phi_E}{\phi_E-1}}, \tag{A.9}$$

  - Bundle of final public good

$$Q_{NA,t}(x) \equiv \left[\left(\frac{1}{n}\right)^{\theta_N} \int_0^n Q_t(i,x)^{\frac{\theta_N-1}{\theta_N}} \, di \right]^{\frac{\theta_N}{\theta_N-1}}, \tag{A.10}$$

  - Bundles of tradables produced in Home

$$Q_{HA,t}(x) = \left[\left(\frac{1}{n}\right)^{\theta_T} \int_0^n Q_t(h,x)^{\frac{\theta_T-1}{\theta_T}} \, dh \right]^{\frac{\theta_T}{\theta_T-1}}, \tag{A.11}$$

  - Bundles of tradables produced in REA

$$Q_{GA,t}(x) = \left[\left(\frac{1}{n^*}\right)^{\theta_T} \int_n^{n+n^*} Q_t(g,x)^{\frac{\theta_T-1}{\theta_T}} \, dg \right]^{\frac{\theta_T}{\theta_T-1}}, \tag{A.12}$$

  - Bundles of tradables produced in RW

$$Q_{FA,t}(x) = \left[\left(\frac{1}{1-n-n^*}\right)^{\theta_T} \int_{n+n^*}^1 Q_t(f,x)^{\frac{\theta_T-1}{\theta_T}} \, df \right]^{\frac{\theta_T}{\theta_T-1}}, \tag{A.13}$$

– Bundles of nontradables produced in Home

$$Q_{NA,t}(x) = \left[\left(\frac{1}{n}\right)^{\theta_N} \int_0^n Q_t(i,x)^{\frac{\theta_N - 1}{\theta_N}} di \right]^{\frac{\theta_N}{\theta_N - 1}}, \tag{A.14}$$

– Demand for intermediate nontradable input i

$$Q_{A,t}(i,x) = \left(\frac{1}{n}\right) \left(\frac{P_t(i)}{P_{N,t}}\right)^{-\theta_N} Q_{NA,t}(x), \tag{A.15}$$

– Cost-minimizing price of one basket of local nontradable intermediates $P_{N,t}$

$$P_{N,t} = \left[\int_0^n P_t(i)^{1-\theta_N} di \right]^{\frac{1}{1-\theta_N}}, \tag{A.16}$$

– Total demand for intermediate nontradable good i

$$\int_0^n Q_{A,t}(i,x)\, dx + \int_0^n Q_{E,t}(i,y)\, dy + \int_0^n C^g_{N,t}(i,x)\, dx = \left(\frac{P_t(i)}{P_{N,t}}\right)^{-\theta_N} \left(Q_{NA,t} + Q_{NE,t} + C^g_{N,t}\right). \tag{A.17}$$

- Firms (intermediate tradable sector)

  – Production function

$$T^S_t(i) = \left((1-\alpha_T)^{\frac{1}{\xi_T}} L_{T,t}(i)^{\frac{\xi_T - 1}{\xi_T}} + \alpha_T^{\frac{1}{\xi_T}} K_{T,t}(i)^{\frac{\xi_T - 1}{\xi_T}} \right)^{\frac{\xi_N}{\xi_N - 1}}, \tag{A.18}$$

  – Labor demand

$$L_{T,t}(i) = (1-\alpha_T) \left(\frac{W_t}{MC_{T,t}(i)}\right)^{-\xi_T} T^S_t(i), \tag{A.19}$$

  – Capital demand

$$K_{T,t}(i) = \alpha_T \left(\frac{R^K_t}{MC_{T,t}(i)}\right)^{-\xi_T} T^S_t(i), \tag{A.20}$$

  – Nominal marginal cost

$$MC_{T,t}(i) = \left((1-\alpha_T) W_t^{1-\xi_T} + \alpha_T \left(R^K_t\right)^{1-\xi_T} \right)^{\frac{1}{1-\xi_T}}, \tag{A.21}$$

– FOC wrt $\bar{P}_t(h)$

$$
\begin{aligned}
0 = &\left(\frac{\bar{P}_{H,\tau}(h) + \eta P_{N,t}}{P_{H,\tau}}\right)^{-\theta_H} \\
&-\theta_H \frac{(\bar{P}^*_{H,\tau}(h) + \eta P_{N,t})^{-\theta_H - 1}}{(P^*_{H,\tau})^{-\theta_H}} \left(\bar{P}^*_{H,\tau}(h) - MC_{\tau,t}(h)\right) \\
&-\kappa^p_H \frac{P_{H,t}/P_{H,t-1}(h)}{\pi^{\alpha_H}_{H,t-1}\bar{\pi}^{1-\alpha_H}} \left(\frac{P_{H,t}(h)/P_{H,t-1}(h)}{\pi^{\alpha_H}_{H,t-1}\bar{\pi}^{1-\alpha_H}} - 1\right) \\
&-\beta\kappa^p_H \frac{\lambda_{t+1}}{\lambda_t} \frac{P_{H,t+1}(h) P_{H,t+1}/P_{H,t}(h)^2}{\pi^{\alpha_H}_{H,t}\bar{\pi}^{1-\alpha_H}\pi_{t+1}} \left(\frac{P_{H,t+1}(h)/P_{H,t}(h)}{\pi^{\alpha_H}_{H,t}\bar{\pi}^{1-\alpha_H}} - 1\right) \frac{Q_{H,t+1}}{Q_{H,t}},
\end{aligned}
\tag{A.22}
$$

– FOC wrt $\bar{P}^*_t(h)$

$$
\begin{aligned}
0 = &\left(\frac{\bar{P}^*_{H,\tau}(h) + \eta P_{N,t}}{P^*_{H,\tau}}\right)^{-\theta^*_H} \\
&-\theta^*_H \frac{(\bar{P}^*_{H,\tau}(h) + \eta P_{N,t})^{-\theta^*_H - 1}}{(P^*_{H,\tau})^{-\theta^*_H}} \left(\bar{P}^*_{H,\tau}(h) - \frac{MC_{\tau,t}(h)}{S^*_\tau}\right) \\
&-\kappa^{p\,*}_H \frac{\bar{P}^*_{H,t}/\bar{P}^*_{H,t-1}(h)}{(\pi^*_{H,t-1})^{\alpha^*_H}(\bar{\pi}^*)^{1-\alpha^*_H}} \left(\frac{\bar{P}^*_{H,t}(h)/\bar{P}^*_{H,t-1}(h)}{(\pi^*_{H,t-1})^{\alpha^*_H}(\bar{\pi}^*)^{1-\alpha^*_H}} - 1\right) \\
&-\beta\kappa^{p\,*}_H \frac{\lambda_{t+1}}{\lambda_t} \frac{\bar{P}^*_{H,t+1}(h)\bar{P}^*_{H,t+1}/(\bar{P}^*_{H,t}(h))^2}{(\pi^*_{H,t})^{\alpha^*_H}(\bar{\pi}^*)^{1-\alpha^*_H}\pi_{t+1}} \left(\frac{\bar{P}^*_{H,t+1}(h)/\bar{P}^*_{H,t}(h)}{(\pi^*_{H,t})^{\alpha^*_H}(\bar{\pi}^*)^{1-\alpha^*_H}} - 1\right) \frac{Q^*_{t+1}}{Q^*_t}\frac{S^*_{t+1}}{S^*_t}.
\end{aligned}
\tag{A.23}
$$

- Firms (intermediate non-tradable sector)

  – Production function

$$
N^S_t(i) = \left((1-\alpha_N)^{\frac{1}{\xi_N}} L_{N,t}(i)^{\frac{\xi_N - 1}{\xi_N}} + \alpha_N^{\frac{1}{\xi_N}} K_{N,t}(i)^{\frac{\xi_N - 1}{\xi_N}}\right)^{\frac{\xi_N}{\xi_N - 1}}, \tag{A.24}
$$

  – Labor demand

$$
L_{N,t}(i) = (1-\alpha_N)\left(\frac{W_t}{MC_{N,t}(i)}\right)^{-\xi_N} N^S_t(i), \tag{A.25}
$$

  – Capital demand

$$
K_{N,t}(i) = \alpha_N \left(\frac{R^K_t}{MC_{N,t}(i)}\right)^{-\xi_N} N^S_t(i), \tag{A.26}
$$

– Nominal marginal cost

$$MC_{N,t}(i) = \left((1-\alpha_N) W_t^{1-\xi_N} + \alpha_N \left(R_t^K\right)^{1-\xi_N}\right)^{\frac{1}{1-\xi_N}}, \tag{A.27}$$

– FOC wrt $\bar{P}_{N,t}(i)$

$$\begin{aligned}0 = (1-\theta_N)\frac{P_{N,t}(i)^{-\theta_N}}{P_{N,t}{}^{-\theta_N}} - \theta_N \frac{P_{N,t}(i)^{-\theta_N-1}}{P_{N,t}{}^{-\theta_N}} MC_{N,t}(i) \\ -\kappa_N^p \frac{P_{N,t}/P_{N,t-1}(i)}{\pi_{N,t-1}^{ind_N}\bar{\pi}^{1-ind_N}} \left(\frac{P_{N,t}(i)/P_{N,t-1}(i)}{\pi_{N,t-1}^{ind_N}\bar{\pi}^{1-ind_N}} - 1\right) \\ -\beta\kappa_N^p \frac{\lambda_{t+1}}{\lambda_t} \frac{P_{N,t+1}(i)\, P_{N,t+1}/P_{N,t}(i)^2}{\pi_{N,t}^{ind_N}\bar{\pi}^{1-ind_N}} \left(\frac{P_{N,t+1}(i)/P_{N,t}(i)}{\pi_{N,t}^{ind_N}\bar{\pi}^{1-ind_N}} - 1\right) \frac{Q_{N,t+1}}{Q_{N,t}}.\end{aligned} \tag{A.28}$$

- Monetary policy
  – Taylor rule

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho R} \left(\frac{\pi_{EA,t}}{\bar{\pi}_{EA}}\right)^{(1-\rho R)\rho\pi} \left(\frac{GDP_{EA,t}}{GDP_{EA,t-1}}\right)^{(1-\rho R)\rho GDP}. \tag{A.29}$$

- Fiscal policy
  – Fiscal rule

$$\frac{TAX_t}{TAX_{t-1}} = \left(\frac{b_{g,t}^S}{\bar{b}_g^S}\right)^{\phi_1} \left(\frac{b_{g,t}^S}{b_{g,t-1}^S}\right)^{\phi_2}. \tag{A.30}$$

- Impatient households
  – Utility function

$$E_t \sum_{\tau=t}^{\infty} \beta_t' \left(\log\left(C_t'(j) - b_c \frac{C_{t-1}'}{gr_t}\right) - \frac{\kappa}{1+\zeta} L_t'(j)^{1+\zeta} + \gamma_t \log(h_t')\right), \tag{A.31}$$

  – Budget constraint

$$\begin{aligned}B_t'(j) \leq (1+i_{t-1})\frac{B_{t-1}'(j)}{\pi_t gr_t} + (1-\tau_{w,t}) w_t(j) L_t'(j) \\ -\frac{\psi}{2}\left(\frac{w_t(j)/w_{t-1}(j)}{\pi_{W,t}^{\alpha}\pi^{1-\alpha}} - 1\right)^2 w_t L_t' + \Pi_t^{prof} - (1+\tau_{c,t})C_t'(j) - q_t\left[h_t' - (1-\delta)h_{t-1}'\right],\end{aligned} \tag{A.32}$$

– FOC wrt consumption

$$\frac{1}{C_t'(1+\tau_{c,t})} = E_t\left(\frac{\beta'(1+i_t)r_t}{\pi_{t+1}C_{t+1}'gr_{t+1}(1+\tau_{c,t+1})}\right) + \lambda_t^{BC} r_t, \tag{A.33}$$

– FOC wrt real estate demand

$$\frac{q_t}{C_t'(1+\tau_{c,t})} = \frac{\gamma_t}{h_t} + E_t q_{t+1}(\lambda_t'^{BC} m\pi_{t+1} - \beta' \frac{(1-\delta)}{C_{t+1}'(1+\tau_{c,t+1})}, \tag{A.34}$$

– FOC wrt labor demand

$$\kappa l_t^{\zeta}\left(\frac{w_t(j)}{w_{t-1}(j)}\right)^{-\sigma_L(1+\zeta)} =$$

$$\frac{1}{C_t'(1+\tau_{c,t})}\left((1-\tau_{w,t})\frac{w_t^{(1-\sigma_L)}(j)}{w_{t-1}^{-\sigma_L}(j)} - \frac{\psi}{2}\left(\frac{w_t(j)/w_{t-1}(j)}{\pi_{W,t}^{\alpha}\pi^{1-\alpha}} - 1\right)^2 w_t\right). \tag{A.35}$$

- Banks - Wholesale sector

– Balance sheet constraint

$$LOANS_t^{entr} + LOANS_t^{bor} + P_{m,t}B_t^{long,bank} = D_t^{bank,d} + equity_t, \tag{A.36}$$

– Technology function

$$equity_t = \left(K_t^{bank,d}\right)^{\alpha}, \tag{A.37}$$

– Profits

$$R_{wh,t}^{LOANS} LOANS_t^{wh} + R_t^{long} P_{m,t} B_t^{long,bank} - R_t^{DEP} D_t^{bank,d} - V_t K_t^{bank,d}$$
$$-\frac{\phi_{LOAN}}{2}\left(LOANS_t - \overline{LOANS}\right)^2 - \frac{\phi_D}{2}\left(D_t^{bank,d} - \overline{D^{bank,d}}\right)^2$$
$$-\frac{\phi_{BK}}{2}\left(V_t K_t^{bank,d} - \kappa LOANS_t\right)^2. \tag{A.38}$$

- Banks - Retail sector

– FOC wrt

$$R_{retail,t}^{LOANS,entr} = mkp_t^{entr} R_{wh,t}^{LOANS} \tag{A.39}$$

– FOC wrt

$$R^{LOANS,bor}_{retail,t} = mkp^{bor}_t R^{LOANS}_{wh,t} \tag{A.40}$$

– FOC wrt deposits D_t

$$\begin{aligned} R^L_t = R^{DEP}_t + \phi_{LOAN}\left(L_t - \overline{L}\right) - \phi_{BK}(BK^{REQ}_t + E(BK^{REQ}_t))R^{LONG}_t BK^{GOV}_{BANK,t} \\ -(BK^{REQ}_t + E(BK^{REQ}_t))L_t \end{aligned} \tag{A.41}$$

– FOC wrt Government capital $B^{GOV}_{BANK,t}$

$$\begin{aligned} R^{LOAN}_t = R^{LONG}_t + \phi_{LOAN} R^{LONG}_t \left(L_t - \overline{L}\right) \\ +\phi_{BK}\left(1 - (BK^{REQ}_t + E(BK^{REQ}_t)\right)(R^{LONG}_t)^2 BK^{GOV}_{BANK,t} \\ -\left(BK^{REQ}_t + E(BK^{REQ}_t)\right)L_t, \end{aligned} \tag{A.42}$$

– FOC wrt $TLTRO_t$

$$\begin{aligned} R^{LOAN}_t = R^{TLTRO}_t + \phi_{LOAN}\left(L_t - \overline{L}\right) \\ -\phi_{BK}(BK^{REQ}_t + E(BK^{REQ}_t)R^{LONG}_t BK^{GOV}_{BANK,t} \\ -\left(BK^{REQ}_t + E(BK^{REQ}_t)\right)L_t + \phi_{TLTRO}(TLTRO_t - 0), \end{aligned} \tag{A.43}$$

– FOC wrt long term sovereign bond B^L_t

$$\begin{aligned} P_{m,t}R^{LOAN}_t = P_{m,t}R^{LONG}_t + R^{TLTRO}_t + \phi_{LOAN}P_{m,t}\left(L_t - \overline{L}\right) \\ -\phi_{BK}(BK^{REQ}_t + E(BK^{REQ}_t)P_{m,t}R^{LONG}_t BK^{GOV}_{BANK,t} \\ -\left(BK^{REQ}_t + E(BK^{REQ}_t)\right)L_t. \end{aligned} \tag{A.44}$$

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Figure 1: Financial structure of the model

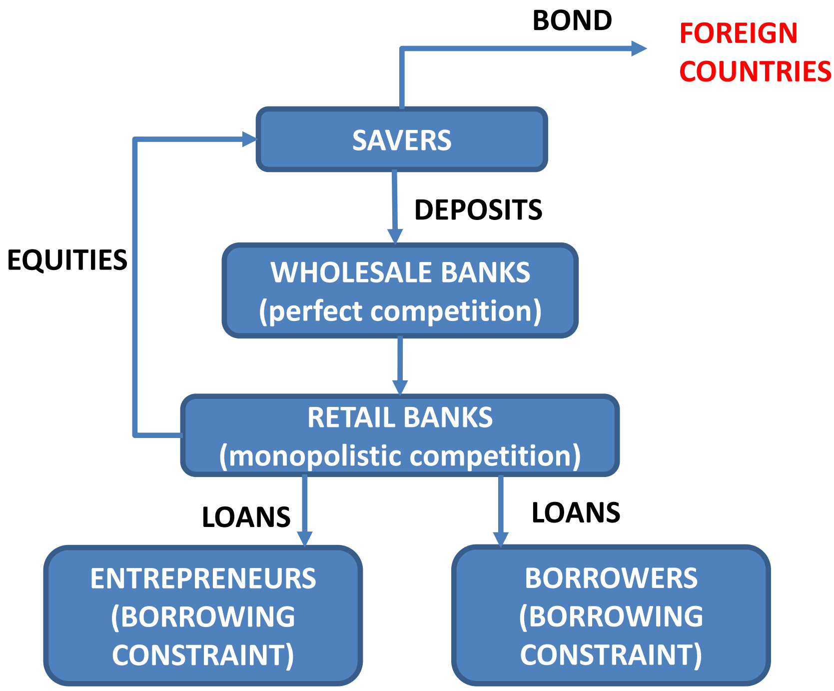


Figure 2: Productive structure of the model

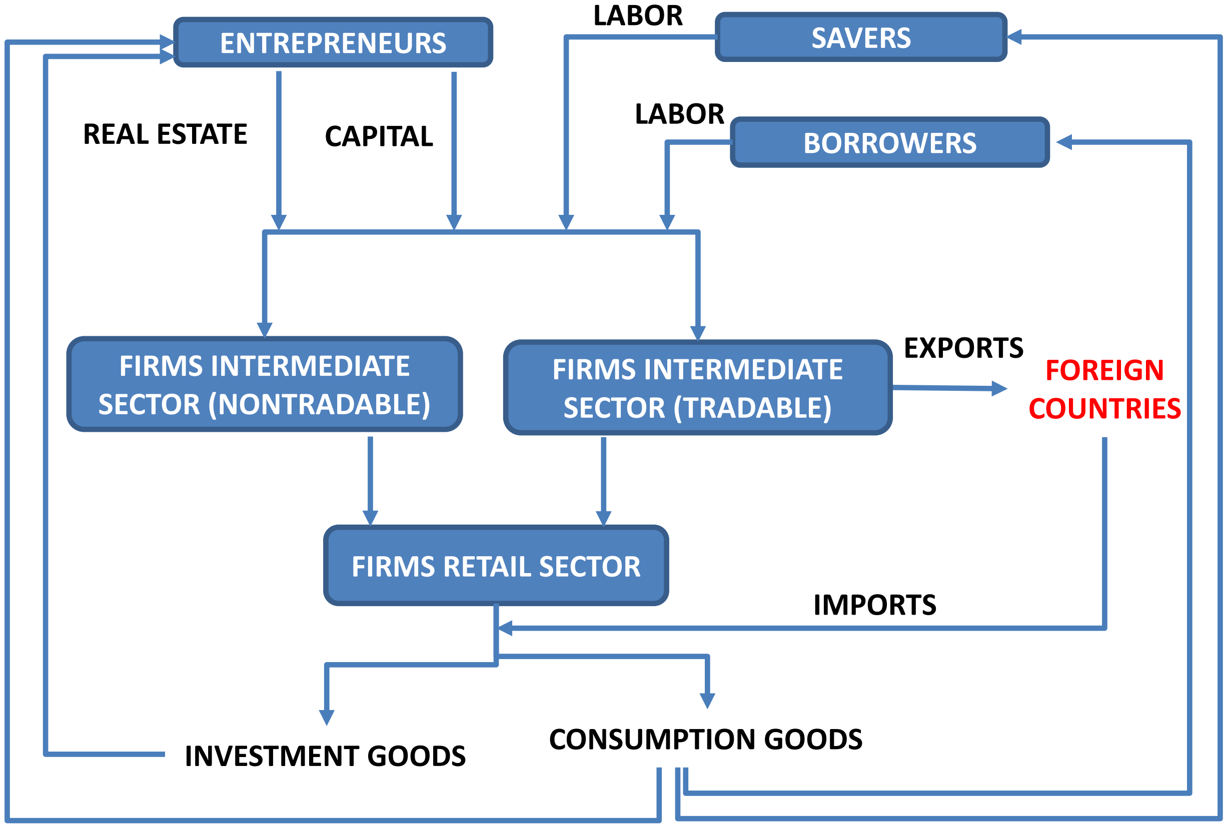


Figure 3: Real estate market

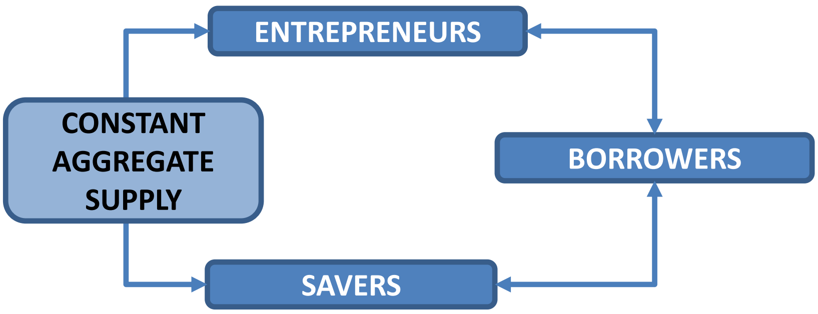
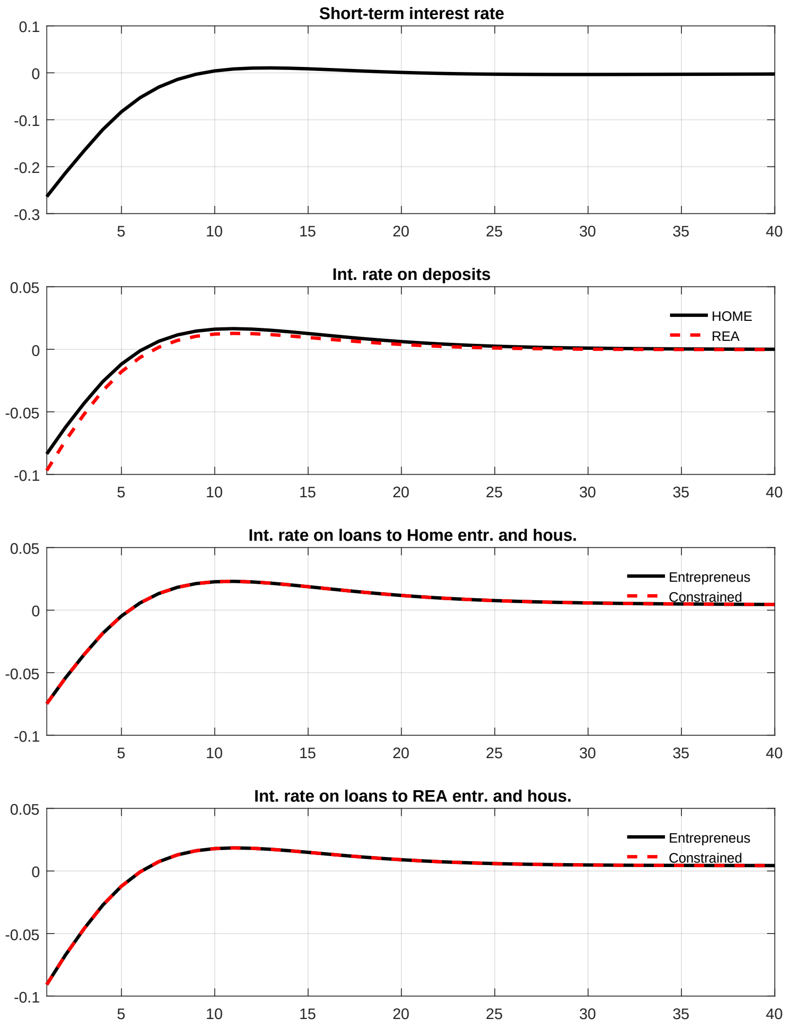
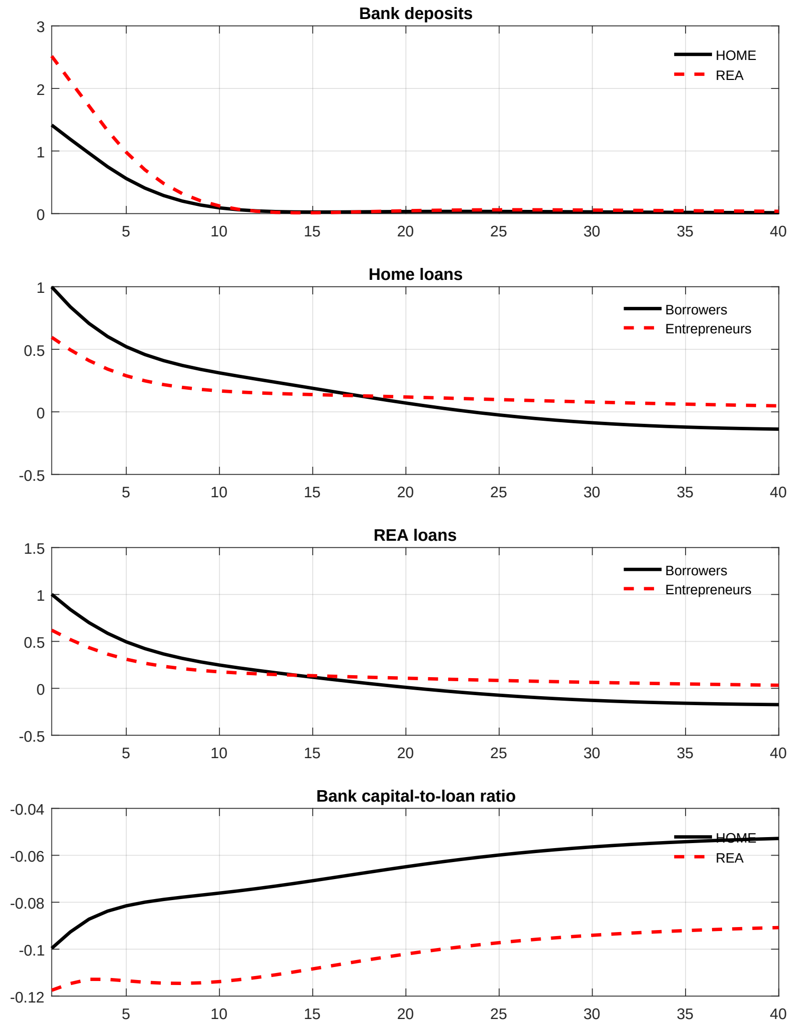


Figure 4: Monetary policy shock. Banking sector: interest rates



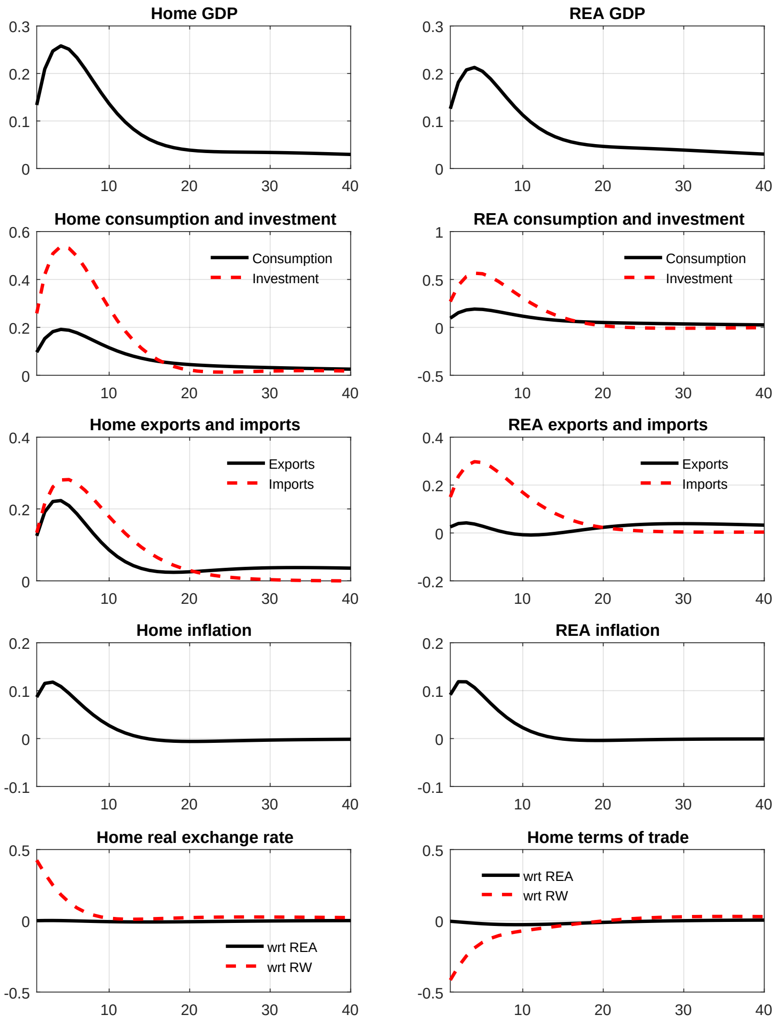
Notes: horizontal axis, quarters; vertical axis, annualized pp dev. from the steady state; for equity prices, % dev.

Figure 5: Monetary policy shock. Banking sector: quantities



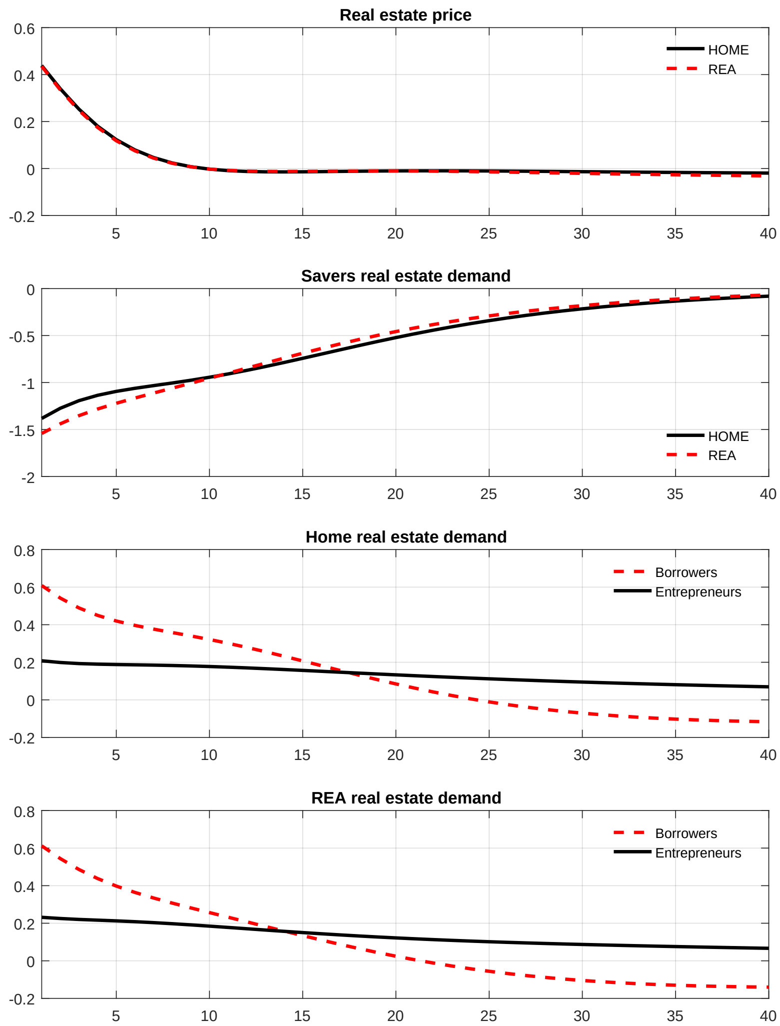
Notes: horizontal axis, quarters; vertical axis, % deviations from the steady state; for the bank capital-to-loan ratio, pp. dev.

Figure 6: Monetary policy shock. Main macroeconomic variables



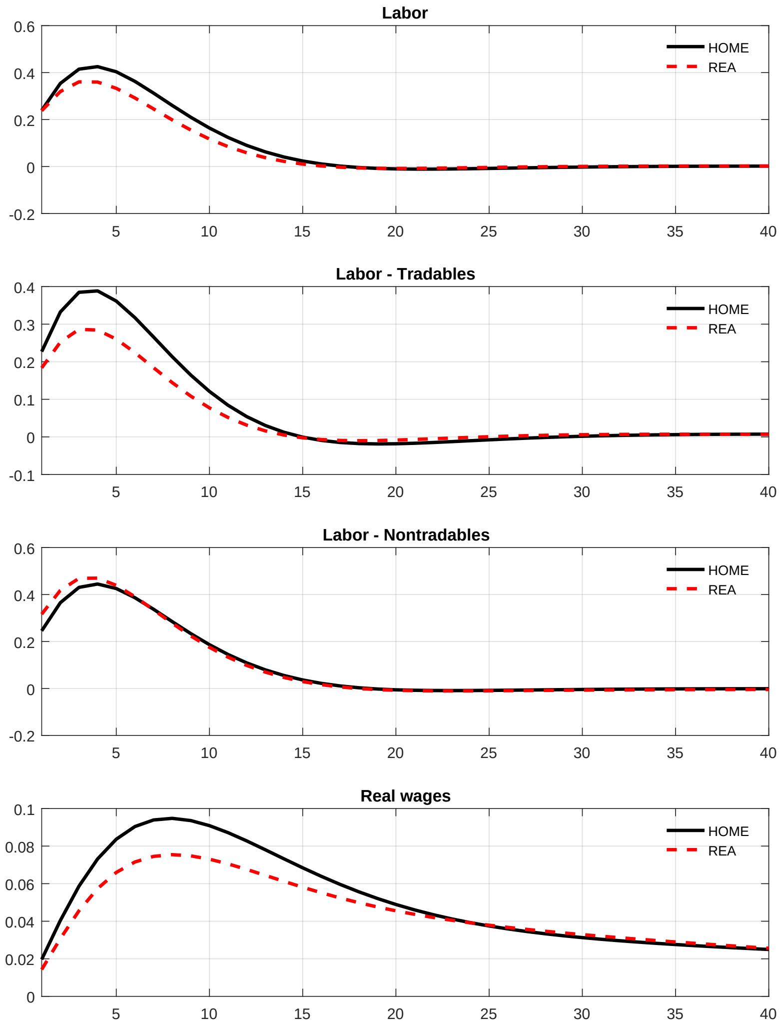
Notes: horizontal axis, quarters; vertical axis, % deviations from the steady state. For inflation, annualized pp. dev.

Figure 7: Monetary policy shock. Labor market



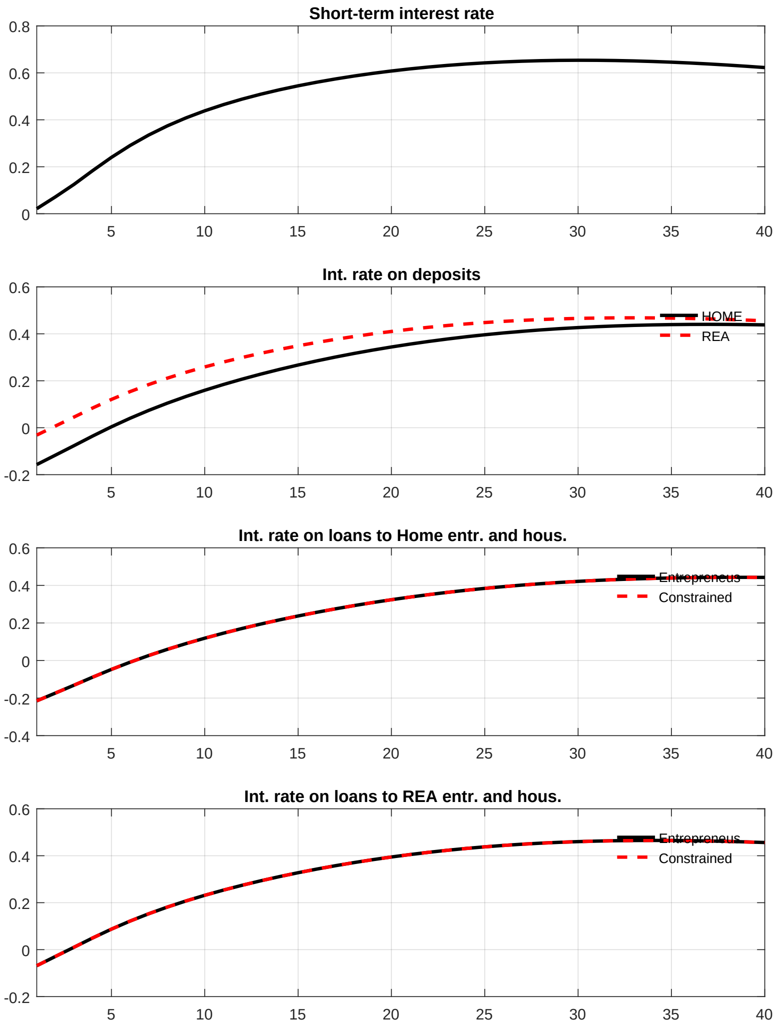
Notes: horizontal axis, quarters; vertical axis, % deviations from the steady state.

Figure 8: Monetary policy shock. Real estate market



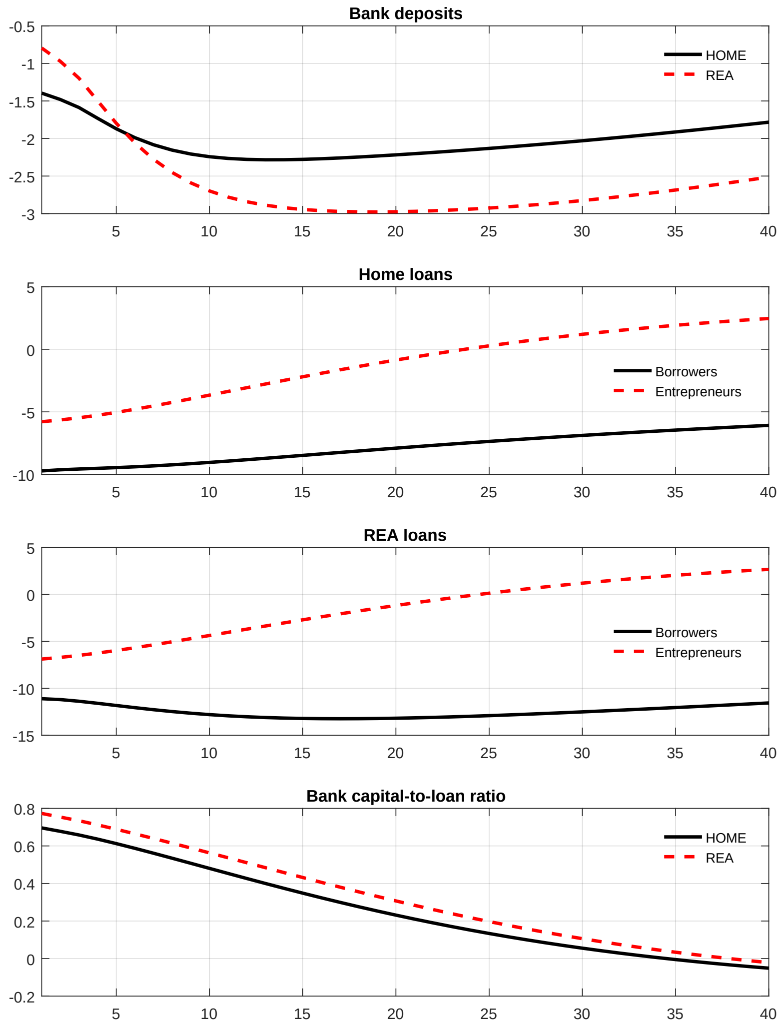
Notes: horizontal axis, quarters; vertical axis, % deviations from the steady state.

Figure 9: Preference shock. Banking sector: interest rates



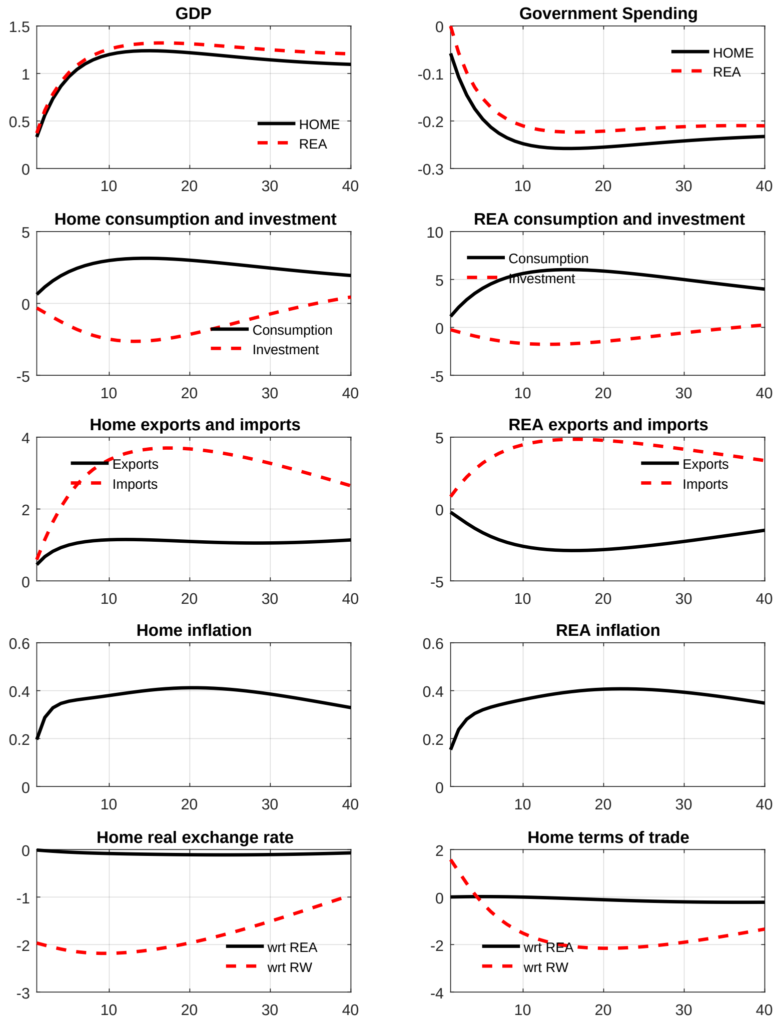
Notes: horizontal axis, quarters; vertical axis, annualized pp dev. from the steady state; for equity prices, % dev.

Figure 10: Preference shock. Banking sector: quantities



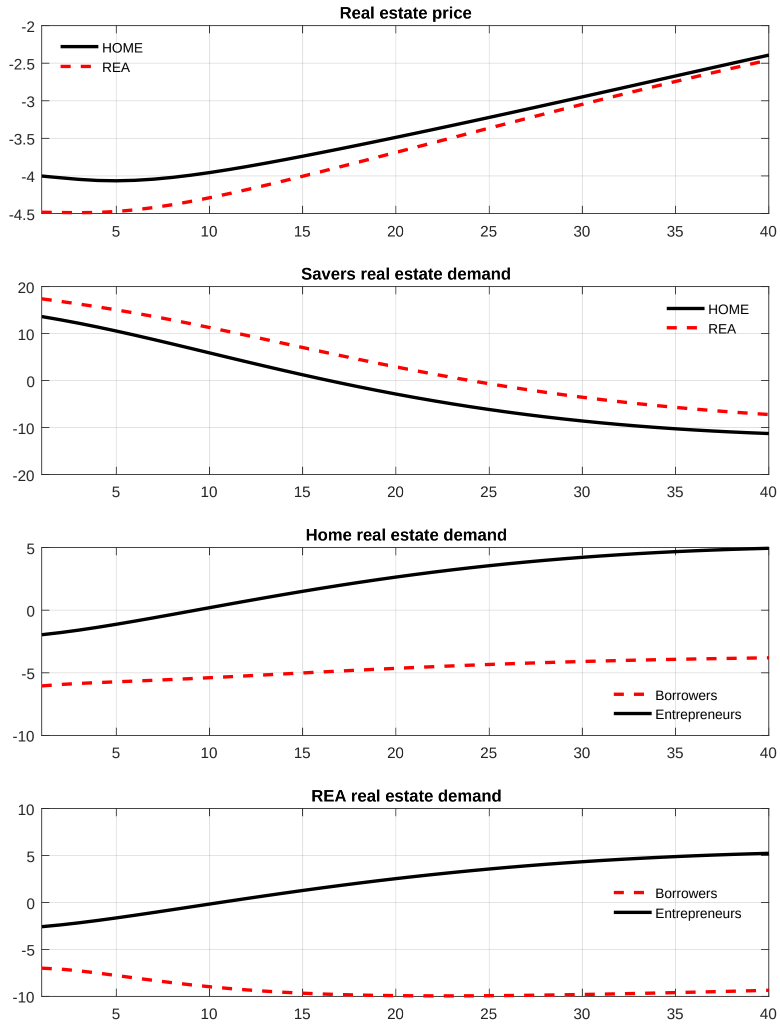
Notes: horizontal axis, quarters; vertical axis, % deviations from the steady state; for the bank capital-to-loan ratio, pp. dev.

Figure 11: Preference shock. Main macroeconomic variables



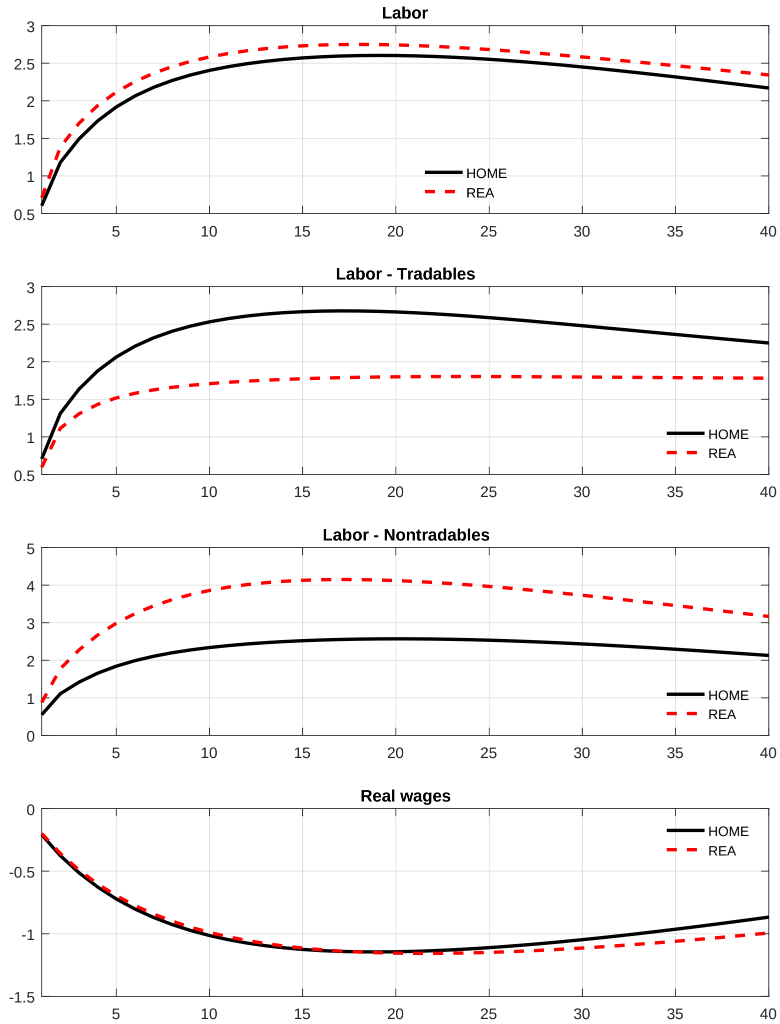
Notes: horizontal axis, quarters; vertical axis, % deviations from the steady state. For inflation, annualized pp. dev.

Figure 12: Preference shock. Real estate market



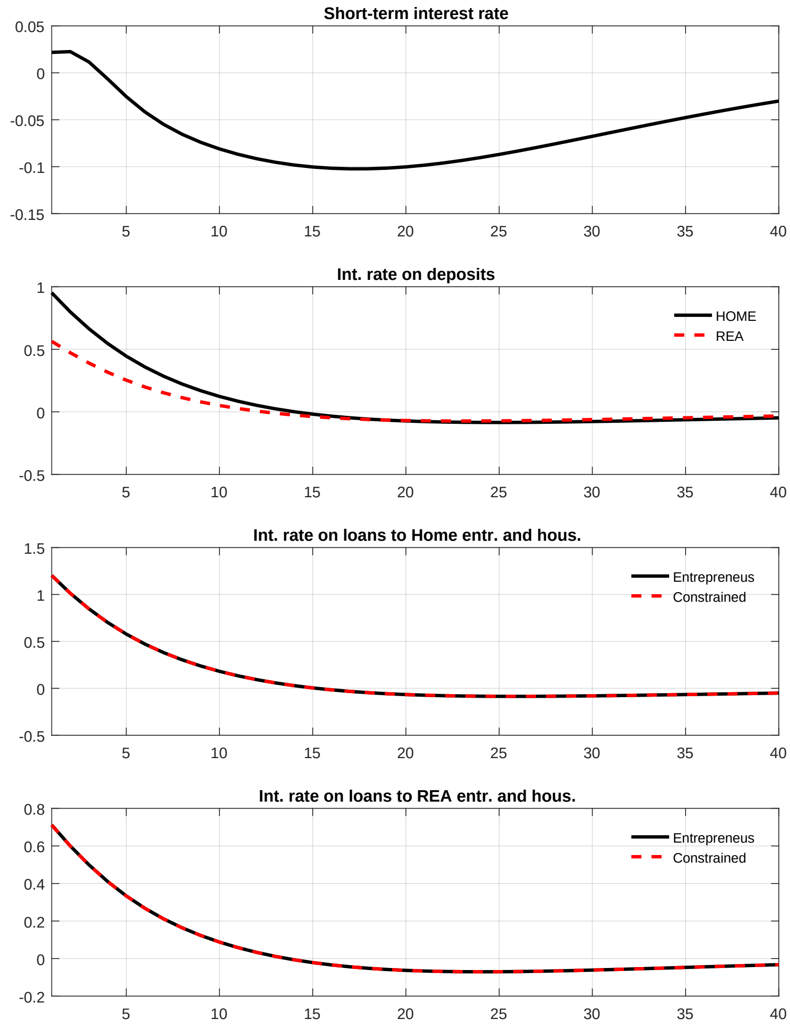
Notes: horizontal axis, quarters; vertical axis, % deviations from the steady state.

Figure 13: Preference shock. Labor market



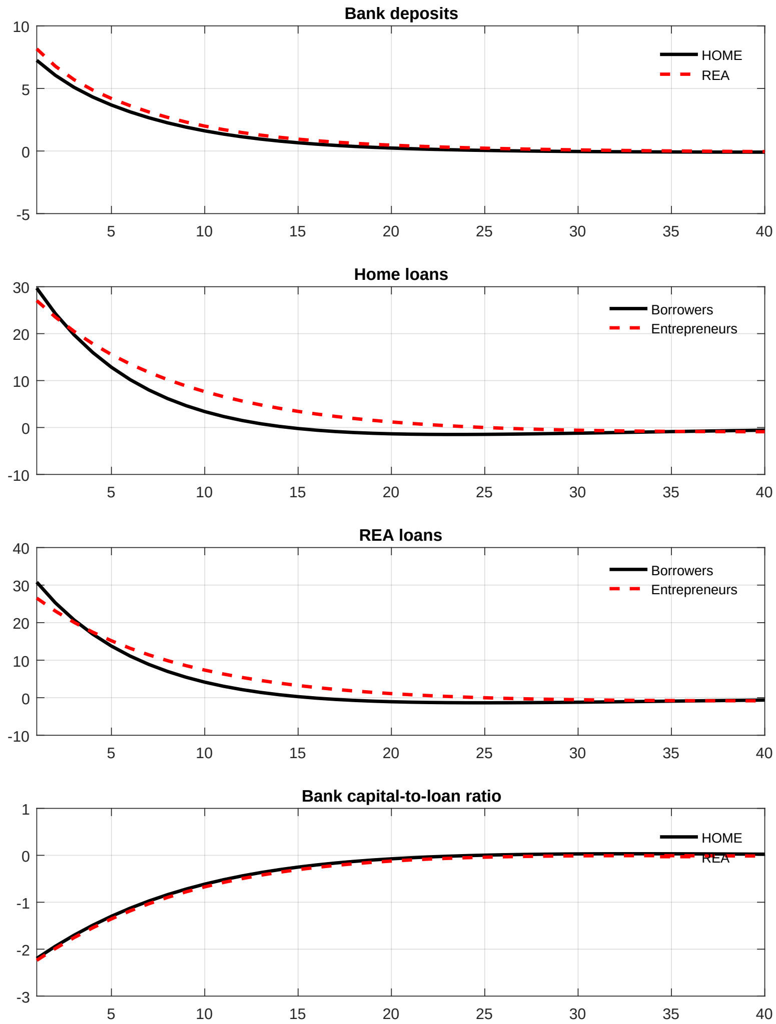
Notes: horizontal axis, quarters; vertical axis, % deviations from the steady state.

Figure 14: Loan-to-value ratio shock. Banking sector: prices



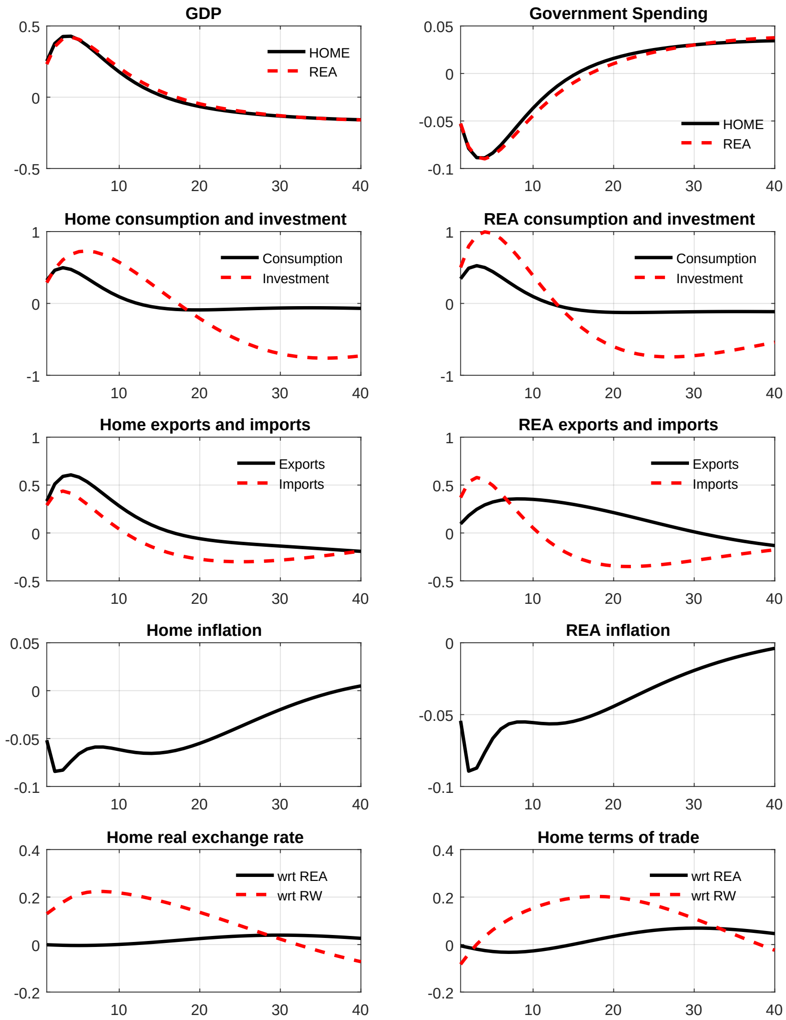
Notes: horizontal axis, quarters; vertical axis, annualized pp dev. from the steady state; for equity prices, % dev.

Figure 15: Loan-to-value ratio shock. Banking sector: quantities



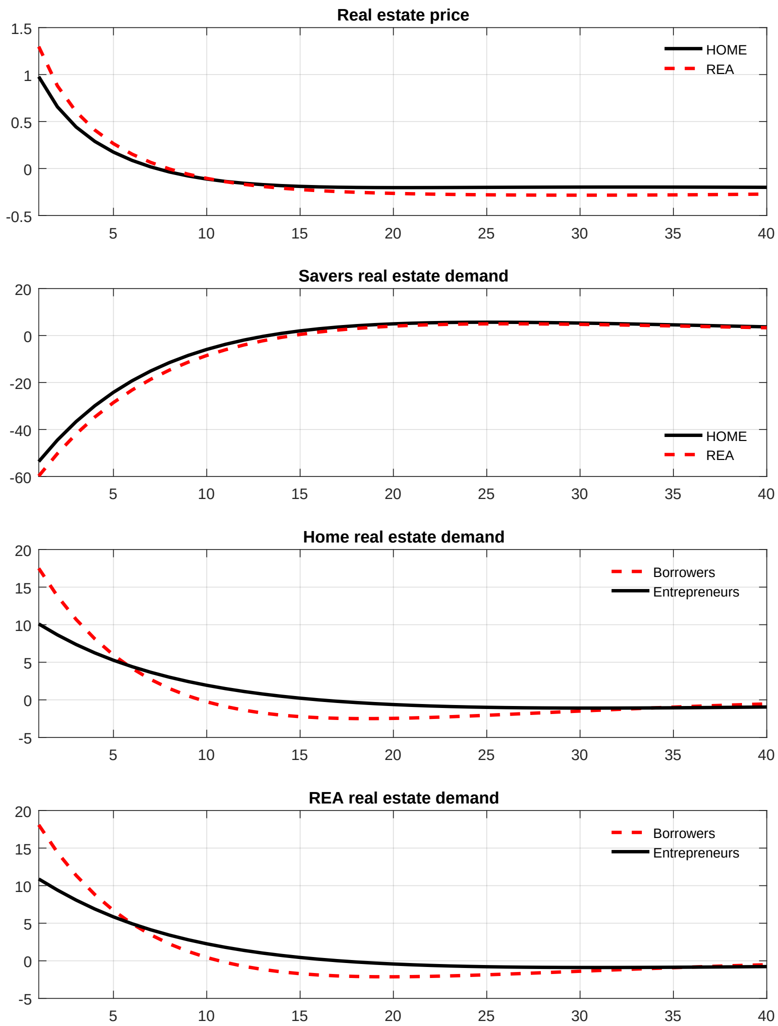
Notes: horizontal axis, quarters; vertical axis, % deviations from the steady state; for the bank capital-to-loan ratio, pp. dev.

Figure 16: Loan-to-value ratio shock. Main macroeconomic variables



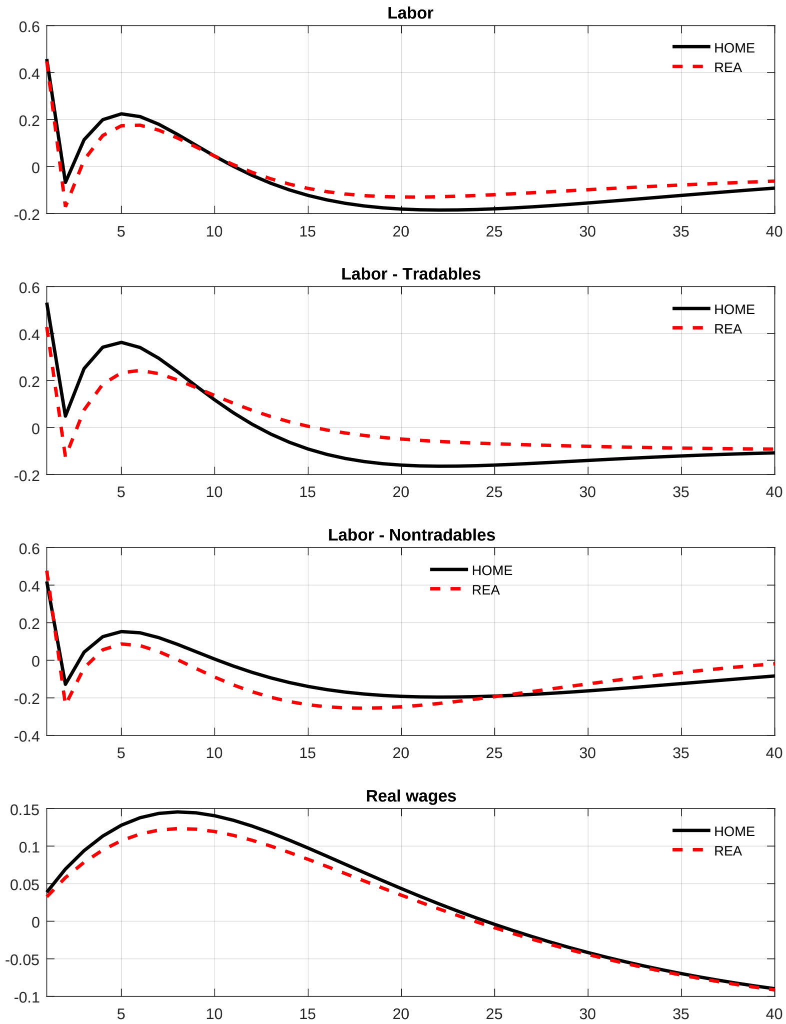
Notes: horizontal axis, quarters; vertical axis, % deviations from the steady state. For inflation, annualized pp. dev.

Figure 17: Loan-to-value ratio shock. Real estate market



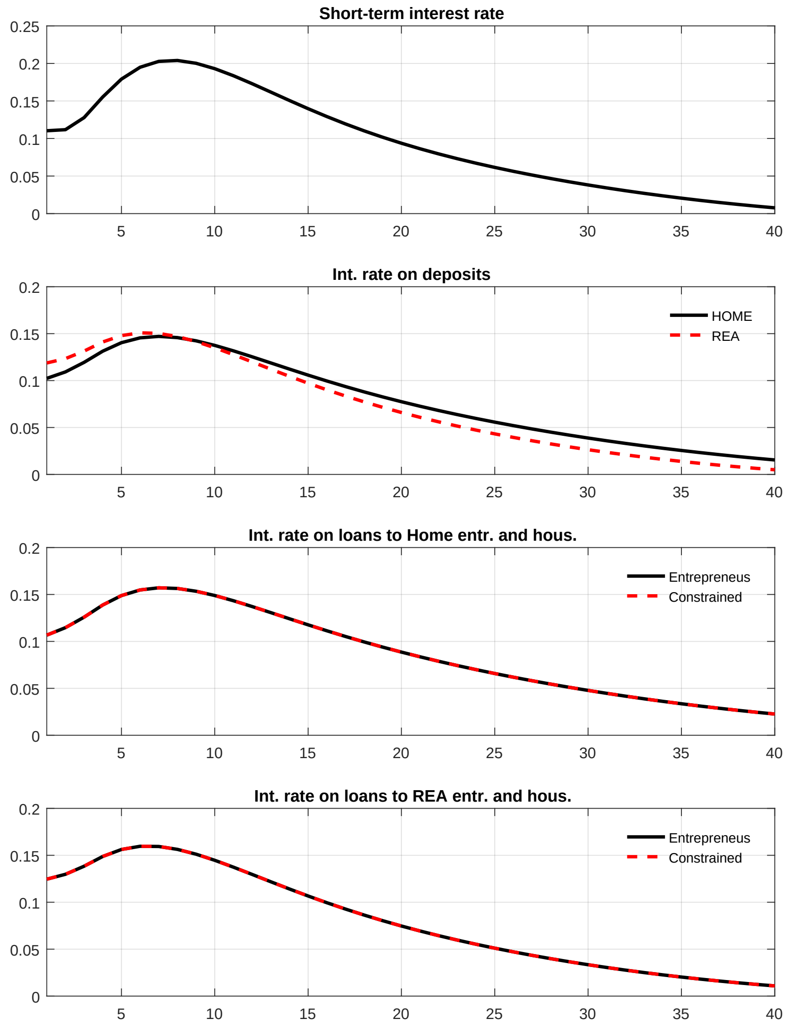
Notes: horizontal axis, quarters; vertical axis, % deviations from the steady state.

Figure 18: Loan-to-value ratio shock. Labor market



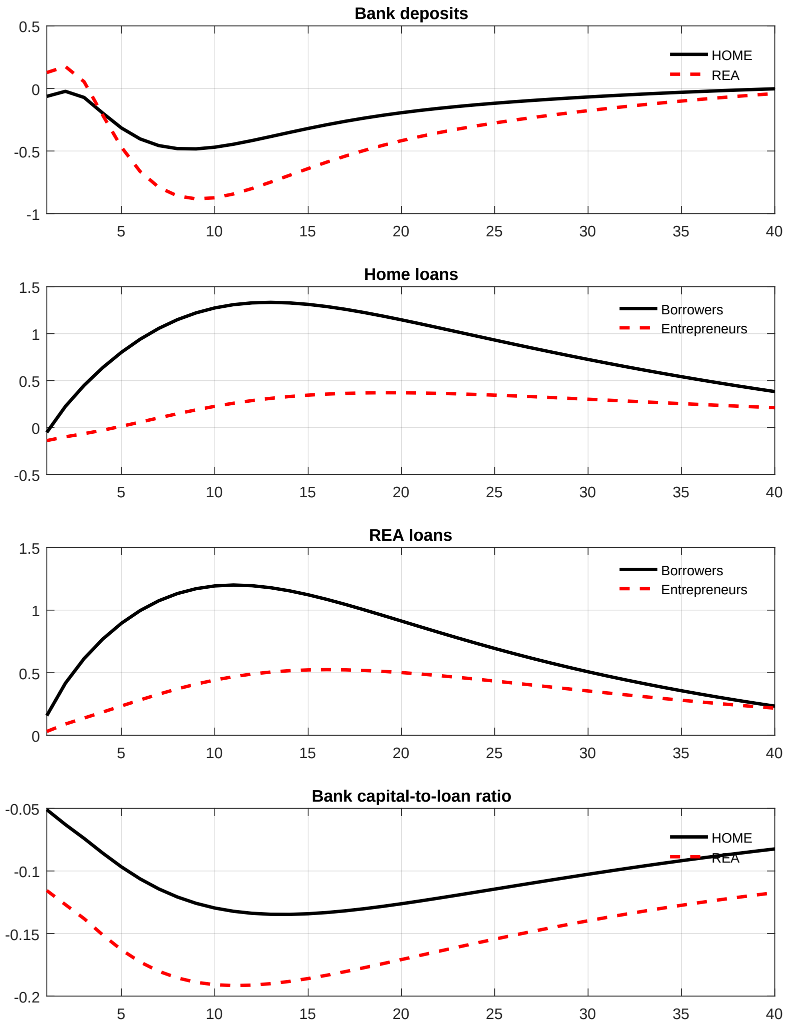
Notes: horizontal axis, quarters; vertical axis, % deviations from the steady state.

Figure 19: Government spending shock. Banking sector: interest rates



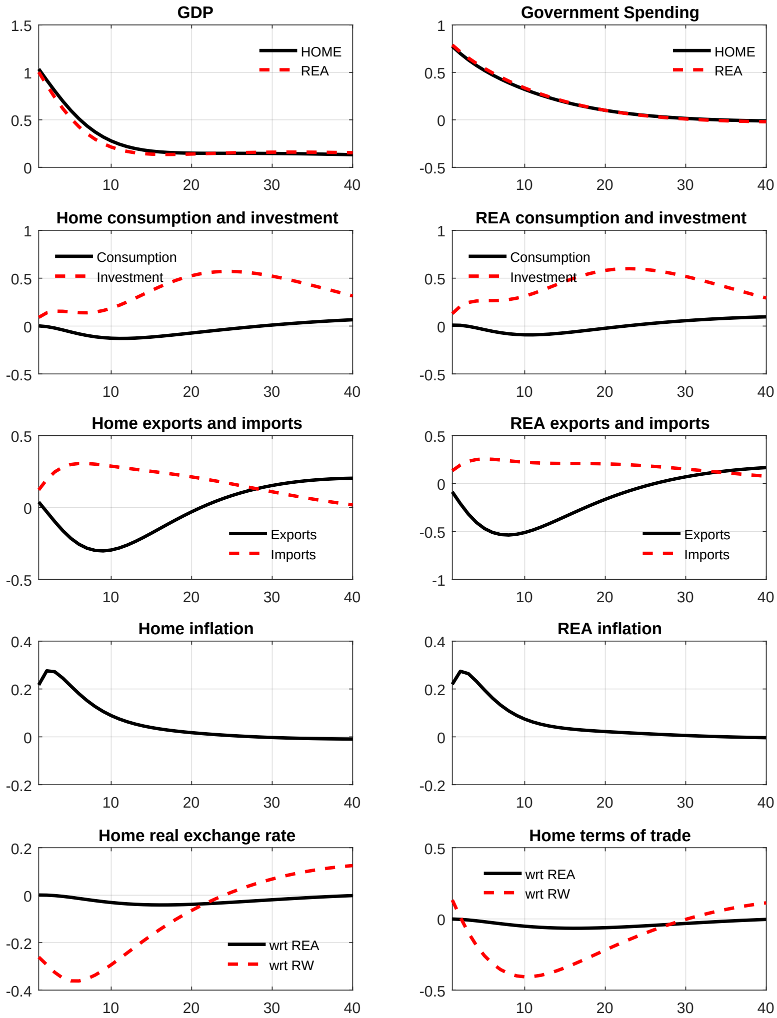
Notes: horizontal axis, quarters; vertical axis, annualized pp dev. from the steady state; for equity prices, % dev.

Figure 20: Government spending shock. Banking sector: quantities



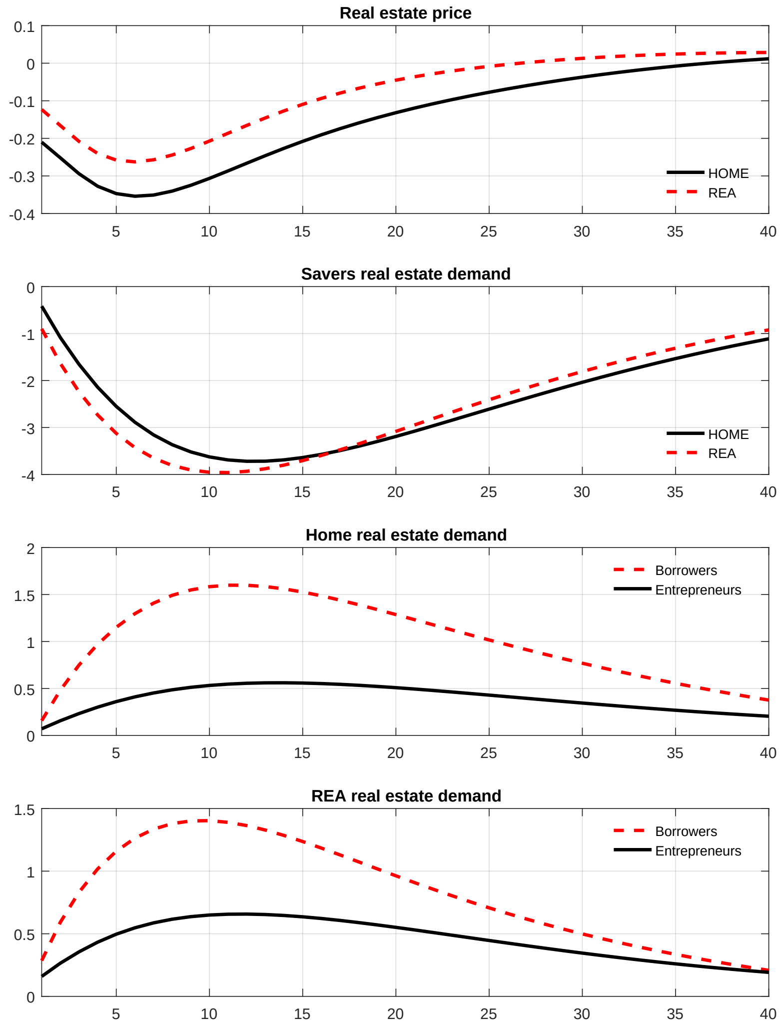
Notes: horizontal axis, quarters; vertical axis, % deviations from the steady state; for the bank capital-to-loan ratio, pp. dev.

Figure 21: Government spending shock. Main macroeconomic variables



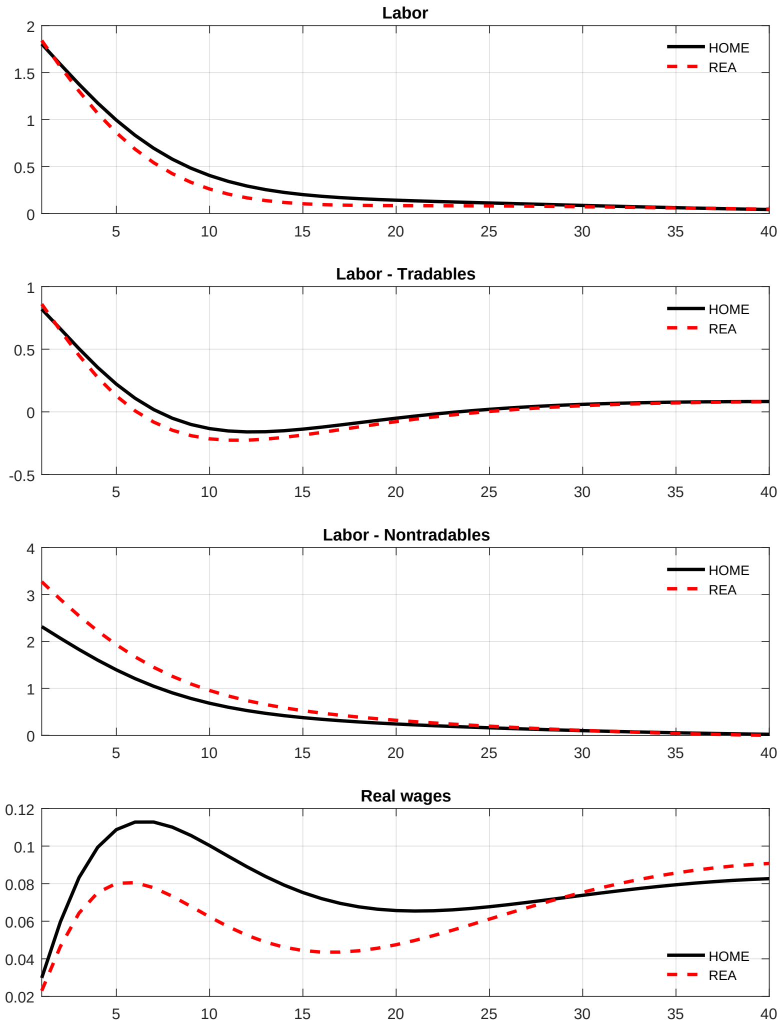
Notes: horizontal axis, quarters; vertical axis, % deviations from the steady state. For inflation, annualized pp. dev.

Figure 22: Government spending shock. Real estate market



Notes: horizontal axis, quarters; vertical axis, % deviations from the steady state.

Figure 23: Government spending shock. Labor market



Notes: horizontal axis, quarters; vertical axis, % deviations from the steady state.