Dynamic fiscal limits and monetary-fiscal policy interactions

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Abstract

We analyze how different types of monetary policy regimes and constraints affect debt limits, defined as the maximum level of debt that an economy can service. We find that a monetary policy stance more reactive to changes in inflation generally raises the debt limit, by reducing the inefficiencies linked to inflation fluctuations. When looking at the impact of the zero-lower bound constraint on debt limits, it becomes important to distinguish the impact of the ZLB constraint in itself (which tends to reduce the debt limits) from the shock that makes the ZLB binding (which tends to increase the debt limit because of the associated reduction in interest rates). On this basis, we then analyze how the transmission of spending shocks changes in presence of fully endogenous debt limits. While spending shocks are generally detrimental for fiscal sustainability away from the ZLB, during periods of binding ZLB their ability to act as a stabilization tool can instead enhance the ability to sustain debt and decrease interest rate spreads. On the other hand, a more reactive monetary policy stance greatly enhances fiscal sustainability as it is better able to peg inflation expectations to the target.

Keywords: fiscal limits, sovereign default, monetary-fiscal policy interactions, zero lower bound

JEL Classification: E52, E61, E63

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1 Introduction

In recent years monetary policy has played a prominent role in the determination of fiscal sustainability in advanced countries. Measures of monetary easing adopted by central banks aimed at sustaining inflation and economic activity have depressed interest rates and thus helped fiscal authorities service their debt. Yet, the zero lower bound (ZLB) has constrained the role of monetary policy as a counter-cyclical tool, calling for an active role of fiscal policy in business cycle stabilization. Despite near zero interest rates, expansionary fiscal policies and fiscal policy shocks due to financial sector bailouts, in some cases have contributed to increase risk premia on government debt and undermine market confidence. Looking forward, the perspective of a normalization of interest rates, raises further concerns on future debt sustainability in many advanced economies, e.g. Beck and Wieland (2017).

In light of these consideration, the impact of monetary policy on the sustainability of public debt and, in turn, the ensuing implications for the transmission of public spending shocks are of particular relevance in the current policy debate. In this paper, we focus on three main questions:

1. How does the degree of monetary policy activeness affect fiscal sustainability?
2. How is fiscal sustainability affected by persistent periods at the ZLB?
3. How does the response of monetary policy affect the sustainability of spending shocks?

This paper explores these issues through the lens of a dynamic stochastic general equilibrium model (DSGE) based on the seminal work of Bi (2012) . Fiscal sustainability is interpreted as the probability that the country will be *able* to service its debt in the future. Central to this class of models is the concept of *fiscal limit*, which is obtained by simulating the present discounted value of all maximum future primary surpluses conditional on the initial state of the economy.

Compared to the existing literature, we introduce two main innovations. First, differently from Bi, Leeper and Leith (2013) , who, in computing the fiscal limit, assume that the monetary authority is always able to peg inflation to the target, we assume that the monetary authority follows a Taylor rule. Inflation dynamics then affect the fiscal limit distributions that become endogenous to monetary policy. Second, we introduce a consumption preference shock as in Erceg and Lindé (2014), in order to mimic the economic developments that made the ZLB binding during the recent crisis. On the one hand, the introduction of a demand-side shock allows us to analyze how changes in the consumption/saving behavior of households affect the government's capacity to service its debt at and away from the ZLB. On the other hand, our framework make it possible to study the consequences on fiscal sustainability of a fiscal policy shock when monetary policy is constrained by the ZLB and government debt is risky.

Our results indicate that the responsiveness of monetary policy to changes in inflation can considerably affect fiscal sustainability. A looser monetary policy stance – i.e., a lower coefficient in the Taylor rule – implies larger fluctuations of inflation away from its target in response to exogenous shocks. In turn, this leads to higher price adjustment costs, lower output and a smaller tax base, which negatively affect fiscal sustainability.

In addition, our analysis suggests that to assess the effect on debt sustainability of aggregate demand shocks is important to distinguish the two channels at play. An aggregate demand shock, by lowering output, decreases the tax revenue (growth channel), but also depresses interest rates reducing interest repayments (interest rate channel). These two channels move in opposite direction. A binding ZLB, by preventing monetary accommodation, amplifies the growth channel, while muting the interest rate channel with detrimental consequences on fiscal sustainability. Indeed, while the government has the possibility to increase the tax rate, using it as an imperfect substitute for monetary policy in order to support inflation, this is costly in terms of output and the resulting tax revenue is depressed.

Finally, we find that making the debt limit endogenous to monetary policy and ZLB considerations is important to better understand the transmission of spending shocks. In normal times, a looser monetary policy stance reduces the margin for expansionary spending shocks, which, by increasing spreads, after the initial expansion may induce a recessionary phase as distortionary taxation has to increase to stabilize debt. In contrast, we find that the positive macroeconomic effects of public spending shocks are larger at the ZLB when monetary policy is constrained and a government spending shock may actually improve debt sustainability. However, the effectiveness of the spending shock depends on the underlying monetary policy stance. A more responsive monetary policy is better able to peg inflation expectations to the target. In turns this dampens the deflationary pressure during periods of binding ZLB, allowing the real interest rate to fall further down, this way sustaining aggregate demand and increasing the fiscal space for the government.

Our analysis is linked to the literature studying the implications of the fiscal theory of the price level on inflation determination, and the different combination of active/passive fiscal and monetary policy (see Davig, Leeper and Walker (2011) and Leeper and Leith (2016) among others). Differently from them, however, we study how *active* monetary policy regimes or temporary suspension of monetary policy rules (i.e. periods of binding ZLB) affect debt limits.

In the spirit of Bi (2012), we model sovereign default as a random event, whose likelihood increases with the level of debt. Hence, we do not propose a theory for sovereign default, as in Uribe (2006), where the focus is on the equilibrium behaviour of default rates and sovereign risk premia. Rather, our objective is to analyse how the maximum amount of debt that a country is able to tolerate changes depending on the type of monetary policy followed and by the fiscal and macroeconomic fundamentals. Moreover, the paper focuses on the *capacity* to pay, rather than the *willingness* to pay, instead a crucial element of the analysis of Arellano (2008). We also abstract from considerations of self-fulfilling dynamics and multiple equilibria as in Lorenzoni and Werning (2013) or on the capacity of monetary policy to prevent them as in Corsetti and Dedola (2016).

The rest of the paper is structured as follows. Section 2 outlines the general equilibrium model. Section 3 presents the methodology for its numerical solution alongside the calibration parameters. Section 4 assesses the role of monetary policy and the ZLB in the determination of the debt limit. Section 5 introduces our forward-looking fiscal indicator. In section 6, finally, we apply our fiscal indicator to evaluate the role of spending shocks in a set of policy scenarios, including the analysis of periods of binding ZLB. Section 7 concludes.

2 The model

The model builds on the works by Bi (2012) and Bi, Leeper and Leith (2013) by introducing preference shocks *á la* Erceg and Lindé (2014) and allowing for the presence of a ZLB on the nominal risk-free interest rate. Time is discrete and denoted as $t = 0, 1, 2, ..., \infty$. The closed economy is populated by a representative household, who consumes, works, owns monopolistically competitive firms producing differentiated intermediate goods and perfectly competitive firms producing a homogeneous final good, and invests in two types of state-noncontingent assets, namely risk-free bonds and risky (i.e., defaultable) government bonds.

2.1 The representative household

The representative household maximizes the following initial utility function:

$$\max_{\left\{C_t, N_t, B_t, B_t^F\right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t - C\nu_t)^{1-\gamma}}{1-\gamma} - \frac{\chi_0}{1+\frac{1}{\chi}} N_t^{1+\frac{1}{\chi}} \right], \tag{1}$$

subject to the flow budget constraint:

$$C_t + \frac{B_t}{R_t} + \frac{B_t^F}{R_t^F} = (1-\tau_t)(W_t N_t + \Upsilon_t) + Z_t + \frac{B_{t-1}^d}{\Pi_t} + \frac{B_{t-1}^F}{\Pi_t}, \tag{2}$$

where E_0 denotes the expectations operator, β the household's discount factor, γ its relative risk aversion and χ its Frisch elasticity. Moreover, C_t denotes private consumption, N_t hours of labour, τ_t the tax rate on wage income and profits, W_t the (real) wage rate, Υ_t the representative firm's profits, Π_t (gross) inflation, Z_t transfers from the government to the households, B_t risky (i.e. defaultable) government bonds, with associated (gross) interest rate R_t, and B_t^F risk-free bonds, with associated (gross) interest rate R_t^F, at time t. Notice that $B_{t-1}^d \equiv (1-\Delta_t)B_{t-1}$ denotes the part of real outstanding debt actually repaid and Δ_t is the haircut on outstanding debt in case of default. As in Erceg and Lindé (2014), the utility function depends on the household's current consumption C_t as deviation from a reference level $C\nu_t$. The exogenous consumption taste shock ν_t lowers the reference level and marginal utility of consumption and follows an $AR(1)$ process

$$\nu_t = (1-\rho_v)\nu + \rho_\nu \nu_{t-1} + \sigma_\nu \epsilon_t, \qquad \epsilon_t \sim N(0,1). \tag{3}$$

The first order conditions of the household problem define the following labor supply schedule

$$N_t = \left[\frac{(1-\tau_t)W_t}{\chi_0 \left(C_t - C\nu_t\right)^{\gamma}} \right]^{\chi}, \tag{4}$$

and the standard consumption Euler equation for riskless bonds

$$\frac{1}{R_t^F} = \beta E_t \left[\left(\frac{C_{t+1} - C\nu_{t+1}}{C_t - C\nu_t} \right)^{-\gamma} \frac{1}{\Pi_{t+1}} \right]. \tag{5}$$

The Euler equation for the (risky) government bonds reads

$$\frac{1}{R_t} = \beta E_t \left[(1 - \Delta_{t+1}) \left(\frac{C_{t+1} - C\nu_{t+1}}{C_t - C\nu_t} \right)^{-\gamma} \frac{1}{\Pi_{t+1}} \right]. \tag{6}$$

where

$$\Delta_t = \begin{cases} 0 & \text{if } B_{t-1} < B_t^* \\ \delta & \text{if } B_{t-1} \geqslant B_t^*, \end{cases} \tag{7}$$

where δ is the size of the haircut if default. As in Bi (2012), the default scheme depends on the effective fiscal limit B_t^*. Each period B_t^* is drawn stochastically from the distribution of fiscal limits conditional on the state of the economy at time t, as explained in section (2).

2.2 Final goods production

The single final output good Y_t is produced using a continuum of differentiated intermediate goods $Y_t(i)$. Competitive final good firms buy the differentiated goods produced by intermediate goods producers and combine them according to an aggregate function which has the CES (constant elasticity of substitution) form

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}. \tag{8}$$

Cost minimization for final good producers results in the demand curve for the generic intermediate good i

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\theta} Y_t \tag{9}$$

and an associated price index for the final good

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \tag{10}$$

2.3 Intermediate goods production

A continuum of intermediate goods $Y_t(i)$ for $i \in [0,1]$ is produced by monopolistically competitive firms, each of which produces a single differentiated good. Intermediate goods firms are subject to Rotemberg adjustment costs that penalise large price changes in excess of steady-state inflation rates. Producer i's maximisation problem reads

$$\max_{P_t(i)} E_0 \sum_{t=0}^{\infty} R_{0,t}^F \left[P_t(i)Y_t(i) - mc_t P_t Y_t(i) - \frac{\phi}{2}\left(\frac{P_t(i)}{P_{t-1}(i)\Pi} - 1 \right)^2 P_t Y_t \right] \tag{11}$$

$$s.t. \quad Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} Y_t, \tag{12}$$

where $R_{0,t}^F \equiv \beta^t \left[\frac{C_t - C\nu_t}{C_0 - C\nu_0} \right]^{-\gamma}$, is the household discount factor, $P_t(i)$ is the price chosen by firm i and P_t is the nominal aggregated price level. Intermediate good producers are endowed with a linear production function

$$Y_t(i) = A_t N_t(i) \tag{13}$$

where A_t is total factor productivity which follows an exogenous AR(1) process of the form

$$\ln\ A_t = (1-\rho_v) \ln\ A + \rho\ _A \ln\ A_{t-1} + \sigma_A \epsilon_t, \qquad \epsilon_t \sim N(0,1) \tag{14}$$

which, in equilibrium, implies the real marginal cost $mc_t = W_t/A_t$.
In a symmetric equilibrium, the first-order condition gives the non-linear New Keynesian Phillips curve under Rotemberg pricing

$$(1-\theta) + \theta mc_t - \phi \frac{\Pi_t}{\Pi}\left(\frac{\Pi_t}{\Pi} - 1 \right) + \phi\beta E_t \left[\left(\frac{C_{t+1} - C\nu_{t+1}}{C_t - C\nu_t} \right)^{-\gamma} \frac{Y_{t+1}}{Y_t} \frac{\Pi_{t+1}}{\Pi} \left(\frac{\Pi_{t+1}}{\Pi} - 1 \right) \right] = 0 \tag{15}$$

where ϕ parametrizes Rotemberg (quadratic) price adjustment costs and θ is the elasticity of substitution between goods. Intermediate goods producers' monopolistic real profits are:

$$\Upsilon_t = Y_t - mc_t Y_t - \frac{\phi}{2}\left(\frac{\Pi_t}{\Pi} - 1 \right)^2 Y_t. \tag{16}$$

2.4 Fiscal and Monetary Policy

The government budget constraint is determined by the following equation

$$\frac{B_t}{R_t} + T_t = \frac{(1-\Delta_t)B_{t-1}}{\Pi_t} + G_t + Z_t$$

where B_t is real debt, Δ_t is the haircut in case of default, T_t is tax revenue, G_t is government consumption and Z_t are fiscal transfers to the household. Public consumption follows an exogenous AR(1) process:

$$\log G_t = (1-\rho_G)\log G + \rho_G \log G_{t-1} + \sigma_G \varepsilon_t, \quad \varepsilon_t \sim N(0,1) \tag{17}$$

while fiscal transfers and labor tax rate respond to debt-targeting rules:

$$Z_t = \max\{Z - \mu^z(B_t - B) + \eta_t^z, \underline{Z}\}, \tag{18}$$

$$\tau_t = \tau + \mu^\tau(B_t - B) \tag{19}$$

where B denotes the steady-state real debt level and $\underline{Z}$ indicates the minimum level of transfers politically feasible,[1] while

$$\eta_t^z = \rho^\tau \eta_{t-1}^z + \sigma^z \varepsilon^z, \quad \varepsilon_t^z \sim N(0,1) \tag{20}$$

Tax revenue is given by:

$$T_t = \tau_t(W_t N_t + \Upsilon_t). \tag{21}$$

Turning to the central bank, it is assumed to follow a (truncated) Taylor rule subject to the zero lower bound (ZLB):

$$R_t^F = \max\left\{R^F\left(\frac{\Pi_t}{\Pi}\right)^\alpha \eta^R, 1\right\} \tag{22}$$

where Π is the target inflation rate and η^R is a monetary policy shock

$$\eta_t^R = \rho^\tau \eta_{t-1}^R + \sigma^R \varepsilon^R, \quad \varepsilon_t^R \sim N(0,1). \tag{23}$$

[1] Parameter $\underline{z}$ aims at capturing the political constraints faced by the government in providing households with a minimum level of transfers.

2.5 Aggregate resource constraint

Closing the model economy, the aggregate resource constraint is given by

$$C_t + G_t = Y_t \left[1 - \frac{\phi}{2} \left(\frac{\Pi_t}{\Pi} - 1 \right)^2 \right], \tag{24}$$

whereby transfers Z_t cancel out as they simply redistribute resources between the household and the government.

2.6 Distribution of the fiscal limits

Following Bi (2012), we quantify the risk of sovereign default starting from fiscal limits that arise from the tax revenue side of the government's budget constraint in presence of distortive taxation. At the peak of the Laffer curve, tax revenues reach their maximum and, for a given level of total government expenditures, the present value of primary surpluses is maximised. Revenues, expenditures and discount rate vary with the shocks hitting the economy, generating a distribution for the maximum debt-GDP level that can be supported.

Differently from Bi, Leeper and Leith (2013) , who, in computing the fiscal limits, assume that the monetary authority is always able to peg inflation to the target, we assume that the monetary authority follows a Taylor rule and we allow inflation to vary around its steady state. Inflation dynamics then affect the fiscal limit distributions that become endogenous to monetary policy. That allows us to study how the fiscal limits depend on the monetary policy stance and respond to the presence of occasionally binding ZLB constraints.

Formally, the stochastic processes governing the exogenous state induce stochastic processes for both the tax rate τ_t^{max} and the associated maximum tax revenue $T_t^{\max}$. Hence, we can write

$$T_t^{\max} = T^{\max}(A_t, \nu_t, G_t, \eta_t^z, \eta_t^R)$$

where the function T^{max} maps the current state into the tax revenue at the peak of the Laffer curve. The fiscal limit is defined as the discounted sum of expected maximum primary surpluses in all future periods.

$$B^* = E \sum_{t=0}^{\infty} \beta^t \beta_p \frac{1}{R_t^F} \left[T^{\max}(A_t, \nu_t, G_t, \eta_t^z, \eta_t^R) - G_t - \underline{Z} \right] \tag{25}$$

where $\frac{1}{R_t^F} = \beta \left[\frac{C(A_{t+1},\nu_{t+1},G_{t+1},\eta_{t+1}^z,\eta_{t+1}^R)-C\nu_{t+1}}{C(A_t,\nu_t,G_t,\eta_{t+1}^z,\eta_{t+1}^R)-C\nu_t} \right]^{-\gamma} \left(\frac{1}{\Pi(A_{t+1},\nu_{t+1},G_{t+1},\eta_{t+1}^z,\eta_{t+1}^R)} \right)$.
The stochastic discount factor is obtained when tax rates are at the peak of the Laffer curve, but modified to allow for a political risk parameter β_p. The distribution of fiscal limits are then computed using Markov Chain Monte Carlo simulations. However, while the assumption of a fixed inflation rate allows to compute debt limits independently from the model solution, letting inflation free to vary requires solving the full non-linear model first before computing Monte Carlo simulations.

3 Numerical solution and calibration

The model is solved in two stages. In the first stage, in order to simulate the debt limit distributions, the model is solved conditional on the government setting the tax rate at the peak of the Laffer curve. In the second stage, the model is solved conditional on the government following the tax rule 19. The debt limit distributions obtained in the first stage are used to compute the state contingent probability of default at each point in time according to 7.

The use of a nominal model presents several problems. The most prominent one is that the computation of the debt limit distributions are not anymore independent on the equilibrium conditions of the model, as they are in Bi (2012) and Bi, Leeper and Leith (2013) . The specific problem lies in the fact that the revenue-maximising tax rate (i.e., the peak of the Laffer curve, which is crucial in the determination of the debt limit distribution) now depends non-linearly on the real wage and the inflation rate. No functional form is available to determine the equilibrium level of these three endogenous variables. Hence, the equilibrium relationships of the three variables need to be solved numerically, given specific values for the state variables and the parameters.

The introduction of a risk-free bond allows us to simplify the computation of the maximum tax rate, the crucial element in the derivation of the debt limit, while preserving the unique equilibrium relation among the endogenous variables, given the state variables and the parameters of the model. Indeed, the introduction of a risk-free bond alongside the government bond allows us to pin down the path of consumption independently of the probability of default. Hence, when we solve the model for the maximum tax rate, the real wage and the inflation rate, we need not consider government debt, with considerable benefits in terms of computational time.

Given this assumption, we limit the number of state variables to five (TFP, government consumption, discount factor shock, transfers, monetary policy shock) and the number of numerically determined control (or jump) variables to three. Notice that a "solution of the model" includes a set (i.e., matrix) of one-to-one relationships between a specific value for the vector of state variables and a specific value for the vector of control variables. So, the model needs to be solved only once before the simulation of the debt limit distribution. When we calculate the debt limit distributions all the control variables are readily available as a function of the state variables either through functional forms or through the one-to-one relationships between state and control variables established in our first step. The solution strategy is presented in Appendix A and relies upon the monotone map method based on Coleman (1991) and Davig, Leeper and Walker (2011) . The use of global solution methods allows us to deal explicitly with the non-linearities associated with the ZLB constraint.

The calibration is presented in table 1. The model is calibrated at quarterly frequencies. The calibration is heavy reliant on Bi, Leeper and Leith (2013) that calibrate fiscal parameters to match average EU-14 data from 1971 to 2007. In steady state, government purchases (public consumption) are 21 percent of GDP and lump-sum transfers are 18 percent of GDP. The tax adjustment parameter (μ_τ) is calibrated to 0.5 at an annual rate, which is close to the average of estimates in EU-14, and the tax rule targets a debt to GDP ratio of 110 percent for 'high-debt countries'. We set the Frisch elasticity to 0.42 which is close to the value used by Linde and Trabandt (2017), who use the same specification for the discount factor shock. Upon default, we assume an haircut of 10 percent as in Cruces and Trebesch (2013) . The other parameters are standard in the literature.

4 Monetary policy stance and debt limit determination

This section studies the role played by monetary policy in the determination of the debt limit distributions. Monetary policy can affect debt limits by reducing interest payments (direct interest rate channel) and by affecting the level of economic activity and, as a consequence, the path of primary balances (indirect growth channel). The analysis will

Parameters		Value	Source
Discount factor	β	0.99	BLL (2013)
Risk aversion	γ	1	BLL (2013)
Public consumption/GDP	g/y	21%	BLL (2013)
Transfers/GDP	z/y	18%	BLL (2013)
Tax rule	μ_τ	0.5/4	BLL (2013)
Inflation	π	3% (annual)	BLL (2013)
Taylor rule	α	1.5	BLL (2013)
Debt/GDP	b/y	110% (annual)	high-debt country
Frisch elasticity	$1/\chi$	0.42	Linde and Trabandt (2017)
Haircut if default	δ	10%	Cruces and Trebesch (2013)
TFP	a	1	standard
Labor supply	n	1	standard
TFP shocks	ρ_a	0.85	standard
TFP shocks	σ_a	0.022	standard
Preference shocks	ρ_ν	0.85	standard
Preference shocks	σ_ν	0.022	standard
Public consumption shocks	ρ_g	0.85	standard
Public consumption shocks	σ_g	0.01	standard
MP shocks	ρ_r	0.85	standard
MP shocks	σ_r	0.01	standard

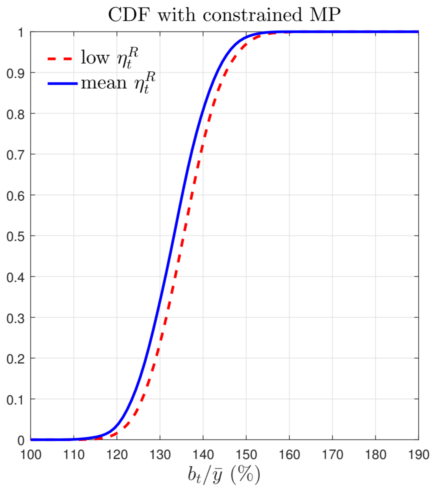
Table 1: Calibration

first show the impact of pure monetary policy shocks, in which the interest channel plays a prominent role. To explore the role of the indirect growth channel we study different degrees of reactivity to changes in inflation in the monetary policy rule (α in eq. 22) and then evaluate the determination of debt limits during periods of binding zero-lower bound constraint.

This analysis sheds light into an important aspect of fiscal-monetary policy interactions, often overlooked in the DSGE literature. Even in most recent studies (see, for example, Bi, Leeper and Leith (2013)), inflation is fixed at its steady-state level when debt limit distributions are computed, so that no role is played by inflation and, thus, monetary policy in determining fiscal sustainability. In contrast, we allow inflation to drift away from its target in response to shocks in fundamentals and policy, so that we can evaluate the impact of these shocks on the sustainability of public debt under different macroeconomic conditions and policy regimes.

Figure 1 shows the impact on the debt limit distribution of both contractionary and expansionary monetary policy shocks as applied to the baseline calibration. The direct interest rate channel and the indirect growth channel move debt limits in the same

Figure 1: Impact of monetary policy shocks on debt limit distributions



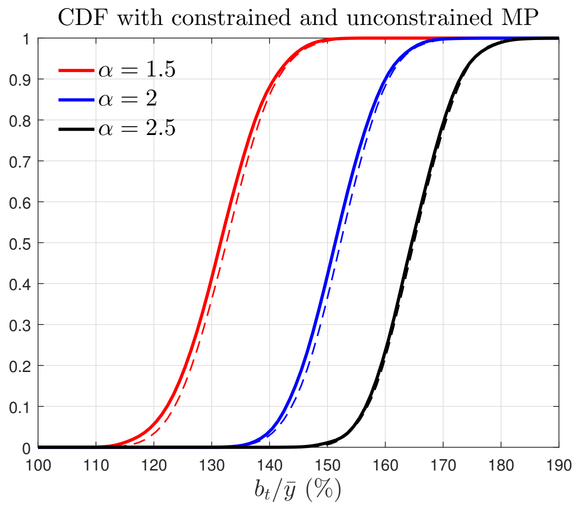
Note: Debt limit distributions after monetary policy shocks reducing (red solid line) and increasing (black dashed/dotted-line) the risk-less interest rate . The blue dashed line indicates the debt distribution when the interest rate shock ϵ_t^R is at steady state.

direction. An expansionary shock is associate with a reduction in interest rates and an increase in output. This increases debt sustainability, as it reduces the probability that the debt limit will be below any given level of the debt-to-GDP ratio.

Figure 2 shows how different degrees of responsiveness of the risk-free interest rate R_t^F to inflation Π_t affect the debt limit distribution at the steady state. We do this by taking into account three different values of α: 1.5 (consensus), 2 (strong) and 2.5 (very strong). The sustainability of debt clearly improves with stronger responsiveness of the Taylor rule to inflation. This is due to the fact that inflation volatility falls the bigger α is, thus reducing the cost associated with deviations of inflation from the target. This, in turn, lifts profits and, with them, tax collection. Notice that the magnitude of the shift in the debt limit distribution is decreasing in α. As α increases, the distribution tends to converge to the one that would emerge if inflation was pegged to the target. Figure 2 also shows the difference between a model with occasionally binding ZLB constraints and a model where the ZLB is assumed to never bind. As expected, the possibility that the ZLB will bind in the future, all else equal, decreases debt sustainability at the steady state. However, the gap between the dotted line (where the ZLB is assumed to never bind) and the solid line is visible for the scenarios with the two lower α, but becomes almost negligible when α reaches 2.5.

To better study the role of a binding ZLB on the debt limits we now introduce a

Figure 2: Impact of monetary policy shocks on debt limit distributions

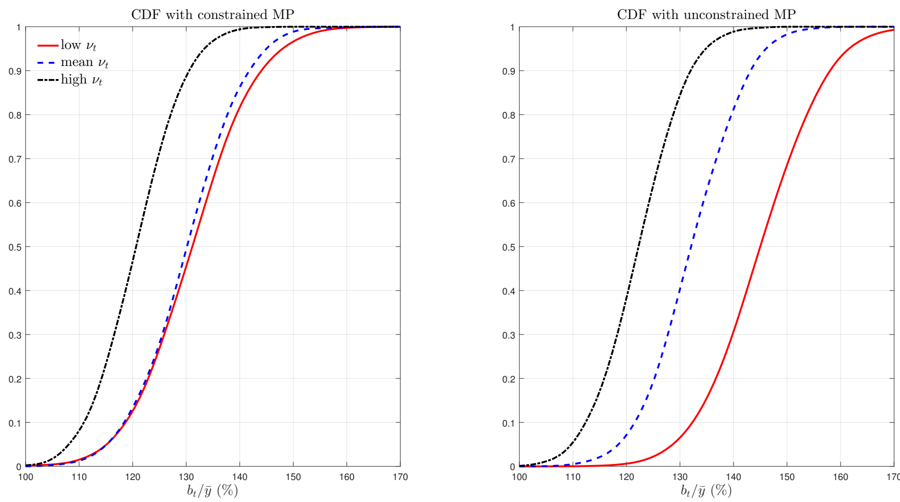


Note: The figure presents how the debt limit distributions change for three different values of the inflation coefficient in the Taylor rule (1.5, 2 and 2.5). The dotted value reports the debt limit distribution without imposing the zero-lower bound constraint. In all these simulations all other variables and shocks are set initially at their steady state value.

consumption preference shocks (ν_t in equation 1). A positive shock reduces the marginal propensity to consume of the representative household, which results in a fall in the interest rate, output and inflation. A shock sufficiently strong can bring the economy to a situation of binding ZLB. In case of a preference shock, the interest rate channel and the indirect growth channel have opposite effects on the debt limits, as a decrease in the risk free interest rate is associated with a fall in output. When studying the debt limits at the ZLB, it is therefore of crucial importance to distinguish the role of the shock itself from its interaction with the ZLB constraint. This is done in Figure 3 by looking at the debt limit distributions under constrained (left panel) and unconstrained (right panel) monetary policy.

In a situation in which interest rates are unconstrained, the negative preference shock actually improves debt sustainability. In such a situation, the reduction in interest rate that accompanies the reduction in output and inflation improves the capacity of the economy to service debt, and thus shifts the debt limit distribution to the right. In other words, the interest channel prevails over the reduction in tax collection triggered by the fall in output. Imposing the zero-lower bound constraint, however, magnifies the growth channel and mutes the interest rate channel. For the calibration used in this paper, the ZLB completely offset the benefits of the reduction in interest rates and brings the debt limit

Figure 3: Impact of consumption preference shocks on debt limit distributions

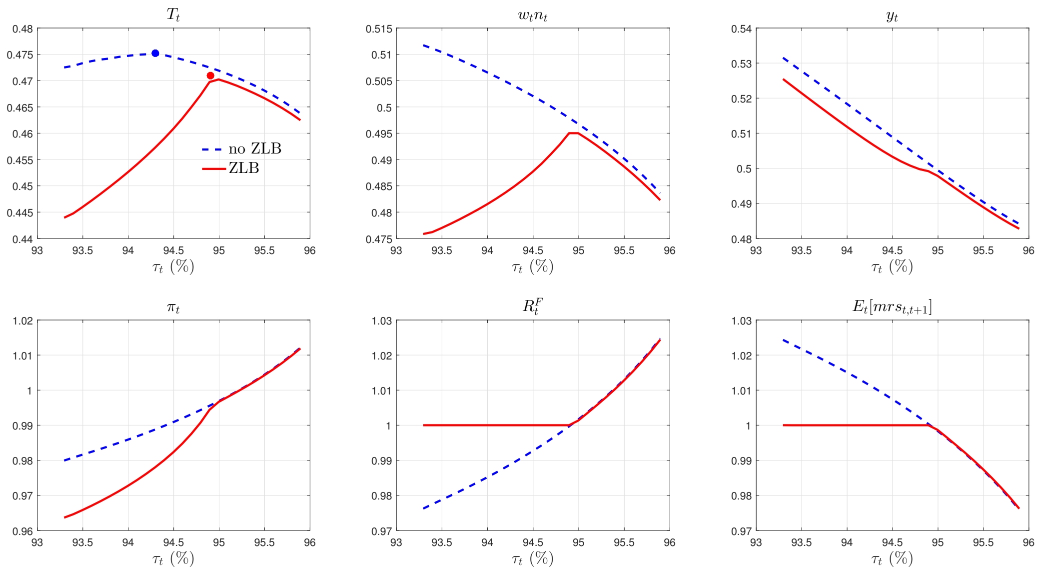


Note: The figure shows how the debt limit distributions change for simulations with an initial positive (high ν_t, black line) and negative (low ν_t, red line) preference shocks. The debt limit distribution with the preference shock at its steady-state value (mean ν_t, blue dotted line) is included for comparison. The left panel shows the debt limit distributions when the ZLB is assumed to bind while the right panel allows interest rate to take also negative values

distribution close to its steady-state value.

Figure 4 inspects in detail the impact of a binding ZLB on debt limit determination, in order to understand the mechanism that leads to the adverse growth effect. The Laffer curve (first chart from the left in the first row) under the ZLB (red solid line) is not anymore bell-shaped as in the case of no ZBL (blue dotted line): it presents a kink in correspondence to its maximum value. The dynamic of inflation is key to understand the shape of the Laffer curve and the dynamic of output. With a binding ZLB, monetary policy is not anymore able to stabilize inflation. The ensuing deflation increases the real interest rate depressing demand further. To clear the market, the wage rate has to decrease in order to induce households to work less. By increasing the tax rate on wage income, the fiscal authority is able to increase the marginal cost for the firms and support inflation. In turn, higher inflation lowers the real interest rate and supports private sector demand reducing the fall in wage. Actually, the ZLB offers to the government a leeway to increase the tax rate, because by doing so, it is actually able to substitute for monetary policy and reduce the distortions generated by deflation. The peak of the Laffer curve is then reached at the point in which the monetary authority is on the edge of the ZLB and inflation is at the same level that would be in place absent the constraint. Such an operation, however, is not cost free.

Figure 4: Binding ZLB in an economy at the debt limit



Note: The chart shows how changes in the tax rate affects the economy after the negative consumption preference shock, both in a situation in which the interest rate is left free to adjust and where the ZLB constrain hold. The first line (from left to right) of the panel report total tax revenue (the real Laffer curve), wage income and output. The second line includes inflation, the risk-free rate and the expected marginal rate of substitution.

Indeed, the tax rate that maximizes tax collection lies at the right of the no-ZLB peak. The higher tax rate puts downward pressure on labor supply, leading to a lower level of output and tax revenue. In practice, in an economy at the ZLB, fiscal policy substitutes for monetary policy: with an active use of the tax rate, the government supports inflation and through it, aggregate demand. The increase in the tax rate, however, is not harmless as it reduces output through a reduction in labor supply. This is due to the reduced disposable income received for every unit of labor supplied in the market. Despite the reduction in output, tax revenues are still substantially higher than what could be collected with tax rates below the maximizing value, because of the elevated cost of inflation fluctuations.

In synthesis, this section presented two important results: first, a more aggressive monetary policy stance can improve debt tolerance through a reduction of the price distortions in the economy; second, fiscal policy tends to replicate the work of monetary policy in an economy where the government wants to maximize tax revenues and the ZLB binds. These two considerations will be important in the simulations presented in section 6 .

5 Forward-looking fiscal space index

From a normative perspective, an important question is how much the government can use fiscal policies to stimulate the economy, without undermining the sustainability of public finances in the future. In other words, policy makers may wander what is the 'fiscal space' of the government. In this section we show how we can use fiscal limits to define a forward-looking measure of the fiscal space.

Any measure of fiscal space needs to be defined in terms of a specific fiscal instrument (i.e., tax rates, government consumption, government investment, etc.). In fact, in general equilibrium, each instrument has a different impact on the economy and, thus, on the debt-to-GDP ratio. Let F be the fiscal instrument in relation to which we want to measure the fiscal space. Let then s_t^{-F} be the state of the economy (the value taken by all exogenous shocks) at time t, with the exclusion of the value of the fiscal instrument F. For all variables other than F, the current state s_t^{-F} determines their future values, in accordance with the respective assumed data generation process.

Let B_t be government debt at time t and $\hat{B}^{\phi}(s_t)$ the debt limit corresponding to a given probability threshold ϕ conditional on the realization of the state s_t. Prior to defining the index, we first need to define an intermediate but important fiscal variable: the fiscal policy signal FS. FS indicates whether the level of debt B_t is expected to reach or go beyond the debt limit $\hat{B}_t^{\phi}$ between time t and time $t+N$. Formally, it is defined as follows:

$$FS_t = \begin{cases} 0 \text{ if } B_{t+i} < \hat{B}_{t+i}^{\phi} \quad \forall\ i \in [0, N] \\ 1 \text{ otherwise} \end{cases} \tag{26}$$

Let then F_t^{FS} be the level (maximum for tax rates, minimum for spending instruments) of the fiscal instrument F at which the fiscal signal FS takes the value of one given the state of the economy s_t^{-F}. [2] The fiscal space available at time t and relative to the fiscal instrument F can then be defined as

$$SP_t^F = (F_t^{FS} - F_t\ | s_t^{-F}) \tag{27}$$

Because of its evident non-linear nature (given the piecewise definition of the fiscal

[2] Of course, in this section we assume that such level actually exists. This needs to be verified.

policy signal FS and of the threshold F_t^{FS}) there is no analytical solution for SP_t^F. It needs to be simulated, based on a specific set of initial shocks.

In synthesis, this fiscal index is completely forward looking and takes into account the general equilibrium implications of changes to the fiscal instrument F. Moreover, it has the interesting property of being state-contingent, as it depends on the state s_t^{-F}.

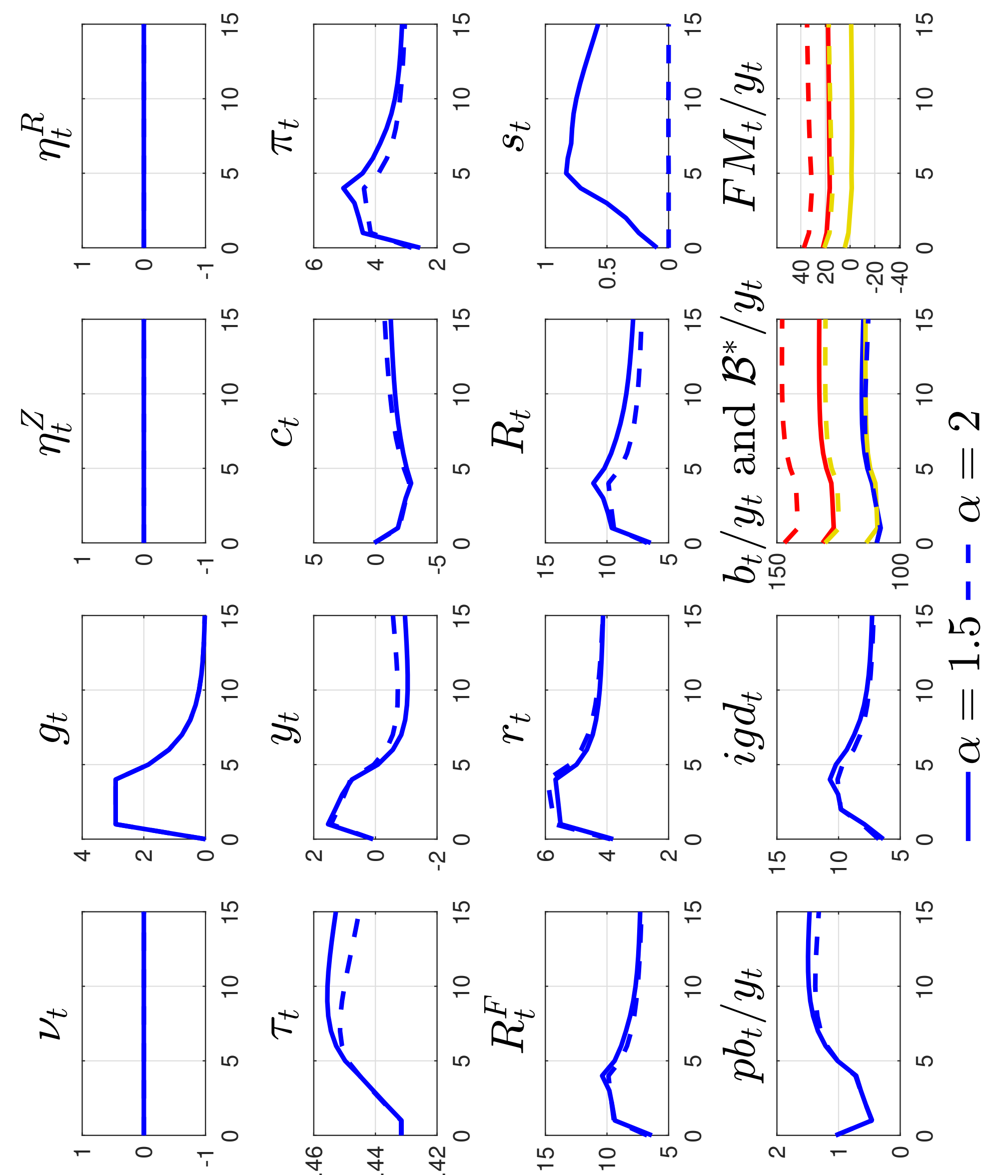
6 The impact and sustainability of government spending shocks

This section presents a concrete application of the tools presented above. The objective is to study the transmission mechanism of government spending shocks in light of the interaction between monetary policy, debt limits and ZLB. We perform three exercises which consider the same government spending shock under different scenarios. In the first we draw impulse-responses in normal times but under different monetary policy stance α; then, we draw impulse-responses at the ZLB, comparing the effect of the increase in government consumption with a scenario in which no fiscal stimulus is provided; finally, we repeat the first exercise, but with binding ZLB.

Figure 5 shows the effect of a positive shock to government consumption. The shock has been assumed to last 3 quarters, after which it follows an AR(1) process with a 0.85 decay parameter. To different α correspond remarkably different dynamics. An increase in government consumption creates inflationary pressure to which the monetary authority responds by raising the nominal risk-free interest rate R_t^F. A more reactive monetary policy (higher α) is better able to stabilize inflation π_t, in turn, the stronger response of the real interest rate r_t dampen the response of output and consumption. Importantly, while for $\alpha = 2$ we do not observe any movement in the spread s_t, for $\alpha = 1.5$ the spread increases up to a maximum of 80 basis points after 5 quarters and then decreases slowly. This is because, as shown in figure 2, debt limits are much lower when $\alpha = 1.5$. Indeed, debt limits are forward looking and take into account that the monetary authority will be less able to stabilize future shocks affecting the economy. This has important consequences: the recessionary phase that follows the initial expansionary effect of the policy, is significantly more pronounced for $\alpha = 1.5$. Indeed, the increase in the spread forces the government to increase the tax rate τ_t, which, since taxes are distortionary, reduces output and crowds out private sector demand.

In figure 6, we study the effect of a positive shock to government consumption at the

Figure 5: Impact of a 3-quarters government spending shock for different α



Note: The figures represent a scenario in which spending is increased by approximately 3% of (steady-state) GDP for 3 quarters. The solid line describes the evolution of the economy for $\alpha = 1.5$, the dotted line describes the evolution of the economy for $\alpha = 2$. In the bottom right panel, the red line indicates the evolution of the 30% debt limit threshold, i.e., the value on the debt limit distribution that corresponds to a 30% default probability.

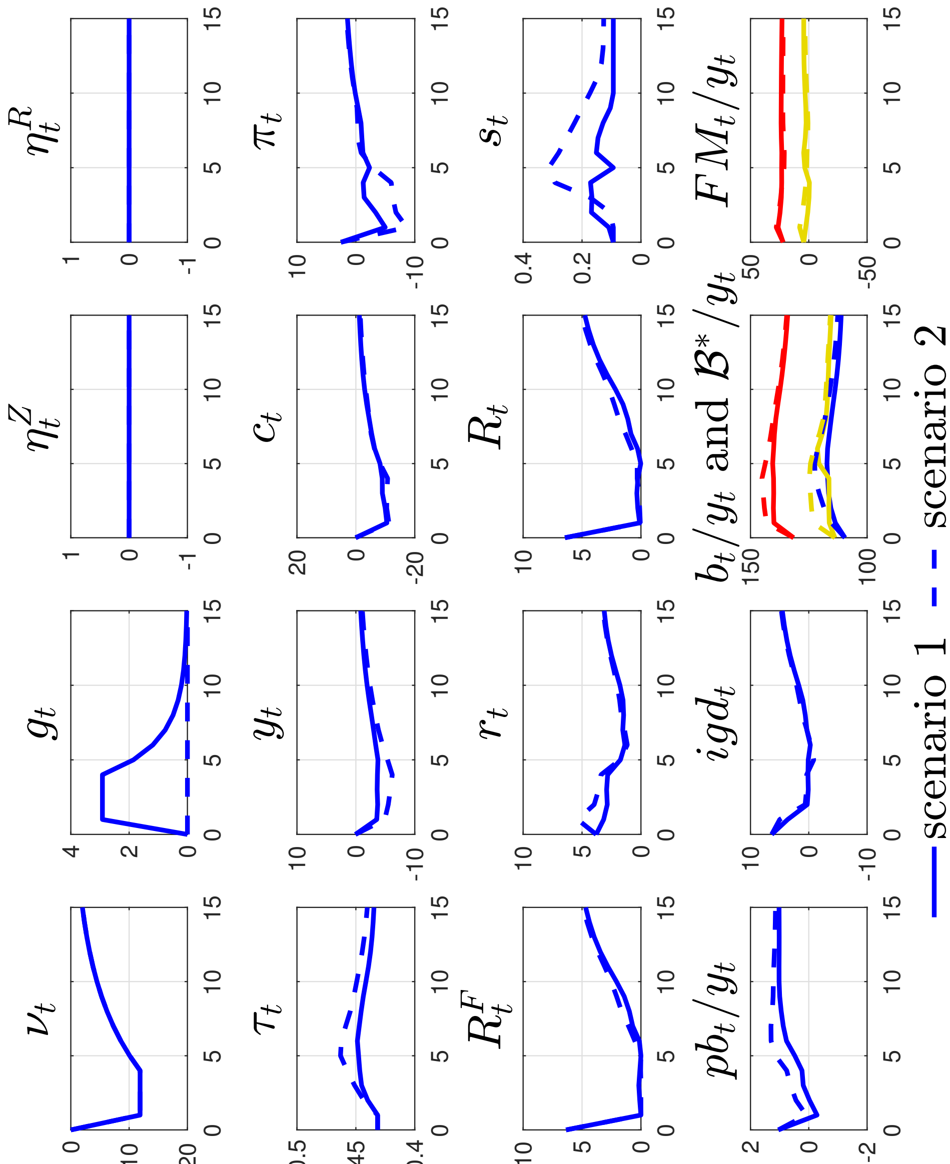
ZLB. The dotted line describes a scenario where no fiscal stimulus is provided, the continuous line describes a scenario where, as in the previous exercise, government consumption is raised of about 3 percent of steady state GDP for 3 quarters, and after that it follows an AR(1) process with a 0.85 decay parameter. Consistent with the recent literature on fiscal expansions at the ZLB, we find that the a positive shock to government consumption is particularly effective in mitigating the recession, with the drop in output y_t significantly dampened. Rather surprisingly, we find that the spending shock improves fiscal sustainability at the ZLB: the spread s_t increases up to a maximum of about 18 basis point with the shock, while it would have increased to about 30 basis point absent the shock. At the heart of the result is the fact that the increase in government expenditure is capable to mitigate significantly the drop in inflation π_t. In turn, the real interest rate r_t falls more than absent intervention. The lower real interest rate on the one hand helps to sustain demand, on the other hand helps the government to finance public expenditure, without significantly affect the debt level. Indeed, despite the government runs a lower primary balance pb/y as compared to a situation without shock, debt increases less than without intervention. In turn, this implies that the government can afford a lower tax rate τ with positive effects on output.

In figure 7, we study the same shock considered in 6; however, here we compare the dynamics that arise respectively with $\alpha = 1.5$ (the case considered in 6) and $\alpha = 2$. The two dynamics are remarkably different. For $\alpha = 2$, the fact that a more reactive monetary authority is better able to peg expectations to the target, implies that the deflation following the preference shock that leads to the ZLB is significantly lower than with $\alpha = 1.5$. As a consequence, the real interest rate r_t on impact falls much more, helping to stabilize the economy. Despite the primary balance to GDP, pb/y, decreases substantially to finance the increase in public expenditure, the stock of debt will actually decrease over time due to the low interest rate environment. Because of that, the government is able to decrease marginally the tax rate τ stimulating the demand further. Both output and consumption fall significantly less. Notice that debt limits actually increase (both the dashed yellow and red line on the last quadrant), which results in the absence of spread s_t.

7 Conclusions

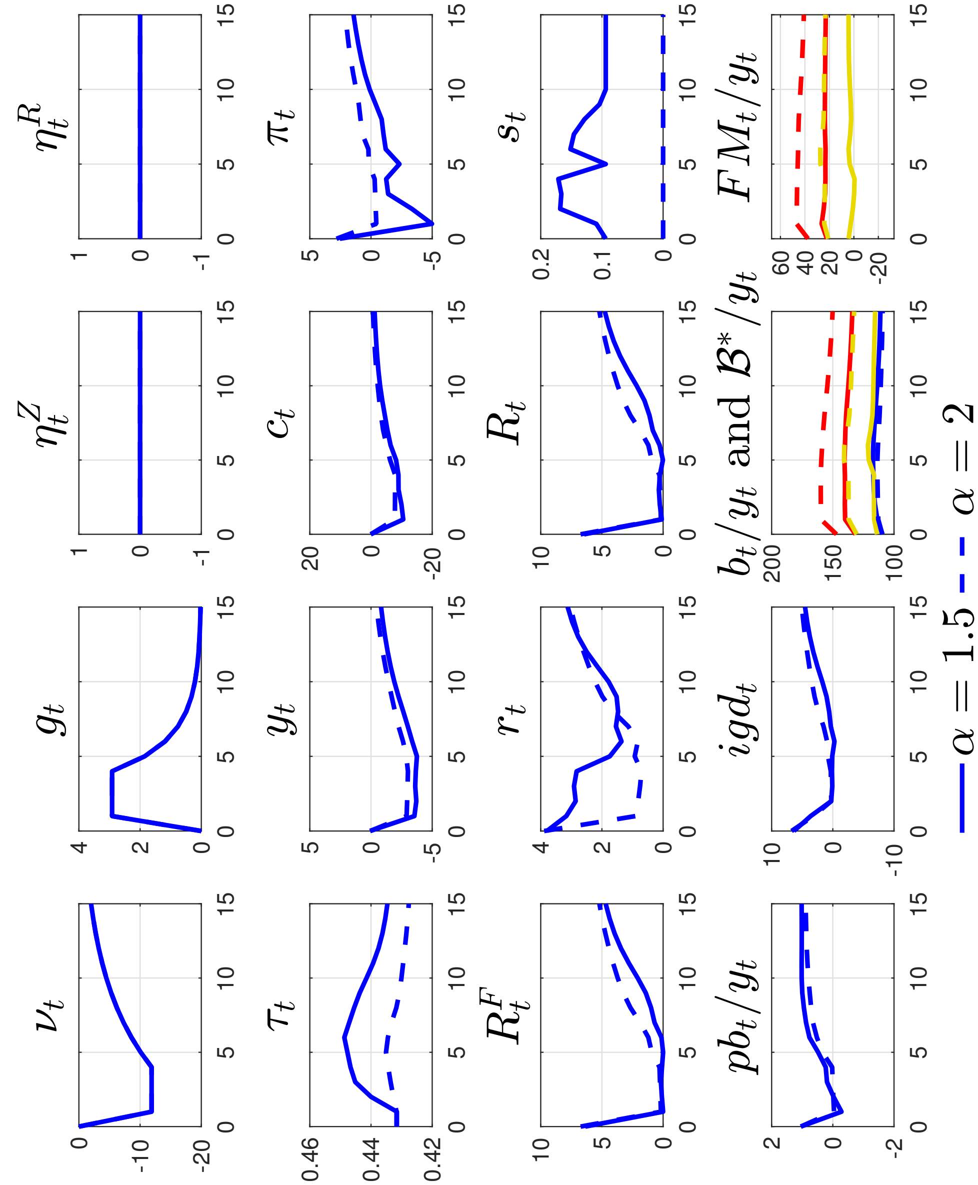
We analyzed fiscal sustainability and monetary fiscal policy interactions through the lens of a DSGE model with risky sovereign debt à la Bi (2012). We found that a looser monetary

Figure 6: Impact of a 3-quarters government spending shock in presence of binding ZLB



Note: The figures represent a scenario in which the ZLB is binding due to a persistent preference shock and $\alpha = 1.5$. The solid line describes the evolution of the economy when spending is increased by approximately 3% of (steady-state) GDP for 3 quarters. The dotted line describes the evolution of the economy absent shock to government consumption. The bottom right panel indicates the evolution of the fiscal margin corresponding to, respectively, a 30%(red line) and 0.4%(yellow line) default probability threshold .

Figure 7: Impact of a 3-quarters government spending shock at the ZLB for different α.



Note: The figures represent a scenario in which the ZLB is binding due to a persistent preference shock and spending is increased by approximately 3% of (steady-state) GDP for 3 quarters. The solid line describes the evolution of the economy for $\alpha = 1.5$. The dotted line describe the evolution of the economy for $\alpha = 2$. The bottom right panel indicates the evolution of the fiscal margin corresponding to, respectively, a 30%(red line) and 0.4%(yellow line) default probability threshold .

policy stance implies larger inflation fluctuations that, by decreasing output and the tax revenue, negatively affect fiscal sustainability. In normal times this reduces the fiscal space available to increase government consumption. Indeed, by engaging in expansionary policies, the government risks to increase spreads that might reverse the initial stimulative effect of the policy.

Consistent with previous findings, at the ZLB an increase in government expenditure is effective in reducing the fall in output and consumption. Notably, it does so without affecting the sustainability of debt, which actually improves in our calibration. By creating inflationary pressure, an increase in government consumption is able to reduce the real interest rate. In turn, this relaxes the ZLB constraint and reduces the cost of servicing the debt. However, the effectiveness of the policy largely depends on the underlying monetary policy stance. A more responsive monetary policy in normal times is better able to peg inflation expectation to the target. In turn, this dampens the deflationary pressure during periods of binding ZLB, allowing the real interest rates to fall further down, this way sustaining aggregate demand and increasing the fiscal space for the government.

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Appendix A Solution algorithm

Before calculating the fiscal limit distribution via Monte Carlo simulations, the absence of functional forms for all the endogenous variables requires the solution of part of the non-linear model (i.e., for the real wage, the inflation rate and the maximum tax rate), with numerical methods.

The solution is based on the monotone mapping method, developed by Coleman (1991) and Davig (2004), which discretizes the state space and conjectures candidate decision rules that reduce the system to a set of first-order expectational difference equations. The decision rules for the real wage $w_t^* = f^w(\psi_t^1)$, the inflation rate $\pi_t^* = f^\pi(\psi_t^1)$ and the corresponding maximum tax rate $\tau_t^* = f^\tau(\psi_t^1)$ are solved in the following steps.

1. Discretize the state space $\psi_t^1 = \{A_t, \nu_t, G_t, \eta^z, \eta^R\}$ [3]
2. For $i = 1, 2, ...$, make a guess for the decision rules $(f_g^w, f_g^\pi, f_g^\tau)$ over the state space. If $i = 1$, set $(f_g^w, f_g^\pi, f_g^\tau)$ to their steady state values; if $i > 1$, set $(f_g^w, f_g^\pi, f_g^\tau)$ to the solutions in the previous iteration $(f_{i-1}^w, f_{i-1}^\pi, f_{i-1}^\tau)$.
3. At each grid point, solve the model and obtain the updated rule $(f_i^w, f_i^\pi, f_i^\tau)$ using the given rule $(f_{i-1}^w, f_{i-1}^\pi, f_{i-1}^\tau)$ as a guess. Given equations 4, 16, 22 and 24 to pin down $(n_t, \Upsilon_t, R_t^F, c_t)$, respectively, use the model equations 5, 15 and 21 to solve the non-linear model and determine the decision rules $(f_i^w, f_i^\pi, f_i^\tau)$. In particular, maximize 21 subject to the non-linear constraints 5 and 15 and non-negativity constraints on endogenous variables as appropriate. The integrals implied by the expectation terms are evaluated using numerical quadrature. The exogenous AR(1) processes are approximated as first-order Markov processes according to the quadrature approach by Tauchen and Hussey (1991).
4. Notice that $(f_{i-1}^w, f_{i-1}^\pi, f_{i-1}^\tau)$ are assumed to be decision rules at t+1 when evaluating expectations, as they provide a set of intra-temporally consistent solutions for the optimising agents. To ensure that the solution is also inter-temporally consistent, establish a rule to check convergence of the decision rules $(f_i^w, f_i^\pi, f_i^\tau)$ and $(f_{i-1}^w, f_{i-1}^\pi, f_{i-1}^\tau)$ as follows:
   (a) if $\max\{|(f_i^w, f_i^\pi, f_i^\tau) - (f_{i-1}^w, f_{i-1}^\pi, f_{i-1}^\tau)| > 1e-6\}$, go back to step 2;
   (b) otherwise, $(f_i^w, f_i^\pi, f_i^\tau)$ are the decision rules.

To solve the full model, the same algorithm is used, but with an enlarged state space that embeds the stock of debt b_t as a state variable.

Finally, notice that a "solution of the model" includes a set (i.e., matrix) of one-to-one relationships between a specific value for the vector of state variables and a specific value for the vector of control variables. So, the model needs to be solved only once before the simulation of the fiscal limit distribution. When we calculate the fiscal limit distributions all the control variables are readily available as a function of the state variables either through functional forms or through the one-to-one relationships between state and control variables established in our first step.

[3] The number of grid points and state variables actually considered can vary depeding on the problem at hand, in order to deal with curse of dimensionality