# Secret vs Public Rings in Private Value Auctions

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## Introduction

- The competition-inducing nature of auctions make them a widely-used allocation mechanism. Increased competition increases the likelihood that the auctioneer receives higher proceeds from sale. Furthermore, auctions by nature are viewed as "fair" mechanisms as they represent public platforms where all interested parties can compete for the item(s) on offer. In this light, auctions are favored by bidders as well [Rothkopf and Harstad, 1994].
- Collusion by a subset of bidders has the consequence of limiting competition, thus reducing the seller's expected revenue.
- The high incidence of rings at auctions suggest that ring presence at auctions is the norm, rather than the exception [Marshall and Marx, 2012].

With this in mind, a seller who decides to conduct an auction is wary of the fact that there may be bidding rings present. Furthermore, individual bidders who also attend the auction also take into account that they may be coming up against a bidding ring.

## Who Knows What?

Regarding whether or not other bidders (henceforth ***nonring bidders***) are aware of ring presence at the auction, we have a number of scenarios (which we call ***Informational Structures***) (these are by no means exhaustive):

– ***Non Cooperative*** : The benchmark case where there is no ring present and no one suspects that a ring is present.

– ***Concealed*** : The ring operates in secret, and the nonring bidders are unaware of their presence, nor do they suspect that a ring will be in operation.

– ***The Private Knowledge Case***: The ring believes it is operating in secret, but the nonring bidders know about the ring's presence.

– ***Public Knowledge*** : The ring is present and it's presence is common knowledge.

– ***Suspicion*** : There is no ring present at the auction, but each participant suspects that there is a ring.

## Questions We Seek to Answer

We analyse the following queries along First Price and Second Price auctions.

- Given that a ring is present at the auction;
  - Will the auctioneer prefer that other participants know about their (the ring) presence, or prefer that the ring remains concealed?
  - How do nonring bidders react to knowing that a ring is present at the auction ?
  - Will the auctioneer prefer that ring presence is public knowledge or private knowledge?
- When there is no ring at the auction;
  - What happens when bidders suspect that a ring will be present, and how does this affect seller's expected revenue?
- Comparing the first price and second price auctions, which format will be preferred by the auctioneer in each of the informational structures?

## A Few Key Insights

- ***Rings are bad for the auctioneer in general. However, loss in expected revenue varies across informational structures.***
- ***A common conclusion is the idea that the first price auction is more robust to collusion than the second price auction. However, we show that this is contingent on the assumed informational structure. In particular, we find that the second price auction is more robust than the first price auction under private knowledge.***
- ***In the first price auction, simply suspecting that a ring is present induces bidders to depress their bids, leading to a loss in revenue for the auctioneer.***

## Setup

- There is a single item for sale to $N \geq 3$ risk-neutral bidders $\mathcal{N} = \{1, 2, \ldots, N\}$, each $i \in \mathcal{N}$ assigning independent private values $X_i \in [0,1]$ drawn from an atomless distribution $F(\cdot)$ with corresponding density $f(\cdot)$.
- We simplify the analysis in certain portions by assuming $F(x) = x$. That is, the underlying distribution will be assumed to be uniform.

**Description of the Ring**

- An ***Almost all-inclusive*** ring, consisting of $N-1$ bidders.
- The ring member with the highest value is designated as the ***ring representative***, and submits the only serious bid on behalf of the ring. Other ring members submit phony bids (normalized to zero). In this regard, the auction becomes asymmetric, with the ring representative having value distribution $G(x) = F(x)^{N-1}$.

Subsequently, we assume that bidders use symmetric increasing equilibrium strategies where possible. The objective of the auctioneer is to maximize revenue. We also adopt the convention of referring to the non-ring bidder with male pronouns, and the ring representative with female pronouns.

We use the term *auctioneer* and *seller* interchangeably.

## The First Price Auction

We state the results in terms of $F(x) = x$ for simplicity.

**The Non Cooperative Case**

- ***Bidding Strategies***: The symmetric equilibrium biding strategy involves a bidder with value $x$ bidding according to the function $\beta(x) = \frac{N-1}{N}x$ with inverse $\phi(b) = \frac{N}{N-1}b$, for any bid $b$.
- ***Seller's Expected Revenue***: $\mathbb{E}_1[\mathbb{R}_{\text{No Ring}}] = \frac{N-1}{N+1}$

**The Concealed Case**

- ***Bidding Strategies***: Denote the nonring bidder's value as $x_N$. Since the nonring bidder is unaware of the ring presence, he still uses the strategy $\beta(x_N) = \frac{N-1}{N}x_N$. The strategy of the ring representative is to best-respond to this. Denote the ring representative's value as $x_R$. We show in general that

**When the ring representative is bidding on behalf of the ring with the belief that the ring presence is concealed, the bid $\tilde{b}$ that he submits is lower in comparison to his bid $b$ when he is bidding non-cooperatively. That is**

$$b \geq \tilde{b}$$

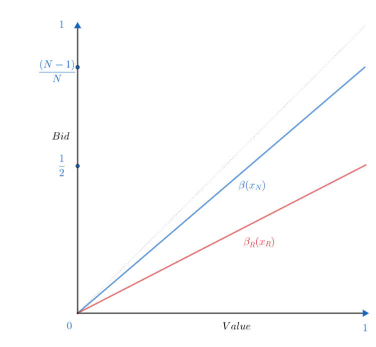
*The intuition for this result is straightforward. Forming the ring reduces the number of active bidders in the auction, hence it is less competitive. So there is a lower chance that the he faces a bidder with a value at least as high as hers. This implies that the ring representative can afford to shade her bid further than in the non-cooperative case.*

- For $F(x) = x$, $\tilde{b} \equiv \beta_R(x_R) = \frac{x_R}{2}$.
- ***Seller's Expected Revenue***: The seller's expected revenue in this case is $\mathbb{E}_1[\mathbb{R}_{\text{Concealed}}] = \frac{5N^2-4}{8N(N+1)}$

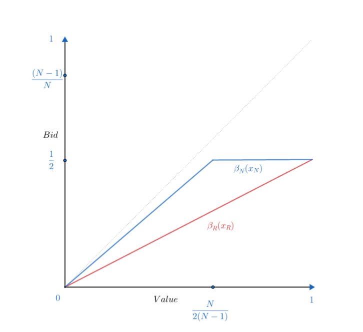
## The First Price Auction

**The Private Knowledge Case**

- ***Bidding Strategies***: The ring believes it is operating in secret, so the ring representative still maintains the strategy $\beta_R(x_R) = \frac{x_R}{2}$. The strategy of the nonring bidder will be to best respond to this. We find that the best response for the nonring bidder is $\beta_N(x_N) = \min\left\{\frac{N-1}{N}x_N, \frac{1}{2}\right\}$.
- ***Seller's Expected Revenue***: The seller's expected revenue is $\mathbb{E}_1[\mathbb{R}_{\text{Private}}] = \frac{2N^2-N-2}{4(N^2-1)}$.



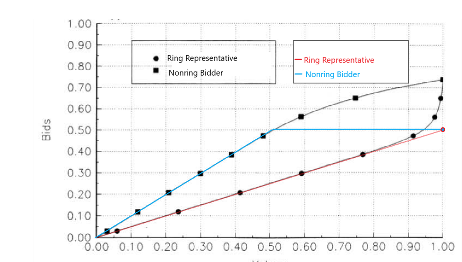
(a) Strategies Under Concealed



(b) Strategies Under Private Knowledge

**The Public Knowledge Case**

- ***Bidding Strategies and Seller's Expected Revenue***: In this case, since the presence of the ring is common knowledge, the auction is effectively a two bidder asymmetric auction, with bidder values drawn from $F(x) = x$ and $G(x) = x^{N-1}$. The strategies are characterised by a system of differential equations given in [Maskin and Riley, 2000],and [Lebrun, 1999], for which no closed form exists in general. However, we use numerical estimates from [Marshall et al., 1994] for $N = 101$ for the comparisons that follow.



(c) Public vs Private Knowledge

| Auction Format | No Ring | Concealed | Private Knowledge | Public Knowledge |
|---|---|---|---|---|
| First Price | 0.9804 | 0.6188 | 0.4975 | 0.6578 |

(d) Revenue Comparison

***In the first Price auction, the best case for the auctioneer is when all bidders bid noncooperatively. However, in the presence of the ring, the auctioneer prefers that their presence be public knowledge. Otherwise, it is better that the ring remains concealed.***

## First Price vs Second Price

Here, we compare the seller's expected revenue across auctions and informational structures.

| Auction Format | No Ring | Concealed | Private Knowledge | Public Knowledge |
|---|---|---|---|---|
| First Price | 0.9804 | 0.6188 | 0.4975 | 0.6578 |
| Second Price | 0.9804 | 0.4999 | 0.4999 | 0.4999 |

***Across all informational structures, the seller is better off under the first price auction, except when the ring presence is private knowlege.***

## What happens when Bidders Suspect a Ring will be Present when there is no ring?

In the first price auction, when individual bidders suspect they may be facing a ring at the auction, each bidder will use the strategy $\hat{\beta}(x) = \min\left\{\frac{N-1}{N}x, \frac{1}{2}\right\}$. This yields lower bids than under noncooperation, where they use $\beta(x) = \frac{N-1}{N}x$. Hence the suspicion of ring presence depresses the seller's expected revenue.

However, since it is still a dominant strategy for bidders to bid truthfully under the second price auction, the seller's expected revenue is unchanged in this case. So we can state the following.

***When bidders come into an auction suspecting that they might be facing a ring, the seller is better off using the second price auction relative to the first price.***

**Remark**: One implication of this is that it may help explain off-equlibrium bids observed in first price auctions. In essence when data on bids show bidding below the non cooperative equilibrium predictions, it may be due to the fact that bidders went into the auction suspecting that they might be facing a ring, and adjust their bids accordingly.

## Bidder Preferences across Auction Formats and Informational Structures

**Below are the expected profits for bidders across auctions and informational structures.**

| Auction Format | No Ring | Concealed | Private Knowledge | Public Knowledge |
|---|---|---|---|---|
| First Price | 0.000097 | 0.002475 | 0.002475 | 0.0025 |
| Second Price | 0.000097 | 0.0049 | 0.0049 | 0.0049 |

(e) Ring Member's Profit Across Auctions

| Auction Format | No Ring | Concealed | Private Knowledge | Public Knowledge |
|---|---|---|---|---|
| First Price | 0.000097 | 0.003712 | 0.1249 | 0.0412 |
| Second Price | 0.000097 | 0.000097 | 0.000097 | 0.000097 |

(f) Nonring Bidder Profit Across Informational Structures

***The second price auction is the most favorable for the ring. As far as informational structures, ring members earn more (weakly) under the public knowledge case compared to all other cases. Meanwhile, conditional on ring presence, the nonring bidder benefits most from the suppressed competition in the first price auction. Furthermore, he is most happy when the ring presence is privately known to him.***

## References

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