Teaching an Old Dog a New Trick: Reserve Price and Unverifiable Quality in Repeated Procurement

Teaching an Old Dog a New Trick:
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in Repeated Procurement[*]

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Abstract

In many circumstances, procurement contracts entail crucial unverifiable dimensions. In a repeated procurement auction framework, we show that by strategically using the reserve price, a buyer is able to elicit the provision of unverifiable quality. Thus the buyer can exploit an effective incentive mechanism, while treating equally all competing firms. We study an infinitely repeated procurement model with many firms and one buyer who is imperfectly informed on the firms' cost. In each period, the buyer runs a standard low-price auction with reserve price. We study the cases of players using grim trigger and stick-and-carrot strategies, analysing both the case of a committed and uncommitted buyer. We find that a competitive process with reserve price is able to elicit the desired level of unverifiable quality provided that the buyer's valuation of the project is not too high and the value of unverifiable quality is not too low; under these conditions, the buyer can credibly threaten the firms to set, in case a contractor fails to deliver the required quality level, a reserve price so low that the selected contractor (if any) makes zero profits. A committed buyer can elicit the desired quality level for a wider range of preference parameters.

Keywords: procurement, relational contracts, unverifiable quality, reserve price.

1   Introduction

During the last two decades public procurement has undergone profound changes. Policy makers, academics and practitioners alike share the broad view that public procurement has evolved from a clerical signoff-ridden set of activities to a strategic tool to pursue a wide array of socially relevant objectives. Among the different procurement systems, there is a wide consensus on the most important objectives being the *efficiency* in the acquisition of required goods, works or services and in the procurement process; *integrity*, that is, avoiding corruption and conflicts of interest; *equality and fairness of treatment* for providers.

In many circumstances, open competition in procurement processes is an effective procedure to select the most efficient contractor, ensuring also that the other above mentioned objectives are met. The efficiency properties of open competition, however, hold when the quality of the procured object is verifiable by a third party at a reasonable cost. The procurement contract can indeed be designed so as to deter the contractor from breaching contract clauses, and from reneging on promised quality levels.[1] There exist cases, though, where procurement contracts are characterised by performance dimensions that are observable by contracting parties, but cannot be objectively measured. Examples comprise Information Technology or management consulting services, where it is virtually impossible to measure a consultant's pro-activeness or his/her ability to provide innovative solutions. Lack of verifiability may also affect quality dimensions such as a software's degree of user-friendliness or the palatability of catering services.

Although, in principle, competitive procedures are deemed to work ineffectively when quality is non-contractible[2], a buyer may enhance a standard competitive process so as to provide the necessary incentives for non-verifiable quality provision. In a companion paper, Albano et al. (2017) show that a public buyer may decide to evaluate firms by using hard information such as bids as well as past performance in executing similar tasks in the past. An alternative solution would be the exclusion from the competitive process of firms in case of a poor past performance (see Calzolari and Spagnolo, 2013). Hence, a *discriminatory* auction is instrumental to enforce the socially optimal level of quality, provided that the overarching regulation allows the buyer to evaluate firms' past performance and deal with different firms on different terms, which is the case, for

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[1]This conclusion would hold under the assumptions of effective contract management and law enforcement.

[2]Manelli and Vincent (1995) show that negotiation-like procedures with a restricted set of highly reputable suppliers perform better than competitive procedures. Bajari et al. (2009) provide empirical evidence in the case of the non-resident private sector building contracts in Northern California from 1995 to 2000.

instance, of the Federal public procurement regulation in the United States.

The main objective of the current paper is to prove that a public buyer can strategically use the reserve price in a standard low-price auction as a *non-discriminatory* tool to select the most efficient supplier and implement the desired level of non-verifiable quality. Thus, we show that the pursue of the objectives of efficiency and equality of treatment in procurement processes is not in contrast with eliciting a required level of unverifiable quality. The relevance of this result especially applies to those procurement systems in which past performance cannot be taken into account in tenders evaluation and the procurer is bound to equality of treatment across bidders, such as in the case of the EU public procurement regulation and of the UNCITRAL Model Law.

In a context of a repeated interaction, the relationship between the buyer and the firms takes the nature of a relational procurement contract (RPC, henceforth). Under this RPC, the buyer selects the most efficient supplier by means of an auction with reserve price and obtains the desired level of quality by setting a high reserve price as the selected contractor delivers the quality level the buyer desires, and a low reserve price otherwise. On the other hand, firms, when awarded the project, provide the required quality as long as the buyer has set a sufficiently large reserve price, and a lower quality otherwise. This relational contract is *non-discriminatory* (in the language of the auction literature) or *multilateral* (in the language of the relational contract literature, see Levin, 2002), in that a deviation by any player results in a breakdown of all existing relationships.

We study a model with $N \geq 2$ firms and one buyer imperfectly informed on the firms' cost. In each period, the buyer awards the contract by means of a standard low-price auction with reserve price. We first study the cases of players using grim trigger strategies and analyse both the case of a *committed* and *uncommitted* buyer; we then consider the case of stick-and-carrot strategies. Quality enforcement is carried out through a RPC whereby the buyer may apply a "cooperative" (that is, high) reserve price to reward the contractor when the required quality is delivered, and threaten to set a "punishment" (that is, low) reserve price in future competitive tendering processes if any contractor fails to provide the required quality.

When the buyer is *uncommitted*, a likely feature of a private buyer, we find two types of equilibria, which are determined by the level of the "cooperative" reserve price. In first type of equilibrium, the "cooperative" reserve price is 'high': Along the equilibrium path, the buyer's desired quality is always delivered and the contractor's rent is limited by competition only. In the second type of equilibrium, the "cooperative" reserve price is 'low': Along the equilibrium path, the buyer runs the risk of not awarding the contract when all competing firms draw the highest possible (fixed) cost. Both types

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of equilibria entail a "punishment" reserve price equal to the lowest (fixed) cost. Given the multilateral nature of the relational contract, when setting the "punishment" reserve price, the buyer anticipates that no quality would be delivered during the "punishment" phase, therefore she sets the reserve price so as to minimise the purchasing cost of the project.

We also characterise the optimal contract(s), that is, the contract(s) yielding the buyer the highest utility. We find two optimal contracts according to the value the buyer attaches to quality. When the buyer cares sufficiently enough about quality, the optimal contract entails a high "cooperative" reserve price, which never constraints firms' bids under any cost configuration. The project is always awarded to the most efficient firm and quality always delivered. When, instead, the buyer cares less about quality, the project is not always delivered as the optimal contract entails a lower "cooperative" reserve price, which only makes the more efficient firms able to participate in the auction. We also find that an optimal contract induces the delivery of the socially optimal quality only when its valuation is sufficiently high. When, instead, the valuation of quality is not too high, only sub-optimal levels of quality are enforceable.

Under the assumption of *commitment*, a more likely feature of a public buyer, the buyer can enforce the provision of quality by threatening the competing firms to use a "punishment" reserve price higher than the cost of the efficient firm. The main difference with the *uncommitted* case is that the committed hypothesis makes it credible for the buyer to threaten a cheating firm with a softer "punishment" reserve price. Since this softer reserve price leaves a strictly positive rent to an efficient cheating firm, the lowest "cooperative" reserve price inducing the delivery of quality is higher than the one under the *uncommitted* assumption. The new higher "punishment" reserve prices makes the conditions on the discount factor more stringent with respect to the *uncommitted* case. In other words, all other things being equal, when the buyer is *committed* the delivery of quality needs more patient firms. Unlike the *uncommitted* case, since the buyer is now unable to deviate form her strategy, an equilibrium exists even for low levels of the preference parameter for quality and for high levels of the intrinsic value of the project. In terms of optimal contract, similarly to the *uncommitted* case, any sufficiently high "cooperative" reserve price is optimal. However, unlike the *uncommitted* case, a *committed* buyer is always able to enforce her desired level, even when her evaluation is low.

We finally check for the robustness of our results by looking at the case of players using stick-and-carrot strategies; these strategies entail a time-limited punishment, implying that players are not willing to give up forever the value of their cooperative

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interaction in case of a deviation. We find that, when firms are close to be infinitely patient, it is possible to induce the delivery of quality even with a punishment of finite length, provided that this length is above a minimal level.

The paper is organised as follows. After reviewing the related literature in Section 2, we describe the model in Section 3. Section 4 analyses the static game, while Section 5 looks at dynamic game with grim trigger strategies, under the different assumptions of an *uncommitted* and a *committed* buyer. Section 6 analyses the case of players using stick-and-carrot strategies. Section 7 concludes. All proofs are relegated to the Appendix.

2   Related literature

Our paper contributes to a growing literature studying the enforcement of unverifiable quality in procurement, first analysed by Lewis and Sappington (1991), Laffont and Tirole (1993) and Che (1993). While these papers and most of the ensuing literature look at this issue in the static setup of incentive and auction theory,[3] we set up our analysis in the context of a repeated interaction between our players, thus investigating relational contracts and the incentives that they create to provide quality. Relational contracts are informal agreements and unwritten codes of conduct that are sustained by the value of future relationships, and are applicable in cases where the outcome of a repeated relationship is based on some unverifiable variables.[4]

Several papers have conducted the analysis of opportunistic behaviour in repeated procurement in the context of a long-term relationship. Klein and Leffler (1981) show that an optimal strategy for the buyer is to promise a rent to the contractor under the threat of terminating the relationship in case of opportunistic behaviour. When the buyer faces more than one potential supplier, following the approach in Levin (2002) relational contracts can be classified as either *bilateral* or *multilateral* contracts: in the former case, any deviation by a player triggers a reaction by his/her counterpart which affects (at least, in a direct way) their relationship only; in the latter case, a deviation by a player affects all the relationships. While Albano et al. (2017) and Doni (2006) study bilateral RPCs allowing the buyer to freely select any magnitude of the handicap levied on an opportunistic contractor, Calzolari and Spagnolo (2013) analyse RPCs under both assumptions, although restricting the buyer's handicapping strategy to its harshest form

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[3]See, for instance, Hanazono et al. (2013) and Giebe and Schweinzer (2015).

[4]Relational contracts have been pioneered by Bull (1987) and MacLeod and Malcomson (1989) and applied in several fields: labour market (MacLeod, 2003; Levin, 2003; and Li and Matouschek, 2013), interaction between/within firms (Baker et al., 2002; and Rayo, 2007), regulation (Cesi et al., 2012) and experimental economics (Fehr and Schmidt, 2007; and Bigoni et al., 2014).

(i.e., *debarment*).

Che (2008) provides and reviews solutions to the problem of unverifiable quality, other than in the context of relational contracts. Among others, Taylor (1993) and Che and Hausch (1999) introduce an option contract whereby the supplier pays a fee to the buyer, who then may accept or reject (at no penalty) the provision at a price equal to its desired level of quality. Che and Gale (2003) show that a buyer may be better off by allowing suppliers to bid on their reward, as in a standard auction. Other papers, in line with Manelli and Vincent (1995), study the enforcing power of competitive procedures versus negotiations when quality is unverifiable; Tunca and Zenios (2006), among the others, study the interaction between a competitive auction and a relational contract. Buyers procure low-quality products by running a competitive price auction and high-quality products by means of a relational contract with a single supplier. They find that, for some values of quality, the use of competitive auctions for low-quality products may ease the enforcement of relational contracts for high-quality products.[5]

In our model, the buyer is able to enforce a *multilateral* RPC by strategically using the *reserve price*. The reserve price plays a major role in the auction literature. However, to our knowledge this is the first paper showing that the reserve price can be used as an effective dynamic incentive device to enforce noncontractible quality. In selling auctions with private values under incomplete information, Riley and Samuelson (1981) show that a profit-maximising seller always sets a level of the reserve price strictly higher than her valuation for the object, thus running the risk of not trading with any of the competing buyers. Hence, a trade-off between revenue and efficiency arises.[6] When bidders' values are affiliated – which is a special form of correlation –, the seller may use the reserve price to signal her private information about the value of the object which can used by competing bidders to fine-tune their bids. This is explored, for instance, by Cai, Riley and Ye (2007). If a cartel may rig the bidding process, the seller faces also the dilemma of whether or not to publicly announce the reserve price. On the one hand, announcing the reserve price does provide a clear focal point to a(n) (all-inclusive) cartel, but maximises the chances of trade taking place; on the other hand, keeping the reserve price secret makes the cartel's choice of a focal point more difficult and thus raises the chances that the object remains unsold (see McAfee and McMillan, 1992). In our paper, we focus on

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[5]See, also, Branco (1997), Wang (2000), Kessler and Lülfesmann (2004) and Milgrom (2004). Empirical analyses provide mixed evidence. Bajari et al. (2009), studying the private-sector building contracts in Northern California from 1995 to 2000, find that those awarded by negotiation perform better. On the other hand, analysing the procurement of regional railway services in Germany, Lalive et al. (2015) find that auctioned lines provide a higher frequency of service (seen as a proxy for quality) compared to lines awarded by direct negotiations.

[6]The result is shown to hold in a more general setting by Myerson (1981).

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the role of the reserve price as an incentive device to elicit noncontractible quality, but we abstract away from the risk of collusion in the relevant procurement market.

3   The model

The players. A buyer wishes to procure a project of fixed size. The project is procured repeatedly in each period of time t, with $t = 0, \ldots, \infty$. The quality of the project, denoted by q, may vary, so that $q \in [0, \infty)$. The gross utility the buyer derives from the project is equal to $v + sq$, where v denotes the value of the project, regardless of its quality, and s is a taste parameter for the intrinsic quality of the project. The buyer's net utility is then $U(v, q, p) = v + sq - p$, where p is the price the buyer pays for the project.

A group of N firms are able to provide the project.[7] At each t, firm i's cost (with $i = 1, \ldots, N$) is given by $C_t(\theta_{it}, q_t) = \theta_{it} + \psi(q_t)$ (we will drop the subscript t whenever possible). The fixed firm-specific cost component θ_i is a discrete random variable whose possible realisations are θ_L and θ_H, with $\theta_L < \theta_H$; we let $\Delta\theta \equiv \theta_H - \theta_L$. At all t, we let $Prob(\theta_{it} = \theta_L) = \beta$, where $\beta \in (0, 1)$. The other cost component $\psi(q)$ is the cost of quality; it is identical across firms when the quality they provide is also identical. We assume that $\psi(q)$ is increasing and convex in q, and satisfies $\psi(0) = 0$; Thus, firm i's profits are given by $\pi_i \equiv \pi(p, q) = p - \theta_i - \psi(q)$ when it provides the project, and 0 otherwise. We assume that firms have no access to credit, so that they face a single-period non-negativity constraint on profits.[8]

To ensure that the procurement activity is always socially beneficial, irrespective to the realisation of the contractor's cost, and that the provision of quality is desirable, we focus on the cases in which $v \geq \theta_H$ and $q > \frac{\psi(q)}{s}$. Accordingly, we set $\underline{s}(q) \equiv \frac{\psi(q)}{q}$ and assume $s \geq \underline{s}(q)$. All players have a common discount factor equal to δ, where $\delta \in [0, 1)$.

The competitive tendering. In each period of time, the buyer awards the project running a low-price auction with reserve price. More specifically, at all t, the competitive procedure is such that the buyer first publicly announces a reserve price r and then firms make their bids. If a bid is above the reserve price, it is excluded from the auction. The project is awarded to the lowest-bid firm; in case of a tie, the project is awarded by using a fair random device.

Bids are mono-dimensional. This is because, despite the project has varying quality, quality is assumed to be not verifiable (see below). Therefore, quality is not contractible

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[7]The buyer is referred to as *she*, while firms are referred to by the gender neutral pronoun.

[8]See Arve (2014) for an analysis of a repeated procurement market with financially constrained firms.

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and cannot be made part of the bid. Firms face no bidding costs.[9]

Informational structure. We look at a game of incomplete information. The buyer has incomplete information on the firm's cost; on the other hand, the rivals' and its own cost parameters are perfectly known by each firm when they are realised. Everyone observes the reserve price the buyer sets and the bids made by all firms. On the other hand, the quality offered by a contractor is observed by the buyer only. Despite quality is perfectly observable, the lack of verifiability makes it non-contractible and non-enforceable by a court of law. This simple informational structure makes our problem tractable and isolates the buyer's main problem, namely to select the most efficient supplier at the (end of the) bidding stage of the procedure, and to induce the required level of the quality of the project at the execution stage.

The game. We analyse an infinitely repeated game resulting from an infinite repetition of the following sequential stage game:

stage 1: the buyer sets and announces a reserve price r;

stage 2: (the *bidding stage*) firms learn their cost parameters and make their bids; the project is awarded by means of a low-price auction with reserve price;

stage 3: (the *execution stage*) the contractor chooses the quality level and delivers the project. The buyer pays the contractor a price equal to its bid, quality is observed and all payoffs are collected.

4   The static game

In this section, we analyse the static constituent game; we show that, in equilibrium, the buyer cannot induce any quality different from 0. For future reference, it is first useful to concentrate on the bidding stage and characterise the firms' bids.

Lemma 1. *Let $\boldsymbol{q} \equiv (q_1, \ldots, q_N)$ be a vector of exogeneously given quality levels to be delivered by firms at the execution stage. Let $\underline{C} \equiv \min_k\{C(\theta_k, q_k)\}$ and $\underline{C}_{-k} \equiv \min_{h \neq k}\{C(\theta_h, q_h)\}$, for all k and h, be the lowest cost among all firms, and among all firms but firm k, respectively. Then, for any $\boldsymbol{q}$, the bidding stage of the static game admits a Nash equilibrium (in undominated strategies) whereby firm i bids $b_i = C(\theta_i, q_i)$ for all i, unless $\underline{C} < r$ and $C(\theta_i, q_i) = \underline{C}$ holds for firm i only, in which case $b_i = \min\{r, \underline{C}_{-i}\}$ and $b_j = C(\theta_j, q_j)$ for all $j \neq i$.*

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[9]This assumption would apply, for instance, when participation costs are of an order of magnitude much lower than the cost of the project itself.

The Lemma is a simple application of standard results on low-price auctions with a reserve price, and the intuition is very simple. Assume that the anticipated quality levels are given and the random components of the firms' costs are realised, so that costs are given (and known) to all firms. When the reserve price is (weakly) lower than the lowest cost, all firms find it optimal to bid their cost and be excluded from the competitive tendering.[10] When, instead, the reserve price is higher than the lowest cost, only the lowest cost(s) and either the reserve price or the second-lowest cost matter for characterising the awarding price. If only one firm has a cost advantage over all the rivals, this firm bids the lowest between the reserve price and the lowest of the rivals' costs, and is awarded the contract; if, instead, more than one firm enjoy the cost advantage, these more efficient firms bid their costs and the contract is awarded randomly between them. In both cases, all rival firms would not gain from placing a bid different from their cost.

The following Proposition characterises the equilibrium of the static game.

Proposition 1. *Let*

$$\overline{v} \equiv \frac{N\beta}{1-\beta}\Delta\theta + \theta_H. \tag{1}$$

The following strategy profile forms a subgame perfect equilibrium of the static game:
- stage 1: if $v \leq \overline{v}$, the buyer sets a reserve price $r = \theta_L$; otherwise, if $v \geq \overline{v}$, the buyer sets a reserve price $r \geq \theta_H$;
- stage 2: firms bid as described in Lemma 1, anticipating to deliver quality equal to 0;
- stage 3: the winning firm, if any, delivers the project, with quality equal to 0.

In equilibrium, all firms but the winning firm, say firm i, make zero profits. As to the winning firm i and the buyer, for all k, $\pi_i = b_i - \theta_i = \min\{r, \min_{k \neq i}\{\theta_k\}\} - \theta_i$ and $U = v - b_i$ if $\theta_i < r$ and $\min_k\{\theta_k\} = \theta_i$ holds for firm i only, and $\pi_i = 0$ and $U = v - \theta_i$ otherwise.

In the final stage of the game, irrespective of the bid and the awarding price, the selected contractor always finds it optimal to deliver quality equal to 0. Hence, in the second stage, *all* firms bid anticipating to deliver no quality. In the first stage of the game, the buyer anticipates that no quality will be provided. When setting the reserve price, the buyer faces the 'standard' trade-off in a single-object low-price auction: she may set a 'high' reserve price so that all firms are willing to participate in the auction, irrespective to their cost, or she may charge a 'low' reserve price, so that only an efficient firm will actively bid. In the first case, clearly, the buyer benefits from the project for

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[10]In fact, because of the zero bidding cost assumption firms are indifferent between bidding their costs and not bidding at all. We assume that firms bid and that, as firms' bids are above the reserve price, the buyer rejects firms' bids.

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any possible realisation of the firms' fixed cost parameter; this comes, however, at the cost of a possibly higher awarding price. Conversely, a 'low' reserve price lowers the final price, but makes a firm unwilling to participate in the auction when inefficient; if *all* firms are inefficient, the buyer does not award the contract and does not benefit from the project.

The resolution of this trade-off hinges on the relationship between v and $\overline{v}$; when the value of the project is large, the buyer prefers to set a high reserve price to make sure that she will reap the utility she derives from the project under all cost realisations. The threshold, in turn, depends on the two fundamental parameters of the model: It is higher the larger the number of firms in the market and the larger their individual probability of having a low fixed cost. Both parameters indeed concur in making less likely the event of all firms being high cost, reducing the probability that the project is not completed. Also, $\overline{v}$ increases with θ_H and decreases with θ_L; indeed, the larger is the cost differential, the larger is the saving accruing to the buyer when the latter sets a low reserve price.

Finally, notice that the buyer's utility does not depend on the reserve price when this is sufficiently large not to constrain the firm's bids. On the other hand, when the reserve price is 'low' and binding, the buyer's utility is decreasing in the reserve price and the buyer clearly prefers to set the lowest admissible reserve price compatible with an efficient firm participating to the auction, i.e. θ_L.

5 The dynamic game with trigger strategies

We now turn our attention to the dynamic game given by an infinite repetition of the constituent game analysed in the previous section. We focus on a relational procurement contract (RPC) which describes, for any history of the game, the reserve price the buyer sets, the bids the firms make and the quality the contractor chooses. This RPC is self-enforcing if it describes a subgame perfect equilibrium of the repeated game.

We assume players adopt the following grim trigger strategies:[11]

> *Buyer*: in each repetition of the game, the buyer sets r_B^C if each firm i (with $i = 1, .., n$) delivered quality equal to $q_{B,i}^C$ in all previous periods in which it was awarded the project (if any). Otherwise, the buyer sets r_B^P.
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> *Firm* i $(i = 1, \ldots, N)$: in each repetition of the game, firm i bids as in

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[11]Clearly, many alternative strategies could be possible and, thus, our game clearly has multiple equilibria; in Section 6, we investigate the case of players using stick-and-carrot strategies.

Lemma 1 with $q_i = q_i^C$ and, if awarded the project, offers quality q_i^C if the buyer sets a reserve price r_i^C in the current and in all previous periods (if any). Otherwise, the firm bids as in Lemma 1 with $q_i = q_i^P$ and, if awarded the project, delivers quality q_i^P.

These strategies can be defined as $\sigma_B(r_B^C, r_B^P, q_{B,1}^C, \ldots, q_{B,N}^C)$ for the buyer, and $\sigma_i(q_i^C, q_i^P, r_i^C)$, for firm i, with $i = 1, \ldots, N$. However, in the rest of the paper we will focus on buyer's 'symmetric' strategies, that is, on strategies in which the quality the buyer requires from the winning firms is identical across firms; in other words, we set $q_{B,i}^C = q_{B,j}^C = q_B^C$, for $i, j = 1, \ldots, N$ and $i \neq j$. This restriction has several justifications: first, the buyer should ensure an 'equality of treatment' across the firms. Since firms are *ex-ante* identical, there are no grounds for differential quality requirements. Secondly, we will look whether and under what conditions the buyer is able to elicit the provision of optimal quality, that is, of the quality which maximises her utility. As we will discuss later, given the nature of the firms' costs, the optimal quality is independent from the identity of the contractor and from its realised cost. Notice also that quality has no value to the firms other than its strategic long-term value in the relationship with the buyer and results only in an additional cost. Hence, should the buyer deviate, the best punishment a firm would put in place is to choose $q_i^P = 0$. We can then unambiguously denote the buyer's and firm i's strategy as

$$\sigma_B(r_B^C, r_B^P, q_B^C) \text{ and } \sigma_i(q_i^C, r_i^C). \tag{2}$$

Before turning to the equilibrium analysis, it is useful to introduce the following definition:

Definition 1. *For any quality level q, let*

$$\rho_{low} \equiv \{r | r \in [\theta_L + \psi(q), \theta_H + \psi(q))\}; \tag{3}$$

$$\rho_{high} \equiv \{r | r \in [\theta_H + \psi(q), +\infty)\}; \tag{4}$$

$$\rho_{low}^0 \equiv \{r | r \in [\theta_L, \theta_H)\}; \tag{5}$$

$$\rho_{high}^0 \equiv \{r | r \in [\theta_H, +\infty)\}. \tag{6}$$

This definition introduces two pairs of interval for the reserve price which will be relevant in the rest of the analysis; the first pair (i.e. ρ_{low} and ρ_{high}) is of relevance in the case of a firm providing quality, while the second pair matters in the case of a firm not providing quality. When the reserve price lays in the interval ρ_{low}, only efficient

firms are able to cover their cost when providing quality; on the other hand, when the reserve price lays in ρ_{high}, all firms, irrespective of their efficiency level, are able to cover their cost when providing quality. A similar argument applies to ρ_{low}^0 and ρ_{high}^0, but in the case of firms not providing quality.

5.1 The case of an uncommitted buyer

In this section, we characterise a self-enforcing RPC whereby the players' strategies form a subgame perfect equilibrium of the repeated game. We characterise the equilibrium of the game by checking the conditions for the absence of profitable one-shot deviations (POSDs, henceforth) for each player (Mailath and Samuelson, 2006).

Our players have several possible deviations from their candidate equilibrium strategies. A firm may deviate at the *execution stage* once awarded the contract, when no previous deviation has occurred, by providing a quality different from the "cooperative" one. A firm may also deviate at the *bidding stage*, by making a bid that does not anticipate the full cost of quality; this raises the chances of winning the contract, but implies that no quality is provided at the *execution stage*. Finally, a firm may also deviate by not punishing a previous deviation by the buyer (i.e. off the equilibrium path); in this case it may provide a quality different from the one prescribed by its strategy in case of a reserve price different from the "cooperative" one. A buyer, too, has several possible deviations from her candidate equilibrium strategy. First, the buyer may prefer to set a reserve price different from the "cooperative" one in the absence of a previous deviation by a firm. Also, off the equilibrium path, it may prefer to forgive a previous deviation by a firm and set a reserve price different from the "punishment" one.

Our equilibrium analysis is illustrated in the following Proposition.

Proposition 2. *Let*

$$\overline{\delta} \equiv \frac{\psi(q)}{\psi(q) + \beta(1-\beta)^{N-1}\left[\min\{r^C - \psi(q), \theta_H\} - r^P\right]}; \tag{7}$$

$$\overline{s}_1 \equiv \frac{\psi(q)}{q} + \frac{(1-\beta)^N}{q}(\overline{v} - v); \tag{8}$$

$$\overline{s}_2 \equiv \frac{\psi(q)}{q} + \frac{N\beta(1-\beta)^{N-1}}{1 - (1-\beta)^N}\frac{r^C - \psi(q) - \theta_L}{q}. \tag{9}$$

If $v > \overline{v}$ (as in (1)), no self-enforcing RPC exists. Assume, instead, $v \leq \overline{v}$; when $\delta \geq \overline{\delta}$, the strategy profile $\sigma_B(r^C, r^P, q)$ and $\sigma_i(q, r^C)$ (with $i = 1, \ldots, N$) defines a self-enforcing RPC under which the project is awarded to (one of) the most efficient firm(s), which delivers quality q, under the following conditions:

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i) $r^P = \theta_L$ and $r^C \in \rho_{high}$, provided that $s \in [\bar{s}_1, +\infty)$;

ii) $r^P = \theta_L$ and $r^C \in \rho_{low}$, provided that $s \in [\bar{s}_2, +\infty)$.

The Proposition illustrates the conditions for the players' strategies to define a RPC under which the buyer is able to select the most efficient firm (or, in case of more than one firm more efficient than the rest, a randomly selected firm among them) and the firm awarded the contract is induced to deliver the required quality level.[12] The role of the different choice variables or market parameters on the conditions for the equilibrium existence is discussed below.

"Cooperative" and "punishment" reserve prices. Depending on the value of the equilibrium r^C, a self-enforcing RPC induces two different types of equilibria. In an equilibrium of the first type, which occurs when r^C is large, the project is always delivered; in the other type of equilibrium, when r^C is small, the project is delivered only under some conditions on the realisation of the firms' cost parameters. More specifically, when $r^C \in \rho_{high}$, the reserve price allows even an inefficient firm to submit a bid covering the cost of quality. When, instead, $r^C \in \rho_{low}$, an inefficient firm would not be able to recover its cost when delivering quality. Therefore, in order to avoid being awarded the project and being forced to renege on quality – something which would trigger a punishment by the buyer –, a firm willing to stick to its "cooperative" strategy has to bid above the reserve price. In the event of *all* firms being inefficient, all firms bid above the reserve price and the buyer is unable to have the project delivered.

Both types of equilibria are sustained by the threat of setting a "punishment" reserve price equal to the cost of an efficient firm, when not delivering quality. Because of the multilateral nature of the relational contract, a deviation by any firm triggers a punishment which affects all firms; the buyer then sets a reserve price which is based on the firms' fixed costs only (that is, not including the cost of quality), anticipating that no firm would be willing to deliver quality. While a reserve price equal or above θ_H is too mild a threat to induce a firm to cooperate, a reserve price above θ_L would induce the buyer to renege on her strategy and further lower the reserve price in order to reduce the contractor's rent.

Critical discount factor. The existence of a self-enforcing RPC requires firms to have an intertemporal preference parameter sufficiently large, that is, a discount factor above the critical level in (7). Notice that the discount factor does not play a role in the buyer's choices; because of the sequential nature of the stage game, the buyer is punished

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[12]These conditions are necessary and sufficient, given the players' strategies.

immediately in case of a deviation and does not, therefore, face a trade-off between short- and long-term effects of her choice. As to the firms, when the "cooperative" reserve price is high (that is, when $r^C \in \rho_{high}$), a firm's reward from a RPC does not depend on the level of the reserve price; thus, the critical discount factor depends on market parameters only. When, instead, the "cooperative" reserve price is tighter (that is, when $r^C \in \rho_{low}$), the firm's "cooperative" profits do depend on the level of the reserve price, which then affects the level of the critical discount factor: the lower the "cooperative" reserve price the more patient a firm has to be to stick to its strategy $\sigma_i(.)$. These features of the threshold level of the discount factor are illustrated in Figure 1, where the critical value of δ is plotted against r^C.

The critical value of δ increases with the cost of quality $\psi(q)$. This is simply because $\psi(q)$ is also the largest gain a firm may obtain when deviating; the larger is the gain from a deviation, the more patience is required from a firm not to deviate. An increase in the number of firms pushes $\overline{\delta}$ up; since, *ceteris paribus*, a higher N makes it less likely that a firm will be selected as a contractor in the future, it reduces the firm's expected payoff in the continuation game, making the condition on the discount rate more stringent. On the other hand, the relationship between the critical discount factor and the probability of firm being efficient is less clear cut. A higher β raises the probability of being efficient both for the current contractor and its rivals. This results in a non-monotone relationship between the expected continuation payoff (and, therefore, the critical discount factor) and β.

Although not explicitly modelled in the paper, varying the length of the procurement contracts affects the frequency of interactions among market participants, which, in turn, alters the value of the discount factor. More precisely, the shorter the duration of the procurement contract the higher the frequency of interaction, and thus the higher the value of the discount factor, which makes a RPC easier to arise. More noticeably, a higher frequency might be the result of the procurement contract being split in multiple similar lots. This might also bring about a higher participation (say, of small and medium-sized firms) in the procurement process through less stringent economic requirements. Since, as discussed more in detail below, a higher N has a positive effect on the buyer's expected utility along the equilibrium path, an immediate policy indication is that the buyer should prefer smaller and more frequently awarded contracts. This conclusion brings about further practical implications for *public* buyers. Public organisations often procure consulting services by resorting to *all-purpose* contracts which comprise a variety of heterogenous tasks to be fully described only when the specific needs arise. There are, though, two alternative procurements strategies, which would be consistent with the
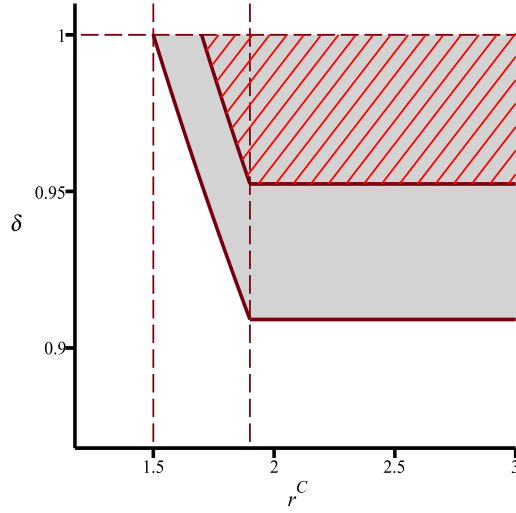
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Figure 1: Equilibrium combinations of δ and r^C (when $\beta = \frac{1}{2}$, $N = 2$, $\psi(q) = q^2$, $q = 1$, $\theta_H = \frac{9}{10}$ and $\theta_L = \frac{1}{2}$); grey shaded area relates to the case of an *uncommitted* buyer setting $r^P = \theta_L$, while the red-striped area to the case of *committed* buyer setting $r^P \in \rho^0_{low}$.

conclusion just derived. The first solution would consist in having each task become a separate procurement contract. The drawback of this solution would be running multiple rather than just one competitive process, which would dramatically increase transaction costs. The second one would consist in bundling the same set of tasks under the same *umbrella* agreement, from which a sequence of separate procurement contracts are to be awarded. This procurement technique is internationally known as *framework agreement*, namely a two-stage procedure whereby a set of firms are selected at the first stage on the basis of a *master contract* and those selected firms compete for a sequence of procurement contracts to be awarded through simplified competitive procedures. When appropriately designed, framework agreements can enlarge the set of potential participants with respect to all-purpose procurement contracts.[13]

Critical preference parameters. The existence of a self-enforcing RPC requires additional conditions on the buyer's preference parameters. The combinations of values of v and s ensuring the existence of a self-enforcing RPC are illustrated in Figure 2 (for given values of the other parameters). First, an equilibrium exists only provided that the buyer's evaluation of the project is not too high (i.e. $v \leq \bar{v}$). The reason is simple: when

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[13]For an extensive non-technical analysis of framework agreements see Albano and Nicholas (2016); Gur et al. (2017) provide a game theoretical analysis of framework agreements.

the buyer has a 'high' valuation of the project (i.e. $v > \overline{v}$), Proposition 1 illustrates that the equilibrium of the static game is with the buyer setting a reserve price equal to θ_H. In the dynamic game, the buyer could threaten to revert to this equilibrium after a deviation: however, a reserve price equal to θ_H entails too mild a punishment for the firms, which then do not have an incentive to "cooperate". Hence, it is only when v is sufficiently low that the buyer can credibly threaten to punish harshly enough a deviation to make the firms willing to cooperate.

Additionally, a high level of the taste parameter of quality is also needed (i.e. $s \geq \overline{s}_1$ or $\overline{s}_2$); this ensures that the buyer sufficiently cares about quality and is willing to set a high reserve price that induces firms to "cooperate", rather than setting a tight reserve price which limits the firms' rent, but which results in the contracting firm not providing the desired quality. Both in the case of a 'low' and 'high' "cooperative" reserve price, for the buyer to prefer to "cooperate", her taste parameter s must be larger than its lower bound, $\frac{\psi(q)}{q}$ increased by an extra term. When the "cooperative" reserve price is 'high', the extra term in $\overline{s}_1$ is equal to 0 when $v = \overline{v}$ and the buyer always prefers to "cooperate." This is because the buyer's "cooperative" and "deviation" utility *net of the cost of quality* (which is always fully repaid to the contracting firm along the equilibrium path) is identical across all cost combinations, while the preference for "cooperation" derives from the basic assumption of quality being socially desirable. When, instead, v takes on a lower value, under defection the buyer suffers from a lower value of the "basic" project under all cost combinations but the one when all firms are inefficient (when the project is not delivered); in expected term, the buyer loses $(1 - (1-\beta)^N)(v - \overline{v})$. On the other hand, under "cooperation", the buyer always suffers from the lower value of the project; in expected term, her losses are simply $(v - \overline{v})$. Therefore, in order to keep "cooperation" desirable, the buyer's valuation of quality must increase at least by the difference between these two expected values.

When the "cooperative" reserve price is 'low', the extra term in $\overline{s}_2$ may be interpreted along a similar line. The buyer's "cooperative" and "deviation" utility *net of the cost of quality* is identical across all cost combinations except when all firms but one are inefficient. The extra term is then larger the larger is the rent left to the contractor to make it deliver quality in this event (which occurs with probability equal to $N\beta(1-\beta)^{N-1}$), normalised by the overall probability of having the project (with quality) delivered (i.e., $1 - (1-\beta)^N$).

Market parameters and choices. In equilibrium, the buyer does benefit from a larger set of competing firms: A higher N raises the probability of the event yielding the buyer's highest payoff, namely the event whereby at two least two firms have drawn low fixed

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costs so competition wipes out firms' profits.

A change in the parameters capturing the features of economic environment affects not only the buyer's expected utility, as discussed above, but also the conditions ensuring the existence of an equilibrium. In what follows, we will conduct some simple comparative statics exercises assessing how a change in the number of firms in the market, N, and the quality level which is part of the equilibrium RPC affect the critical values of s and v; notice that, since, at this stage, the reserve price is exogenously set, it is assumed not to be affected by any change in these market parameters. In the next section, we shall relax this assumption.

A first effect of the change of these market parameters is on the higher threshold for v, $\overline{v}$; an increase in the number of firms, in the probability of a firm of being efficient, and also in the potential cost differential across firms pushes $\overline{v}$ up and enlarges the space of parameters under which a cooperative equilibrium is possible. The reason is clear: these changes all have a positive effect on the possibility of the existence of a static equilibrium under which the buyer sets a low reserve price. Since this equilibrium is the only self-enforcing threat in case of a firm's deviation, they also make cooperation more likely to occur.

We now turn to look at the effect of these market parameters on the threshold level for the buyer evaluation of quality, $\overline{s}_1$ and $\overline{s}_2$. An increase in the quality level part of an equilibrium RPC may push $\overline{s}_1$ and $\overline{s}_2$ either up or down. The reason is due to the twofold effect of an increase in q. First, it increases the cost of quality, as illustrated by the convex and increasing function $\psi(q)$; second, it increases the benefit the buyer receives when quality is delivered. Thus, in both cases of a low and high reserve price, an increase in q pushes the relevant threshold up (making cooperation more difficult) when the associated increase in the cost of quality is sufficiently large, and the associated increase in the benefits to the buyer is small enough.

A change in N affects the buyer's expected utility from cooperation and deviation only through the probability of the different events, since the buyer's utility under the different realisation of the firm's costs are independent from N. Consider first the case of a low reserve price. Under cooperation, an increase in N always increases the expected utility of the project, since it makes more likely the outcomes more favourable to the buyer. The 'size' of this increase in utility depends on the different levels of utility the buyer can obtain under the different cost combinations; it turns out that only the largest difference matters, which coincides with the difference between the firms' cost parameters, $\Delta\theta$. The buyer's utility in case of a deviation also increases when N goes up, since it makes it less likely that all firms are inefficient and the project is not delivered,

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which is the least favourable event for the buyer. In this case too, the 'size' of this effect turns out to depend only on the largest difference in the buyer's utility across the different events, which is now $v - \theta_L$. When N is small, the effect on the utility from a deviation is always larger and an increase in N always makes cooperation more difficult (that is, it requires a larger taste parameter for quality). When instead N is large relatively to β, if the value of the project is sufficiently small, the 'size' of the positive effect on the utility from a deviation is small and may be dominated by the equally positive effect on the "cooperation' utility.[14]

In the case of a high reserve price, the effects behind the relationship between $\bar{s}_1$ and N are the same as in the case of a low reserve price. However, the 'size' of the positive effect of a change in N on the buyer's expected utility – that is, the difference in the buyer's utility across the different events – is much larger. This ensures that the positive effect on the expected utility in case of a cooperation always dominates and an increase in N always pushes $\bar{s}_1$ downwards, making cooperation less difficult.

Overall, it is worth noting that varying any of the market parameters does not have a clear and unique effect on the space of parameters under which an equilibrium exists. For example, it is not possible to argue that an increase in N always makes cooperation easier to sustain; the actual outcome of a change of one market parameter on the existence condition for a cooperative equilibrium are dependent on the other market parameters. As a result, while some clear policy prescriptions can be derived in terms of the desirability of the change in some market parameter - for instance, pointing towards the desirability of enlarging the number of potential participants in the competitive procedures –, all these changes need to be carefully examined in terms of their effect on the equilibrium existence conditions.

5.1.1 Optimal contract and optimal quality with an uncommitted buyer

The previous section illustrates that there exist many RPCs which implement the the buyer's desired quality. We now turn our attention to the issue of the *optimality* of the equilibrium. The reference to optimality may be interpreted in two different ways. First, one may ask which RPC ensures that the buyer obtains the highest utility for a given quality level: in the rest of the discussion, we define this RPC the *optimal* contract. Alternatively, one may ask if and under what conditions a RPC may induce the *optimal quality level*, that is, the buyer's most preferred quality level; in case of a public benevolent buyer, this is also the socially optimal level.

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[14]More precisely, when $N < -\frac{1}{\ln(1-\beta)}$, then it is always the case that $\frac{d\bar{s}_1}{dN} > 0$; on the other hand, when $N > -\frac{1}{\ln(1-\beta)}$, then $\frac{d\bar{s}_1}{dN} > 0$ if and only if $v > \bar{v} + \frac{\beta}{(1-\beta)\ln(1-\beta)}\Delta\theta$.
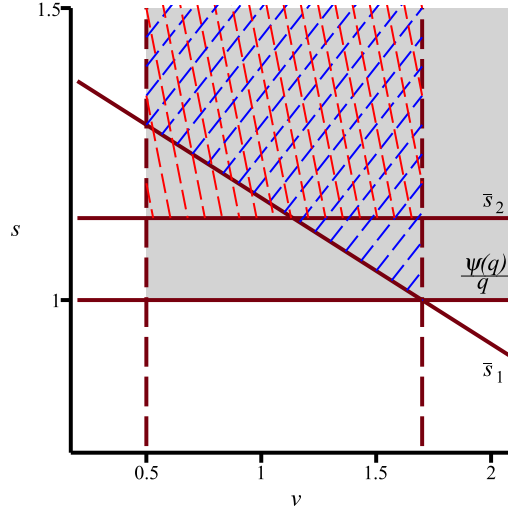
Figure 2: Combinations of v and s for which an equilibrium exists (when $\beta = \frac{1}{2}$, $N = 2$, $\psi(q) = q^2$, $q = 1$, $\theta_H = \frac{9}{10}$ and $\theta_L = \frac{1}{2}$)). In the case of an *uncommitted* buyer, an equilibrium entails r^P and r^C such that $r^P = \theta_L$ always and $r^C \in \rho_{low}$ for combinations of the parameters in the red-striped area and $r^C \in \rho_{high}$ for combinations of the parameters in the blue-striped area. In the case of a *committed* buyer, an equilibrium entails any r^P and r^C such that $r^P \in \rho^0_{low}$ and $r^P + \psi(q) \leq r^C$ for combinations of the parameters in the grey area.

We will address these two questions in the rest of this section. A preliminary result, which is instrumental to formulate our answers, is illustrated in the following Lemma.

Lemma 2. *Let*

$$\hat{r}^C \equiv \psi(q)\left(1 + \frac{1-\delta}{\delta}\frac{1}{\beta(1-\beta)^{N-1}}\right) + r^P. \tag{10}$$

Then,
i) when $r^C \in \rho_{high}$, all self-enforcing RPCs yield the buyer the same utility;
ii) when $r^C \in \rho_{low}$, the self-enforcing RPC yielding the buyer the highest utility entails $r^C = \hat{r}^C$.

This Lemma illustrates a very simple result. When the buyer sets a 'high' "cooperative" reserve price ($r^C \in \rho_{high}$), the reserve price does not constrain the firms' bids and, therefore, does not affect the buyer's utility; this implies that any sufficiently large reserve price gives a "cooperative" buyer the same utility level. On the other hand, when the buyer sets a 'low' "cooperative" reserve price ($r^C \in \rho_{low}$), this caps the firms' bids and, therefore, the price the buyer ends up paying for the project. In this case, given the

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firms' discount factor, the buyer finds it optimal to set the lowest reserve price consistent with the firm having an incentive to deliver the required quality. In other words, the reserve price which is part of an optimal RPC is the one that makes the firms' incentive compatibility constraint just binding.

Notice that, unsurprisingly, when the optimal reserve price is low, it is decreasing in the firm's discount factor and increasing in the number of firms in the market; when firms are less patient, or their continuation payoff is lower because of higher number of competitors, only a sufficiently high reserve price induces them not to deviate. The effect of β is, instead, indeterminate due to the non-monotone effect of a change in β on the probability for the firm to make a profit in future repetitions of the game.

The following Proposition illustrates the nature of the optimal RPC.

Proposition 3. *Let*

$$\bar{s}_3 \equiv \frac{\psi(q)}{q} - \frac{1}{q}\left(v - \bar{v} - \frac{1-\delta}{\delta}\frac{N\psi(q)}{(1-\beta)^N} \right) \tag{11}$$

Then, the self-enforcing RPC which gives the buyer the highest utility entails $r^P = \theta_L$ and
i) when $s \in [\bar{s}_2, \bar{s}_3)$, $r^C = \hat{r}^C$;
ii) when $s \in [\max\{\bar{s}_1, \bar{s}_3\}, \infty)$, $r^C \in \rho_{high}$.

When the buyer sufficiently cares about quality (i.e., $s > \bar{s}_3$), an optimal contract entails a 'high' (and never binding) "cooperative" reserve price: *all firms*, irrespective to their efficiency levels, always bid to be awarded the project, having their bids constrained by competition only, and the project is awarded to the most efficient firm. The project is always delivered and firms are induced to deliver quality by the threat of a stricter reserve price only. When the buyer cares less about quality (i.e., $\bar{s}_3 > s$), the optimal contract induces the buyer to apply a 'low' reservation price during the "cooperative" phase; its level is just high enough to make *only the efficient firms* willing to (actively) participate to the auction and win the project.[15] When all firms are inefficient, they are not willing to (actively) participate to the auction and the buyer is not able to procure the project.

When the buyer chooses the reserve price optimally, it is possible to perform comparative statics exercises to investigate the effect that exogenous variable have on the players payoffs and on the existence conditions. These exercises are similar in nature to

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[15]We intend that a firm does not "actively" participate to the auction when it bids above the reserve price in order to be excluded from the procedure; see also footnote 10.
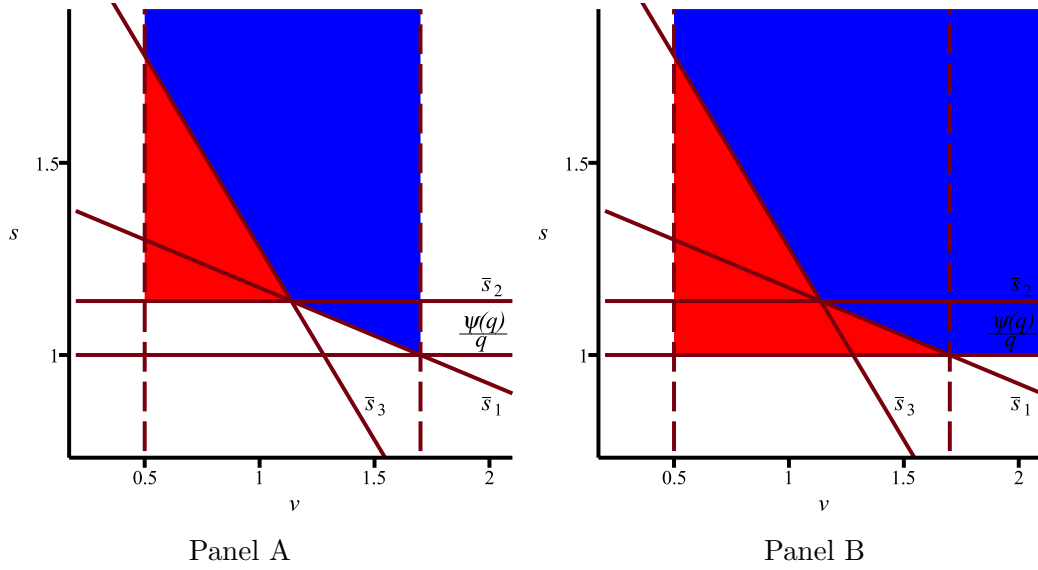
Panel A   Panel B

Figure 3: Optimal contracts for combinations of parameters v and s (when $\beta = \frac{1}{2}$, $N = 2$, $\psi(q) = q^2$, $q = 1$, $\theta_H = \frac{9}{10}$ and $\theta_L = \frac{1}{2}$)). In the case of an *uncommitted* buyer (Panel A), an optimal contract entails r^P and r^C such that $r^P = \theta_L$ always and $r^C = \hat{r}^C$ for combinations of the parameters in the red area and $r^C \in \rho_{high}$ for combinations of the parameters in the blue area. In the case of a *committed* buyer (Panel B), an optimal contract entails r^P and r^C such that $r^P \in \rho_{low}^0$ always and $r^C = \hat{r}^C$ for combinations of the parameters in the red area and $r^P + \psi(q) \leq r^C$ for combinations of the parameters in the blue area.

those performed in the previous Section, yet they may give different results. This is due to the effect that any change in an exogenous parameter of the model has on the reserve price and, through this, on the players' payoffs. Quite unsurprisingly, even if an increase in N makes the optimal contract (weakly) more costly, it is confirmed that the larger is the number of potential competitors, the higher is the buyer's expected utility along the equilibrium path. In terms of equilibrium existence conditions, the most relevant differences with respect to the analysis of the previous section are that an increase in the required quality level or in the number of firms in the market always push $\bar{s}_2$ upwards, making cooperation more difficult. In both cases, the motivation lays in the increase in the optimal reserve price, which outweighs all other effects and thus makes cooperation more costly to the buyer.

The other issue introduced at the beginning of this section regards the ability of a RPC to induce the optimal quality. This is simply defined as the one that maximises the buyer's utility $v + sq - p$, subject to a non-negativity constraint on the profits of the firms which delivers the project, say, firm i. Denoting this optimal quality level by

q^*, this is implicitly defined by the standard optimality condition for quality, $\frac{d\psi(q^*)}{dq^*} = s$; the convexity of $\psi(q)$ ensures that $\underline{s}(q^*) < s$ always holds. An immediate corollary of Proposition 2 is that it may not be possible to enforce the optimal quality level when its valuation is too low. This is because a RPC, as illustrated in the previous section, does exist only for sufficiently high levels of the taste parameter for quality. Hence, when s is too low, only sub-optimally high levels of quality are typically enforceable.

5.2 The value of commitment

What if the buyer could commit to a certain level of the reserve price, depending on the level of the quality provided by the contractor? This is more than a theoretical question. When the procuring entity carrying out the procurement procedure is also the final beneficiary/user of the project, the assumption of lack of commitment seems quite appropriate as the procurer-user is, at least in principle, in a position of fully exploiting the strategic value of the reserve price, thus responding at any point in time to firms' strategies. When awarding a contract on behalf of another public organisation, instead, the procuring entity is less likely to strategically manipulate the reserve price as some of the rules of the procurement game might be directly determined by the final user(s).[16] The highest price that the procuring entity is willing to accept might then be closely linked to (if not coincide with) the budget made available by the final user(s), which, in turn, may be related to the observed quality of the project. Hence, the reserve price might inherit the budget's commitment feature stemming from national public finance rules or by the financial resources being transferred by international donors. In terms of the buyer's strategy, when awarding a project on behalf of another public organisation, this relationship with the funding entity may prevent the buyer from deviating from her "cooperative" strategy, which prescribes she sets a high reserve price in response to the required quality level, and a low reserve price otherwise. In other words, the buyer may be prevented from setting a low reserve price when she receives the desired quality level or from forgiving a deviating contractor, by setting a low reserve price when she receives a quality level lower than expected.

In this section, we analyse our dynamic model under the hypothesis that our buyer is then able to commit to her strategy $\sigma_B(.)$. In other words, for any given choice of r^C and r^P, we assume that she will set the reserve prices exactly as instructed by $\sigma_B(.)$, even if a different choice were preferable. The assumption of a committed buyer implies

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[16]This is the case, for instance, of centralised procurement agencies awarding public contracts on behalf of other public bodies or procuring organisations specifically created for awarding big infrastructure projects.

that we need to characterise a self-enforcing RPC whereby the firm' strategies form a subgame perfect equilibrium of the repeated game, taking the buyer's strategy as given. Our equilibrium analysis is illustrated in the following Proposition.

Proposition 4. *Assume the buyer is committed to her strategy. Provided that $\delta \geq \overline{\delta}$, for any admissible q, s and v, and for any $r^P \in \rho^0_{low}$, and r^C such that $r^P + \psi(q) \leq r^C$, the strategy profile $\sigma_i(q, r^C)$ (with $i = 1, \ldots, N$) defines a self-enforcing RPC under which the project is awarded to (one of) the most efficient firm(s), which delivers quality q.*

The Proposition illustrates the nature of the self-enforcing RPCs in the case of a *committed* buyer. These equilibrium RPCs share many features with the ones under the assumption of an *uncommitted* buyer; hence, it is worth emphasising the differences between the two cases only. Consider first the "punishment" reserve price. Unlike the *uncommitted* case, any reserve price below the cost of an inefficient firm (not providing quality) is a sufficient threat to induce the provision of quality. While an *uncommitted* buyer would have reneged on any "punishment" reserve price above the minimal level (i.e. θ_L), the commitment hypothesis makes it credible for her to threaten using a "punishment" reserve price which leaves a positive rent to the contractor. The (possible) rent given to an (efficient) contractor in case of a deviation, however, changes the nature of the "cooperative" reserve price. Only a "cooperative" reserve price sufficiently high – relatively to the "punishment" one – ensures that the firm's benefits from cooperation are large enough for the firms to cooperate.

In terms of the critical discount factor, the combinations of δ and r^C consistent with the existence of a self-enforcing RPC are the same as in the *uncommitted* case, provided that r^P is set to its minimal value (i.e. θ_L). Higher values of r^P, instead, make the condition on δ more stringent; this is illustrated in Figure 1, where the diagonally striped region illustrates the combinations of δ and r^C under which a self-enforcing RPC exists, when r^P is higher than θ_L.

Another important difference relative to the case of an *uncommitted* buyer is now that a much wider set of combinations of the buyer's utility parameters are consistent with the existence of a self-enforcing RPC. Indeed, an equilibrium exists even for values of v above $\overline{v}$. The reason is simply that in order to ensure that a high-valued project is always delivered, the buyer is not able to renege on the punishment of a deviating firm. Additionally, an equilibrium exists even when the taste parameter of quality is low (i.e., below $\overline{s}_1$ and $\overline{s}_2$). Contrary to what happens in the case of a *uncommitted* buyer, she cannot deviate from the "cooperative" strategy even if her taste parameter for quality is not high enough.

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5.2.1 Optimal contract and optimal quality with a committed buyer

Mirroring the analysis in Section 5.1.1, we now characterise the contract which gives the highest utility to a *committed* buyer among the many self-enforcing ones characterised in Proposition 4. This is done in the following Proposition.

Proposition 5. *Assume the buyer is committed to her strategy. Then, the self-enforcing RPC which gives the buyer the highest utility entails $r^P = \theta_L$, and*
i) when $s \in [\underline{s}(q), \overline{s}_3]$, $r^C = \hat{r}^C$, where $\hat{r}^C$ is as in (10);
ii) when $s \in [\overline{s}_3, \infty)$, $r^C \in \rho_{high}$.

Although optimal contracts look similar to the ones described in the case of an *uncommitted* buyer, one important difference stands out. With a *committed* buyer, the optimal self-enforcing RPC is available also for combinations of the buyer's utility parameters for which such a RPC would not exist for an *uncommitted* buyer. In other words, enforcing the optimal RPC is now possible also when the intrinsic value of the project is high and the taste parameter of quality is sufficiently low. Differently from the case of an *uncommitted* buyer, the immediate corollary of Proposition 5 is that commitment makes it possible for the buyer to always enforce the optimal level of quality, even when this is low.

6 The dynamic game with stick-and-carrot strategies

In this section, we check to robustness of our result to the change of the hypothesis regarding the players' strategies. We assume now that players use stick-and-carrot strategies: any player deviating from the "cooperative" path is punished for a limited number of periods only. The punishment ends if, during the "punishment" phase, the punished players maintain an appropriate behaviour. The everlasting punishment typical of trigger strategies may indeed be not very realistic, since it implies that players give up forever the value of their "cooperative" interaction in case of a deviation. Thus, especially in public procurement, the analysis of more forgiving strategies seems also appropriate.

We concentrate on players adopting the stick-and-carrot strategies described below. Notice that, in describing the strategy of the buyer, we restrict our attention to the case of symmetric strategies, as already assumed and discussed in the case of trigger strategies; this implies that all firms are required the same quality level, both during the "cooperative" and the "punishment" phase.

> *Buyer*: the buyer sets a reserve price equal to r_B^C as long as the firm awarded the project delivered quality q_B^C in the previous period. Otherwise, the reserve

price is set equal to r_B^P for T_B periods; if during these T_B periods, the quality delivered is q_B^P, in period $T_B + 1$ the buyer reverts to reserve price equal to r_B^C. Otherwise, if during any of these T_B periods, a quality different from q_B^P is delivered, the buyer imposes a reserve price equal to r_B^P again for the next T_B periods.

Firm i ($i = 1, \ldots, N$): firm i bids as in Lemma 1 with $q_i = q_i^C$ and, if awarded the project, offers quality q_i^C as long as the buyer sets a reserve price r_i^C in the current period. Otherwise, the firm bids as in Lemma 1 with $q_i = q_i^P$ and, if awarded the project, delivers quality q_i^P in the same and in the following $T_i - 1$ periods; if during these $T_i - 1$ periods, the reserve price is set equal to r_i^P, in period T_i the firm reverts to the initial choice of $q_i = q_i^C$. Otherwise, if during any of these $T_i - 1$ periods, a reserve price different from r_i^P is set, the firm bids as in Lemma 1 with $q_i = q_i^P$ and, if awarded the project, delivers quality q_i^P in the same and in the following $T_i - 1$ periods.

When referring to these strategies, we will denote the buyer's strategy as $\tilde{\sigma}_B(r_B^C, r_B^P, q_B^C, q_B^P, T_B)$. As to firm i's strategy, we write it as $\tilde{\sigma}_i(r_i^C, r_i^P, q_i^C, q_i^P, T_i)$.

In the rest of this section, we restrict our analysis to the case in which, during the "punishment" phase, the buyer's stick-and-carrot strategy entails a quality level equal to 0 (i.e., $q_B^P = 0$), so that the quality level required after a deviation is lower than in equilibrium. This restriction implies that we need not worry about a deviation by firm i during the "punishment" phase. Note, also, that a firm, after a buyer's deviation, optimally sets the "punishment" quality equal to zero; this is because, in this case, a quality choice different from zero has no strategic or market value, hence no quality provision ensures that the firms do not bear unnecessary costs. Finally, to avoid messy notation, we restrict our analysis to buyer's and firms' strategies such that the intended length of the punishment in case of a deviation by another player is identical for all players, so that $T_B = T_i = T$.

We concentrate on the case of the discount factor arbitrarily close to 1. This simplifies the technical analysis and is less restrictive than it appears: if an equilibrium exists when $\delta \to 1$, by continuity, there always exists a critical discount factor sufficiently close to 1 but strictly smaller than 1, such that a self-enforcing RPC exists.

We can now state the main result of this section.

Proposition 6. *Assume* $\delta \to 1$. *Let*

$$\overline{T} \equiv \max \left\{ \frac{\psi(q)}{\beta(1-\beta)^{N-1} \left[\min \{ r^C - \psi(q), \theta_H \} - r^P \right]}, 1 \right\}$$

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Then, provided that $v \leq \overline{v}$ and $\overline{T} \leq T$, the strategy profile $\tilde{\sigma}_B(r^C, r^P, q, 0, T)$ and $\tilde{\sigma}_i(r^C, r^P, q, 0, T)$ (with $i = 1, \ldots, N$) defines a self-enforcing RPC whereby the project is awarded in each period to (one of) the most efficient firm(s), which delivers quality q, under the following conditions:

i) $r^P = \theta_L$ and $r^C \in \rho_{high}$, provided that $s \geq \overline{s}_1$;

ii) $r^P = \theta_L$ and $r^C \in \rho_{low}$, provided that $s \geq \overline{s}_2$

The Proposition illustrates the sufficient conditions such that, when players are close to be infinitely patient, it is possible to induce the delivery of quality even with a punishment of finite length, provided that this length is above a minimal level.

Together with conditions on v, q, s and the "punishment" price r^P, by now familiar from the analysis with trigger strategies, the Proposition provides the sufficient conditions for an equilibrium to exist in terms of a lower bound, $\overline{T}$, for the punishment length. This lower bound is strictly decreasing in r^C when this is 'low' and does not depend on r^C when this is 'high'. When the reserve price is 'low', the punishment length and the reserve price are substitutes to each other; a 'low' r^C reduces the incentive to cooperate that, in turn, must be restored by a higher minimum punishment length. When, instead, r^C is 'high', it does not affect the firms bids, whose behaviour is only determined by the competitive pressure of rivals. The minimum punishment length is then only a function of the expected profits a firm may make along the equilibrium path, which in turn depends on the possible competitive advantage.

7 Conclusions

Unverifiable quality is known to cause severe problems in procurement, especially if the buyer is constrained to use competitive procedures. When the overarching regulatory allows the buyer to evaluate firms' past performance, such as the federal public procurement regulation in the U.S., a *discriminatory* scoring auction is instrumental to enforce the socially optimal level of quality. When past performance cannot be taken into account in tenders evaluation, such as in the case of the EU public procurement regulation and of the UNCITRAL Model Law, a public buyer can still exploit one dimension of the tender design, namely the reserve price, to enforce unverifiable quality.

In this paper, we have shown that a public buyer can credibly threaten to punish competing firms with a low reserve price if quality is not delivered. Such a "punishment" reserve price affects equally the expected profit of *all* firms, thus it represents a *non-discriminatory* enforcement mechanism. The analysis of both the case of uncommitted and committed buyer – which arguably capture the differences between a private and

public buyer – has highlighted that commitment do have a strong value in that it makes a RPC more likely to occur as well as it makes it always possible that the buyer is able to implement the socially optimal quality level.

Our paper then provides a clear policy recommendation. Public buyers – particularly those abiding by the EU and UNCITRAL public procurement regulations – are often perceived too constrained in their ability to reward/punish firms for those aspects of past performance that cannot be accurately described by contractual clauses. A remedy in fact exists. Using the reserve price as incentive mechanism may, at least, soothe the problem of unverifiable quality in public procurement. While this represents a new justification for urging public buyers to use a reserve price, it remains nonetheless open the question of the interaction between the reserve price as a quality-enforcing mechanism and as cartel-fighting tool, especially in those markets with few competitors and little entry.

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Appendix

We provide here the proofs of all Lemmata and Propositions in the paper.

Proof of Lemma 1. Trivial, then omitted. ∎

Proof of Proposition 1. In stage 3, since quality is costly and does not affect the firms' revenues, the firm awarded the contract chooses to deliver quality equal to 0. In stage 2, the N firms, anticipating that the one winning the contract will deliver quality equal to 0, bid as in Lemma 1 anticipating to deliver qualities $q_i = 0$, for $i = 1, \ldots, N$. In stage 1, the buyer's utility depends on the level of the reserve price r. Before turning to analyse the buyer's expected utility, it is useful to remind that i) β^N is the probability that all firms draw θ_L; ii) $(1 - \beta)^N$ is the probability that all firms draw θ_H; iii) $N\beta(1 - \beta)^{N-1}$ is the probability that only one firm draws θ_L; iv) $1 - N\beta(1 - \beta)^{N-1} - (1 - \beta)^N$, is the probability that at least two firms draw θ_L.

Let ρ_{low}^0 and ρ_{high}^0 be defined as in (5) and (6), respectively. Consider a reserve price $r' \in \rho_{low}^0$. Depending on the realisations of the firms' cost parameters, the buyer's utility is as follows:

$$
U(r') = \begin{cases} v - r' & \text{with probability } N\beta(1 - \beta)^{N-1}; \\ 0 & \text{with probability } (1 - \beta)^N; \\ v - \theta_L & \text{with probability } 1 - (1 - \beta)^N - N\beta(1 - \beta)^{N-1}. \end{cases} \tag{12}
$$

The buyer's expected utility is then $EU(r') = (v - r')N\beta(1 - \beta)^{N-1} + (v - \theta_L)(1 - (1 - \beta)^N - N\beta(1 - \beta)^{N-1})$. Since $\frac{\partial EU(r')}{\partial r'} = -\beta(1 - \beta)^{N-1} < 0$, it is dominant for the buyer to set r' equal to its lower bound, θ_L; hence, at the buyer's optimal choice, $EU(r') = (v - \theta_L)(1 - (1 - \beta)^N)$.

Consider now a reserve price $r'' \in \rho_{high}^0$. Depending on the realisations of the firms' cost parameters, the buyer's utility is as follows:

$$
U(r'') = \begin{cases} v - \theta_H & \text{with probability } N\beta(1 - \beta)^{N-1}; \\ v - \theta_H & \text{with probability } (1 - \beta)^N; \\ v - \theta_L & \text{with probability } 1 - (1 - \beta)^N - N\beta(1 - \beta)^{N-1}. \end{cases} \tag{13}
$$

The buyer's expected utility is then $EU(r'') = (v - \theta_H)(N\beta(1 - \beta)^{N-1} + (1 - \beta)^N) + (v - \theta_L)(1 - (1 - \beta)^N - N\beta(1 - \beta)^{N-1})$.

Comparing the buyer's expected utility in the two cases, then $EU(r') \geq EU(r'')$ when $v \geq \bar{v}$. ∎

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Proof of Proposition 2. We begin by establishing a Lemma which describes a firm's static optimal deviation, given the rivals' strategies.

Lemma 3. *Consider the static game of Section 4. Assume that all firms, but firm i, bid as in Lemma 1, anticipating to offer a common quality $\tilde{q}$ if awarded the project. Then, for any r and any realisation of the firms' cost parameters, bidding its best reply to its rivals' bids yields firm i a gain bounded from above by $\psi(\tilde{q})$ relatively to Lemma 1 where firm i anticipates to deliver quality $\tilde{q}$ as all other competitors.*

Proof. We have to consider three cases:

i) $C_i(\theta_i, \tilde{q}) \leq \min\{r, \underline{C}_{-i}\}$. Firm i bids as in Lemma 1 and delivers $\tilde{q} = 0$. In this case firm i is the most efficient even when anticipating to deliver quality $\tilde{q} > 0$. It then bids as in Lemma 1 and optimally deviates at the execution stage, thus reaping an additional profit of $\psi(\tilde{q})$.

ii) $C_i(\theta_i, 0) \leq \min\{r, \underline{C}_{-i}\} < C_i(\theta_i, q)$. Firm i would lose the auction if bidding according to Lemma 1, but would win it if it were to anticipate to deliver zero quality. Thus firm i's optimal deviation consists in bidding $b_i' = \min\{r, \underline{C}_{-i}\}$ - thus winning the auction - and delivering $\tilde{q} = 0$. Firm i's additional profit is equal to $\psi(\tilde{q})$ if $\min\{r, \underline{C}_{-i}\} = \underline{C}_{-i}$, and strictly lower than $\psi(\tilde{q})$ if $\min\{r, \underline{C}_{-i}\} = r$.

iii) $\min\{r, \underline{C}_{-i}\} < C_i(\theta_i, 0) < C_i(\theta_i, \tilde{q})$. In this case, any deviation at the bidding stage would not alter the outcome of the stage game, thus firm i makes zero additional profit. $\qquad\square$

Next, we check whether firm i $(i = 1, \ldots, N)$ or the buyer have a POSD from the strategies described in the Proposition, on and off the equilibrium path.

i) conditions for no POSDs for a firm off the equilibrium path: when facing a reserve price $\theta_L \leq r^P$ because of a previous deviation from the equilibrium path, quality provision has no strategic value and it is only a cost; hence, no POSD exists whenever a firm bids as in Lemma 1, anticipating to deliver a quality level equal to 0;

ii) conditions for no POSDs for a firm on the equilibrium path: when no previous deviation has occurred, no POSD exists for firm i if

$$\pi_i^C + \frac{\delta}{1-\delta}E\pi_i^C \geq \pi_i^D + \frac{\delta}{1-\delta}E\pi_i^P \tag{14}$$

where π_i^C denotes the firm's profits when it stick to its "cooperative" strategy $\sigma_i(.)$, π_i^D denotes the firm's profits when it (optimally) deviates from its "cooperative" strategy $\sigma_i(.)$ and π_i^P denotes the firm's profits when the buyer sets a "punishment" reserve

price; E denotes the expectation operator, before the cost parameters are realised. This constraint can be re-expressed as

$$\delta \geq \frac{\pi_i^D - \pi_i^C}{(\pi_i^D - \pi_i^C) + (E\pi_i^C - E\pi_i^P)} \tag{15}$$

(provided that the denominator is positive). Since the RHS of this inequality is increasing in the numerator, the constraint is verified to always hold when it holds for the largest value of the difference $\pi_i^D - \pi_i^C$ which, from Lemma 3, is equal to $\psi(q)$. As to the expressions for expected profits, they may take different values depending on the values of r^C and r^P, as described in the subcases below. In the analysis of all these subcases below, following the previous argument, we set $\pi_i^D - \pi_i^C = \psi(q)$ and derive the values of $E\pi_i^C$ and $E\pi_i^P$ making use of standard results on Bertrand games (the derivation is trivial and it is omitted):

$ii.i)$ $r^C \in \rho_{high}$ and $r^P \in \rho_{high}^0$: we have that $E\pi_i^C = E\pi_i^P = \beta(1-\beta)^{N-1}\Delta\theta$. Hence, (15) becomes $\delta \geq 1$, which, clearly, can be satisfied only in the limit case of $\delta = 1$.

$ii.ii)$ $r^C \in \rho_{high}$ and $r^P \in \rho_{low}^0$: we have that $E\pi_i^C = \beta(1-\beta)^{N-1}\Delta\theta$ and $E\pi_i^P = \beta(1-\beta)^{N-1}(r^P - \theta_L)$. These imply that (15) becomes

$$\delta \geq \frac{\psi(q)}{\psi(q) + \beta(1-\beta)^{N-1}(\theta_H - r^P)} \equiv \delta_a. \tag{16}$$

$ii.iii)$ $r^C \in \rho_{low}$ and $r^P \in \rho_{low}^0$: we have that $E\pi_i^C = \beta(1-\beta)^{N-1}(r^C - \theta_L - \psi(q))$ and $E\pi_i^P = \beta(1-\beta)^{N-1}(r^P - \theta_L)$. These imply that (15) becomes

$$\delta \geq \frac{\psi(q)}{\psi(q) + \beta(1-\beta)^{N-1}(r^C - \psi(q) - r^P)} \equiv \delta_b. \tag{17}$$

Notice that for δ_b to be smaller than 1, it must be that $r^C \geq \psi(q) + r^P$.

$ii.iv)$ $r^C \in \rho_{low}$ and $r^P \in \rho_{high}^0$: we have that $E\pi_i^C = \beta(1-\beta)^{N-1}(r^C - \theta_L - \psi(q))$ and $E\pi_i^P = \beta(1-\beta)^{N-1}\Delta\theta$. These imply that (15) becomes

$$\delta \geq \frac{\psi(q)}{\psi(q) + \beta(1-\beta)^{N-1}(r^C - \psi(q) - \theta_H)} \equiv \delta_c. \tag{18}$$

Since $\delta_c > 1$, this is, clearly, never satisfied.

Notice that δ_a and δ_b in (16) and (17) differ only for the term in the denominator which multiplies $\beta(1-\beta)^{N-1}$. Also, δ_a is derived under the assumption that $r^C \geq$

$\theta_H + \psi(q)$, which implies $r^C - \theta_H < \psi(q)$; on the other hand, δ_b is derived under the assumption that $r^C < \theta_H + \psi(q)$, which implies $r^C - \theta_H \geq \psi(q)$. Hence, we can write

$$\overline{\delta} \equiv \max\{\delta_a, \delta_b\}$$

$$= \frac{\psi(q)}{\psi(q) + \beta(1-\beta)^{N-1}[\min\{r^C - \psi(q), \theta_H\} - r^P]}. \tag{19}$$

iii) conditions for no POSDs for the buyer off the equilibrium path: when facing a previous deviation by a firm from the equilibrium path, the reserve price has no strategic value to the buyer and only affects her single-period utility; hence, from Proposition 1, no POSD exists provided that the buyer sets a reserve price $r^P = \theta_L$ if $v \leq \overline{v}$ and $r^P \geq \theta_H$ if $v \geq \overline{v}$;

The analysis in *i)*, *ii)* and *iii)* restricts the candidate equilibria only to those strategy profiles in which $r^C \lessgtr \theta_H + \psi(q)$ and $r^P = \theta_L$, for any level of q and provided that $v \leq \overline{v}$ and $\delta \geq \overline{\delta}$. In the rest of the proof, we restrict our attention to these candidate equilibria.

iv) conditions for no POSDs for the buyer on the equilibrium path: when no previous deviation has occurred, no POSD exists for the buyer i if

$$EU^C + \frac{\delta}{1-\delta}EU^C \geq EU^D + \frac{\delta}{1-\delta}EU^P, \tag{20}$$

where U^C denotes the buyer's utility when she sticks to her "cooperative" strategy $\sigma_B(.)$, U^D denotes the buyer's utility when she (optimally) deviates from her "cooperative" strategy $\sigma_B(.)$ and U^P denotes the buyer's utility when she sets a "punishment" reserve price; E denotes the expectation operator, before the cost parameters are realised. Notice that, since the buyer chooses her action before the realisation of the firms' cost parameters, all utilities are expected.

From Proposition 1, when $v \leq \overline{v}$, the optimal deviation is setting a reserve price equal to θ_L. Therefore, $EU^D = EU^P$. This allows to rewrite (20) simply as

$$EU^C \geq EU^P, \tag{21}$$

where EU^C may take different values depending on the value of r^C, as described in the subcases below. In the analysis of all these subcases below, we derive the values of EU^C and EU^P making use of standard results on Bertrand auctions (the derivation is trivial and it is omitted); since $r^P = \theta_L$ always, EU^P is constant and such that $EU^P = (1 - (1-\beta)^N)(v - \theta_L)$.

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iv.i) $r^C \in \rho_{high}$: depending on the realisations of the firms' cost parameters, the single period buyer's utility is as follows:

$$
U(r^C) = \begin{cases} v + sq - \psi(q) - \theta_H & \text{with probability } N\beta(1-\beta)^{N-1}; \\ v + sq - \psi(q) - \theta_H & \text{with probability } (1-\beta)^N; \\ v + sq - \psi(q) - \theta_L & \text{with probability } 1 - (1-\beta)^N - N\beta(1-\beta)^{N-1}. \end{cases}
$$
(22)

The buyer's expected utility is then

$$
EU^C = v + sq - \psi(q) - \theta_H(N\beta(1-\beta)^{N-1} + (1-\beta)^N) - \theta_L(1 - (1-\beta)^N - N\beta(1-\beta)^{N-1}).
$$
(23)

Simple algebra shows that $EU^C \geq EU^P$ when $s \geq \bar{s}_1$.

iv.ii) $r^C \in \rho_{low}$: depending on the realisations of the firms' cost parameters, the single period buyer's utility is as follows:

$$
U(r^C) = \begin{cases} v + sq - r^C & \text{with probability } N\beta(1-\beta)^{N-1}; \\ 0 & \text{with probability } (1-\beta)^N; \\ v + sq - \psi(q) - \theta_L & \text{with probability } 1 - (1-\beta)^N - N\beta(1-\beta)^{N-1}. \end{cases}
$$
(24)

The buyer's expected utility is then

$$
EU^C = (v + sq - r^C)N\beta(1-\beta)^{N-1} + (v + sq - \psi(q) - \theta_L)(1 - (1-\beta)^N - N\beta(1-\beta)^{N-1}).
$$
(25)

Simple algebra shows that $EU^C \geq EU^P$ when $s \geq \bar{s}_2$. ∎

Proof of Lemma 2. In equilibrium, when $r^C \in \rho_{low}$, $EU(r^C)$ is as given in (25), where $\frac{\partial EU(r^C)}{\partial r^C} = -N\beta(1-\beta)^{N-1} < 0$. Then, r^C is set to its lowest level compatible with the firms' incentive to play their "cooperative" strategies. This is found by solving $\delta = \bar{\delta}$, from (7) w.r.t. r^C, which gives $r^C = \hat{r}^C$, as in (10). Notice that $\psi(q) + r^P \leq r^C$ always holds, as it is requested to ensure that $\delta \leq 1$.

In equilibrium, when $r^C \in \rho_{high}$, , $EU(r^C)$ is as given in (23), and does not depend on r''. ∎

Proof of Proposition 3. Trivial, by comparing $EU(\hat{r}^C)$, with $\hat{r}^C$ evaluated at $r^P = \theta_L$, and $EU(r'')$ from Lemma 2. ∎

Proof of Proposition 4. The result follows directly from points *i)* and *ii)* in the proof of Proposition 2 where we have shown that the cases in which $\theta_H < r^P$ induce a deviation by the firm. ∎

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Proof of Proposition 5. Trivial and, therefore, omitted. ∎

Proof of Proposition 6. We check whether firm i $(i = 1, \ldots, N)$ or the buyer have a POSD from the strategies described in the Proposition, on and off the equilibrium path. We restrict the analysis to punishment lengths with $T \geq 1$ since for obvious reasons $T = 0$, meaning "forgiveness", will not be self enforcing.

Notice that a deviation on the length of the punishment T, is actually a deviation on the reserve price (for the buyer) or from the quality (bid or actual, for the firm) prescribed by the equilibrium strategy during the "punishment" (in case of a punishment shorter than T) or during the "cooperative" path (in case of a punishment longer than T). Hence, checking for a POSD on the length of the punishment is equivalent to a check for a POSD in terms of those actions; hence, no specific attention is devoted in the rest of this proof to deviation on the length of the punishment T.

i) conditions for no POSDs for a firm off the equilibrium path: when facing a reserve price $\theta_L \leq r^P$ because of a previous deviation from the equilibrium path, quality provision has no strategic value and it is only a cost; hence, no POSD exists whenever a firm bids as in Lemma 1, anticipating quality equal to 0;

ii) conditions for no POSDs for a firm on the equilibrium path: when all rival firms sticks to their "cooperative" strategies, when no previous deviation has occurred, no POSD exists for firm i if:

$$\pi_i^C + \sum_{t=1}^{\infty} \delta^t E\pi_i^C \geq \pi_i^D + \delta\Pi_i^P, \tag{26}$$

where π_i^C, π_i^C and π_i^D are defined as in the proof of Proposition 2; also, Π_i^P, which gives the expected future profits after a deviation (taking as given a punishment of T periods), is given by

$$\Pi_i^P = \sum_{t=0}^{T-1} E\pi_i^P + \sum_{t=T}^{\infty} E\pi_i^C \tag{27}$$

where π_i^D is defined as in the proof of Proposition 2.

When $\delta \to 1$, equation (26) becomes

$$T \geq \frac{\pi_i^D - \pi_i^C}{E\pi_i^C - E\pi_i^P} \equiv \overline{T} \tag{28}$$

Since $\overline{T}$ is increasing in the numerator, (26) is verified to always hold when it holds for the largest value of the difference $\pi_i^D - \pi_i^C$ which, from Lemma 3, is equal to $\psi(q)$. As to the expressions for expected profits, they may take different values depending on the

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value of r^C and r^P. For all the relevant combinations of r^C and r^P, we will derive the threshold value for $\overline{T}$ in (28), by setting $\pi_1^D - \pi_1^C = \psi(q)$ and borrowing the values of $E\pi_i^C$ and $E\pi_i^P$ from the four cases i)-iv) in the proof of the Proposition 2.

Case i): $r^C \in \rho_{high}$ and $r^P \in \rho_{high}^0$. Since $E\pi_i^C = E\pi_i^P = \beta(1-\beta)^{N-1}\Delta\theta$, then the value of $\overline{T}$ is not defined.

Case ii): $r^C \in \rho_{high}$ and $r^P \in \rho_{low}^0$. Since $E\pi_i^C = \beta(1-\beta)^{N-1}\Delta\theta$ and $E\pi_i^P = \beta(1-\beta)^{N-1}(r^P - \theta_L)$, then

$$T \geq \frac{\psi}{\beta(1-\beta)^{N-1}(\theta_H - r^P)} \equiv T_a; \tag{29}$$

Case iii): $r^C \in \rho_{low}$ and $r^P \in \rho_{low}^0$. Since $E\pi_i^C = \beta(1-\beta)^{N-1}(r^C - \theta_L - \psi)$ and $E\pi_i^P = \beta(1-\beta)^{N-1}(r^P - \theta_L)$, then

$$T \geq \frac{\psi}{\beta(1-\beta)^{N-1}(r^C - \psi(q) - r^P)} \equiv T_b. \tag{30}$$

Notice that for the denominator to be positive it must be that $\psi(q) + r^P \leq r^C$.

Case iv): $r^C \in \rho_{low}$ and $r^P \in \rho_{high}^0$. Since $E\pi_i^C = \beta(1-\beta)^{N-1}(r^C - \theta_L - \psi(q))$ and $E\pi_i^P = \beta(1-\beta)^{N-1}\Delta\theta$, then

$$T \geq \frac{\psi(q)}{\beta(1-\beta)^{N-1}(r^C - \theta_H - \psi)} \tag{31}$$

which clearly gives a negative $\overline{T}$.

Since we only consider punishment length at least longer than one period ($\overline{T} \geq 1$), note that $T_a \geq 1$ if $\psi(q) \geq \beta(1-\beta)^{n-1}(\theta_H - r^P)$ and $T_b \geq 1$ if $\psi(q) \geq \beta(1-\beta)^{n-1}(r^C - \psi(q) - r^P)$.

iii) conditions for no POSDs for the buyer on the equilibrium path: when no previous deviation has occurred, no POSD exists for the buyer if

$$EU^C + \frac{\delta}{1-\delta}EU^C \geq EU^D + \sum_{t=1}^{T-1}\delta^t EU^P + \sum_{t=T}^{\infty}\delta^t EU^C \tag{32}$$

where U^C, U^P and U^P are defined as in the proof of Proposition 2. This inequality reduces to (21) and the analysis carried out in the proof of Proposition 2 applies

iv) conditions for no POSDs for the buyer off the equilibrium path: as shown in the proof of the Proposition 2 no POSD from the "punishment" reserve price exists provided that the buyer sets a reserve price $r^P = \theta_L$ if $v \leq \overline{v}$ and $r^P \geq \theta_H$ if $v \geq \overline{v}$. However,

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as already shown, any punishment with $r^P \geq \theta_H$ will induce a deviation by the firm on the equilibrium path therefore such a 'mild' punishment characterizes neither a self enforcing equilibrium in trigger strategies nor in stick-and-carrot strategies. ∎