On the evolution of individual preferences and family rules

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June 12, 2018

Abstract

In this paper we study the interplay between the evolution of individual preferences and the persistence of family rules. Members of each sex are differentiated by an inherited trait. Assuming random matching, we show that the distribution of traits evolves—by means of mixed marriages—from generation to generation towards a fully *mixed-trait* society in a finite time, almost surely. Eventually, in the long run, everyone will also display the same trait. For the case where the trait is a parameter measuring a person's taste for receiving filial attention in old age, we also show that, if the pair of parameter values characterizing a couple satisfy a certain condition, it is in the couple's common interest to obey a rule requiring them to give specified amounts of attention to their respective parents. Couple asymmetry and individual compensation are discussed. As the distribution of this trait changes because of mixed marriages, the share of the population who obey the rule in question changes too. In the long run, everybody has the same preferences, and either everybody obeys the same rule, or nobody obeys any. The consequences of immigration and the implications for welfare policy are discussed.

Keywords: Marriage, evolution, random matching, family rules, immigration, welfare, Markov chain

JEL Codes: C78, D13, J12

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1 Introduction

The way individual preferences and family rules are transmitted and evolve from one generation to the next was traditionally the concern of ethnic or religious minorities intent on preserving their identity. More recently, it has become the concern of natives worried about loosing their identity to immigrants. Beyond that, the evolution of individual preferences and family rules has implications for the design of welfare policy. A strand of economic literature stemming from Bisin and Verdier (2001), and Tabellini (2008), assumes that optimizing parents motivated either by a paternalistic form of altruism, or by a social conscience, undertake costly actions in order to transmit their preferences or values on to their offspring. Preferences or values evolve over time as a result of social interaction among individuals who received different inputs from their respective parents. The implicit assumption underlying these models is that the parental couple think and act as if they were one person. What happens if mother and father have different preferences?

The issue is taken up by Cigno et al. (2017) in a more complex setting where at least some individuals respond rationally not only to the economic and legal environment, but also to a family rule (a "family constitution") that is itself a collectively rational response to the environment.[1] The rule explicitly considered in that paper requires every working-age adult to give a certain amount of attention (a good without perfect market substitutes) to each of her or his retirement-age parents, conditional on the receiver having done the same for her or his own parents.[2] If a married person complies with some such rule, that will affect the domestic resource allocation negotiated between this person and her or his spouse, who may be complying with a different rule, or with no rule at all. That article demonstrates that, if a person's preferences satisfy the condition for the existence of a family constitution,

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[1] The approach was originally developed by Cigno (1993, 2006) under the assumption that individuals reproduce asexually. Cigno et al. (2017) demonstrate that it works also in a world where sexually differentiated individuals marry, have children and bargain with their respective spouses over the allocation of domestic resources.

[2] By complying with such a rule, a person implicitly threatens to punish her or his parents if they fail to comply, because the latter know that, if they do not comply, their children will be legitimated to give them nothing without forfeiting the entitlement to receive attention from the grandchildren.

it is in that person's interest to seek out a like-minded marriage partner, and for the couple thus formed to pass their common preferences on to their children.

In the present paper, we look at what happens if young men and women are randomly matched. In Section 2, we show how the distribution of a generic trait evolves from generation to generation as a result of random matching under the assumption that a person's trait is a convex combination of her or his parents' traits. In Section 3, we interpret the trait in question as a preference parameter measuring a person's taste for filial attention, and show how this affects the existence and evolution of a rule entitling the old to receive filial attention on condition that they obeyed the same rule when they were young. The concluding section compares our results with those of others who view the same issue from different standpoints.

2 Evolution

Assuming that a population is initially characterized by two different values of a generic trait δ, and that couples are randomly formed, we now show that these initial values quickly disappear as men and women are randomly matched. For a time, men and women will display a variety of mixed traits (convex combinations of the two initial ones), and the distribution of these mixed traits will be binomial. In the long run, the distribution will tend to approximate the normal distribution, and the variance of this distribution will tend to zero. Eventually, therefore, all individuals will have the same mixed trait.

For simplicity we assume that, in generation $t = 0$ there are $n_0 = n_0^L + n_0^H$ men, and $n_0 = n_0^L + n_0^H$ women, where $n_0^L, n_0^H \geq 2$. Here, n_0^j denotes the number of adults of each sex with trait δ^j, where $j = H, L$, and $\delta^H > \delta^L$. The number n_0 is large in the sense of the Law of Large Numbers. We assume that everybody marries, and that each couple has a son and a daughter, so that the number of men and the number of women will remain equal to n_0 also in subsequent generations. Marriages between siblings are not allowed. We assume that siblings have the same trait, equal to the mean of their parents' traits.

Nothing of substance changes if we assume instead that a child's trait is a random variable, symmetrically distributed over the interval between the father's and the mother's trait. We are interested in studying how the two original traits, δ^H and δ^L, are replaced by mixed traits as time goes on.

Definition 1 *A trait* $\delta(\alpha)$ *is a mixed trait if it is a strictly convex combination of the two native traits,* δ^L *and* δ^H*, i.e.,* $\delta(\alpha) := \alpha\delta^L + (1-\alpha)\delta^M$*, with* $\alpha \in (0, 1)$*.*

2.1 The absorbing process of mixed traits

In this section we show how the number of mixed-trait individuals evolve over time. The analysis builds on Camera and Gioffré (2014) and exploits the properties of the Markov matrix that describes the probabilities for each individual to become "mixed" across two periods. At each $t \geq 0$, the state of the world is described by the number of individuals, say $2m$, who have inherited a mixed trait. As each couple is replaced by another as we move from a generation to the next, we can think of t as a generation. Given that each individual with a mixed trait

$$\delta(\alpha) = \alpha\delta^H + (1-\alpha)\,\delta^L, 0 \leq \alpha \leq 1,$$

has a sibling with the same trait $\delta(\alpha)$, we can limit ourselves to tracking the spread of mixed traits in the female half of the population, because the same applies to the male half. At $t = 0$, there are $m = 0$ females with a mixed trait. At $t = 1$, there is a nonzero probability that some new born individuals have a mixed trait. This probability depends on the initial distribution of traits. Notice that, if m_1 women exhibit a mixed trait at $t = 1$, then m_1 of the women who exhibited either the trait δ^H or the trait δ^L at $t = 0$ will have been replaced by them. At $t = 1$ the female population is composed of m_1 individuals with mixed traits and $n_0 - m_1$ individuals with one of the original traits. Given that, with random matching, the number of persons with a mixed trait cannot decrease, the number of women displaying a mixed trait at $t = 2$ will be $m_2 \geq m_1$. The contagion of mixed traits continues over time. Its evolution is described by the $n_0 + 1 \times n_0 + 1$ upper-triangular Markov matrix $\mathcal{M}$, where

$$\mathcal{M} := \begin{pmatrix} M_{00} & M_{01} & M_{02} & M_{03} & \dots & M_{0,n_0-2} & M_{0,n_0-1} & M_{0,n_0} \\ 0 & M_{11} & M_{12} & M_{13} & \dots & M_{1,n_0-2} & M_{1,n_0-1} & M_{1,n_0} \\ 0 & 0 & M_{22} & M_{23} & \dots & M_{2,n_0-2} & M_{2,n_0-1} & M_{2,n_0} \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & M_{n_0-2,n_0-2} & M_{n_0-2,n_0-1} & M_{n_0-2,n_0} \\ 0 & 0 & 0 & 0 & \dots & 0 & M_{n_0-1,n_0-1} & M_{n_0-1,n_0} \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 1 \end{pmatrix}. \tag{1}$$

The generic element $M_{mm'}$ of this matrix represents the probability to move the female population from a state where m women have mixed traits to a state where m' women do. Put another way, $M_{mm'}$ is the probability that $m' - m$ women no longer have a pure trait (δ^L or δ^H). Matrix $\mathcal{M}$ is upper-triangular because once someone inherits a mixed trait, the possibility that one of her or his children has a pure trait is lost forever. The contagion is irreversible. To better understand the meaning of matrix $\mathcal{M}$, consider the first row. M_{00} is the probability that, starting from a population with only pure traits (i.e., with $m = 0$), no offspring inherits a mixed trait after the first matching (i.e., $m' = 0$). Similarly, M_{01} is the probability that, starting from $m = 0$, only one woman exhibits a mixed trait, $m' = 1$; and so on.

More generally, let $m \in \kappa := (0, 1, \dots, n_0)^\intercal$ be the number of women with a mixed trait. For each $t \geq 1$, define $\mu_k(t) := e_m^\intercal \mathcal{M}^t \kappa$ as the expected number of mixed-trait individuals t generations ahead, where e_m is a column vector with 1 in the m^{th} position, and 0 everywhere else. Appendix 1 demonstrates the following.

Lemma 1 *The expected number of mixed-trait individuals satisfies*

1. $\mu_k(t+1) \geq \mu_k(t) \geq k$, *for all* $k = 1, \dots, n_0 - 1$ *and all* $t \geq 1$*;*

2. $\mu_{k+1}(t) \geq \mu_k(t)$, *for all* $k = 1, \dots, n_0 - 1$ *and all* $t \geq 1$*;*

3. The state $k = n_0$ is absorbing, i.e., $\lim_{t\to\infty} \mu_k(t) = n_0$.

The contagion process eventually leads to a state where all individuals display a mixed trait. This absorbing state is reached almost surely in a finite number of generations. We can compute the average number of generations τ_k required for n_0 individuals to have mixed traits when the initial state is $m = 0, 1, \ldots, n_0$,

$$\begin{aligned} \tau_k &= 1 + \sum_{m'=m}^{n_0} M_{mm'} \tau_{k'} \qquad \text{for} \quad m = 0, 1, \ldots, n_0 - 1 \\ \tau_{n_0} &= 0. \end{aligned}$$

With probability M_{mm}, the number of mixed-trait individuals does not increase between two generations. Following a a round of matchings, therefore, we expect to have to wait τ_m generations before a state with only mixed-trait individuals is reached. With probability $M_{mm'}$, the population of mixed-trait women will increase by $m' - m$ units. Hence, we expect to have to wait $\tau_{k'}$ generations before there are n_0 mixed-trait women. Clearly, $\tau_{n_0} = 0$.

Now let $\mathcal{M}_0$ denote the matrix obtained when the last row of $\mathcal{M}$ is a vector of zeros. The elements of vector $\tau := (\tau_0, \tau_1, \ldots, \tau_{n_0})^\mathsf{T}$ are solutions to the system of equations

$$\tau = \mathbf{1}_0 + \mathcal{M}_0 \cdot \tau \qquad \Rightarrow \qquad (\mathcal{I} - \mathcal{M}_0) \cdot \tau = \mathbf{1}_0,$$

where $\mathcal{I}$ is the identity matrix and $\mathbf{1}_0$ is the (n_0+1)-dimensional unit vector whose $(n_0+1)^{th}$ component is zero. Since $\mathcal{I} - \mathcal{M}_0$ is upper-triangular with non-zero diagonal elements, then $\mathcal{I} - \mathcal{M}_0$ is invertible, and $\tau = (\mathcal{I} - \mathcal{M}_0)^{-1} \cdot \mathbf{1}_0$ is the unique solution.

2.2 Distribution and convergence of individual traits

Having assumed that generation $t = 0$ is characterized by only two (pure) traits, δ^L and δ^H, the number of possible traits (both mixed and pure) characterizing generation t is $S(t) = 2^t + 1$. To see why, take generation $t = 0$. Here, the number of traits is $S(0) = 2^0 + 1 = 2$. In generation $t = 1$ there are three possible traits, δ^L, $\frac{\delta^L+\delta^H}{2}$, δ^H. Therefore, $S(1) = 2^1 + 1 = 3$. In generation $t = 2$ five traits are possible, δ^L, $\frac{3\delta^L+\delta^H}{4}$,$\frac{2\delta^L+2\delta^H}{4}$,$\frac{\delta^L+3\delta^H}{4}$, δ^H. Hence, $S(2) = 2^1 + 1 = 5$. More generally, in generation t, the possible traits are

$$\delta_t(j) := \frac{(2^t - j)\delta^L + j\delta^H}{2^t} = \delta^L + \frac{\delta^H - \delta^L}{2^t} j, \qquad \text{with } j = 0, 1, \ldots, 2^t. \tag{2}$$

Note that each trait is identified by the index j. Define $\pi_0 = (\pi_0(0), \pi_0(1))$ the initial distribution of δ^L and δ^H, with $\pi_0(0) = \frac{n_0^L}{n_0}$, and $\pi_0(1) = \dfrac{n_0^H}{n_0}$.

The distribution evolves across generations due to random matching. In generation t, the distribution will be

$$\pi_t = (\pi_t(0), \pi_t(1), \ldots, \pi_t(2^t + 1)), \quad \text{with} \sum_{j=0}^{2^t+1} \pi_t(j) = 1 \quad \text{for all } t \geq 0$$

Hence, the average trait of generation t is

$$\delta_t := \sum_{j=0}^{2^t+1} \pi_t(j)\delta_t(j)$$

How does the distribution evolve from generation to generation? Appendix 1 demonstrates the following.

Proposition 1 *In each period* t*, the distribution of mixed traits* $\delta_t(j)$ *is binomial, with mean* $\delta^L + \frac{\delta^H-\delta^L}{2^t}(2^t + 1)\pi$ *and variance* $\left(\frac{\delta^H-\delta^L}{2^t}\right)^2 (2^t + 1)\pi(1 - \pi)$.

As $t \to \infty$*, the expected trait held by all agents is*

$$\delta^* := (1 - \pi)\delta^L + \pi\delta^H.$$

Take the simple case where $\delta^H = 1$ and $\delta^L = 0$. In the long run, everybody will have the same trait, $\delta = \pi$, where

	$\pi = 0.1$	$\pi = 0.2$	$\pi = 0.3$	$\pi = 0.4$	$\pi = 0.5$
$\sigma = 0.01$	9.81	10.64	11.03	11.22	11.28
$\sigma = 0.05$	5.20	6.02	6.41	6.60	6.66

Table 1: Number of generations (periods) t needed to reach a distribution of the population whose standard deviation is $\sigma \in \{0.01, 0.05\}$. The initial distribution $\pi = \frac{n_0^H}{n_0}$ takes values in the set $\{0.1, 0.2, \ldots, 0.5\}$. For simplicity, we have also used $\delta^H = 1$ and $\delta^L = 0$.

$$\pi = \frac{n_0^H}{n_0}.$$

But how long is the long run? A sensible way to address this question is to calculate in how many generations t the standard deviation of the binomial distribution of δ will become $\sigma \in \{0.01, 0.05\}$ for $\pi \in \{0.1, 0.5\}$. The answer is found solving the equation

$$\Big(\frac{\delta^H - \delta^L}{2^t}\Big)^2 (2^t + 1)\pi(1 - \pi) = \sigma^2, \qquad \text{for } \pi \in \{0.1, 0.2, \ldots, 0.9\}.$$

The value of t associated with each (π, σ) is shown in Table 1. Of course, the limit δ (the mean of the distribution) will vary with (π, σ) too.

The first column of this table says that, if 10 percent of the population is initially characterized by $\delta = 1$, and the remaining 90 percent by $\delta = 0$, so that the limit value of δ is 0.1, it will take 5.2 generations for the standard deviation to become equal to 0.05, and another 4.61 generations for it to fall to 0.01. assuming that generations overlap every 20 years, this means that it will take 130 years for approximately 68 percent of the population to have a δ comprised between 0.095 and 0.105, and more than 245 years for that same share of the population to have a δ comprised between 0.099 and 0.101.The remaining columns show how the convergence slows down, and the limit value of δ gets closer to zero, as the initial share of individuals with $\delta = 1$ rises from one tenth to a half of the total population.

To appreciate the implications of these numbers, imagine that the entire population was originally characterized by δ^L. Suppose that there is then a once-for-all influx of immigrants, equal in size to one ninth of the native population, and that all the newcomers are characterized by δ^H. After between five and ten generations, the population will be fairly homogeneous again, and its characteristics will be very similar to those of the original population. In other words, the immigrants will have been absorbed by the native population. If the number of immigrants is larger than one ninth, but no larger than one half of the native population (i.e., not so large that the immigrants outnumber the natives), it will take longer for the population to become homogeneous again, and the future inhabitants will not look much like the original ones. In other words, there will be convergence, but not absorption.

3 Preferences, wages and family rules

We now consider a specific model where δ is a preference parameter that may give rise to a family rule. The model is a stripped-down version of the one in Cigno et al. (2017).[3] We assume that individuals live two periods. A person is young in period 1, old in period 2. The young can work, marry and have children, the old cannot. If a young woman marries a young man, they have a daughter and a son. Individuals derive utility from their consumption of market goods, and from filial attention. Market goods (including the personal services of professional helpers) are not perfect substitutes for filial attention. When a marriage takes place, the wage rates of the directly interested parties (and their respective parents) are known, but those of their children are not.

Let c_{pi} denote i's consumption in period $p = 1, 2$. Let $a^i_{k_i}$ be the amount of filial attention that i receives from $k_i = D_i, S_i$, where D_i is i's daughter and S_i is i's son, in period 2. The utility function is

$$U_i = c_{1i} + \ln c_{2i} + \max\left(0,\ \delta_i\left(\ln \beta a^i_{D_i} + \ln \beta a^i_{S_i}\right)\right), \tag{3}$$

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[3] The original model is more complicated than the present one because it allows for descending altruism, parental investment in the children's education, and bequests. Where the issues addressed in the present paper are concerned, nothing of substance would change if we used that model.

where the parameter δ_i measures i's taste for filial attention, and the constant β is a scaling factor designed to make $\ln \beta a_k^i$ positive for a_k^i sufficiently large.[4] Notice that parents are not altruistic towards their children, and that children are not altruistic towards their parents. Allowing for a modicum of altruism on either side would make the analysis less sharp without altering the results in any substantive way.

If i stays single, he or she maximizes (3) subject to the period budget constraints

$$c_{1i} + s_i = w_i$$

and

$$c_{2i} = rs_i.$$

Having assumed that only married couples have children,

$$a_D^i = a_S^i = 0. \tag{4}$$

The pay-off of singlehood is then

$$R_i := \max_{s_i} (w_i - s_i + \ln rs_i) = w_i - 1 + \ln r.$$

3.1 Bargaining in the absence of family rules

Will i marry? As assumed in Section 1, a match is a random draw from the entire population of n_0 males and n_0 females (further down we shall allow for the possibility that it is a draw from a particular sub-population). One way to interpret this is to say that the matching process is driven by passion rather than calculation. When a match is drawn, the couple may either marry, or split; there is no re-sampling. If they marry, they Nash-bargain over the allocation of their time and income. Having assumed that people are not altruistic, parents

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[4] Otherwise, $\ln t_k^i$ would be negative for any t_k^i smaller than unity.

will not get filial attention as a present. They could buy it off their children. Given that the good in question does not have a perfect market substitute, however, the children would form a cartel, and set the price so high that the entire surplus generated by the transaction would go to them.[5] Parents are thus indifferent between buying and not buying attention from their children. We assume that they will not. Could children be *obliged* by some family rule to provide the good? In the present subsection we assume that no such a rule exists, and thus that (4) applies even if i is married.

Take the (f, m) couple. Having ruled out re-sampling, and thus that f's (m's) best alternative to marrying m (f) is to remain single, the Nash-bargaining equilibrium maximizes

$$N = (U_f - R_f)(U_m - R_m), \tag{5}$$

subject to the period budget constraints

$$c_{1f} + s_f = w_f + T,$$

$$c_{2f} = rs_f,$$

$$c_{1m} + s_m + T = w_m$$

and

$$c_{2m} = rs_{m,}$$

where T is defined as a transfer from m to f in period 1.

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[5] Bernheim et al. (1985) argue that, as an alternative to paying cash, a parent could commit to bequeathing her entire fortune either to the child who has given her the most attention or, if that attention falls below a certain minimum, to a third party. According to this argument, the surplus would go to the parent, rather than to the children. Cigno (1991) points out, however, that the children could counter the parent's strategy by drawing-up a perfectly legal contract committing only one of them to give the parent the minimum amount of attention required to inherit the lot, and then to share the inheritance (minus a specified amount as compensation for the attention given) equally with the others. That would give the entire surplus back to the children.

We show in Appendix 2 that the equilibrium is

$$\widehat{s}_f = \widehat{s}_m = 1,\ \widehat{T} = 0.$$

The equilibrium pay-offs are

$$\widehat{U}_i = w_i - 1 + \ln r = R_i,\ i = f, m. \tag{6}$$

Strictly speaking, therefore, the couple are indifferent between marrying or splitting. For consistency with the assumption made in Section 2, we assume that they marry.

3.2 Bargaining in the presence of family rules

Now let F_i and M_i denote, respectively, i's father and mother (both defunct when i's children, D_i and S_i, are born). Cigno et al. (2017) demonstrate that a rule requiring every young person to give a certain amount of attention to each of her or his elderly parents,

$$a_i^{h_i} = \frac{\delta^{h_i}}{w_i},\ a_k^i = \frac{\delta^i}{w_{k_i}},\ h_i = F_i, M_i,\ i = f, m,\ k_i = D_i, S_i, \tag{7}$$

conditional on the beneficiary having done the same a period earlier, is self-enforcing and renegotiation-proof[6] if (a) $\delta_{h_i} = \delta_i = \delta_{k_i}$, (b) $w_{h_i} = w_i = w_{k_i}$,and (c) these parameters satisfy a certain condition. They also show that, in equilibrium, every adult whose preferences and wage rate would satisfy the said condition if he or she marry a member of the opposite sex who holds the same preferences, and has the same wage rate, will indeed do so. The couples thus formed will have an interest in transmitting or inculcating their common preferences on to their children. In the present model, however, couples are formed at random, and there is thus the possibility that the children's preferences will differ from those of either or both parents. Consequently, it does not make much sense to look for a self-enforcing,

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[6] Meaning that the rule supports a Nash-equilibrium and is not Pareto-dominated by any other rule also supporting such an equilibrium.

renegotiation-proof rule that will remain in place for ever. We shall then limit ourselves to looking for conditions such that it is in i's interest to give $a_i^{h_i}$ to h_i in order to receive $a_{k_i}^i$ from k_i. Like Cigno et al. (2017), we assume that w is a random variable,[7] taking value w^H with probability ψ, and value w^L with probability $1-\psi$, $w^H > 1 > w^L > 0$.

Given (7), i's reservation utility is now $\widehat{U}_i$. The Nash-bargaining equilibrium then maximizes

$$N' = \left(U_f' - \widehat{U_f}\right)\left(U_m' - \widehat{U_m}\right), \tag{8}$$

subject to

$$c_{1f} + s_f = \left(1 - a_f^{F_f} - a_f^{M_f}\right) w_f + T,$$

$$c_{2f} = r s_f,$$

$$c_{1m} + s_m + T = \left(1 - a_m^{F_m} - a_m^{M_m}\right) w_m$$

and

$$c_{2m} = r s_m.$$

Assuming an interior solution (or the rule would be inoperative), and recalling that k's wage rate is an expectation at the time when the (f, m) couple is formed, we show in Appendix 3 that the equilibrium is

$$s_f^* = s_m^* = 1,\ T^* = \frac{w_m - w_f}{2} + 2\left[\delta_m\left(\frac{\ln\beta\delta_m}{E\ln w} - 1\right) - \delta_f\left(\frac{\ln\beta\delta_f}{E\ln w} - 1\right)\right],$$

where

$$E\ln w := \psi\ln w^H + (1-\psi)\ln w^L.$$

In contrast with the case without family rules, the compensatory transfer from m to f may

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[7] There, however, the distribution of a person's wage rate is conditioned by the amount of educational investment made by that person's parents. In our simplified model there is no educational investment.

thus be different from zero (positive or negative). The equilibrium pay-offs are now

$$U_f^* = \frac{3w_f + w_m}{2} + \delta_f \left(\frac{\ln \beta \delta_f}{E \ln w} - 1 \right) + \delta_m \left(\frac{\ln \beta \delta_m}{E \ln w} - 1 \right) - 1 + \ln r \tag{9}$$

and

$$U_m^* = \frac{w_f + 3w_m}{2} + \delta_f \left(\frac{\ln \beta \delta_f}{E \ln w} - 1 \right) + \delta_m \left(\frac{\ln \beta \delta_m}{E \ln w} - 1 \right) - 1 + \ln r. \tag{10}$$

It will be in the couple's common interest to comply with (7) if and only if

$$U_f^* - \widehat{V_f} = U_m^* - \widehat{V}_m = \frac{w_f + w_m}{2} + \delta_f \left(\frac{\ln \beta \delta_f}{E \ln w} - 1 \right) + \delta_m \left(\frac{\ln \beta \delta_m}{E \ln w} - 1 \right) \geq 0 \tag{11}$$

Therefore, some couples will comply with (7), and some will not. Those who do not comply will neither give, nor receive filial attention. Those who comply will do both, but the amounts given or received will not be necessarily the same for all of them. Given that filial attention has no perfect market or government provided substitutes, this has implications for the design of welfare policy.

3.3 Evolution of preferences and family rules under restricted and unrestricted random matching

We saw in Section 2 that, if a fraction π of the initial population is characterized by δ^H, and the rest by δ^L, and provided that any young person is free to marry any young person of the opposite sex, the distribution of δ will tend to approximate the normal distribution and ultimately collapse to a single value,

$$\delta = \pi \delta^H + (1 - \pi) \delta^L.$$

In the long run, therefore, (11) reduces to

$$Ew + 2\delta\left[\ln\beta\pi - E\ln w - 1\right] \geq 0 \tag{12}$$

where

$$Ew := \psi w^H + (1-\psi)\, w^L.$$

This implies that, eventually, either everybody will obey the same rule, or nobody will obey any.

But suppose that, in a subset of the population distinguished by a visible characteristic (religion, ethnicity) θ, the share of δ^H is initially π_θ, while in the remaining part of the population it is initially π_0, with $\pi_0 < \pi < \pi_\theta$. For an i that belongs to the θ segment of the initial population, the expected benefit of obeying (7) is then

$$E\left(U_f^* - \widehat{V_f}\right) = \frac{3w_f + Ew}{2} + \delta_i\left(\frac{\ln\beta\delta_f}{E\ln w} - 1\right) + E\left[\delta_m\left(\frac{\ln E\beta\delta_m}{E\ln w} - 1\right)\right], \tag{13}$$

with

$$E\left[\delta_m\left(\frac{\ln E\beta\delta_m}{E\ln w} - 1\right)\right] = \pi\delta^H\left(\frac{\ln\beta\delta^H}{E\ln w} - 1\right) + [1-\pi]\,\delta^L\left(\frac{\ln\beta\delta^L}{E\ln w} - 1\right)$$

if f is free to marry any young person of the opposite sex, and

$$E\left[\delta_m\left(\frac{\ln E\beta\delta_m}{E\ln w} - 1\right)\right] = \pi_\theta\delta^H\left(\frac{\ln\beta\delta^H}{E\ln w} - 1\right) + [1-\pi_\theta]\,\delta^L\left(\frac{\ln\beta\delta^L}{E\ln w} - 1\right)$$

if she is restricted to marrying another θ. Clearly, $E\left(U_f^* - \widehat{U}_f\right)$ is higher in the second case than in the first. Symmetrical expressions apply to m.

In the long run, if the ban on marrying outside the θ community is maintained, the condition for every member of this subpopulation to obey (11) becomes

$$Ew + 2\delta\left[\ln\beta\delta_\theta - E\ln w - 1\right] \geq 0, \tag{14}$$

where

$$\delta_{\pi} := \pi_{\theta}\delta^{H} + (1 - \pi_{\theta})\,\delta^{L}. \tag{15}$$

As δ_{θ} is greater than δ, (15) is clearly less stringent than (12). It may then be the case that all members of the θ community will obey the same family rule, while the rest of the population will either obey no rule, or obey a watered-down version of the one in force in the θ subpopulation. That would strengthen the identity of the latter, and widen their distance from the rest of the population.

Given that people are better-off if a self-enforcing family rule is in operation – because that gives them access to otherwise unattainable non-market goods – there may thus be a rationale for members of a subpopulation whose δ is on average higher than in the rest of the population, to marry among themselves.[8] This proposition clashes, however, with the assumption that young people are driven by passion rather than calculation. Calculating parents may then try to stop their passionate children following their inclinations, *for their own good*, by restricting their contacts with persons of the *wrong* type.[9] But it may also be the case that parents wrongly believe θ to be positively correlated with δ (simply because they have only hearsay information about what happens outside their community), while in reality there is no such correlation. Or it may even be that – no matter whether there is any association between θ and δ – parents characterized by θ regard marrying outside their community as a betrayal or even a sin, and that they will thus feel obliged to stop this happening by any means, including physical force or even murder.

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[8] If θ-types marry only θ-types, the distribution of δ among members of this subpopulation will converge to

$$\delta(\theta) = \pi(\theta)\,\delta^{H} + [1 - \pi(\theta)]\,\delta^{L} > \delta^{*}.$$

[9] Indeed, calculating parents will insist on their children marrying someone with a w at least as high as their own.

4 Discussion

Assuming that the members of each sex are differentiated by an inherited trait, and that couples are formed at random, we have shown that the variance of the trait will gradually diminish as a result of mixed marriages, and that everybody will ultimately have the same trait. This has implications for the effects of an inflow of immigrants characterized by a common trait that is different from the one which is common to the residents. The population that results from this inflow will tend to have a new common trait (melting-pot effect). Unless the original immigrants are unrealistically numerous (more numerous than the original residents), however, the new common trait will be closer to that of the original residents, than to that of the immigrants. If members of a subpopulation distinguishable by some visible characteristic correlated with the trait in question are constrained to marry among themselves, however, there will be no universal convergence towards a common trait. This subpopulation will converge to a common trait, and the rest towards another. This will only make the cleavage between the two parts of the population more evident.

Using a specific model of family decisions where the inherited trait is a parameter measuring a person's taste for receiving filial attention in old age, we have also shown that, if the parameter values characterizing a couple satisfy a certain condition, it is in the couple's common interest to obey a rule requiring each of them to give specified amounts of attention to their respective parents. These amounts depend on the beneficiaries' taste for filial attention. As the distribution of the preference parameter evolves as a result of mixed marriages, so does the share of the population who obey a family rule, and the amount of filial attention that is given and received by those who obey it. This has implications for the design of welfare policy, because filial attention has no market or government provided perfect substitutes. The model differs from the one in Cigno et al. (2017), where couples are assortatively matched according to their preferences and wage rates, and it is in the interest of anyone whose preferences are compatible with the existence of a rule like the one we have mentioned to seek out and marry a like-minded member of the opposite sex. In that paper, the preferences of those who comply with some such rule remain the same for ever.

The present paper bears similarities with a strand of economic literature that also aims to predict how preferences evolve from one generation to the next, but it differs from it in at least one fundamental respect. All the contributions to that literature assume in fact that the preferences (or "values") parents transmit to their children are, at least in part, altruistic towards society as a whole. Some of these contributions, like Bisin and Verdier (2001), and Tabellini (2008), assume that individuals reproduce asexually, and that preferences are modified by peer influence. In those papers, therefore, evolution occurs through socialization rather than mixed marriages. Others, like Alger and Weibull (2013), assume sexual reproduction and random matching as in the present paper. Unlike us, however, they use the evolutionary stability notion developed in Weibull (1995) to show that pro-social behaviour will tend to prevail. All these contributions are complementary rather than antithetical to ours, and help explain why contemporary behaviour appears to be influenced by ancient occurrences as reported, for example, by Alesina et al. (2013), or Galor and Özak (2016).

Appendix 1. Evolution: Proofs

Proof of Lemma 1. *1.* The average number of mixed-trait individuals in the group after $t \geq 1$ periods is

$$\mu_k(t) = e_k^\intercal \mathcal{M}^t \kappa = \sum_{k'=1}^{n_0} M_{kk'}^t k' = \sum_{k'=k}^{n_0} M_{kk'}^t k' \geq k,$$

We have

$$\mu_k(t+1) - \mu_k(t) = e_k^\intercal \mathcal{M}^t (\mathcal{M} - I)\kappa \geq 0$$

because each element of vector $(\mathcal{M} - I)\kappa$ is non-negative.

2. Consider two different initial states of the world that differ according to their initial number of mixed-trait individuals, one state is k, while the other is $k+1$. Let agent ℓ have a native trait in the $k-$state and a mixed-trait in the $(k+1)-$state. Recall that the matching process is independent of k. Let $\mathcal{P}(t)$ be the set of native-trait individuals at date t when the group starts with k mixed-trait individuals in period t_0. We have

$$|\mathcal{P}(t_0)| = n_0 - k \quad \text{and} \quad \mathcal{P}(t+1) \subseteq \mathcal{P}(t), \text{ for } t \geq t_0$$

Consider $\ell \in \mathcal{P}(t_0)$, and denote by $\mathcal{D}_\ell(t)$ the set of new mixed-trait individuals generated by making ℓ a initial mixed-trait individual, instead of a native-trait one. Clearly $\mathcal{D}_\ell(t_0) = \{\ell\}$ and $\mathbb{E}|\mathcal{D}_\ell(t)| \geq 1$, where $\mathbb{E}|\mathcal{D}_h(t)|$ denotes the expected number of additional mixed-trait individuals on date t as a consequence of agent ℓ being an initial mixed-trait individual in the $(k+1)-$group. We have

$$\mu_{k+1}(t) - \mu_k(t) = \mathbb{E}|\mathcal{D}_\ell(t)|$$

because $\mu_k(t)$ is the expected cardinality of the set of defectors present on date t, given k initial defectors.

3. It is sufficient to show that, starting from any state $k < n_0$, the probability of not being absorbed is zero as t goes to infinity. Let $\sum_{k'=k}^{n_0-1} M_{kk'}^{(t)} = 1 - M_{kn_0}^{(t)}$ be the probability that, starting from k the process will not reach the absorbing state in t steps. Then $1 - M_{kn_0}^{(t)} < 1$.

The probability of not being absorbed in $2t$ steps is

$$1 - M_{kn_0}^{(2t)} = 1 - \sum_{k'=k}^{n_0} M_{kk'}^{(t)} M_{k'n_0}^{(t)} = 1 - \sum_{k'=k}^{n_0} M_{kk'}^{(t)} M_{k'n_0}^{(t)} + (M_{kn_0}^{(t)})^2 - (M_{kn_0}^{(t)})^2 < (1 - M_{kn_0}^{(t)})^2$$

because $\sum_{k'=k}^{n_0} M_{kk'}^{(t)} M_{k'n_0}^{(t)} + (M_{kn_0}^{(t)})^2 > 2M_{kn_0}^{(t)}$. Eventually, $\lim_{t\to\infty}(1 - M_{kn_0}^{(t)}) = 0$.

■

Proof of Proposition 1. In period $t = 0$, each group (male of female) is partitioned in two subgroups $n_0\pi_0(0)$ individuals have the trait δ^L, while $n_0\pi_0(1)$ individuals have the trait δ^H. Given that siblings cannot marry each other, there are $n_0(n_0 - 1)$ possible couples, where $n_0 = n_0^L + n_0^H$.

In period $t = 1$, $S(1) = 3$ traits are possible. The probability to have a match between two L-types, which gives birth to a male and a female with the trait δ^L, is

$$\pi_1(0) = \frac{n_0^L(n_0^L - 1)}{n_0(n_0 - 1)} = \frac{\pi_0(0)(n_0\pi_0(0) - 1)}{n_0 - 1} \overbrace{\approx}^{n_0 \text{ large}} \pi_0^2(0)$$

Similarly, the probability to have a match between two H-types, which gives birth to a male and a female with the trait δ^H, is

$$\pi_1(2) = \frac{n_0^H(n_0^H - 1)}{n_0(n_0 - 1)} = \frac{\pi_0(1)(n_0\pi_0(1) - 1)}{n_0 - 1} \approx \pi_0^2(1)$$

Finally, the probability to have a match between a L-type and an H-type, which generates two individuals with the mixed trait $\frac{\delta^L + \delta^H}{2}$, is

$$\pi_1(1) = \frac{2n_0^H n_0^L}{n_0(n_0 - 1)} = \frac{2\pi_0(1)\pi_0(0)n_0}{n_0 - 1} \approx 2\pi_0(1)\pi_0(0).$$

At the end of period $t = 1$ there are still n_0 males and n_0 females (grandchildren replace grandparents), however, for each of these groups $\pi_1(0)n_0$ individuals will have now a trait $\delta_1(0) = \delta^L$, $\pi_1(1)n_0$ individuals will have a trait $\delta_1(1) = \frac{\delta^L + \delta^H}{2}$, while $\pi_1(2)n_0$ individuals will inherit a trait $\delta_1(2) = \delta^H$.

In period $t = 2$, $S(2) = 5$ traits are possible. The probability to have a match between two L-types, which will preserve the native trait δ^L, is now

$$\pi_2(0) = \frac{\pi_1(0)n_0[\pi_1(0)n_0 - 1]}{n_0(n_0-1)} \approx \pi_1^2(0) = \pi_0^4(0)$$

The probability to generate a trait $\delta_2(1) = \frac{3\delta^L + \delta^H}{4}$ is the probability that a L-type meets a type with a trait $\frac{\delta^L+\delta^H}{2}$, that is

$$\pi_2(1) = \frac{2\pi_1(0)n_0\pi_1(1)n_0}{n_0(n_0-1)} \approx 2\pi_1(0)\pi_1(1) = 4\pi_0^3(0)\pi_0(1)$$

The probability to generate a trait $\delta_2(2) = \frac{2\delta^L+2\delta^H}{4}$ is the probability that two types with trait $\frac{\delta^L+\delta^H}{2}$ meet each other plus the probability that L meets H, that is

$$\pi_2(2) = \frac{\pi_1(1)n_0[\pi_1(1)n_0-1]}{n_0(n_0-1)} + \frac{2\pi_1(0)\pi_1(2)n_0}{n_0-1} \approx \pi_1^2(1) + 2\pi_1(0)\pi_1(2) = 6\pi_0^2(0)\pi_0^2(1)$$

The probability to generate a trait $\delta_2(3) = \frac{1\delta^L+3\delta^H}{4}$ is the probability that an H-type meets a type with a trait $\frac{\delta^L+\delta^H}{2}$, that is

$$\pi_2(3) = \frac{2\pi_1(2)n_0\pi_1(1)n_0}{n_0(n_0-1)} \approx 2\pi_1(2)\pi_1(1) = 4\pi_0^3(1)\pi_0(0)$$

and finally, the probability to generate a match between two H-types, which generates again a native trait δ^H, is

$$\pi_2(4) = \frac{\pi_1(2)n_0[\pi_1(2)n_0 - 1]}{n_0(n_0-1)} \approx \pi_1^2(2) = \pi_0^4(1)$$

Hence, the distribution of j in period t is given by a binomial distribution $B(2^t+1, \pi)$, with mean $(2^t+1)\pi$ and variance $(2^t+1)\pi(1-\pi)$, where without loss of generality we have defined $\pi := \pi_0(1)$, with $\pi_0(0) = 1-\pi$. In other words, the probability to have j in period t is

$$\pi_t(j) = \binom{2^t+1}{j}\pi^j(1-\pi)^{2^t+1-j}$$

From the definition of $\delta_t(j)$ in (2), since $\delta_t(j)$ is a linear transformation of the random variable j, the distribution of $\delta_t(j)$ in each period t is given by a binomial distribution with mean

$\delta^L + \frac{\delta^H - \delta^L}{2^t} \cdot (2^t + 1)\pi$ and variance $\left(\frac{\delta^H - \delta^L}{2^t}\right)^2 (2^t + 1)\pi(1-\pi)$. Clearly, as $t \to \infty$ the variance goes to zero, while the mean converges to $\delta^L(1-\pi) + \delta^H \pi$. ■

Appendix 2. Nash-bargaining without family rules

Using the FOCs for the maximization of (5),

$$\frac{\partial N}{\partial T} = (U_m - R) - (U_f - R) = 0,$$

$$\frac{\partial N}{\partial s_f} = \left(-1 + \frac{1}{s_f}\right)(U_m - R) = 0$$

and $\frac{\partial N'}{\partial b} = -\frac{2}{rs_m - 2b}\left(V_f' - \hat{V}_f\right) + \frac{2\alpha_f}{b}\left(V_m' - \hat{V}_m\right) + \frac{2\alpha_m}{b}\left(V_f' - \hat{V}_f\right) = 0.$

$$\frac{\partial N}{\partial s_m} = \left(-1 + \frac{1}{s_m}\right)(U_f - R) = 0,$$

we find

$$\hat{s}_f = \hat{s}_m = 1 \text{ and } \hat{T} = 0.$$

Substituting $\hat{s}_f$, $\hat{s}_m$ and $\hat{T}$ into the expression for U_f or U_m gives us the common pay-off $\widehat{U}$.

Appendix 3. Nash-bargaining with family rules

Using the FOCs for the maximization of (8),

$$\frac{\partial N'}{\partial T} = \left(V_f' - \hat{V}_f\right) - \left(V_m' - \hat{V}_m\right) = 0,$$

$$\frac{\partial N'}{\partial s_f} = \left(-1 + \frac{r}{rs_f}\right)\left(V_m' - \hat{V}_m\right) = 0$$

and

$$\frac{\partial N'}{\partial s_m} = \left(-1 + \frac{r}{rs_m - 2b}\right)\left(V_f' - \hat{V}_f\right) = 0,$$

we find the Nash-bargaining equilibrium for the case in which f and m obey family rules, and the solution to the Nash-maximization problem subject to these rule is interior (i.e., the amount of filial attention received by f and m is large enough to add to their utility),

$$\widehat{s}_f' = \widehat{s}_m' = 1,\ \widehat{T}' = \frac{\delta_{F_f} + \delta_{M_f} - \delta_{F_m} - \delta_{M_m}}{2} + \delta_m \ln \beta \frac{\delta_m}{w} - \delta_f \ln \beta \frac{\delta_f}{w}.$$

T' is determined so that $\left(V_f' - R\right) = (V_m' - R)$, and thus that $V_f' = V_m'$. Substituting $\widehat{s}_f'$ and $\widehat{T}'$ into the expression for V_f' or V_m', we find $\widehat{V}.'$

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