

The footprint of evolutionary processes of learning and selection upon the statistical properties of industrial dynamics

Giovanni Dosi^{*1}, Marcelo Pereira^{†2} and Maria Enrica Virgillito^{‡1}

¹*Sant'Anna School of Advanced Studies*

²*Universidade Estadual de Campinas*

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Abstract

Evolutionary theories of economic change have identified as the two main drivers of the dynamics of industries the mechanisms of *market selection* and of *idiosyncratic learning* by individual firms. In this perspective, the interplay between these two engines shapes the dynamics of entry-exit and the variations of market shares and, collectively, the patterns of change variables such as average productivities, size and growth rates.

In the following contribution we shall address the construction of an agent based model that tries to take into account, via simple behavioural rules, the most relevant micro stylised facts, with particular attention devoted to the analysis of the condition under which fat tail distributions emerge.

Keywords

Firms Growth Rate, Productivity, Fat Tail Distributions, Learning Processes, Market Selection Mechanism.

JEL Classification

C63-L11-L6

1 Introduction

Evolutionary theories of economic change have identified as the two main drivers of the dynamics of industries the mechanisms of *market selection* and of *idiosyncratic learning* by individual firms. In this perspective, the interplay between these two engines shapes the dynamics of entry-exit and the variations of market shares and, collectively, the patterns of change variables such as average productivities, size and growth rates.

Learning (what in the empirical literature is sometimes called the *within effect*) entails a

*gdosi<at>sssupsup.it

†marcelocpereira<at>uol.com.br

‡m.virgillito<at>sssupsup.it (corresponding author)

various processes of idiosyncratic innovation, imitation, changes in technique of production. Selection (what in the empirical literature is called the *between effect*) is the outcome of processes of market interaction where more competitive firms gain market shares at the expense of less competitive ones. A stream of analysis – from the pioneering work by Ijiri and Simon (1977) all the way to Bottazzi and Secchi (2006a) – studies the result of both mechanisms in terms of the ensuing exploitation of new business opportunities, captured by the stochastic process driving growth rates.

A second stream, including several contributions by Metcalfe (see among others Metcalfe (1998)), focuses on the processes of competition/selection often represented by means of a replicator dynamics where shares vary as a function of the relative competitiveness or “fitness”.

Finally, many evolutionary models unpack the two drivers of evolution distinguishing between some idiosyncratic processes of change in the techniques of production, on the one hand, and the dynamic of differential growth driven by differential profitabilities and the ensuing rates of investment (such as in Nelson and Winter (1982)). Or, on the other hand by an explicit replicator dynamics, (such Silverberg et al. (1988) and Dosi et al. (1995)).

Whatever the analytical perspective, the purpose is to account for one or several of the many empirical regularities that emerge from the statistical analysis of the industrial dynamics (for a critical survey, see Dosi (2007)). In particular, the stylized facts include:

- persistent heterogeneity in productivity and all other performance variables;
- persistent market turbulence due to change in market shares and entry-exit phenomena;
- fat tail distribution of growth rates;

In the following contribution we shall address the construction of an agent based model that tries to take into account, via simple behavioural rules, the main findings discussed above, with particular attention devoted to the analysis of the condition under which fat tail distributions emerge. In particular in section 2 we will briefly summarize the empirical stylized facts, in section 3 we will discuss the main theoretical models oriented at explaining the shape of the firm growth rate, in section 4 we will present our model, in section 5 the related analysis of properties and results, in section 6 our conclusions.

2 Empirical stylised facts: productivity and size

During the last two decades, many studies in industrial organization and particularly in industrial dynamics have pointed out the emergence of a rich ensembles of stylised facts related to *productivity* and *size* distribution. In the next section we will explore in details the related findings.

2.1 Productivity distribution

Firstly, as extensively discussed in Doms and Bartelsman (2000), Syverson (2011), Dosi (2007) and Foster et al. (2008) among many others, *productivity dispersion*, at different level of disaggregation, is a rather striking and robust phenomenon. Secondly, the heterogeneity across firms shows to be *persistent over time*, as mainly described in Bartelsman and Dhrymes (1998), Dosi and Grazzi (2006) and Bottazzi et al. (2008) with autocorrelation coefficients in the range 0.8–1. A clear-cut evidence is provided in figure 1.a, that illustrates the dynamic of labour productivity. The distribution shows a wide support that goes from 1 to 5 in log terms suggesting huge dispersion in terms of the ability to transform input in output among producers. The distribution is stable over time and the support, as suggested by the high autocorrelation coefficient, does not shrink. Table 1.b describes how strong is

the persistence nature of productivity level. These findings, robust to the use of parametric and non-parametric tools, empirically discard any idea of a unified production process among firms, where a unique, optimal combination between inputs and output is in place.

An other relative interesting feature is the *asymmetry* of the distribution at sectoral level. Dosi et al. (2012) suggest how the thicker left tail narrates an history where low productive firms persistently survive in a market structure characterized by an high degree of tolerance. Also the asymmetry, as the wide support, is time invariant, suggesting that there is no trace of convergence toward higher degree of selection in the competitive arena.

A some how less explored phenomenon related to the dynamic of productivity is the *double exponential nature* of its growth rate distribution. Extensive evidence is provided in Bottazzi et al. (2005) and Dosi et al. (2012). Figure 1.c supplies a better visual explanation. The double exponential nature of growth rate in productivity, which in a log-log scale reads as a Laplace distribution, hints at an underlining multiplicative process that determines efficiency performance.

2.2 Size distribution

The *power-law nature* (Pareto or Zipf law according to whether the slope of the straight line, in a log-log plot, is less than 2 or equal to 1 respectively)¹ of the firm size distribution, has been investigated by many authors² since the pioneering work by Simon and Bonini (1958), where a modified version of the Gibrat Law (that per se converges to a log-normal limit distribution), with a constant rate of entry is explored as generating mechanism of Pareto distribution in size. The authors interpret the parameter of the Yule distribution, which asymptotically converges to a Pareto distribution, as the rate of entry in a given industry. Hence it provides a degree of concentration in the market. But, differently from the other empirical stylised facts previously discussed, the Pareto distribution is not robust to the level of disaggregation. Particularly, being the Pareto a scale free distribution, in principle it should be scale invariant, or equivalently, it should be detectable irrespectively of the considered level of aggregation. Bottazzi et al. (2007), Dosi et al. (2008) find that the size distributions fairly differ across sectors in terms of shape and fatness of the tails. Technological factors, the cumulative process of innovation, the predominance of process or product innovation strongly affect sector specific size distribution (Marsili, 2005). It seems that the empirical findings of the Zipf (Pareto) law distribution is a mere effect of the aggregation as already discussed in Dosi et al. (1995). Nonetheless, the skewness of the distribution, due the larger presence of small units relative to big units, seems to be a rather robust phenomenon also at a sectoral decomposition level (see figure 1.d).

A huge empirical literature testifies the emergence of *Laplace distribution* in growth rates. A typical empirical finding is illustrated in 1.e. Differently from the Pareto distribution, this applies across different levels of sectoral disaggregation, across countries, over different historical periods for which there are available data and it is robust to different measures of growth, e.g. in terms of sales, value added or employment, (for more details see Bottazzi et al. (2002), Bottazzi and Secchi (2006a), Bottazzi et al. (2008) and Dosi (2007)).

Finally, how the level and the size growth rate are related one to each other? Since Stanley et al. (1996) an extensive literature of empirical papers found a negative relation between the variance of growth rates and size (see Sutton (2002), Lee et al. (1998), Bottazzi and Secchi (2006b)). An illustration of the phenomenon is provided in figure 1.f. The underlining idea is that firms can be described as a collection of independent units, each of them characterised by a growth process. The higher the firm size, the higher the number of its components

¹See Newman, 2005 for a succinct overview.

²See Stanley et al., 1995 and Axtell, 2001 for US manufacturing data.

(plants, departments); hence under the assumption of independent growth processes for each of the unit, according to the CLT, the variance of the growth rate decreases proportionally to the inverse square root of size.

2.3 Market turbulence

As discussed in Dosi et al. (2008), the emergence of the above mentioned invariances in the market structure, that clearly depict a puzzling idea of some sort of coordination, coexists with the occurrence of a persistent change in the dynamic of market shares mainly due to entry-exit phenomena. This persistent turbulence, due to entry-exit flows, with changes in incumbents market shares are well discussed in Baldwin and Rafiquzzaman (1995) and Doms and Bartelsman (2000). Firms do not maintain a static position in the market arena: they benefit from period of expansion and they are worn out by severe contractions, where this may even occur anti-cyclically with respect to the trend of the related industry. Bartelsman et al. (2005) reports a turnover rates around 15%-20% across countries. Additionally entry and exit cross-correlate. They suggest that cross-correlation hints at a phenomenon of “creative destruction” where obsolete firms are replaced by newer one, but without significantly affecting the total number of firms in each time period. The responsible for the big turbulence are smaller and younger firms that challenge the market arena, where 20% – 40% of enters die in the first two years and only 40% – 50% survive beyond the seventh year in a given cohort. It means that to an high entry rate corresponds a low degree of penetration: high probability of entry but low probability of surviving. Finally, weak empirical support has been finding for profitability as a key variable in the determination of the entry choice (for a detailed survey on entry stylised fact see Geroski, 1995).

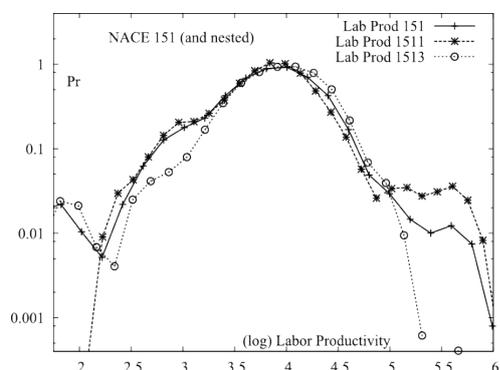
The aim of this paper is building a truly simple evolutionary process able to reproduce the listed robust empirical findings that, to summarize, comprise: (i) heterogeneity, (ii) persistence and (iii) asymmetry related to the productivity distribution, (iv) skewness and (v) Laplace shape respectively for the level and the growth rate of the size distribution, (vi) negative variance-size relation, (vii) market turbulence.

3 Theoretical interpretation

In this section we will argue upon the theoretical reasons under the emergence of the above mentioned ensemble of stylised facts discussing more in depth the most relevant theoretical models in the literature that try to replicate at least one of the proposed stylised fact. After all, paraphrasing Ijiri and Simon (1977), stating that an economic variable is distributed according to a given distribution “would appear, in common sense terms, to be less an explanation than a relocation of the mystery”.

The introduction of longitudinal micro-level data has actually strongly broken down the paradigm of the existence of a representative firm whose increases in productivity derives from the shift of the aggregate production function common to all firms. The why of this persistent heterogeneity across firms has been investigated by many contributions. The envisaged reasons span from internal firms factors as capabilities, managerial practices, labour conditions, innovation and R&D, learning by doing to environmental market conditions as different degrees of competition, spillover and regulation.

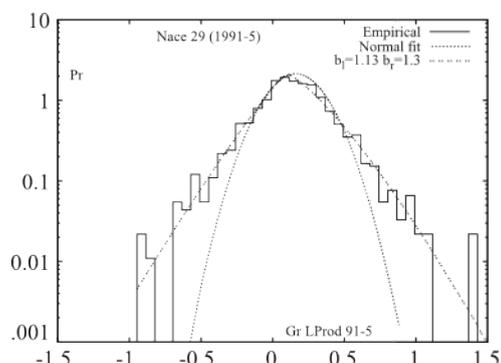
In theoretical models like the ones by Jovanovic (1982), Ericson and Pakes (1995), Hopenhayn (1992a), Hopenhayn (1992b) and Pakes and Ericson (1998) there is the explicit attempt to link heterogeneous productive firms with performance, selection and survival of the “fittest” (we would say from an evolutionary perspective). Dosi et al. (1995) and Bottazzi et al. (2001)



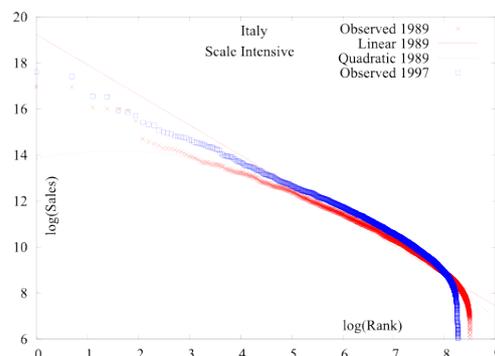
(a) Source: Dosi et al. (2013)
Empirical distribution of labour productivity

SECTOR	ISIC Code	Labor Prod.		Π_i Growth rates	
		AR(1)	Std.Dev.	AR(1)	Std.Dev
Production & processing of meat	151	1.0021	0.0016	-0.3446	0.0915
Knitted & crocheted articles	177	1.0056	0.0023	-0.2877	0.1005
Wearing apparel & acc.	182	1.0035	0.0012	-0.3090	0.0871
Footware	193	1.0029	0.0019	-0.3903	0.0793
Articles of paper and paperboard	212	1.0053	0.0008	-0.3027	0.0603
Printing and services related to printing	222	0.9962	0.0011	-0.4753	0.1103
Plastic products	252	1.0030	0.0010	-0.3150	0.0557
Articles of concrete, plaster & cement	266	0.9985	0.0016	-0.4572	0.0979
Metal products	281	1.0034	0.0012	-0.4125	0.0715
Treatment, coating of metal & mech. engin.	285	1.0051	0.0013	-0.1846	0.0679
Special purpose machinery	295	1.0011	0.0011	-0.3040	0.0495
Furniture	361	0.9994	0.0001	-0.4472	0.0808

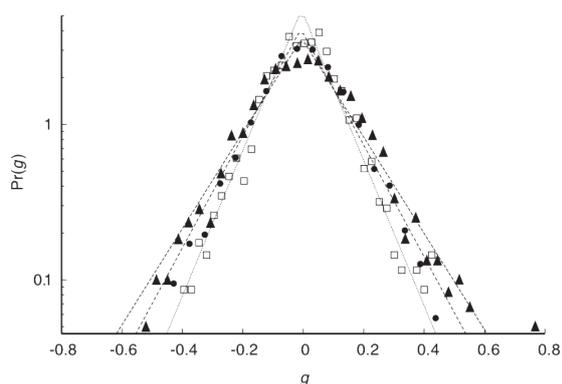
(b) Source: Dosi and Grazzi (2006)
AR(1) coefficients for Labour Productivity in levels and first differences. Dataset: Istat Micro.1.



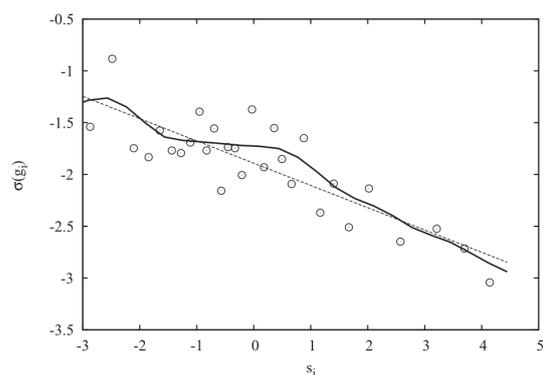
(c) Source: Dosi et al. (2012)
Tent shape productivity growth rate.
Dataset: Istat Micro.3



(d) Source: Dosi et al. (2008).
Skewness in the size distribution. Dataset: Fortune 500



(e) Source: Bottazzi and Secchi (2006a).
Tent shape size growth rate. Dataset: Istat Micro.1



(f) Source: Bottazzi and Secchi (2006b).
Variance-size relation. Dataset: Pharmaceutical Industry (PHID).

Figure 1: Empirical stylised facts on productivity and size distribution

define this stream of literature as “rational evolutionary models”. The evolutionary aspects lies in the idiosyncratic productivity process that leads more productive firms to expand their own capacity, or equivalently their market shares, and less productive firms to shrink their weight in the market up their death. An inherent process of selection that reward the more efficient and penalize the less efficient is in act. These models, at a first glance distinguishable in passive (as Jovanovic (1982)) and active (as Ericson and Pakes (1995)) learning models, address issue as growth/death rates conditional on age, the dependence or not of the current size on the initial one, the entry-exit rate in equilibrium. The “rational” attributes stems from the fact that all of them are characterized by profit-seeking maximizing agents over an infinite time horizon that can in each time step decide or not whether to stay in the market according to their technological rational expectation. Furthermore, the idea that the selection mechanism acts so well that every observed variable is an equilibrium one, is really demanding.

Conversely, according to the evolutionary perspective, the micro-patterns of the industrial dynamic are the inherent outcome of two processes of *learning* and *selection* among *boundedly-rational agents*. Upon those mechanisms, the macro regularities emerge as the result of the continuum coupling of *change* and *coordination*. Models like the ones proposed in Nelson and Winter (1982), Silverberg and Lehnert (1993), Dosi et al. (1995), Dosi et al. (2000), Bottazzi et al. (2001) and Winter et al. (2003) are some different and complementary examples of the evolutionary-modelling approach. Evolutionary models that address the pattern of industry evolution with particular reference to the learning and selection process, can be subdivided into three different categorizations: [i] the group of models that mainly investigate the pattern of firm growth as a process of cumulation of learning opportunities (from Ijiri and Simon (1977) to Bottazzi and Secchi (2006a)) folding together the two different processes; [ii] the group of models that unpack and treat separately the two processes (Silverberg et al. (1988) and Dosi et al. (1995)), [iii] the group of models that mainly focus on the selection process (see Metcalfe (1998) for an extensive discussion).

Besides productivity, size is the other variable of interest, in both levels and growth rate. Related to its scale invariant structure, Pareto distributions suggest that the industrial system is only the results of interaction, without any role played by the firm individual behaviour and the institutional-external factors. As above discussed, being the pure power law shape not robust across different level of disaggregation, and revealing to be very sector-specific, we will concentrate our focus on a briefly discussion of theoretical models focused on growth rate dynamics, and particularly to the strikingly robust empirical findings of fat-tail growth rate distribution.

Firms grow and decline by relatively lumpy jumps which cannot be accounted by the cumulation of small, - “atom-less”-, independent shocks. Rather “big” episodes of expansion and contraction are relatively frequent. More technically, this is revealed by fat tail distributions (in log terms) of growth rates. What determines such property?

In general, such fat tail distributions are powerful evidence of some underlying correlation mechanism. Intuitively, new plants arrive or disappear in their entirety, and, somewhat similarly, novel technological and competitive opportunities tend to arrive in “packages” of different “sizes” (i.e. economic importance). This is what Bottazzi (2014) calls the *bosonic nature* of firm growth, in analogy with the correlating property of a family of elementary particles – indeed the bosons –.

In the literature one of the first model that addresses the issue of size growth rate is proposed by Ijiri and Simon (1967). Starting from a cumulative process for firms size, the authors decompose the total growth rate as the sum of an idiosyncratic component and an industry time-variant component. The growth rate at a sectoral level is assumed to be constant, as the initial size of the firm. Finally the idiosyncratic shock is modelled as an $AR(1)$ process

capturing [i] the independence of growth rate from size (Gibrat Law), [ii] one-period autocorrelation of growth rates (or single period Markov process), [iii] a mean reversion behaviour. The growth rate is described as the sum of independent micro-shocks. The size path turns out to be a random walk model with a drift.

In turn, firm specific increasing returns in business opportunities, as shown by Bottazzi and Secchi (2003) are a source of such correlations. In particular, the authors build upon the “island” model by Ijiri and Simon (1977) and introduce the hypothesis of path-dependent exploitation of business opportunity via a Polya Urn scheme, wherein in each period “success breeds success”. It is a two step model, where in the first step an assignment procedure of the *fixed number* of business opportunities is realised. In the second step, these business opportunities act as source of growth rate. The dynamic of firms growth rate is still a *Gibrat-type* but with the strong difference that the number of opportunities M are not assigned with a constant probability $1/N$, but proportionally to the number of opportunities that in *each period* the firm already has. At each time step a micro-shocks of type $i \in 1, \dots, N$ is extracted from an urn. Once it is extracted the ball is replaced and, additionally, a new ball of the same colour is introduced. This implies that, once one type i has been extracted, the probability of being re-extracted increases. This procedure is repeated M times, the number of the total business opportunities. Indeed this cumulative process is at the core of the emergence of fat tails distributions. The authors demonstrate that when N and the ratio M/N increase, the limit distribution of this scheme is Laplace distributed, and this occur independently from the distribution function of the shocks. Their explanation of the tent shape relies on the idea that a big chunk of microshocks M are concentrated in few firms N . Being the growth rate the sum of micro-shocks, in order to avoid the possibility of an infinite variance, is necessary that the micro-shocks hit the firms with infinitesimal variance.³ The last remark is that the assignment procedure occurs once a year: dynamic increasing returns displace in *space*, as a cumulation of many shocks in few firms, but never in *time*. It turns to be rather difficult imagine that firms update their expertise every year, when the urn is open.

Our conjecture, however, is that spatial cumulative processes are only one of the drivers of the apparent correlations underlying the “tents”. Indeed, we suggest that a rather large ensemble of evolutionary processes, characterized by different forms of *idiosyncratic* (i.e. firm-specific) *learning* and *competitive interactions* yields the observed distributions of growth. This is the conjecture we are going to explore in this work.

The value added of this work is hence two-fold. By far in the literature there have been attempts to address learning and selection separately. Our contribution mainly means to capture the two footprints of industrial dynamics together, wherein heterogeneous productivity and fat tail distribution of growth rated coexist.

4 The model

The model is an evolutionary agent-based model microfounded upon simple behavioural heuristics. The absence of any rational technological expectation is intentionally pursued. The most part of human decisions, and among them economic decisions as putted by Gigerenzer and Selten (2002) are made under low degree of information, time pressure, uncertainty and low computational effort. For these reasons the low of motion that govern our economy are not derived from the profit seeking maximizing agents. They are conversely, empirically grounded.

The three processes that takes place in the market are learning, selection and entry. The

³Also the model by Buldyrev et al. (2007) finds its root in Ijiri and Simon (1977), and attempts to find those exponential mixtures of Gaussian distributions able to reproduce the fat tail properties of the observed growth data.

model, as we will show, is able to provide a rich ensemble of stylised facts of the pattern of industrial evolution.

4.1 Idiosyncratic learning process

We build upon a simplified version of Dosi et al. (1995) whereby learning is represented by some multiplicative stochastic process upon firms productivity or more generically “level of competitiveness” a_i of the form:

$$a_i(t) = a_i(t-1)(1 + \theta_i(t)) \quad (1)$$

where the $\theta_i(t)$ are the realization of the firm-specific process. Formally θ_i are realization of a sequence of random variables $\{\Theta_i\}_{i=1}^N$ where N are the fixed number of firms. This equation aims to capture the dynamic of capabilities formation within each firm. According to the emerging-capabilities literature (see Teece et al. (1994)) firms stuck in their attitude to do innovation, searching, problem solving and so on. The internal capability structure reflects into the external productivity dynamics, embedded in the ability to be more competitive (lower prices via process innovation) or to introduce new products. Different capability structures are actually considered as the main source of heterogeneity among firms. The choice of a multiplicative process to model the dynamic of productivity is basically meant to grasp its persistent heterogeneous nature across firms and it turns to be a *Gibrat-type* dynamic not as usual in size but in the level of competitiveness.

We experiment with different learning processes:

- $\theta_i(t)$ is drawn from a set of possible alternative distributions namely Normal, Lognormal, Poisson, Laplace and Beta, namely a *Baseline Regime*;
- $\theta_i(t) = 0$ under *Schumpeter Mark I*;
- $\theta_i(t) = \pi_i(t) \left(\frac{a_i(t-1)}{\sum_i a_i(t-1)s_i(t-1)} \right)^\gamma$ under *Schumpeter Mark II*, where $\pi_i(t)$ is the same draw as under the *Baseline Regime*

At one extreme, in the first case, incumbents do not learn after birth. Advances are only carried by new entrants. At the opposite extreme, in the third case, incumbents do not only learn, but do it in a cumulative way so that a “draw” by any firm is scaled by its extant relative competitiveness. This captures what Paul David, quoting Robert Merton, calls the “Matthew effect”:

“For unto every one that hath shall be given, and he shall have abundance: but from him that hath not shall be taken even that which he hath.” (*Matthew 25:29, King James Version*)⁴.

Finally the Baseline Regime is the “prototypical” scenario upon which we will perform extensive experiments.

4.2 Market selection and birth-death processes

Competitive interactions are captured by a “stochastic quasi-replicator” dynamics:

$$\Delta s_i(t, t-1) = A s_i(t-1) \left(\frac{a_i(t)}{\bar{a}_t} - 1 \right) \quad (2)$$

where:

$$\bar{a}_t = \sum_i a_i(t) s_i(t-1) \quad (3)$$

⁴For more details see Dosi and Sylos Labini (2007)

where $s_i(t)$ is the market share of firm i which changes as a function of the ratio of the firm's productivity (or "competitiveness") to the weighted average of the industry. It is a "quasi-replicator" since a genuine replicator lives on the unit simplex. The "quasi" one may well yield negative shares, in which case the firm is declared dead and market shares are accordingly recomputed. Being A an elasticity parameter that captures the intensity of the selection mechanism operated by the market, the death rule implies that whenever it is weak, firms survive irrespectively of their market shares (fitness) and their competitiveness. However, empirically, firms with strikingly low relative competitiveness do die even in environments characterized by low degree of competition. Concerning equation 2 we shall study the effect of different degrees of *market selectiveness*, as captured by the A parameter. In that, note that the *competitive process as such* induces ex-post correlation in growth rates: the growth of the share of any one firm induces the fall of the total share of its complement to one! Finally, entry of new firms occurs proportionally to the number of incumbents present in the market:

$$E(t) = \omega(t)N(t-1) \quad (4)$$

where $E(t)$ is the number of entrants at time t , $N(t-1)$ is the number of incumbents in the previous period and $\omega(t)$ is a random variable uniformly distributed on a finite support (which in the following, for simplicity, we assume drawn from a uniform distribution). The idea that the number of entrants is proportional to the number of incumbents is strongly empirically verified (see e.g Geroski (1991) and Geroski (1995) that finds a significant cross-correlation between entry and exit, but also the preferential attachment scheme in Buldyrev et al. (2007)). The number of firms at each time steps is maintained constant, that is the number of dying is offset by an equal amount of rising firms.

The productivity attributed to the entrants follows the same incumbent rule, according to the Market regime where it takes place, multiplied by the average productivity in the market. What happens is that entrants productivity diverges from the average market productivity of a stochastic component, that is again a random extraction from alternative distributions (Normal, Lognormal, Poisson, Laplace and Beta):

$$a_j(t) = (1 + \theta_j(t)) \sum_i a_i(t) s_i(t-1) \quad (5)$$

where $\theta_j(t)$ is a random variable which parametrizes barriers to learning by entrant, or conversely the advantage of "newness".

4.3 Timeline of the events

- There are N initial incumbent firms. They have at time 0 equal productivity (but in the Schumpeter Mark I regime) and equal market shares .
- At the beginning of each period, if not under regime Mark I, firms learn according to the dynamic of the specified process on productivity.
- Firms acquire or loose market share according to the quasi-replicator.
- Firms exit the market according to the rules of death: $s_i(t) \leq 0$.
- Market shares growth rate are calculated.
- The number of entrants is drawn as a function of the total number of firms at the beginning of the period, and market shares of incumbents are adjusted accordingly.

5 Model properties

Our conjecture is the the replicator dynamics put in act a mechanism of correlation equivalent to the Polya urn mechanism. In particular we can read the stochastic replicator as a

	Value
Number of firms	150
Number of time steps	500
Number of MC runs	50
Initial productivity	1
Initial market share (1/N)	0.006667
Age Entrants	1
A	1
γ	1
$Beta(\beta_1, \beta_2)$	[1, 5]
$Normal(\mu, \sigma)$	[0.05, 0.8]
$Lognormal(\mu_1, \sigma_1)$	[-3.5, 1]
$Laplace(\alpha_1, \alpha_2)$	[0.01, 0.015]
$Uniform(\nu_1, \nu_2)$	[0, 0.1]

Table 1: Parameters initialization.

Polya urn model. As demonstrated by Schreiber (2001) and discussed in Pemantle (2007), *the stochastic replicator is a generalized Polya urn scheme*. Particularly, based on the description proposed by the latter, at each time $t \geq 0$ there is a population $N(t)$ made by firms whose only attribute is the type i of micro-shocks, being $\{1, \dots, i\}$. These firms are represented by an urn with colors $\{1, \dots, i\}$, the productivity $a_i(t)$ is the random fitness of firm i and the market share $s_i(t)$ its representation. The size of the population is determined as follows. At each time step t a ball (firm) of color (micro-shock) i is extracted (with replacement) from the urn and returned to that along with $a_i(t)$ extra balls of color i . Being $a_i(t)$ a measure of the fitness of type i in the population, its representation (market share) will change of an amount proportional to its fitness against the others $N(t) - 1$ balls. Repeating this mechanism will allow the growth of type i to be proportional to its own success against all the other types weighted by their own representation in the population, clearly: $\Delta s_i(t, t-1) = s_i(t-1)a_i(t) / \sum_i a_i(t)s_i(t-1)$. The finite number of opportunities of the Bottazzi and Secchi (2006a) model, reads in our case as the finite, given dimension of the market, that is assumed to be stationary, where the following constrain holds: $\sum_i s_i(t) = 1, \forall t \geq 0$. When the fitness function is not influenced by any random noise, that is, in the *Schumpeter Mark I* regime, the model collapses into a deterministic discrete replicator for the first time step. We will show in the following section that this type of process is able to robustly reproduce fat-tail distribution of growth rates under the three learning regime, and even to generate Laplace distribution under the *Schumpeter Mark II*. In our replicator process, the cumulativeness at the origin of fat-tail, occurs both in space as in Bottazzi and Secchi (2006a) (finite dimension of the market) and in time (autocorrelated productivity).

5.1 Baseline Regime

We will start by analysing the *Baseline Regime* that amount to an intermediate set-up between the two more “extreme” market configurations. Particularly we set:

- The market selection parameter $A = 1$;
- The cumalativiness parameter $\gamma = 0$

- A Beta (1,5) distribution for the extraction of the micro-shocks.

In table 2 the aggregate descriptive statistics, across 500 time steps in the Baseline regime, are presented.

	Average	Min	Max	Sigma
Number of entrants	12.172	0	35	5.72
Average age	8.52	1	12.5	2.06
Average productivity growth	0.046	0.034	0.060	0.004
Average shares growth	-0.088	-0.26	0.016	0.045

Table 2: Descriptive Statistics for 500 time steps. Baseline Regime

In the following the dynamic of productivity and its persistence, size and growth rate distribution, and market turbulence we will presented. In figure 4 the results from the Baseline Regime are reported. Figure 4.a shows the log normalized productivity distribution that reads as:

$$\log n_i(t) = \log a_i(t) - \log \sum_i a_i(t) s_i(t-1) \quad (6)$$

The width of the distribution appears to be rather disperse. Figure 4.b shows the autocorrelation structure of the productivity (not normalized) distribution. It appears rather strong with a coefficient of 0.97 at $t-1$. Proceeding with the analysis of the size distribution, figure 4.c shows the $\log rank - \log size$ plot fitted against a LogNormal distribution. As already discussed, if the size distribution follows a Power Law:

$$sr^\beta = A \quad (7)$$

linearising we have:

$$\log r = \alpha + \beta \log s \quad (8)$$

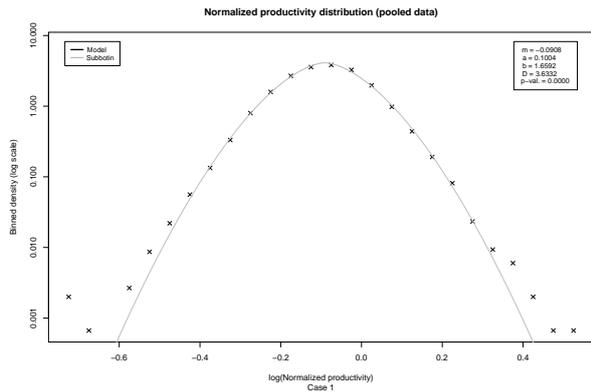
where s and r are respectively the size and the rank of the distribution. β is the slope parameter, and under the Zipf Law (that is a restriction of the Pareto law) it is equal to one. In our case, the slope is clearly different from one, presenting a cut-off point above which the Lognormal distribution is not any more a well approximation of the size distribution. Our emphasis, more than to detect the emergence of a Zipf distribution, which as above discussed, it's not robust under sectoral level of disaggregation (and consider our regimes as sectors characterised by a different innovative profile), is devoted to the emergence of the clear-cut skewness and a strong departure from the Lognormal distribution. Finally figure 4.d shows the fat-tail distribution of growth rates, plotting the distribution against a Normal (red line) and a Laplace (green line) one. The growth rate of firm size distribution is defined as:

$$\log g_i(t) = \log s_i(t) - \log s_i(t-1) \quad (9)$$

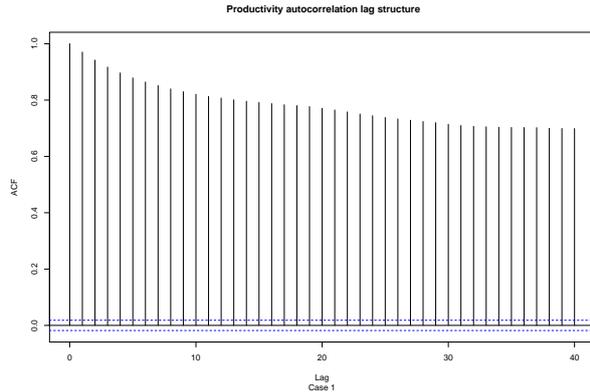
where market shares represent our proxy for size. It is not necessary to normalize the size by the average growth rate of the market, being the latter equal to zero. In order to understand how fat the tails are, we estimate a symmetric Subbotin function, which is defined by three parameters m , a and b . In particular m is a location parameter, a is a scale parameter and b tells how fat the tails are. The Subbotin is a big family of distributions. According to the value of the parameter b , the Subbotin:

$$f_S(x) = \frac{1}{2ab^{1/b}\Gamma(1/b+1)} e^{-\frac{1}{b}|\frac{x-\mu}{a}|^b} \quad (10)$$

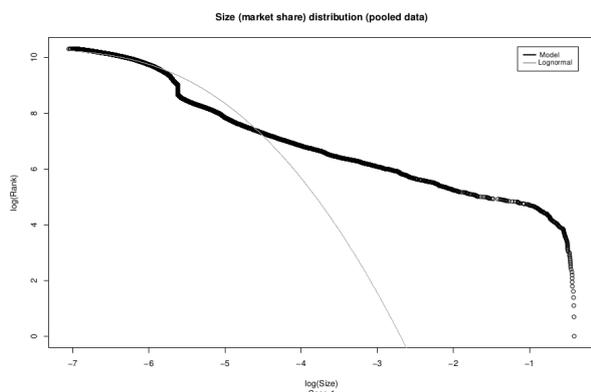
can yield:



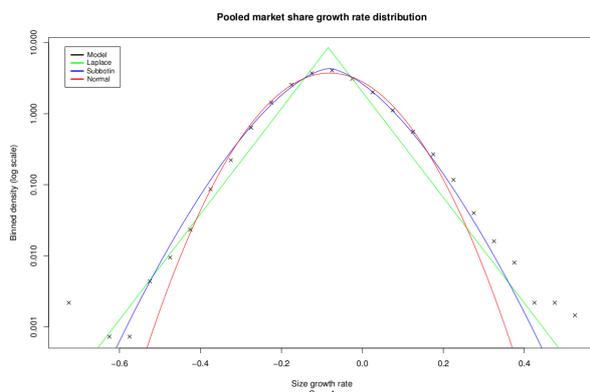
(a) Normalized productivity distribution.



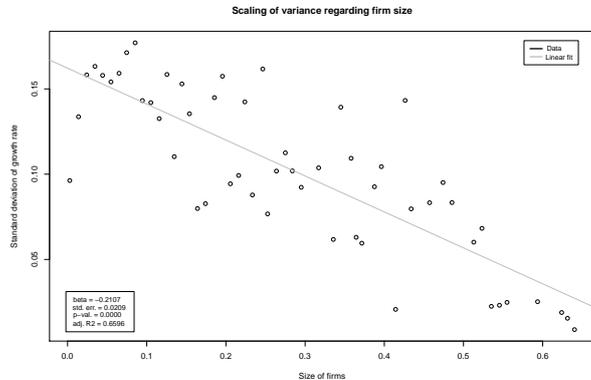
(b) Autocorrelation structure of productivity.



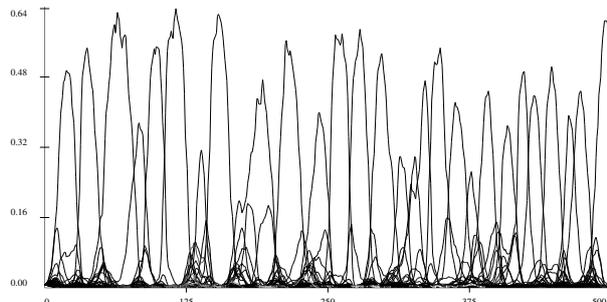
(c) Pooled size distribution.



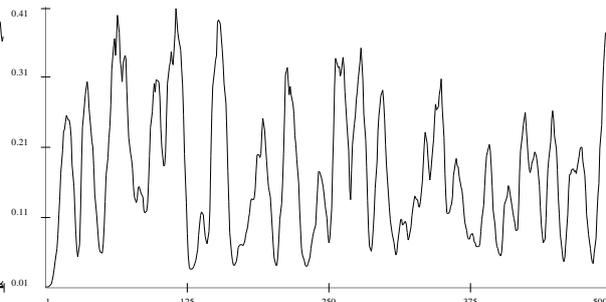
(d) Growth rates distribution.



(e) Scaling variance relation.



(f) Market Shares.



(g) Herfindahl-Hirschman Index.

Figure 2: Baseline Scenario

	β
<i>Baseline Regime</i>	-0.2562 (0.0453)
<i>Schumpeter Mark I</i>	-0.190 (0.0736)
<i>Schumpeter Mark II</i>	-0.4121 (0.058)

Table 3: Scaling variance relation. Estimation of the slope coefficient across regimes

- Gaussian Distribution: $b = 2$
- Laplace Distribution: $b = 1$

The figure shows a rather strong departure from Normality, with the b parameter on average around 1.5, even though, in this baseline configuration we cannot reject the hypothesis of Normality in many cases. The presence of fat-tails in the distribution of growth rates is a sign of the shocks correlation across firms. Correlations among shocks is an hint that my growth erodes yours, that is the selection effect of the replicator dynamic in action. On the contrary, normality means absence of any selection pressure. Figure 4.e shows the negative relationship between the variance of growth rate and size, together with an *OLS* fit:

$$\sigma(g_i) = \alpha + \beta S_i \quad (11)$$

It is important to underline how, differently from the empirical literature, here we are using shares and not revenues as size proxy. The estimation of the slope coefficient is presented in table 3, together with the R^2 estimation averaged across fifty runs (in brackets the standard deviation). Across the three regimes, the negative relation is always present.

Finally figures 4.f and 4.g depict the turbulence of the market showing the dynamics over time of market shares and Herfindahl-Hirschman index. It is rather clear how the market is characterised by persistence fluctuations that are *endogenously* determined by the entry-exit process.

We will proceed now with the exploration of the effect of different shapes of the microshock distributions upon the growth rates. Recall that in this baseline regime the innovation process is carried on both by entrants and incumbents. The model exhibits a strong qualitative invariance to the shape of the input distributions of the innovation shocks, confirming the findings by Bottazzi and Secchi (2006a). In table 4 the results of the average parameter values across fifty runs of Monte Carlo simulations (in brackets their standard deviation). The range of the parameter values is between 1.637 – 1.467. This suggest how the tails of the distribution are very far from being normal.

5.2 Schumpeter Mark I Regime

The Schumpeter Mark I regime where, just to recall, there is no learning process and innovation is carried on by entrants only, is a quite extreme case, that tries to basically isolate the effect of market selection and to test the emergence of fat tail distribution. In table 5 the descriptive statistics are shown. This market it's relatively more calm, with respect to the Baseline Regime, with a lower number of entrants, with more lasting age, and as expected very low shares and productivity growth.

Figure 4.a and 4.b show the dynamic of productivity and its persistent nature. In this particular case, having turned off the extraction of the incumbent growth rates, their initial

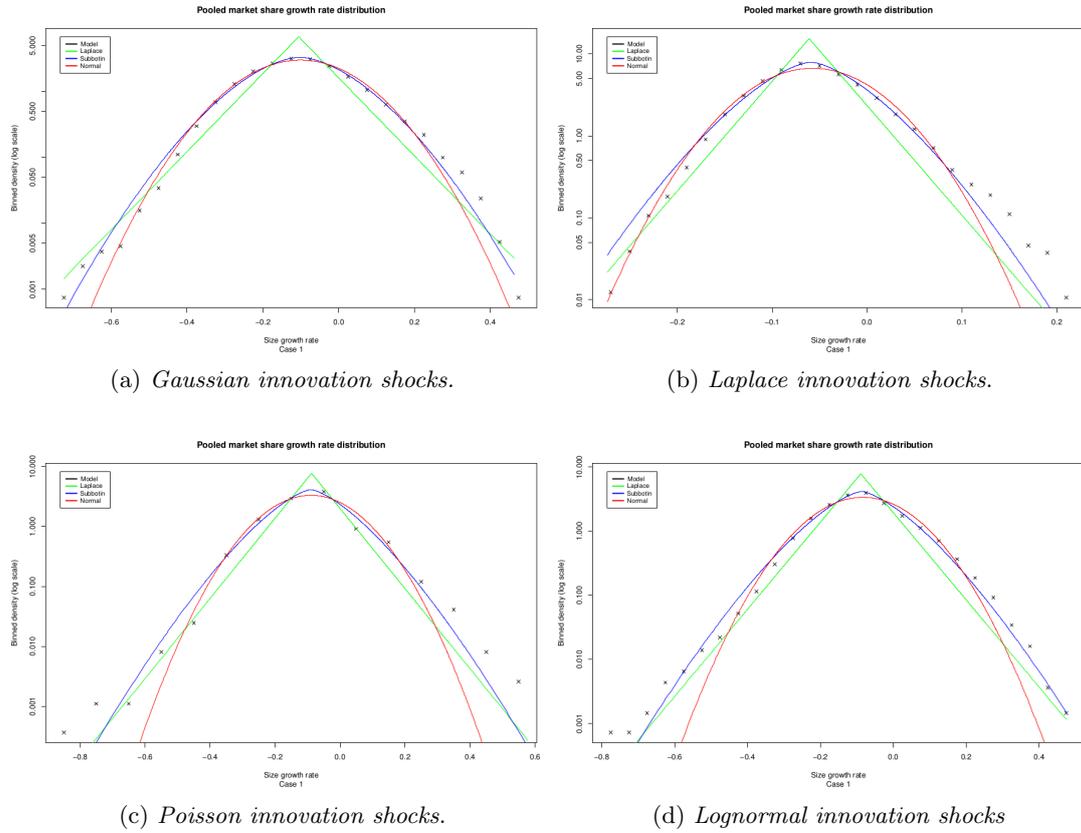


Figure 3: Baseline Regime. Firm growth rates under different innovation shocks.

	m	a	b
<i>Gaussian shocks</i>	-0.102 (0.029)	0.128 (0.003)	1.637 (0.038)
<i>Laplace shocks</i>	-0.057 (0.004)	0.0566 (0.002)	1.489 (0.048)
<i>Poisson shocks</i>	-0.0843 (0.0257)	0.099 (0.002)	1.327 (0.048)
<i>Beta shocks</i>	-0.081 (0.004)	0.096 (0.003)	1.539 (0.040)
<i>Log-normal shocks</i>	-0.0896 (0.0039)	0.1027 (0.0025)	1.467 (0.040)

Table 4: Baseline Regime. Parameters estimation across different innovation shocks.

	Average	Min	Max	Sigma
Number of entrants	3.73	0	13	2.09
Average age	22.07	1	30.36	4.14
Average productivity growth	0.000	0	0.37	0.016
Average shares growth	-0.024	-0.11	-0.01	0.01

Table 5: Descriptive Statistics for 500 time steps. Schumpeter Mark I Regime

	m	a	b
<i>Gaussian shocks</i>	-0.0198 (0.00066)	0.0502 (0.0011)	1.426 (0.026)
<i>Laplace shocks</i>	-0.013 (0.0018)	0.045 (0.0012)	1.360 (0.027)
<i>Poisson shocks</i>	-0.046 (0.0008)	0.0389 (0.0013)	0.900 (0.0056)
<i>Beta shocks</i>	-0.025 (0.001)	0.053 (0.001)	1.397 (0.023)
<i>Log-normal shocks</i>	-0.0307 (0.001)	0.0543 (0.0013)	1.329 (0.020)

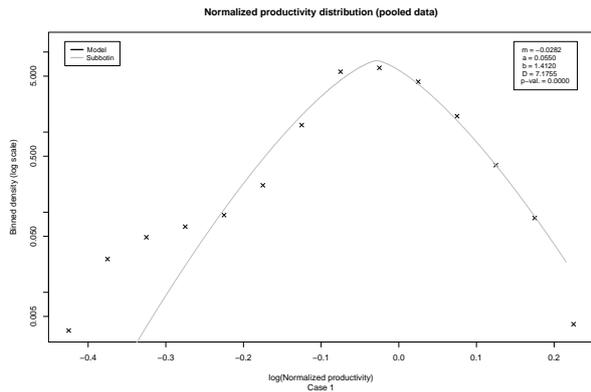
Table 6: Scumpeter Mark I. Parameters estimation across different innovation shocks.

productivity has been heterogeneously initialized: from a range between 1 – 1.5, each firm is endowed by an initial amount of productivity that differs from the others. Figure 4.c shows the skew distribution of firm size. It is worthy to note how in this regime an higher fraction of firm size with respect to the baseline regime is characterized by a lognormal distribution. Figure 4.d illustrates the distribution of growth rates. Even in absence of any process of learning, the only effect of market selection operated by the replicator dynamics accompanied by an entry process, is able to determine fat-tail distribution. Particularly, as in the other regime, we test for a possible invariant property across distributions. Figure 4 represents the firm growth rates wherein the entrants productivity is extracted from different distribution. Also in this case the persistent nature of fat tails is presented in table 6 where the maximum value of the b parameter is recorded for Normal shocks up to the 0.9 value of the Poisson distributed shocks.

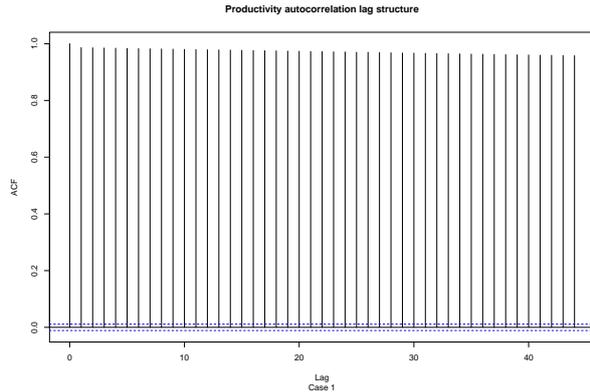
5.3 Schumpeter Mark II Regime

Finally we shall explore a purely cumulative regime in the learning process. In table 7 the aggregate descriptive statistics. Contrary to the Mark I, in this Regime we have an higher degree of turbulence, with many entrants in the market, with an average age of five periods (relatively close to the empirical recorded values).

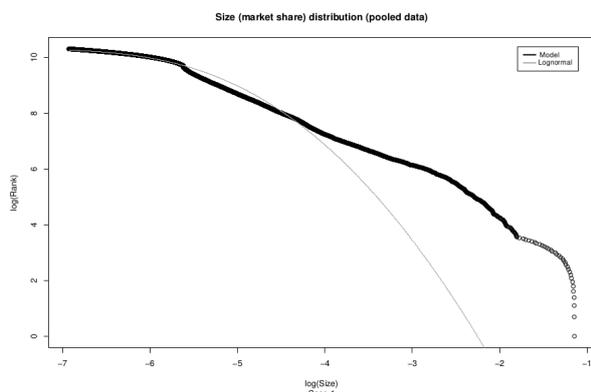
In figure 6 the dynamics of productivity, size and market shares are presented. As in the baseline regime the persistent heterogeneity of productivity is shown in figures 6.a and 6.b. The skewness of the size distribution and the fat-tail nature of growth rates are illustrated in figures 6.c and 6.d. The scaling variance relation in 6.e. In figures 6.f and 6.g the behaviour of market shares and the market index concentration conclude the description of the cumulative regime. Compared to the baseline scenario, the distribution of the growth rate in the



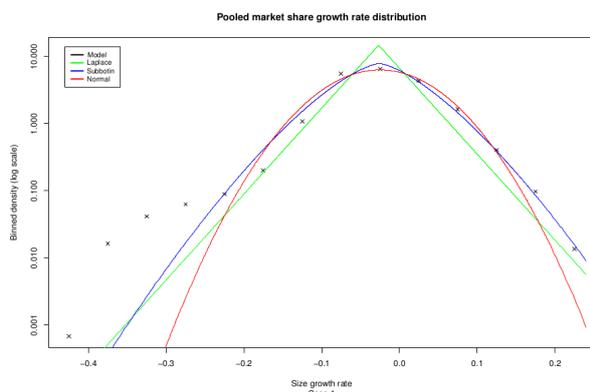
(a) Normalized productivity distribution.



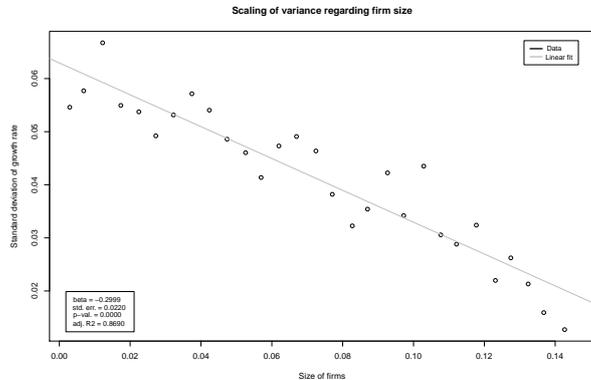
(b) Autocorrelation structure of productivity.



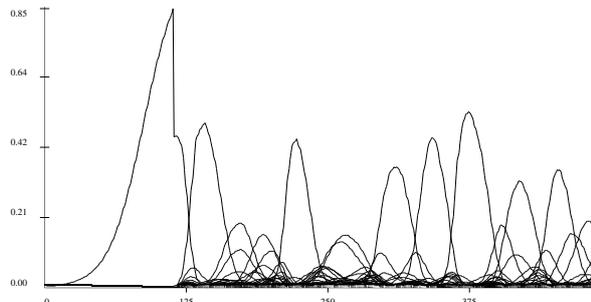
(c) Pooled size distribution.



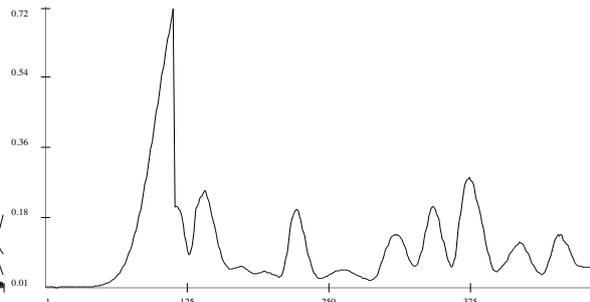
(d) Growth rates distribution.



(e) Scaling variance relation.



(f) Market Shares.



(g) Herfindahl-Hirschman Index.

Figure 4: Schumpeter Mark I Regime

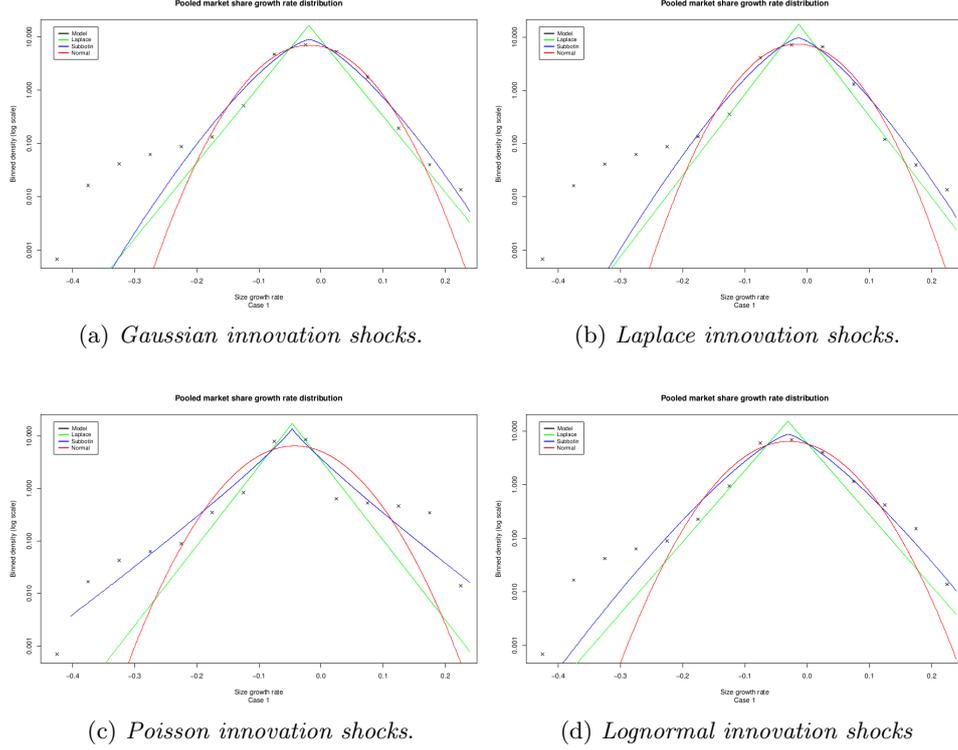


Figure 5: Schumpeter Mark I. Firm growth rates under different innovation shocks

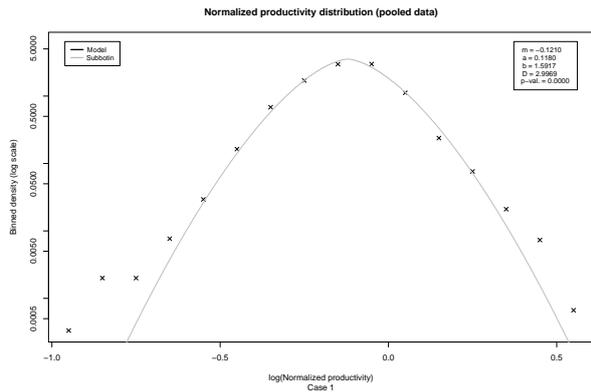
	Average	Min	Max	Sigma
Number of entrants	17.4	0	67	9.6
Average age	5.3	1	10.43	1.79
Average productivity growth	0.039	0.013	0.060	0.006
Average shares growth	-0.13	-0.49	0.032	0.083

Table 7: Descriptive Statistics for 500 time steps. Schumpeter Mark II Regime

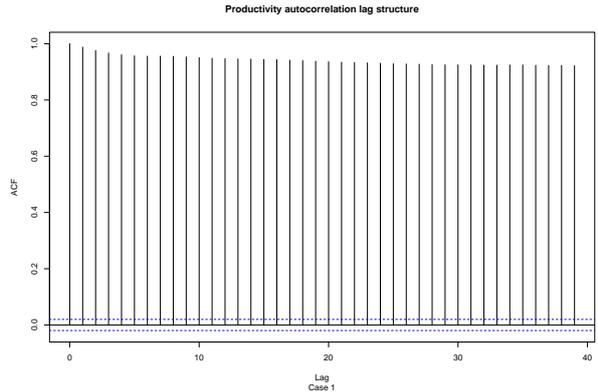
Schumpeter Mark II manifests a closer shape to the Laplace, with a b parameter on average equal to 1.3. This suggests that an higher cumulateness in the learning process hence an higher memory of the past performances, increases the autocorrelation in time, and make the shape of the growth rates more Laplacian. Figure 4 shows the invariant shape of the growth rates under different innovation shock distributions. Table 8 reports the estimated parameter values and the relative standard deviation.

5.4 Cumulateness and selection

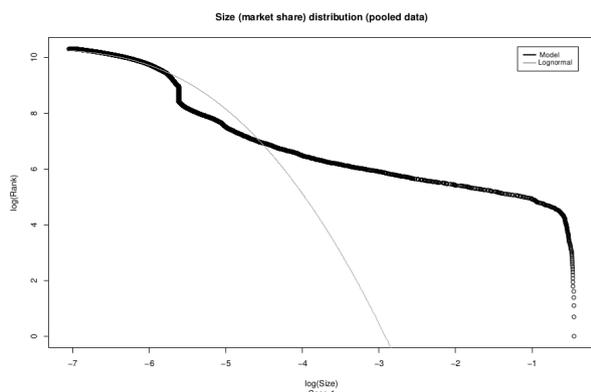
In the Schumpeter Mark II regime we explore the effects on the distribution of firms growth rates of two different parameters: the γ parameter which captures the degree of cumulateness in the learning process and the A parameter which embodies the degree of selectivity in the market. The default distribution of the innovation shocks is a usual a Beta(1,5). We start analysing the effect of cumulateness. As expected, the increase in the γ parameter, shown in figure 8 induces a more tent-shaped distribution in the growth rates up to becoming “super Laplacian” (see 8.c). Table 9 shows the negative relation between the γ and the b parameters. This result is extremely important: the empirical distribution of the firm



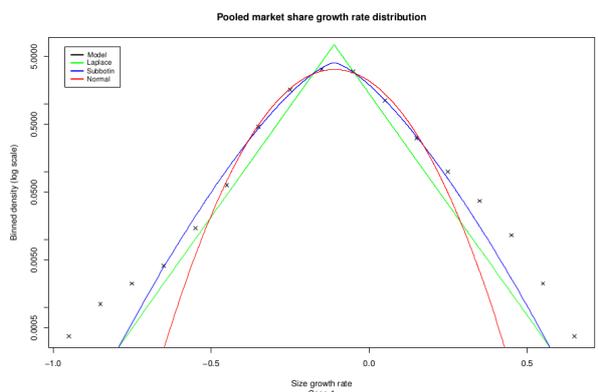
(a) Normalized Productivity distribution



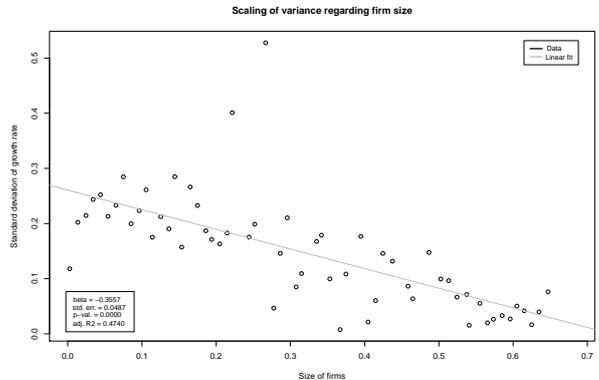
(b) Autocorrelation structure of productivity



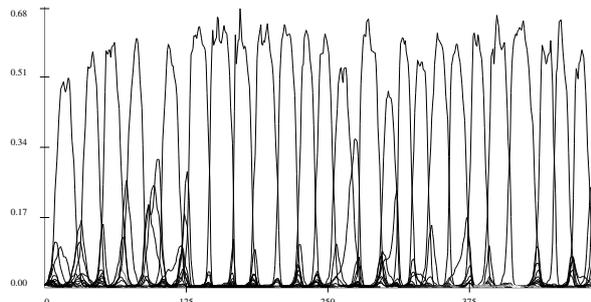
(c)



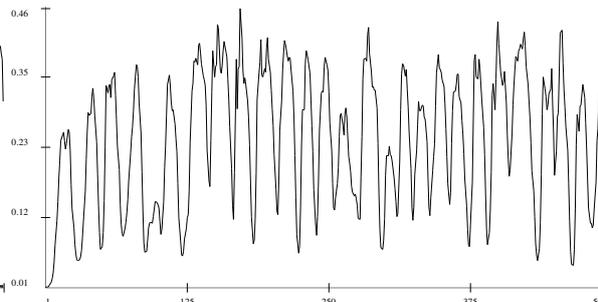
(d)



(e) Scaling variance relation.



(f) Market Shares.



(g) Herfindahl-Hirschman Index.

Figure 6: Schumpeter Mark II Regime

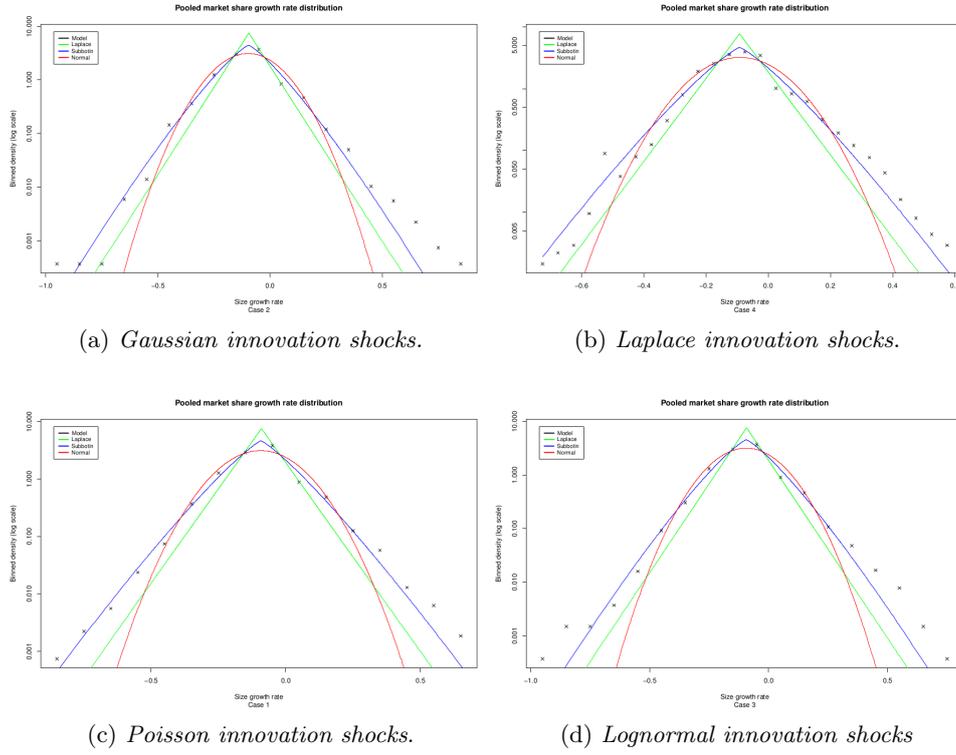
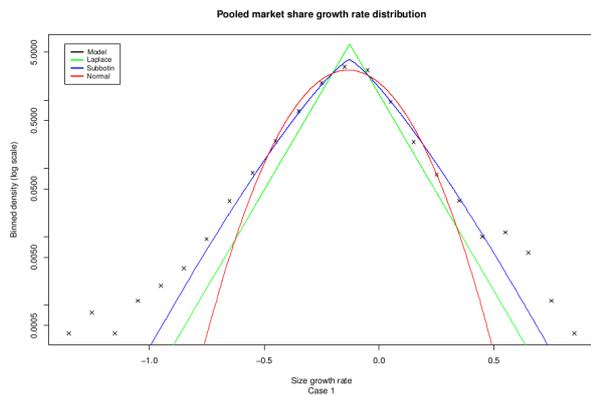


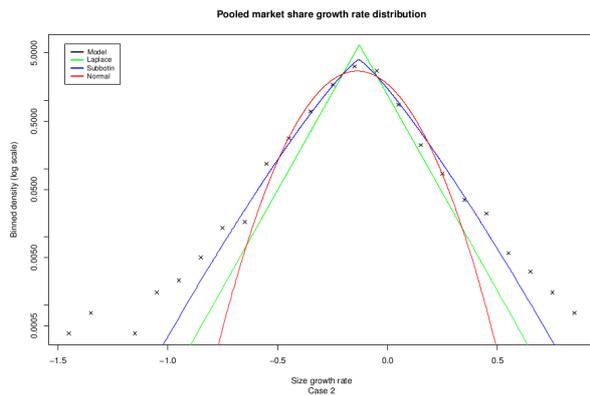
Figure 7: Schumpeter Mark II Regime. Firm growth rates under different innovation shocks.

	m	a	b
<i>Gaussian shocks</i>	-0.162 (0.00543)	0.147 (0.007)	1.402 (0.053)
<i>Laplace shocks</i>	-0.087 (0.0038)	0.068 (0.0043)	1.284 (0.049)
<i>Poisson shocks</i>	-0.096 (0.0039)	0.1008 (0.0026)	1.231 (0.0523)
<i>Beta shocks</i>	-0.113 (0.0047)	0.1074 (0.0046)	1.367 (0.0409)
<i>Log-normal shocks</i>	-0.118 (0.0060)	0.1132 (0.0046)	1.327 (0.045)

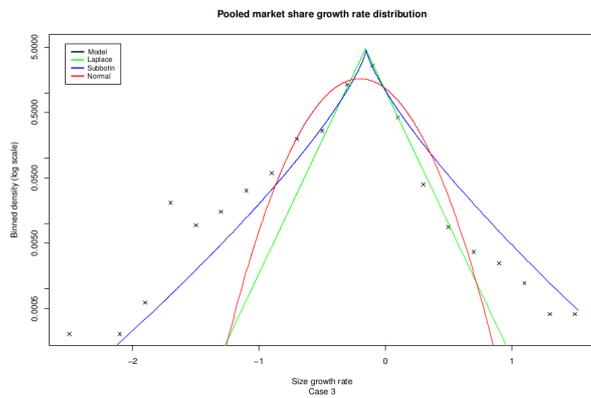
Table 8: Schumpeter Mark II Regime. Parameters estimation across different innovation shocks.



(a) $\gamma = 1.5$

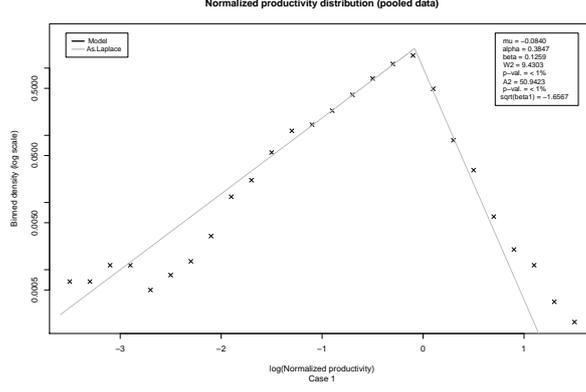


(b) $\gamma = 2.$



(c) $\gamma = 3.$

Figure 8: Schumpeter Mark II Regime. Firm growth rates under different cumulateness.



(a) *Normalized productivity distribution under low selection pressure.*

Figure 9: Schumpeter Mark II Regime. $A = 0.2$

b	
$\gamma = 1.5$	1.293 (0.0568)
$\gamma = 2$	1.176 (0.0797)
$\gamma = 3$	0.811 (0.100)

Table 9: Schumpeter Mark II Regime. The effect of cumuliveness

growth rates has been extensively proven to be tent-shaped. Actually, the Schumpeter Mark II appears to be the regime that best replicates the empirical growth rates distribution. This is equivalent to say that in the real markets a strong “Matthew effect” in the accumulation of capabilities takes place. It is worthy to underline that the transmission mechanism start from the cumulative learning process and affects selection.

Regarding the effect of the A parameter, it is rather clear how a low selection pressure allows firms with lower growth rates to survive, moving the mass of the distribution on the lower part of the support (see figures 10.a and 10.b), where the high growth firms, the “gazzellas” occupies the upper-left tail. Conversely, when the selection pressure increases (see 10.c and 10.d), the more inefficient firms are frozen out by the market, the mass of the distribution shifts in the left-part, whit just few alive inefficient firms that occupy the left tail. We then deduce that the selection parameter is responsible for the symmetry of the distribution. What happens to the the productivity distribution under a low selection pressure? Figure 9 shows how, under a low selectivity, the support of the distribution increases, becoming closer to the empirical ones and a more asymmetric distribution of productivity, with a thicker left tail emerges. This means that, given the same conditions on productivity, the low degree of selection, allows to a big portion of low productive firms to operate in the market.

6 Conclusions

Empirically one ubiquitously observes a large ensemble of micro-stylised facts. In this paper we address the possible causes of these phenomena. Here we investigate what kind of

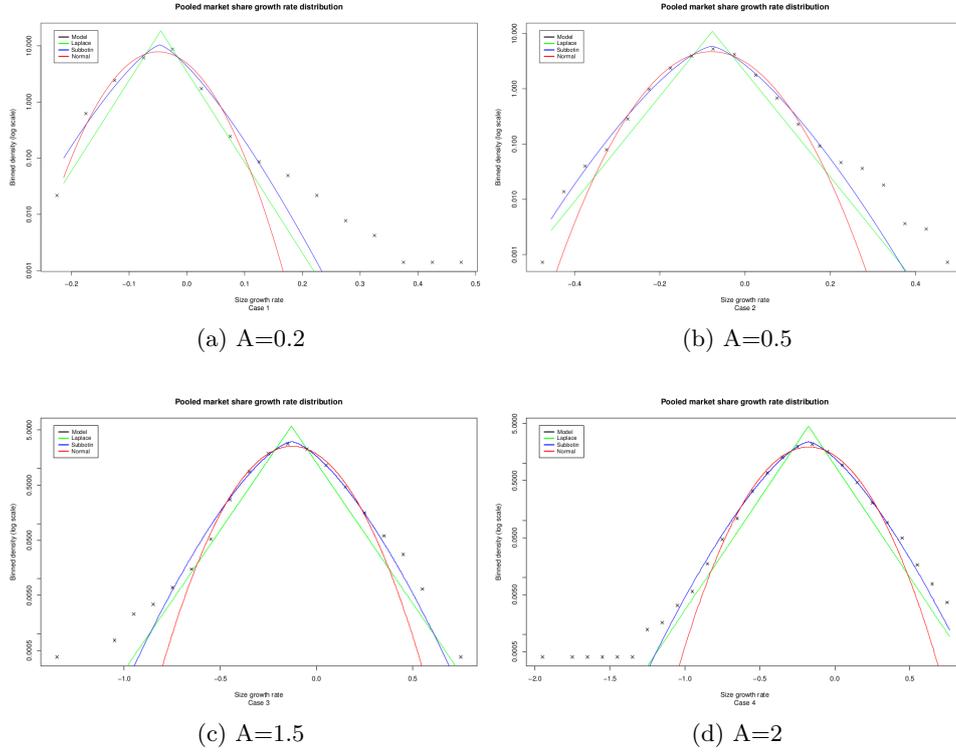


Figure 10: Schumpeter Mark II Regime. Firm growth rates under different selection pressure.

	m	a	b
$A=0.2$	-0.0478 (0.00194)	0.044 (0.00192)	1.423 (0.0697)
$A=0.5$	-0.0760 (0.00316)	0.069 (0.0030)	1.355 (0.0540)
$A=1.5$	-0.144 (0.0063)	0.1412 (0.00621)	1.419 (0.048)
$A=2$	-0.174 (0.0085)	0.1758 (0.0064)	1.458 (0.0289)

Table 10: Schumpeter Mark II Regime. The effect of different selection pressure

economic process can generate fat tail distribution of the firm growth rates, together with persistent heterogeneity in productivity, skewness in firm size distributions negatively related with the growth rate standard deviation. In particular we focus on a bare-bone evolutionary model where the two pillars of evolution, namely, learning and market selection, interact. We examine three alternative market regimes: a *Schumpeter Mark I*, wherein no learning for incumbents take place, an *intermediate* regime where incumbents do learn, and a *Schumpeter Mark II* where the learning process is cumulative. The learning regimes interacted with a “market regime” captured by some form of replicator dynamic. The quite remarkable finding is that under all regimes competitive interactions induces correlation in the growth dynamics of firms and thus the absence everywhere of a normal distribution of growth rates. Additionally, persistent heterogeneity across firms and skewed size distribution are recovered by the model. Fat tails emerge everywhere together with the scaling variance relation. Moreover with cumulative learning the distributions of growth rates turn out to be Laplacian. However even under the Schumpeter I regime the very process of competitive selection generates fat tails, also in absence of any learning.

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