Unspanned macroeconomic factors in the yield curve

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Abstract

In this paper, we extract common factors from a cross-section of U.S. macro-variables and Treasury zero-coupon yields. We find that two macroeconomic factors have an important predictive content for government bond yields and excess returns. These factors are not spanned by the cross-section of yields and are well proxied by economic growth and real interest rates.

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1 Introduction

Government bond yields with different maturities and macroeconomic variables are both characterized by a high degree of comovement, indicating that the bulk of their dynamics is driven by a few common forces. Three common factors, usually interpreted as the level, slope and curvature of the yield curve, can explain changes and shifts of the entire cross-section of yields, see Litterman and Scheinkman (1991). Although there is less consensus on the number and nature of macroeconomic factors, two factors, one nominal and one real, summarize well the dynamics of a large variety of macroeconomic indicators for the United States, see Sargent, Sims et al. (1977), Giannone, Reichlin and Sala (2005) and Watson (2005).

Macroeconomic factors and yield curve factors are also characterized by a strong interaction. The short end of the yield curve moves closely to the policy instrument under the direct control of the central bank, which responds to changes in inflation, economic activity, or other economic conditions, see Taylor (1993). The average level of the yield curve is usually associated with the inflation rate and the spread between long and short rates with temporary business cycles conditions, see Diebold, Rudebusch and Aruoba (2006). For these reasons, macroeconomic information has been shown to help forecasting future interest rates and excess bond returns, see Ang and Piazzesi (2003), Mönch (2008), De Pooter, Ravazzolo and Van Dijk (2007), Favero, Niu and Sala (2012) and Ludvigson and Ng (2009).

In this paper, we aim at identifying the factors summarizing macroeconomic information that is not spanned by the traditional yield curve factors. The economic literature so far has not addressed this problem since in existing studies macroeconomic factors are either proxied by preselected observable variables, see Bianchi, Mumtaz and Surico (2009), Dewachter and Lyrio (2006), Diebold et al. (2006), Joslin, Priebsch and Singleton (2014), Rudebusch and Wu (2008), and Wright (2011), or extracted from a large set of macroeconomic indicators and treated separately from the yield curve factors, see Ang and Piazzesi (2003), Favero et al. (2012), Ludvigson and Ng (2009), Mönch (2008) and Mönch (2012).

We estimate a macro-yield model that treats macroeconomic factors as unobservable components that we extract simultaneously with the traditional yield curve factors. Following Diebold and Li (2006) and Diebold and Rudebusch (2013), the factors affecting the yield curve are identified by constraining the loadings to follow the smooth pattern proposed by Nelson and Siegel (1987). More specifically, our empirical model is a Dynamic Factor Model (DFM) for Treasury zero-coupon yields and a representative set of macroeconomic variables with restrictions on the factor loadings. Following Doz, Giannone and Reichlin (2012) the model is estimated by quasi maximum likelihood, i.e. we maximize the likelihood of a potentially miss-specified model. Precisely, the likelihood is computed assuming that the dynamic factor model is Gaussian and exact (the idiosyncratic errors are assumed to be cross-sectionally orthogonal). Doz et al. (2012) have shown that, when estimation is carried out with a large number of highly collinear variable, the estimator is consistent and robust to non Gaussianity and to weak correlation among idiosyncratic components.

Using monthly U.S. data from January 1970 to December 2008, we find that a significant component of macroeconomic information is not captured by the yield curve factors and, at the same time, is unspanned by the yield curve, in the sense that it does not affect contemporaneously the cross-section of yields. The unspanned macroeconomic information is driven by two factors that are well proxied by economic growth and real interest rates. These factors have substantial predictive information for bond yields and excess bond returns, in spite of the fact that they do not affect contemporaneously the shape of the yield curve. The macro-yields model explains up to 55% of the variation in excess bond returns and outperforms all existing models in forecasting bond yields and excess returns.

The paper is organized as follows. Section 2 presents the macro-yields model. Section 3 describes the data, the estimation procedure and the information criteria used for model selection. Section 4 describes the empirical results and in-sample validation of the model. Section 5 assesses the forecasting performance of the model in real time. Section 6 concludes.

2 The macro-yields model

We propose a dynamic factor model for the joint behavior of government bond yields and macroeconomic indicators. Bond yields at different maturities are driven by the traditional level, slope and curvature factors. Macroeconomic variables load on the yield curve factors as well as on some additional macro factors that capture the information in macroeconomic variables over and above the yield curve factors. We assume that these additional macro factors do not provide any information about the contemporaneous shape of the yield curve. According to this model, the level, slope, and curvature factors are spanned by both the bond yields and macroeconomic variables. The additional macro factors, instead, are contemporaneously loaded only by the macroeconomic variables and, thus, are unspanned by the cross-section of the yields. The remaining of this section, describes the model in details.

The cross section of bond yields is modeled using the Dynamic Nelson-Siegel framework of Diebold and Li (2006). Denoting by y_t the $N_y \times 1$ vector of yields with N_y different maturities at time t, we have

$$y_t \;=\; a_y + \;\Gamma_{yy}\; F_t^y + v_t^y, \tag{1}$$

where F_t^y is a 3×1 vector containing the latent yield-curve factors at time t, Γ_{yy} is a $N_y \times 3$ matrix of factor loadings, and v_t^y is an $N_y \times 1$ vector of idiosyncratic components. The yield curve factors F_t^y are identified by constraining the factor loadings to follow the smooth pattern proposed by Nelson and Siegel (1987) (hereafter NS)

$$a_y = 0; \quad \Gamma_{yy}^{(\tau)} = \left[1 \;\; \frac{1-e^{-\lambda\tau}}{\lambda\tau} \;\; \frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right] \equiv \Gamma_{NS}^{(\tau)}, \tag{2}$$

where $\Gamma_{yy}^{(\tau)}$ is the row of the matrix of factor loadings corresponding to the yield with maturity τ months and λ is a decay parameter of the factor loadings. Diebold and Li (2006) show that this functional form of the factor loadings, implies that the three yield curve factors can be interpreted as the level, slope, and curvature of the yield curve. Indeed, the loading equal to one on the first factor, for all maturities, implies that an increase in this factor increases all yields equally, shifting the level of the yield curve. The loadings on the second factor are large for short maturities, decaying to zero for the long ones. Accordingly, an increase in the second factor decreases the slope of the yield curve. Loadings on the third factor are zero for the shortest and the longest maturities, reaching the maximum for medium maturities. Therefore, an increase in this factor augments the curvature of the yield curve. The specific shape of the loadings depends on the decay parameter λ, which we calibrate to the value that maximizes the loading on the curvature factor for the yields with maturity 30 months, as in Diebold and Li (2006).

Given these particular functional forms for the loadings on the three yield curve factors, one can summarize movements in the term structure of interest rates into three factors which have a clear-cut interpretation. The NS factors are just linear combinations of yields. The level factor can be proxied by the long term yield, the slope by the spread between the long and short maturity yield (first derivative) and the curvature by sum of the spreads between a medium and a long term yield, and between a medium and the short term yield (second derivative), see Diebold and Li (2006).[1] Due to its flexibility and parsimony, the NS model accurately fits the yield curve and performs well in out-of-sample forecasting exercises, as shown by Diebold and Li (2006) and De Pooter et al. (2007). For these reasons, fixed-income wealth managers in public organizations, investment banks and central banks rely heavily on NS type of models to fit and forecast yield curves, see BIS (2005), ECB (2008), Gürkaynak, Sack and Wright (2007) and Coroneo, Nyholm and Vidova-Koleva (2011).

Macroeconomic variables, are assumed to be potentially driven by two sources of co-movement, the yield curve factors F_t^y and macro specific factors. Denoting by x_t the $N_x \times 1$ vector of macroeconomic variables at time t, we have

$$x_t \; = \; a_x + \Gamma_{xy} \; F_t^y + \Gamma_{xx} \; F_t^x + v_t^x, \tag{3}$$

where F_t^x is an $r \times 1$ vector of macroeconomic latent factors, Γ_{xy} is a $N_x \times 3$ matrix of factor

[1] Similar proxies are used by Ang, Piazzesi and Wei (2006) and Duffee (2011a).

loadings on the yield curve factors, Γ_{xx} is a $N_x \times r$ matrix of factor loadings on the macro factors, and v_t^x is an $N_x \times 1$ vector of idiosyncratic components.

The yield curve and the macroeconomic factors are extracted by estimating (1) and (3) simultaneously

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} 0 \\ a_x \end{pmatrix} + \begin{bmatrix} \Gamma_{yy} & \Gamma_{yx} \\ \Gamma_{xy} & \Gamma_{xx} \end{bmatrix} \begin{pmatrix} F_t^y \\ F_t^x \end{pmatrix} + \begin{pmatrix} v_t^y \\ v_t^x \end{pmatrix}, \ \Gamma_{yy} = \Gamma_{NS}, \ \Gamma_{yx} = 0, \tag{4}$$

where Γ_{NS} is defined according to (2).

The joint dynamics of the yield curve and the macroeconomic factors follow a VAR(1)

$$\begin{pmatrix} F_t^y \\ F_t^x \end{pmatrix} = \begin{pmatrix} \mu_y \\ \mu_x \end{pmatrix} + \begin{bmatrix} A_{yy} & A_{yx} \\ A_{xy} & A_{xx} \end{bmatrix} \begin{pmatrix} F_{t-1}^y \\ F_{t-1}^x \end{pmatrix} + \begin{pmatrix} u_t^y \\ u_t^x \end{pmatrix}, \quad \begin{pmatrix} u_t^y \\ u_t^x \end{pmatrix} \sim N\left(0, \begin{bmatrix} Q_{yy} & Q_{yx} \\ Q_{xy} & Q_{xx} \end{bmatrix}\right). \tag{5}$$

The idiosyncratic components collected in $v_t = [v_t^y \quad v_t^x]'$ are modelled to follow independent autoregressive processes

$$v_t = B v_{t-1} + \xi_t, \quad \xi_t \sim N(0, R) \tag{6}$$

where B and R are diagonal matrices, implying that the common factors fully account for the joint correlation of the observations. The residuals to the idiosyncratic components of the individual variables, ξ_t, and the innovations driving the common factors, u_t, are assumed to be normally distributed and mutually independent. This assumptions implies that the common factors are not allowed to react to variable specific shocks.

The assumptions of Gaussianity and of independence among idiosyncratic components might be sources of miss-specification. It is hard to relax these restrictions since they are necessary to retain parsimony, insure identification of the common and idiosyncratic components and limit computational complexity. However, Doz et al. (2012) have shown that, if the factor structure is strong, the Maximum Likelihood estimates are robust not only to non Gaussianity but also to the presence of limited correlations among idiosyncratic components.

Allowing Γ_{xy} to be different from zero is crucial to insure that the macroeconomic factors F_t^x capture only those source of co-movement in the macroeconomic variables that are not already spanned by the yield curve factors. Existing studies, instead, have imposed a block-diagonal structure of the factor loadings ($\Gamma_{xy} = 0$ and $\Gamma_{yx} = 0$), either explicitly, as in Mönch (2012), either implicitly by extracting the macro factors exclusively from macroeconomic variables, as in Ludvigson and Ng (2009).

Assuming that macroeconomic factors do not provide any information about the contemporaneous shape of the yield curve ($\Gamma_{yx} = 0$) restricts the macroeconomic factors F_t^x to be unspanned not only by the yield factors but also by the entire cross-section of yields. This restriction is expected to be immaterial since, as stressed above, the yield factors F_t^y are notoriously effective at fitting the entire yield curve.

In the remainder of the paper we will maintain the restriction $\Gamma_{yx} = 0$ and leave Γ_{xy} unrestricted, unless otherwise mentioned.[2]

3 Estimation and preliminary results

3.1 Data

We use monthly U.S. Treasury zero-coupon yield curve data spanning the period January 1970 to December 2008. The bond yield data are taken from the Fama-Bliss dataset available from the Center for Research in Securities Prices (CRSP) and contain observations on three months and one through five-year zero coupon bond yields. The macroeconomic dataset consists of 14 macroeconomic variables, which include five inflation measures, seven real variables, the federal funds rate and a money indicator. Appendix B contains a complete list of the macroeconomic variables along with the transformation applied to ensure stationarity. Following Ang and Piazzesi (2003), De Pooter et al. (2007), Diebold et al. (2006) and Mönch (2008), we use annual growth rates for all variables, except for capacity utilization, the federal funds rate, the unemployment rate

[2]Results for the block-diagonal model ($\Gamma_{xy} = 0$, $\Gamma_{yx} = 0$) and the unrestricted model ($\Gamma_{xy} \neq 0$) are available upon request.

and the manufacturing index which we keep in levels.[3]

3.2 Estimation

Equations (4)–(6) describe a restricted state-space model with autocorrelated idiosyncratic components for which maximum likelihood estimators of the parameters are not available in closed form. Conditionally on the factors, the model reduces in a set of linear regressions. As consequence, the Maximum Likelihood estimates can be easily computed using the Expectation Maximization (EM) algorithm, as described in detail in Appendix A.[4]

We initialize the yield curve factors with the NS factors using the two-steps OLS procedure introduced by Diebold and Li (2006). We then project the macroeconomic variables on the NS factors and use the principal components of the residuals of this regression to initialize the unspanned macroeconomic factors. Γ_{yy} is restricted to be equal to the NS loadings. All the other parameters are initialized with the OLS estimates obtained using the initial guesses of yield and macro factors described above. Given the initial parameters, a new set of factors is obtained using the Kalman smoother. If we stop at this stage, we have the two-step procedure of Doz, Giannone and Reichlin (2011).[5] Maximum Likelihood estimates are obtained by iterating these two steps until convergence provided that OLS regressions are modified in order to take into account the fact that the common factors are estimated.[6]

For comparison, we also estimate an only-yields model, which uses only the information contained in the yields. This is a restricted version of the macro-yields model in equations (4)–(6) with $Q_{yx} = A_{yx} = \Gamma_{xy} = 0$ and can hence be estimated using the same procedure.

[3]Since the selection of variables has an element of arbitrariness, we have performed robustness checks with an alternative databases constructed by Banbura, Giannone, Modugno and Reichlin (2012) that includes all the variables that are constantly monitored by market participants. Results, available upon request, show that the main findings are confirmed.

[4]Using the Expectation Conditional Restricted Maximization (ECRM) algorithm is also possible to estimate λ, but, despite the increase in the computation burden, the empirical results remain qualitatively similar to those obtained by setting λ to the value that maximizes the loading of the the yields with maturity 30 months on the curvature factor.

[5]Interestingly, using final or initial estimates delivers similar results (available upon request). This is not surprising since Doz et al. (2011) and Doz et al. (2012) show that, if the factor structure is strong, the two-step and the maximum likelihood approach have similar properties, both asymptotically and in small sample.

[6]See Appendix A for details.

3.3 Model selection

The macro-yields model decomposes variations in yields and macroeconomic variables into yield curve factors, unspanned macroeconomic factors and idiosyncratic noises. The yield curve factors are identified as the NS factors which have a clear interpretation as level, slope, and curvature. However, the true number of unspanned macroeconomic factors is unknown. We select the optimal number of factors using an information criteria approach. The idea is to choose the number of factors that maximizes the general fit of the model using a penalty function to account for the loss in parsimony.

Bai and Ng (2002) derive information criteria to determine the number of factors in approximate factor models when the factors are estimated by principal components. They also show that their IC_3 information criterion can be applied to any consistent estimator of the factors provided that the penalty function is derived from the correct convergence rate. For the quasi-maximum likelihood estimator, Doz et al. (2012) show that it converges to the true value at a rate equal to

$$C_{NT}^{*2} = \min\left\{\sqrt{T}, \frac{N}{\log N}\right\} \tag{7}$$

where N and T denote the cross-section and the time dimension, respectively. Thus, a modified Bai and Ng (2002) information criterion that can be used to select the optimal number of factors when estimation is performed by quasi-maximum likelihood is as follows

$$IC^*(s) = \log(V(s, \hat{F}_{(s)})) + s\, g(N,T), \quad g(N,T) = \frac{\log C_{NT}^{*2}}{C_{NT}^{*2}} \tag{8}$$

where s denotes the number of factors, $\hat{F}_{(s)}$ are the estimated factors and $V(s, \hat{F}_{(s)})$ is the sum of squared idiosyncratic components (divided by NT) when s factors are estimated. The penalty function $g(N,T)$ is a function of both N and T and depends on C_{NT}^{*2}, the convergence rate of the estimator, in our case given by (7).

To select the number of factors in the macro-yields model, we estimate the macro-yields model

Table 1: Model selection

Number of factors	IC^*	V
3	0.02	0.44
4	-0.03	0.31
5	-0.11	0.22
6	0.01	0.18
7	0.23	0.17
8	0.43	0.16

This table reports the information criterion IC^*, as shown in (8) and (7), and the sum of the variance of the idiosyncratic components (divided by NT), V, when different numbers of factors are estimated.

in equations (4)–(6) allowing from three up to a total of eight factors, where the first three are identified as the yield curve factors and the others are unspanned macro factors. Table 1 reports the information criterion, as shown in Equation (8), and the sum of the variance of the idiosyncratic components for these different specifications of the macro-yields model. The information criterion selects the model with five factors, i.e. three yield curve factors plus two unspanned factors. This is also confirmed by the fact that the strongest reduction in the sum of the variances of the idiosyncratic components is obtained passing from the four to the five factors specification. Thus our macro-yields model is a latent factor model with three factors that explain the cross-section of yields and two unspanned macroeconomics factors.

4 In sample results

4.1 Model fit

Table 2 reports the share of variance of the macroeconomic variables explained by the macro-yields factors. Results show that, as expected, the yield curve factors explain most of the variance of the federal funds rate and the yields at different maturities. They also explain the part of the variance of price indices, unemployment, nominal earnings, nominal consumption and money, in

Table 2: Cumulative variance of yields and macro variables explained by the macro-yields factors

	Level	Slope	Curv	UM1	UM2
Government bond yield with maturity 3 months	0.59	0.94	1.00	1.00	1.00
Government bond yield with maturity 1 year	0.61	0.83	0.99	0.99	0.99
Government bond yield with maturity 2 years	0.65	0.78	0.99	0.99	0.99
Government bond yield with maturity 3 years	0.70	0.79	1.00	1.00	1.00
Government bond yield with maturity 4 years	0.74	0.80	0.99	0.99	0.99
Government bond yield with maturity 5 years	0.78	0.82	0.99	0.99	0.99
Average Hourly Earnings: Total Private	0.07	0.29	0.33	0.33	0.67
Consumer Price Index: All Items	0.19	0.48	0.48	0.50	0.85
Real Disposable Personal Income	0.00	0.02	0.03	0.34	0.36
Effective Federal Funds Rate	0.53	0.93	0.96	0.96	0.97
House Sales - New One Family Houses	0.00	0.19	0.19	0.23	0.23
Industrial Production Index	0.02	0.02	0.03	0.69	0.69
M1 Money Stock	0.17	0.25	0.25	0.25	0.31
ISM Manufacturing: PMI Composite Index (NAPM)	0.03	0.05	0.05	0.61	0.65
Payments All Employees: Total nonfarm	0.00	0.02	0.10	0.70	0.70
Personal Consumption Expenditures	0.16	0.23	0.33	0.46	0.78
Producer Price Index: Crude Materials	0.03	0.14	0.14	0.20	0.43
Producer Price Index: Finished Goods	0.03	0.32	0.32	0.33	0.80
Capacity Utilization: Total Industry	0.02	0.16	0.21	0.63	0.64
Civilian Unemployment Rate	0.44	0.54	0.55	0.65	0.68

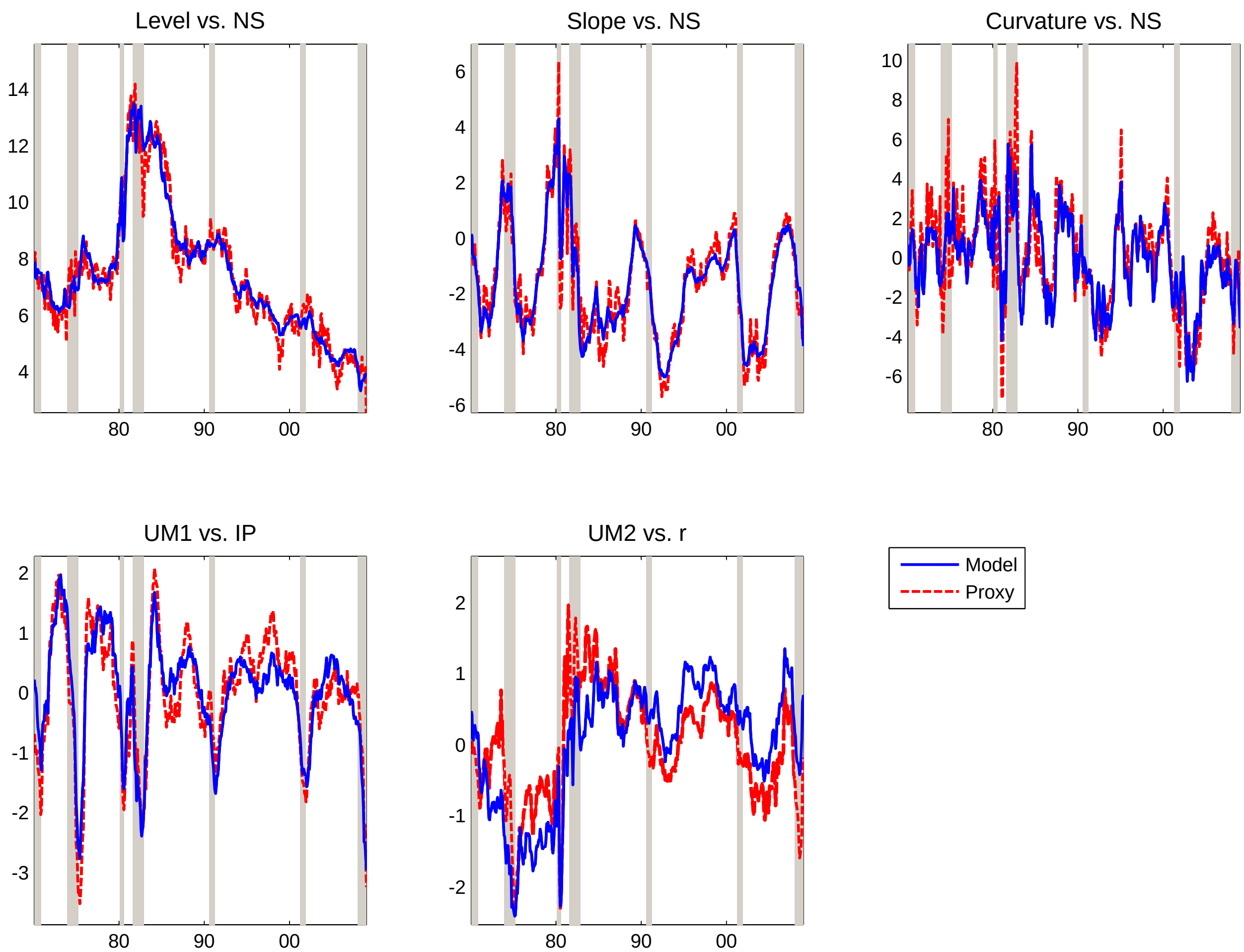
This table reports the cumulative share of variance of yields and macro variables explained by the macro-yields factors. The first three columns refer to the yield curve factors (level, slope and curvature) and the last two to the unspanned macroeconomic factors ($UM1$ and $UM2$).

line with previous studies, see Diebold et al. (2006). The first unspanned macro factor captures the dynamics of industrial production and other real variables, while the second unspanned factor mainly explains inflation and other nominal variables.[7]

Figure 1 displays the estimated factors of the macro-yields model. The top three plots report the yield curve factors, while the bottom two refer to the unspanned factors. The estimated yield curve factors of the macro-yields model are highly correlated with the NS factors, which we estimate by ordinary least squares as in Diebold and Li (2006) and report in dashed red lines in the top plots. The differences between the NS factors and the first three macro-yields factors are due to the fact that, in the macro-yields model, the yield curve factors are common to both yield curve and macroeconomic variables. In fact, in the macro-yields model, we extract the yield curve factors from both yields and macroeconomic variables and impose the NS restrictions on the factors loadings of the yields to identify them as yield curve factors. The two bottom plots of Figure 1 show the unspanned macro factors. The bottom left plot reports the first unspanned macro factor along with the industrial production index, while the bottom right plot reports the second unspanned macroeconomic factor along with the real interest rate (computed as the difference between the federal funds rate and the consumer price index). As it is clear from the plots, the first unspanned macroeconomic factor closely tracks the industrial production index, with a correlation of 90%, and the second unspanned macroeconomic factor proxies the real interest, with a correlation of 74%. This is in line with the fact that, as reported in Table 2, the first unspanned macroeconomic factor explains mainly measures of real economic activity, while nominal variables are explained partly by the yield curve factors and partly by the second unspanned factor. We can thus conclude that the macro-yields models identifies two unspanned macroeconomic factors: real economic activity and real interest rate. In the next Section we assess the quantitative importance of the unspanned

[7]The two macroeconomic factors are not identified since any transformation HF_t^x, with H non-singular, gives an observationally equivalent model. In order to achieve identification additional restrictions are required. We do not impose such restrictions and the EM algorithm converges to the Maximum Likelihood solution that is "close" to the initialisation, i.e. the principal components of the residuals of the macro variables after regressing them on the NS factors. Identification can be achieved by assuming that the first macro factor has a loading of one for industrial production, and that the second macro factor has a loading of one for CPI and is not loaded by IP. Once we impose this restriction, results, available upon request, do not change.

Figure 1: Macro-yields factors



This figure displays the estimated factors of the macro-yields model. The dashed red lines in the three top graphs refer to the NS yield curve factors estimated by ordinary least squares as in Diebold and Li (2006). The red dashed line in the bottom left plot refers to the industrial production index (IP), while the red dashed line in the bottom plot refers to the real interest rate (FFR-CPI). The grey-shaded areas indicate the recessions as defined by the NBER.

macroeconomic factors in explaining bond risk premia.

4.2 Bond risk premia

The bond risk premium measures the compensation required by risk averse investors to hold long-term government bonds for facing capital loss risk, if the bond is sold before maturity.

Long-term yields are determined by market expectations for the short rates over the holding period of the long-term asset plus a yield risk premium. Assuming a minimum investment horizon of one year, we have

$$y_t^{(\tau)} = \left(\frac{\tau}{12}\right)^{-1} \sum_{i=0,12,\ldots,\tau-12} E_t[y_{t+i}^{(12)}] + yrp_t^{(\tau)}. \tag{9}$$

An alternative measure for the bond risk premium can be obtained by looking at bond returns. The one-year holding period bond return for a bond with maturity τ months is the return of buying a bond with τ months to maturity at time t, selling it one year later, at time $t+12$, as a bond with $\tau - 12$ months to maturity, i.e.,

$$r_{t+12}^{(\tau)} = -(\tau - 12) y_{t+12}^{(\tau-12)} + \tau y_t^{(\tau)}. \tag{10}$$

The expected one-year holding period return on long term bonds equals the expected return on the short term bond plus the return risk premium

$$E_t[r_{t+12}^{(\tau)}] = y_t^{(12)} + rrp_t^{(\tau)}, \tag{11}$$

accordingly the return risk premium is the one-year expected return in excess of the one-year rate

$$rrp_t^{(\tau)} = E_t[r_{t+12}^{(\tau)}] - y_t^{(12)} \equiv E_t[rx_{t+12}^{(\tau)}]. \tag{12}$$

The relation between the return risk premium and the yield risk premium is as follows

$$yrp_t^{(\tau)} = \frac{1}{\tau} E_t \left[rrp_t^{(\tau)} + rrp_{t+12}^{(\tau-12)} + \ldots + rrp_{t+\tau-24}^{(24)} \right], \tag{13}$$

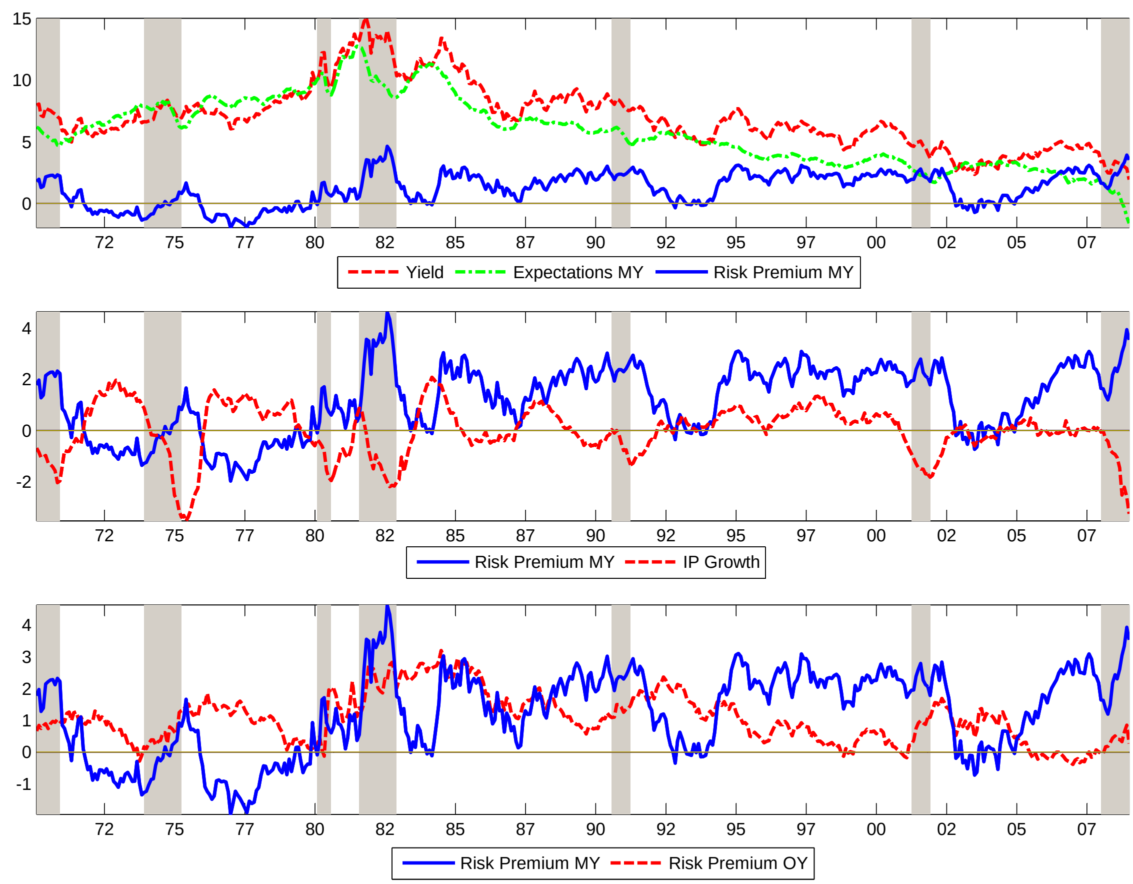
which means that the yield risk premium is the average of expected future return risk premia of declining maturity. This implies that the statements in Equations (9) and (11) are equivalent, if one equation holds with zero (constant) bond risk premium, the other equation holds with zero (constant) bond risk premium as well.

The expectations hypothesis of the term structure of interest rates states that the yield risk premium is constant. This implies that expected excess returns are time invariant and, thus, excess bond returns should not be predictable with variables in the information set at time t. However, the expectations hypothesis has been empirically rejected since Fama and Bliss (1987) and Campbell and Shiller (1991). They find that excess returns can be predicted by forward rate spreads and by yield spreads, respectively. More recent evidence by Cochrane and Piazzesi (2005) shows that a linear combination of forward rates (the CP factor) explains between 30% and 35% of the variation in expected excess bond returns. Moreover, Ludvigson and Ng (2009) find that macroeconomic factors constructed as linear and non-linear combinations of principal components extracted from a large data-set of macroeconomic variables (the LN factor) have important forecasting power for future excess returns on U.S. government bonds, above and beyond the predictive power contained in forward rates and yield spreads. Cooper and Priestley (2009) also find that the output gap has in-sample and out-of-sample predictive power for U.S. excess bond returns.

The top panel of Figure 2 shows the 5 years to maturity yield along with the corresponding components as in Equation (9), where the sum of expectations is the sum of forecasts produced with our macro-yields model and the risk premium is the difference between the 5 years to maturity yield and the sum of the forecasts of the 1 year to maturity yields. The expectation component is larger than the risk premium but the graph shows that there is substantial variation of the risk premium over time, which is not compatible with the expectations hypothesis.[8] The middle

[8] At the end of the sample the expectation component is negative due to the fact that in this period short rates

Figure 2: Yield risk premium, 5-year bond



This figure displays the yield risk premium using the 5 years to maturity bond. The top panel shows the 5 years to maturity yield (red dashed line) along with the corresponding expectation (green dot-dashed line) and the yield risk premium (blue line) components, computed as in Equation (9) using the macro-yields model. The middle panel reports the yield risk premium according to the macro-yields model (blue line) and the standardized industrial production growth (red dashed line). The bottom plot shows the yield risk premium obtained from the macro-yields model (blue line) and the only-yields model (red dashed line). The grey-shaded areas indicate the recessions as defined by the NBER.

graph plots the risk premium against the industrial production index growth and it reveals that the yield risk premium obtained from the macro-yields model displays a clear counter-cyclical pattern. Its correlation with the industrial production index growth is -0.33. This is consistent with the fact that investors want to be compensated for bearing risks related to recessions. Conversely, the bottom graph in Figure 2, shows the risk premium obtained from the only-yields model. This model delivers an acyclical risk premium, with a correlation of only -0.07 with the industrial production index growth. This indicates that using macro variables greatly improves the estimates of the risk premium.

Given that, as shown in Equation (13), the yield risk premium is the average of expected future return risk premia of declining maturity, we analyze the predictive ability of the macro-yields model for excess returns and compare it with the predictions of the only-yields model. We also compare our results with predictions obtained using the CP factor, the LN factor and the CP and LN factors combined.

We implement predictive regressions for the CP and LN factors by regressing excess bond returns on the predictive factors $X_t = \{CP_t, LN_t\}$, as follows

$$rx_{t+12}^{(\tau)} = \beta X_t + \varepsilon_{t+12}^{(\tau)}. \tag{14}$$

We construct the predictive factors X_t by pooling the predictive regression for the individual maturities

$$\overline{rx}_{t+12} = \gamma x_t + \overline{\varepsilon}_{t+12}, \tag{15}$$

where $\overline{rx}_{t+12} = \frac{1}{4}\sum_{\tau=24,36,48,60} rx_{t+12}^{(\tau)}$ and x_t contains the predictor variables. To construct the CP factor we use the following predictor variables $x_t^{CP} = [1, y_t^{(12)}, f_t^{(24)}, \ldots, f_t^{(60)}]$, where $f_t^{(\tau)}$ denotes the τ-month forward rate.[9] We estimate equation (15) using x_t^{CP} as predictor variables

reached the zero lower bound. Our macro-yields model does not impose a zero-lower bound to the predicted yields, but one could interpret the negative expectation component as a shadow rate.

[9]The τ-month forward rate for loans between time $t+\tau-12$ and $t+\tau$ is defined as

$$f_t^{(\tau)} = -(\tau-12)y_t^{(\tau-12)} + \tau y_t^{(\tau)}.$$

Table 3: In-sample fit of excess bond returns

Maturity	MY	OY	CP	LN	LN+CP
2y	0.55	0.12	0.22	0.33	0.41
3y	0.53	0.12	0.24	0.33	0.43
4y	0.50	0.14	0.27	0.32	0.43
5y	0.46	0.15	0.24	0.30	0.40

This table reports the R^2 for one-year ahead one year holding period excess bond returns from different models. The columns MY and OY refer to the model-implied expected excess bond returns from the macro-yields model (MY) and the only-yields model (OY) respectively. The columns CP, LN and CP+LN refer to the predictive regression using the Cochrane and Piazzesi (2005) factor (CP), the Ludvigson and Ng (2009) factor (LN), and both the Cochrane and Piazzesi (2005) and the Ludvigson and Ng (2009) factors jointly.

and construct the CP factor as $CP_t = \hat{\gamma}^{CP} x_t^{CP}$. To construct the LN factor, we use as predictor variables $x_t^{LN} = [1, PC1_t, \ldots, PC8_t, PC1_t^3]$, where PC denotes principal components extracted from a large dataset of 131 macroeconomic data series.[10] We then estimate equation (15) using x_t^{LN} as predictor variables and construct the LN factor as $LN_t = \hat{\gamma}^{LN} x_t^{LN}$.

Notice that the LN factors are constructed aggregating principal components extracted from a set of macroeconomic and financial variables without imposing that they are unspanned by the cross section of the yields similarly to the factors extracted by assuming a block-diagonal structure on the factor loadings. As a consequence, those factors duplicate information that is already spanned by the yield factors.

Results in Table 3 show that the macro-yields model explains about 46-55% of the variation of one-year ahead excess returns, while the only-yields model can explain only the 12-15% of the variation of the one-year ahead excess returns. Table 3 reports also the R-squared from the predictive regressions of excess bond returns on the CP and the LN factors. Results show that the CP factor explains 22-27% of the variation in one-year ahead excess returns, slightly lower than

[10]The 131 macroeconomic data series used to construct the LN factor have been downloaded from Sydney C. Ludvigson's website at http://www.econ.nyu.edu/user/ludvigsons/Data&ReplicationFiles.zip.

the value reported in Cochrane and Piazzesi (2005). This is due to the fact that our predictive regressions are estimated on a more updated sample, and the performance of the CP factor has deteriorated over time, as also shown by Thornton and Valente (2012). The LN factors explain a third of the variation of future excess bond returns, while the CP and LN factors jointly explain 40-43% of the variation in one-year ahead excess bond returns, lower than what is explained by our macro-yields model. We can thus conclude that, in-sample, the macro-yields model outperforms the CP and the LN factors even combined.

Figure 3 shows the predicted and realized average excess bond returns from the macro-yields and the only-yields model, and also from the predictive regressions using the CP and the LN factors. The figure shows that the predicted excess bond returns from the only-yields model are quite flat, indicating that the yield curve factors poorly predict excess bond returns. The CP factor seems doing a better job than the only-yields model, but does not improve over the macro-yields model. The macro-yields model is able to better predict the average excess return, also with respect to the LN factor.

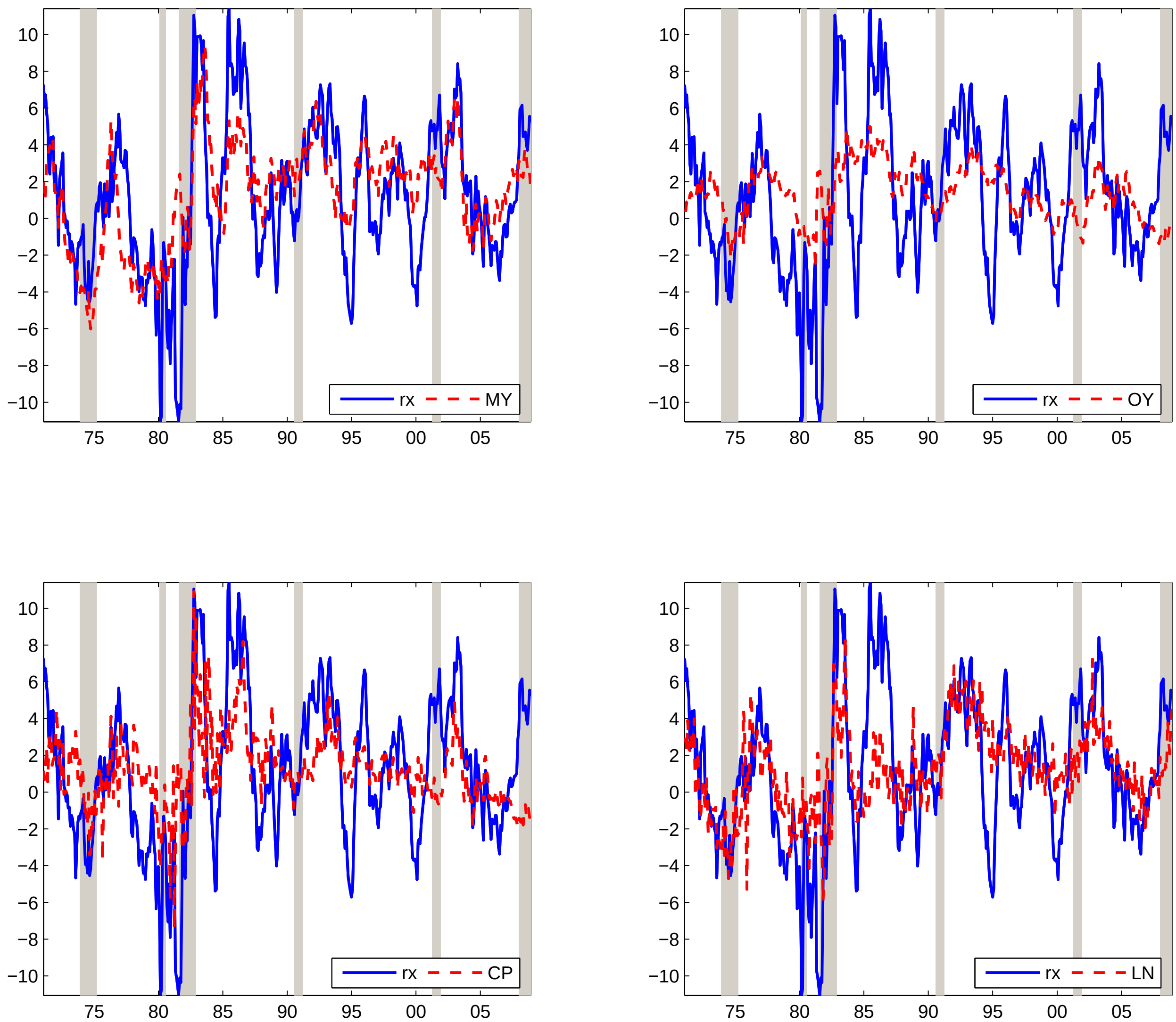
4.3 Unspanning conditions

Results in the previous section show that the unspanned macro factors play an important role in explaining the term premium, despite being constrained to not affect current yields. In the context of Equation (9), this can only happen if the unspanned macro factors have offsetting effects on average expected future short rates and term premia, see Duffee (2011b).

To understand whether our macro factors are truly unspanned by the yield curve, we compute the risk premium of an unrestricted macro-yields model which does not impose zero restrictions on the factor loadings of the yields on the macro factors, i.e. $\Gamma_{yx} \neq 0$ in Equation (4).[11] The estimates of the bond premium delivered by this model are practically indistinguishable from the estimates obtained using the macro-yields model which instead imposes the restriction $\Gamma_{yx} = 0$ (the correlation between the estimates is 0.99). The fact that imposing the unspanning restrictions

[11] More extensive results for the unrestricted macro-yields model are available upon request.

Figure 3: Average 1-year holding period excess return: realized and predicted



This figure displays the average excess return $\overline{rx}_{t+12}$ (blue continuous line) and the corresponding predicted values from different models (dashed red line). The dashed red line in the top plots refer to the model-implied predicted values from the macro-yields MY model (top right) and only-yields OY model (top left). The dashed red line in the bottom plots refer to the predicted values from the predictive regressions using the CP factors (bottom left) and the LN factor (bottom right). The grey-shaded areas indicate the recessions as defined by the NBER.

Table 4: Likelihood ratio test statistic for the unspanning restrictions

H_0	Test statistic	p-value
$\Gamma_{yx} = 0$	12.85	0.38
$A_{yx} = 0$	79.03	0.00

This table reports the likelihood ratio test statistic for the unspanning restrictions and the corresponding p-values, computed using a chi-squared distribution with degrees of freedom equal to the number of restrictions tested. The first line refers to the null hypotheses $\Gamma_{yx} = 0$ in Equation (4) while the second line refers to the null hypotheses $A_{yx} = 0$ in Equation (5).

has no effect on the yield risk premium indicates that the macro factors are unspanned by the yield curve. In practice, this means that, in periods of recession, the unspanned macro factors increase the risk premium and decrease the expected future short rates by the same amount, without contributing to a steepening of the current yield curve. Conversely, in periods of economic expansion, the unspanned macro factors decrease the bond premium and increase the expected future short rates by the same amount, without contributing to a flattening of the current yield curve. Changes in the current shape of the yield curve can only be determined by changes in the yield curve factors.

To formally test for the unspanning properties in the context of our state-space macro-yields model, we define a factor as unspanned by the yield curve if it satisfies the following two conditions. First, it doesn't affect the current cross-section of yields, i.e. it is not loaded contemporaneously by the yields ($\Gamma_{yx} = 0$ in Equation (4)). Second, it has predictive ability for the yield curve factors ($A_{yx} \neq 0$ in Equation (5)), see also Joslin et al. (2014).

The unspanning conditions can be tested performing likelihood ratio tests, as follows

$$LR = 2 \times (L_u - L_r) \tag{16}$$

where LR has a chi-squared probability distribution with degrees of freedom equal to the number of restrictions imposed. To compute the likelihood ratio test for the zero restrictions on the factor loadings, L_u denotes the loglikelihood of an unrestricted macro-yield model that does not impose the restriction $\Gamma_{yx} = 0$ and L_r is the loglikelihood of our macro-yields model. The test statistic in Table 4 shows that we cannot reject the null hypothesis of factor loadings of the yields on the macro factors equal to zero. This implies that, indeed, the macro factors do not affect the current shape of the yield curve.[12]

To test the predictive ability of the macro factors obtained from macro-yields model in Equations (4)–(6) for the yield factors, and therefore the yield curve of interest rates, we perform the likelihood ratio test statistics in Equation (16), where, in this case, L_u is the loglikelihood of our macro-yield model and L_r is the restricted loglikelihood obtained imposing $A_{yx} = 0$ in Equation (5). Results in Table 4 show that we can reject the null hypothesis of no Granger causality from the macro factors to the yield curve factors.

The result of the test shows that the macroeconomic factors identified by the macro-yields model do not explain the cross-section of yields but have predictive ability for the future evolution of the yield curve. As a consequence, they satisfy both conditions for being truly unspanned macroeconomic factors.[13]

5 Out-of-sample forecast

To evaluate the predictive ability of the macro-yields model, we generate out-of-sample iterative forecasts of the factors, as follows

$$E_t(F^*_{t+h}) \equiv \hat{F}^*_{t+h|t} = (\hat{A}^*_{|t})^h \hat{F}^*_{t|t},$$

[12]This result is due to the fact that almost all the bond yields variation is explained by the Nelson and Siegel factors. The same result may not hold when the yield factors provide a poorer fit of the yields, as in Joslin, Le and Singleton (2013).

[13]Moreover, looking at the coefficients and their relative standard errors, available upon request, we can infer that the first unspanned factor, proxied by economic growth, Granger causes the slope and the curvature, while the second unspanned factor, proxied by the real interest rate, Granger causes the level.

where h denotes the forecast horizon and $\hat{A}^*_{|t}$ is estimated using the information available till time t.[14] We then compute out-of-sample forecasts of the yields given the projected factors, in this way

$$E_t(z_{t+h}) \equiv \hat{z}_{t+h|t} = \hat{\Gamma}^*_{|t} \hat{F}^*_{t+h|t},$$

where $\hat{\Gamma}^*_{|t}$ is estimated using data up to time t.

Collecting the excess returns for bonds with maturities from two to five years in the vector rx_t, we compute the out-of-sample predictions of excess bond returns as follows

$$E_t(rx_{t+12}) \equiv rx_{t+12|t} = \Pi_1 y_{t+12|t} + \Pi_2 y_t = \Pi_1(\hat{\Gamma}^*_{|t} F^*_{t+12|t}) + \Pi_2 y_t, \tag{17}$$

where $\Pi_1 = \begin{bmatrix} D_{[-1:-K]} & 0_{[K\times 1]} \end{bmatrix}$, $\Pi_2 = \begin{bmatrix} -1_{[K\times 1]} & D_{[2:K+1]} \end{bmatrix}$, $D_{[-1:-K]}$ denotes a diagonal matrix with elements $-1, -2, \ldots, -K$ in the diagonal and $K+1$ denotes the total number of maturities. Notice that Equation (17) implies that the forecast errors made in forecasting the excess returns are proportional to the ones made in forecasting the yields, i.e. $rx_{t+12|t} - rx_{t+12} = \Pi_1(y_{t+12|t} - y_{t+12})$, see Carriero, Kapetanios and Marcellino (2012).

We forecast yields and excess returns recursively using data from January 1970 and evaluating the forecast performances on the sample from January 1990 to December 2008.

5.1 Yields

To evaluate the prediction accuracy of the macro yields model for out-of-sample forecasts of yields, we use the Mean Squared Forecast Error (MSFE), i.e. the average squared error in the evaluation period for the h-months ahead forecast of the yield (or excess return) with maturity τ

$$\text{MSFE}_{t_0}^{t_1}(\tau, h, M) = \frac{1}{t_1 - t_0 + 1} \sum_{t=t_0}^{t_1} \left(\hat{y}^{(\tau)}_{t+h|t}(M) - y^{(\tau)}_{t+h} \right)^2, \tag{18}$$

[14]See Appendix A for the definitions of F^*_t, Γ^* and A^*.

Table 5: Out-of-sample performance for yields

Macro-Yields

Maturity	3m	1y	2y	3y	4y	5y
h=1	1.17	1.05	1.06	1.00	1.05	1.14
h=3	0.79*	0.93	0.99	0.96	0.99	1.02
h=6	0.78**	0.89	0.94	0.93	0.93	0.94
h=12	0.69**	0.74**	0.79**	0.80***	0.80***	0.80***
h=24	0.62***	0.66***	0.74**	0.82**	0.88*	0.97

Only-Yields

Maturity	3m	1y	2y	3y	4y	5y
h=1	0.93	1.09	1.17	1.11	1.07	1.11
h=3	0.96	1.13	1.20	1.14	1.10	1.13
h=6	0.99	1.18	1.25	1.21	1.15	1.16
h=12	1.04	1.16	1.26	1.27	1.25	1.26
h=24	1.06	1.12	1.27	1.39	1.49	1.62

This table reports the relative MSFE of the macro-yields model and the only-yields model over the MSFE of the random walk for multi-step predictions of the yields. The first column reports the forecast horizon h. The sample starts on January 1970 and the evaluation period is January 1990 to December 2008. *, ** and *** denote significant outperformance at 10%, 5% and 1% level with respect to the random walk according to the White (2000) reality check test with 1,000 bootstrap replications using an average block size of 12 observations.

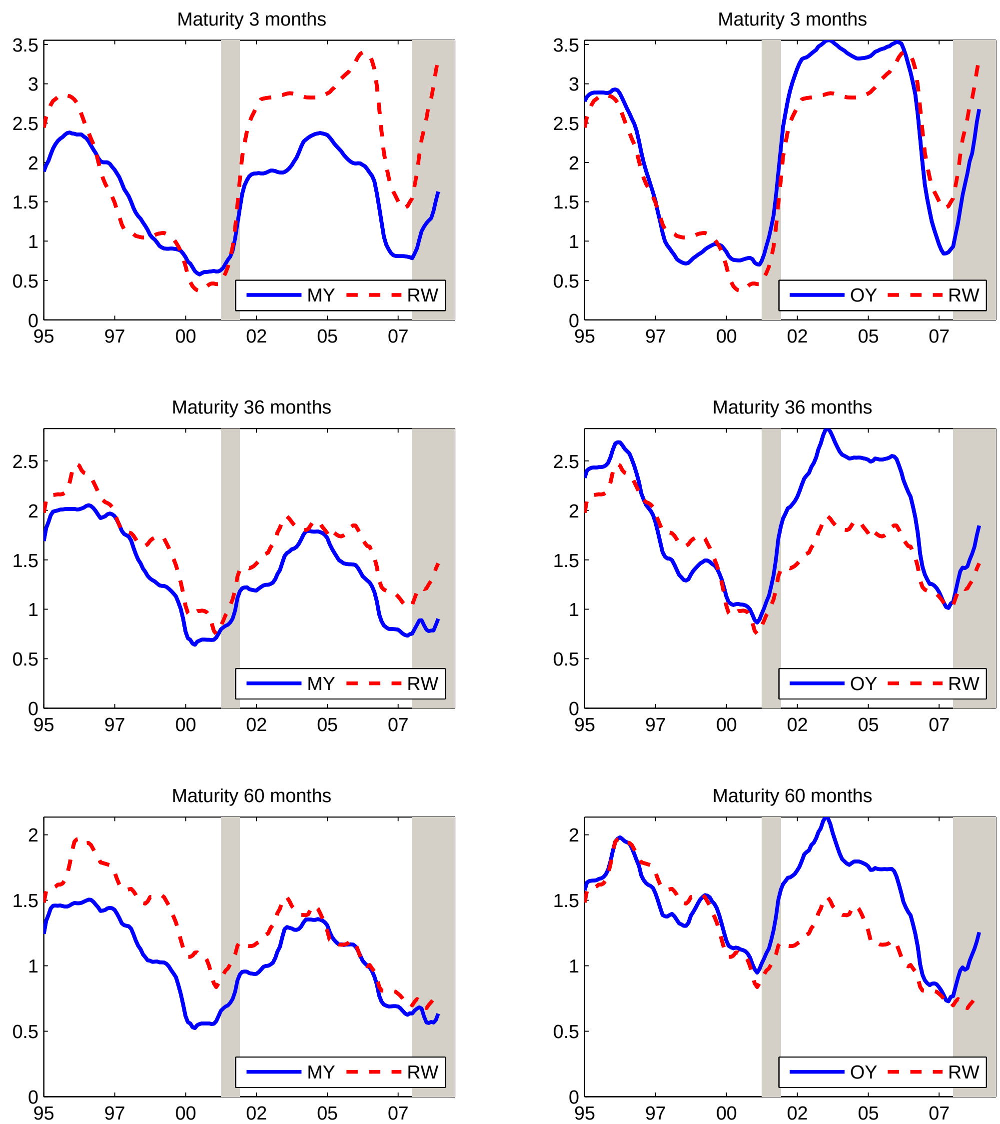
where t_0 and t_1 denote, respectively, the start and the end of the evaluation period, $y_{t+h}^{(\tau)}$ is the realized yield with maturity τ at time $t+h$ and $\hat{y}_{t+h|t}^{(\tau)}(M)$ is the h-step ahead forecast of the yield with maturity τ from model M using the information available up to t.

Forecast results for yields are usually expressed as relative performance with respect to the random walk, which is a naïve benchmark for yield curve forecasting very difficult to outperform, given the high persistency of the yields. The random walk h-steps ahead prediction at time t of the yield with maturity τ is

$$E_t(y_{t+h}^{(\tau)}) \equiv \hat{y}_{t+h|t}^{(\tau)} = y_t^{(\tau)},$$

where the optimal predictor does not change regardless of the forecast horizon. To measure the relative performance of the macro-yields model with respect to the random walk, we use the relative

Figure 4: 12-months ahead smoothed squared forecast errors for yields



This figure displays the 5-years rolling 12-months ahead squared forecast error for the yields with 3, 36 and 60 months to maturity. The blue continuous line refers to the 5-years rolling squared forecast error of the macro-yields MY model (left plots) and of the only-yields OY model (right plots). The dashed red line refers to 5-years rolling squared forecast error of the random walk. The dates on the horizontal axis refer to the end of the rolling window period. The grey-shaded areas indicate the recessions as defined by the NBER.

MSFE computed as

$$\text{rMSFE}_{t_0}^{t_1}(\tau, h, M) = \frac{\text{MSFE}_{t_0}^{t_1}(\tau, h, M)}{\text{MSFE}_{t_0}^{t_1}(\tau, h, RW)}.$$

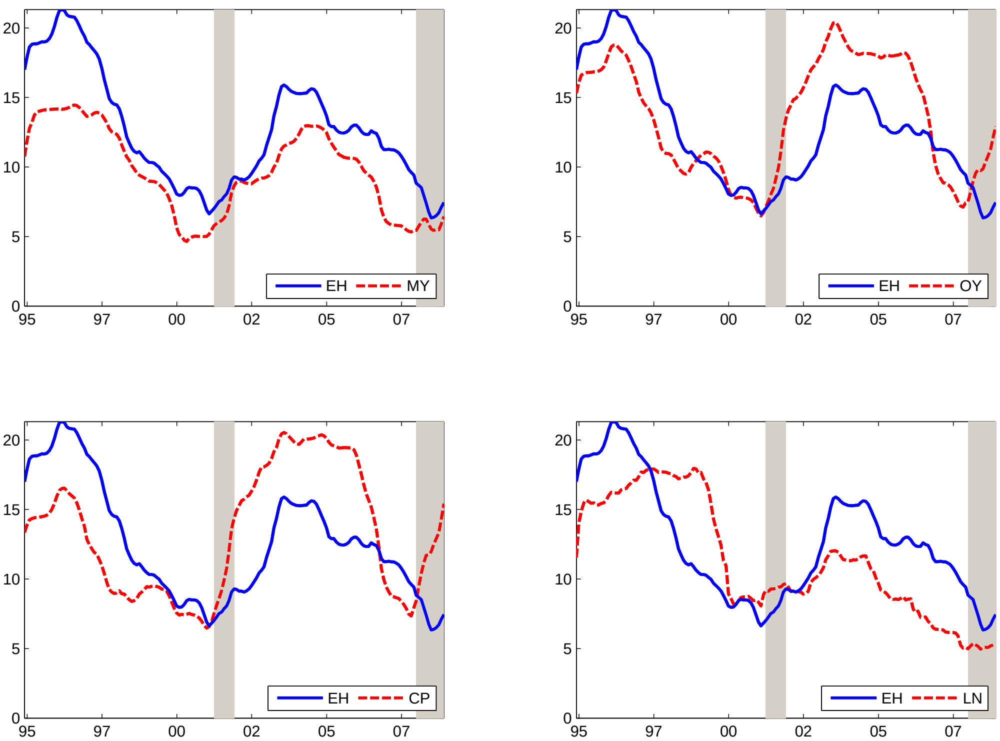
Table 5 reports the rMSFE with respect to the random walk for the macro-yields model the only-yields model. Results in Table 5 show that the macro-yields model outperforms the only-yields model for all but 1-month horizon. Moreover, the macro-yields model outperforms the random walk at 3-, 6-, 12- and 24-month ahead for all the maturities, with significant a out-performance, according to the White (2000) reality check test, for the 12- and 24-month ahead forecasts.[15] This evidence is corroborated by Figure 4, which reports the 12-month ahead smoothed squared forecast errors of the macro-yields, the only-yields and the random walk models for yields with 3-, 36- and 60-month to maturity. The figure highlights how the macro-yields model systematically outperforms the random walk especially in the last part of the evaluation sample for the short maturities, and in the first part of the sample for long maturities. The only-yield model, instead, performs as well as the random walk in the first part of the evaluation sample. However, its performance deteriorates in the last part of the evaluation sample, significantly underperforming the random walk. These results indicate that the unspanned macroeconomic factors, while not important for explaining the contemporaneous variation of the yields curve, contain useful information to predict the future values of the yield curve factors and, thus, the future evolution of the yield curve.

5.2 Excess bond returns

Out-of-sample forecast results for excess bond returns are reported in Table 6, which contains the relative MSFE of the macro-yields model with respect to the constant excess return benchmark, where one-year holding period excess returns are unforecastable at one year horizon, as in the expectation hypothesis. We use the expectation hypothesis since, because of its simplicity, represents a benchmark of unpredictability. The macro-yields model outperforms the constant excess return benchmark for all maturities and the outperformance is significant for all maturities according to the White (2000) reality check test.

[15] For more details about the reality check test see Appendix C.

Figure 5: Smoothed mean squared forecast errors for excess bond returns



This figure displays the 5-years rolling mean squared forecast error for one-year holding period excess bond returns from the expectation hypothesis EH (blue continuous line) and the corresponding values from different models (dashed red line). The dashed red line in the top plots refer to 5-years rolling mean squared forecast error of the macro-yields MY model (top right) and only-yields OY model (top left). The dashed red line in the bottom plots refer to the 5-years rolling mean squared forecast error from the predictive regressions using the CP factors (bottom left) and the LN factor (bottom right). The dates on the horizontal axis refer to the end of the rolling window period. The grey-shaded areas indicate the recessions as defined by the NBER.

Table 6: Out-of-sample predictive performance for excess returns

Maturity	MY	OY	CP	LN	LN+CP
2y	0.76**	1.20	1.17	0.80	0.80
3y	0.75**	1.20	1.21	0.79	0.83
4y	0.74**	1.18	1.21	0.78	0.83
5y	0.75**	1.18	1.18	0.81	0.83

This table reports the relative MSFE of the macro-yields model (MY), the only-yields model (OY), the Cochrane and Piazzesi (2005) factor (CP), the Ludvigson and Ng (2009) (LN) factor, the Cochrane and Piazzesi (2005) and the Ludvigson and Ng (2009) factors combined ($LN+CP$) with respect to the expectation hypothesis for excess returns. The sample starts on January 1970 and the evaluation period is January 1990 to December 2008. * and ** denote significant outperformance at 10% and 5% level with respect to the expectation hypothesis according the White (2000) reality check test with 1,000 bootstrap replications using an average block size of 12 observations.

Table 6 also reports the out-of-sample relative MSFEs of the excess bond returns forecasts using the CP factor, the LN factor, and the CP and LN factors combined obtained from the predictive regressions in equation (14). The worst performing models are the ones that do not use macroeconomic variable, i.e., the only-yield model and the CP factors. In line with the predictive regressions of excess bond returns and with the 12-month ahead out-of-sample forecast performance of the macro-yields model for the yields, results in Table 6 show that the macro-yields model is the best performing model for the prediction of the 1-year excess bond returns for all maturities followed by the combination of the CP and LN factors. However, although the unspanned model significantly outperforms the naïve benchmark while the CP+LN does not, we cannot reject the hypothesis that the forecasts of these two models are statistically equally accurate.

To further understand the performance of the macro-yields model to predict 1-year holding period excess bond returns, Figure 5 plots the 5-year rolling mean squared forecast error of the macro-yields model, the only-yields model, the CP and LN factors along with the 5-year rolling mean squared forecast error under the expectation hypothesis (EH). The figure shows that the performance of the only-yield model and the CP factors are similar: both models outperform the expectation hypothesis in the first part of the evaluation sample but display large forecast errors in the second part. Also the performance of the macro-yields model and the LN factors are similar, they both provide more accurate predictions than the expectation hypothesis, in particular in the last part of the evaluation period. The better accuracy of the macro-yields model relative to LN factors in Table 6 is coming mainly from the first half of the evaluation sample, up to the end of the 90's. In that period the macro-yields model significantly outperforms the EH, while the LN factors do not. Afterward both the macro-yields and the LN models outperform significantly the EH and are equally accurate.

However, the figure shows that the macro-yields model, apart from being the best performing model on average, as shown in Table 6, it is the best performing model for the whole evaluation period. This is a clear evidence that the unspanned macroeconomic factors identified by the proposed macro-yields model have predictive ability for the yield curve factors and, thus, for excess bond returns.

6 Conclusions

In this paper we analyze the predictive content of macroeconomic information for the yield curve of interest rates and excess bond returns in the United States. We find that two macroeconomic factors characterizing economic growth and real interest are unspanned by the cross-section of government bond yields and have significant predictive power for the bond yields and excess returns.

In future research, we plan to extend our empirical specification to allow for the zero lower bound of interest rates, non-synchronicity of macroeconomic data releases and mixed frequencies. The macro-yields model presented in this paper cannot be estimated on a sample that includes the great recession, as it does not honor the zero lower bound for the interest rates. However, our model model can be easily extended to deal with this issue by anchoring the shorter end of the yield curve using market expectation, along the lines of Altavilla, Giacomini and Ragusa (2014).

Data revisions and jagged edges due to the non-synchronicity of macroeconomic data releases are important characteristics to be taken into account when extracting macroeconomic information, see Giannone, Reichlin and Small (2008). In addition, bond yields are available at higher frequencies than macroeconomic variables. These features can be easily incorporated into our empirical model along the line described in Banbura et al. (2012).

A Estimation procedure

We can rewrite the macro-yields model in equations (4)–(6) in compact form as

$$
\begin{aligned}
z_t &= a + \Gamma F_t + v_t, & & (19)\\
F_t &= \mu + A F_{t-1} + u_t, \quad u_t \sim N(0, Q) & & (20)\\
v_t &= B v_{t-1} + \xi_t, \qquad \xi_t \sim N(0, R) & & (21)
\end{aligned}
$$

where $z_t = \begin{pmatrix} y_t \\ x_t \end{pmatrix}$, $F_t = \begin{pmatrix} F_t^y \\ F_t^x \end{pmatrix}$, $a = \begin{bmatrix} 0 \\ a_x \end{bmatrix}$, $\Gamma = \begin{bmatrix} \Gamma_{yy} & \Gamma_{yx} \\ \Gamma_{xy} & \Gamma_{xx} \end{bmatrix}$, $A = \begin{bmatrix} A_{yy} & A_{yx} \\ A_{xy} & A_{xx} \end{bmatrix}$, $Q = \begin{bmatrix} Q_{yy} & Q_{yx} \\ Q_{xy} & Q_{xx} \end{bmatrix}$, $\mu = \begin{bmatrix} \mu_y \\ \mu_x \end{bmatrix}$ and $\Gamma_{yy} = \Gamma_{NS}$ is the matrix whose rows correspond to the smooth patterns proposed by Nelson and Siegel (1987) and shown in equation (2). In addition $\Gamma_{yx} = 0$, as the macroeconomic factors F_t^x are unspanned by the cross-section of yields $\Gamma_{yx} = 0$. We also estimate the only-yields model using the same procedure, as it implies the following restrictions in (19)–(20): $z_t = y_t$, $F_t = F_t^y$, $a = 0$, $\Gamma = \Gamma_{NS}$, $\mu = \mu_y$.

The macro-yields model in (19)–(20) can be put in a state-space form augmenting the states F_t with the idiosyncratic components v_t and a constant c_t as follows

$$
\begin{aligned}
z_t &= \Gamma^* F_t^* + v_t^*, \quad v_t^* \sim N(0, R^*)\\
F_t^* &= A^* F_{t-1}^* + u_t^*, \quad u_t^* \sim N(0, Q^*)
\end{aligned}
$$

where $\Gamma^* = \begin{bmatrix} \Gamma & a & I_N \end{bmatrix}$, $F_t^* = \begin{bmatrix} F_t \\ c_t \\ v_t \end{bmatrix}$, $A^* = \left[\begin{array}{cc|c|c} A & \mu & \dots & 0 \\ \hline \vdots & \iddots & 1 & \vdots \\ \hline 0 & \dots & \dots & B \end{array}\right]$, $u_t^* = \begin{bmatrix} u_t \\ \nu_t \\ \xi_t \end{bmatrix}$, $Q^* = \left[\begin{array}{c|c|c} Q & \dots & 0 \\ \hline \vdots & \varepsilon & \vdots \\ \hline 0 & \dots & R \end{array}\right]$ and $R = \varepsilon I_n$, with ε a very small fixed coefficient. In this state-space form, c_t an additional state variable restricted to one at every time t.

The restrictions on the factor loadings Γ^* and on the transition matrix A^* can be written as

$$H_1 \ \mathrm{vec}(\Gamma^*) = q_1, \qquad H_2 \ \mathrm{vec}(A^*) = q_2,$$

where H_1 and H_2 are selection matrices, and q_1 and q_2 contain the restrictions.

We assume that $F_1^* \sim N(\pi_1, V_1)$, and define $y = [y_1, \dots, y_T]$ and $F^* = [F_1^*, \dots, F_T^*]$. Then denoting the parameters by $\theta = \{\Gamma^*, A^*, Q^*, \pi_1, V_1\}$, we can write the joint loglikelihood of z_t and F_t, for $t = 1, \dots, T$, as

$$\begin{aligned} L(z, F^*; \theta) \quad = \quad & -\sum_{t=1}^{T} \left(\frac{1}{2} \left[z_t - \Gamma^* F_t^* \right]' (R^*)^{-1} \left[z_t - \Gamma^* F_t^* \right] \right) + \\ & -\frac{T}{2} \log |R^*| - \sum_{t=2}^{T} \left(\frac{1}{2} [F_t^* - A^* F_{t-1}^*]' (Q^*)^{-1} [F_t^* - A^* F_{t-1}^*] \right) + \\ & -\frac{T-1}{2} \log |Q^*| + \frac{1}{2} [F_1^* - \pi_1]' V_1^{-1} [F_1^* - \pi_1] + \\ & -\frac{1}{2} \log |V_1| - \frac{T(p+k)}{2} \log 2\pi + \lambda_1' \left(H_1 \, \mathrm{vec}(\Gamma^*) - q_1 \right) + \lambda_2' \left(H_2 \, \mathrm{vec}(A^*) - q_2 \right) \end{aligned}$$

where λ_1 contains the lagrangian multipliers associate with the constraints on the factor loadings Γ^* and λ_2 contains the lagrangian multipliers associated with the constraints on the transition matrix A^*.

The computation of the Maximum Likelihood estimates is performed using the EM algorithm. Broadly speaking, the algorithm consists in a sequence of simple steps, each of which uses the Kalman smoother to extract the common factors for a given set of parameters and closed form solutions to estimate the parameters given the factors. In practice, we use the restricted version of the EM algorithm, the Expectation Restricted Maximization, since we need to impose the smooth pattern on the factor loadings of the yields on the NS factors. The ERM algorithm alternates Kalman filter extraction of the factors to the restricted maximization of the likelihood. At the j-th iteration the ERM algorithm performs two steps:

1. In the Expectation-step, we compute the expected log-likelihood conditional on the data and

the estimates from the previous iteration, i.e.

$$\mathcal{L}(\theta) = E[L(z, F^*; \theta^{(j-1)})|z]$$

which depends on three expectations

$$\begin{aligned} \hat{F}_t^* &\equiv E[F_t^*; \theta^{(j-1)}|z] \\ P_t &\equiv E[F_t^*(F_t^*)'; \theta^{(j-1)}|z] \\ P_{t,t-1} &\equiv E[F_t^*(F_{t-1}^*)'; \theta^{(j-1)}|z] \end{aligned}$$

These expectations can be computed, for given parameters of the model, using the Kalman filter.

2. In the Restricted Maximization-step, we update the parameters maximizing the expected log-likelihood with respect to θ:

$$\theta^{(j)} = \arg\max_{\theta} \mathcal{L}(\theta)$$

This can be implemented taking the corresponding partial derivative of the expected log likelihood, setting to zero, and solving.

The procedure outlined above can be extended to estimate also the decay parameter λ controlling for the shape of the loadings of the yields on the slope and curvature factors. Since the factor loadings are a non-linear function λ, an additional step consisting in the numerical maximization of the conditional likelihood with respect to λ is required. The procedure is know as Expectation Conditional Restricted Maximization (ECRM) algorithm.

B Data

Table 7: Macroeconomic variables

Series N.	Mnemonic	Description	Transformation
1	AHE	Average Hourly Earnings: Total Private	1
2	CPI	Consumer Price Index: All Items	1
3	INC	Real Disposable Personal Income	1
4	FFR	Effective Federal Funds Rate	0
5	HSal	House Sales - New One Family Houses	1
6	IP	Industrial Production Index	1
7	M1	M1 Money Stock	1
8	Manf	ISM Manufacturing: PMI Composite Index (NAPM)	0
9	Paym	All Employees: Total nonfarm	1
10	PCE	Personal Consumption Expenditures	1
11	PPIc	Producer Price Index: Crude Materials	1
12	PPIf	Producer Price Index: Finished Goods	1
13	CU	Capacity Utilization: Total Industry	0
14	Unem	Civilian Unemployment Rate	0

This table lists the 14 macro variables used to estimate the macro-yields. Most series have been transformed prior to the estimation, as reported in the last column of the table. The transformation codes are: 0 = no transformation and 1 = annual growth rate.

C Reality check test

To compare the out-of-sample predictive ability of a model with respect to the benchmark, we use the reality check test of White (2000), as we compare only non-nested models.

If we denote by $e_t(b)$ the forecast errors of the benchmark and by $e_t(M)$ the forecast errors of the model under consideration. Then we can define the null hypothesis of no predictive superiority over the benchmark as

$$H_0 : f = E(f_t) \equiv E(e_t(b)^2 - e_t(M)^2) \leq 0 \tag{22}$$

The test is then based on the statistic

$$\overline{f} = \frac{1}{t_1 - t_0} \sum_{t=t_0}^{t_1} \hat{f}_t \tag{23}$$

where t_0 and t_1 denote, respectively, the start and the end of the evaluation period, and hats denote estimated statistics.

To approximate the asymptotic distribution of the test statistic, we use block-bootstrap as follows:

1. We generate bootstrapped forecast errors $\hat{e}_t^*(b)$ and $\hat{e}_t^*(M)$ using the stationary block-bootstrap of Politis and Romano (1994) with average block size of 12. This procedure is analogous to the moving blocks bootstrap, but, instead of using blocks of fixed length uses blocks of random length, distributed according to the geometric distribution with mean block length 12. Also to give the same probability of resampling to all observations, we use a circular scheme.

2. Construct the bootstrapped test statistic as

$$\overline{f}^* = \frac{1}{t_1 - t_0} \sum_{t=t_0}^{t_1} (\hat{e}_t^*(b)^2 - \hat{e}_t^*(M)^2)$$

3. Repeat steps 1 and 2 for 1,000 times to obtain an estimate of the distribution of the test statistic $\overline{f}^* = [\overline{f}^*_{(1)}, \ldots, \overline{f}^*_{(1,000)}]$.

4. Compare $V = (t_1 - t_0)^{1/2}\overline{f}$ with the quantiles of $V^* = (t_1 - t_0)^{1/2}(\overline{f}^* - \overline{f})$ to obtain the p-value.

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