Mothballing in a Duopoly

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September 10, 2018

Contents

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Abstract

The combination of horizontal drilling and hydraulic fracturing to produce oil and natural gas has grown dramatically over the last few years taking the market by surprise with the name of "Shale Revolution". The first effects of shale revolution have been an increase of oil and gas supply, especially for US and a dop in crude oil prices in the 2014-2016. One of the remaining open question is why OPEC did not react reducing production to maintain high prices. In literature the answer falls into three main categories: (1) OPEC tried to defend its market share by flooding the market in an attempt to drive out shale producers; (2) the shale oil revolution changed the market weights leaving the only choice to accept low prices; (3) OPEC was uncertain about the potential of shale oil and needed to test its resilience under low prices. In order to better study and understand the market strategies, we have built a real option model between leader and a follower producers.

1 Introduction

The global oil market is experiencing many changes especially due to new technologies used to extract crude oil and natural gas. The combination of horizontal drilling and hydraulic fracturing to produce oil and natural gas was developed in the 1970s (Mănescu and Nuňo, 2015) but has grown dramatically over the last few years taking the market by surprise with the name of "Shale Revolution"[1]. In 2013 the Unites States production is estimated to have produced 3.5 mb/d of shale oil which is three times higher than the amount it produced in 2010 (Energy Information Administration-EIA, 2014). The level of US crude oil production reached almost that of Saudi Arabia and Russia in 2015 (Bataa and Park, 2017). By 2020, US shale oil is estimated to reach 4.8 mb/d, representing about a third of total US supply (Mănescu and Nuňo, 2015). Therefore, the first effect of shale revolution has been (and probably will be) an increase of oil and gas supply, especially for US. The second (indirect) effect is on the 2014-2016 drop in crude oil prices[2]. Indeed, as pointed out by Ansari (2017), "although results have given evidence for a variety of drivers, including demand and geopolitical circumstances, the shale oil revolution is widely considered to be the main driver of price developments". This have also been profund implication for global stategical competition in the oil market. Let us remind that the fluctuations of international crude oil prices could have huge impacts (Chen et. al, 2016) on the economic output (Wei et al., 2008;Wang and Zhang, 2014), inflation and unemployment (Uri, 1996; Du et al., 2010), stock market (Cong et al., 2008) and fundamental industries (Jiao et al., 2012). Therefore it is strongly important also for policy makers to study price fluctuation and the comodities market strategies. Given the increase of US oil supply and so of the competition in the oil market, the expected reaction of OPEC was a reduction of supply to maintain price and profit shares. Surprisingly, OPEC maintained stable production.

Therefore, many analysts predicted that a new normal era for the global oil market had begun, and that the oil price would remain somewhere between U.S.$35 and U.S.$50 per barrel in the future. [See Hartmann and Sam (2016)

and Barnato (2016) 'Oil's new normal may be lower than you think,' CNBC May 31, 2016.]

One of the remaining open question is why OPEC did not react reducing

[1]Formally, shale oil refers to a subset of unconventional oil (know as " tight oil") in which conventional oil (light oil with low sulphur content) is trapped in very low-permeability tight formations (known as shales) which makes extraction difficult. However, as stressed by Mănescu and Nuňo, (2015) "the convention in the press is to use the terms *shale oil* and *tight oil* interchangeably when referring to oil extracted from all low-permeability formations (i.e. not only oil from shale formations)" . Therefore, we follow this convention throughout the paper and refer to the entire tight oil category as "shale oil".

[2]The Western Texas Intermediate (WTI) crude oil price reached U.S.$26.21 per barrel in February 2016, which is a record low since July 2002, while the U.S. real import price fell more than 73% June 2014–February 2016, making it the most rapid decline within this time frame since 1973.

production to maintain high prices. There is no consensus regarding the reason behind OPEC's initial decision. Researchers' results in these regards fall into three main categories: (1) OPEC tried to defend its market share by flooding the market in an attempt to drive out shale producers (Behar and Ritz, 2017; Brown and Huntington, 2017; Coy, 2015; Gause, 2015; Mănescu and Nuño, 2015); (2) the shale oil revolution nullified OPEC's market power (shale oil has taken OPEC's role as the swing producer), leaving its members no choice but to accept low prices (Baffes et al., 2015; Baumeister and Kilian, 2016; Dale, 2016; Kaletsky, 2015; The Economist, 2015); and (3) OPEC was uncertain about the potential of shale oil and needed to test its resilience under low prices (Fattouh et al., 2016; Huppmann and Livingston, 2015).

The first category is in line with a standard market entry game: in a dynamic environment, if a new player wants to enter in the market, it may be rational for the incumbent firm to enforce a downward-pressure on prices in order to drive out the contestant, despite short-run losses for the incumbent. In this line, Behar and Ritz (2017) show that increasing the conventional oil supply is the dominant strategy when shale costs are high. The algebraic analysis of their approach reveals discontinuous best responses. This could mean that a switch to the flooding strategy occurs when parameter thresholds are crossed, which is why market prices may jump as a response to even small parameter shocks. Nevertheless, in order that the incumbent's threat could work, it requires (i) effectiveness, (ii) credibility, and (iii) temporal sustainability.

The second category claims that these conditions could not hold and that OPEC had to accept the presence and dominance of US shale oil. The quick expansion of shale resulted in a a chenge of the weights in the market: the quick responsiveness of shale oil creates competition in which shel oil can substitute conventional OPEC oil. The effect is that the only remaining choice for OPEC is to follow the rules of competition. Baffes et al. (2015) note that OPEC's decision to freeze output, "implies that OPEC will no longer act as the swing oil producer [and that] ... marginal cost of unconventional oil producers may play this role."

Lastly, a possible third explanation is that OPEC was driven by uncertainty and a desire for industry consolidation. Indeed, a pragmatic OPEC most likely attempted to gain crucial information regarding shale's performance in lower price ranges (Huppmann and Livingston; 2015). In a more formal way, Fattouh et al. (2016), by using a parametrised static game under uncertainty show that, without sufficient knowledge about US shale elasticity, it is rational for a Saudi Arabia not to cut output. Hence, there is a strong incentive for Saudi Arabia to learn which game it is playing.

Also recently the relationship among shale revolution, oil prices and strategies in the energy markets has been an interesting topic[3] that has pushed research and discussion. In order to understand better the strategic behaviour of

[3]In the Economist Espresso from November 30, 2017, we read about an OPEC meeting in Vienna. Among others, it states that
"Soaring prices would further stimulate American shale production".

conventional and unconventional oil producers in a context of uncertainty, we develop a real option framework between leader and a follower producers.

2 The Model

Firm 1 is the market leader and the more efficient producer (OPEC in the example). Upon investing I_1, Firm 1 is flexible in the sense that it is able to produce any production amount $q_1(t)$ it wants. Firm 1's production costs are zero.

Remark 1 *Alternatively, Firm 1 invests in capacity K_1. The investment cost is δK_1 and q_1 is limited from above by K_1, i.e. $q_1(t) \leq K_1$.*

Firm 2 is less efficient (Shale-Gas producer in the example). Whenever it is in the production stage, it incurs a positive cost C. To be able to produce, Firm 2 must invest I_2. Firm 2 is a dedicated (non-flexible) producer in the sense that it either can produce a fixed $\bar{q}_2$ or zero. The firm is able to mothball. Then it does not produce and maintenance costs are $M < C$. To go from the production to the mothballing stage, Firm 2 incurs a sunk cost E_M, and to restart production, i.e. going from the mothballing to the production stage, requires spending a sunk cost being equal to R.

Remark 2 *Firm 2's situation is simpler than in the mothballing models by Guerra et al. (2017) and Dixit and Pindyck (1994, Chapter 7) in that Firm 2 cannot exit. This step seems to be less relevant in analyzing the "OPEC problem".*

The inverse demand function is given by

$$p = X(\alpha - \eta Q),$$

in which

$$dX = \mu X dt + \sigma X dz,$$

in which dz is the increment of a Wiener process. Furthermore, Q is market output, i.e. either

$$Q = 0,$$

if none of the firms are active producer, or

$$Q = q_1(t),$$

In the Economist Espresso from December 1, 2017, it is stated that "OPEC agreed to extend its oil-production cut of 1.8m barrels per day by nine months, to the end of 2018. The cartel is walking a fine line in trying to draw down global surpluses and nudge prices up without sparking new production by nimble American shale producers" if only Firm 1 produces actively, or

$$Q = \bar{q}_2,$$

if only Firm 2 produces actively, or

$$Q = q_1(t) + \bar{q}_2,$$

if both firms produce actively.

In setting the output, Firm 1, the market leader, is the Stackelberg leader and announces its output first. Then Firm 2 reacts. It either chooses to be active. i.e. $q_2 = \bar{q}_2$, or q_1 is so large, and thus price p is so low, that Firm 2 refrains from production so that $q_2 = 0$ (this is what happened in Vienna on November 30).

3 Model derivations

As a first step we **assume that suspension and resumption of operation is costless for the follower**. We assume in what follows that firm 1 is the leader and firm 2 is the follower.

3.1 Model derivations - Output Game

For detailed derivations see Section 5.1.

The leader can choose between two strategies referred to as *accommodate* and *squeeze* strategy, respectively. When applying the *squeeze* strategy, the leader chooses its output quantity in order to force the follower into mothballing. If *squeezeing* the follower out of the market is too expensive for the leader, i.e. leads to suboptimal profit, it will accommodate the follower in the market. For changing levels of X over time the leader will choose the optimal strategy.

First we look at the follower: For a given level of q_1 the follower will produce as long as $\pi_F(X, q_1) > -Mq_1$ and suspend otherwise. Therefore,

$$\pi_F(X, q_1) = \begin{cases} X(\alpha - \eta(q_1 + \bar{q}_2))\bar{q}_2 - C\bar{q}_2 & \text{if } q_1 < \frac{1}{\eta}\left(\alpha - \eta\bar{q}_2 - \frac{C-M}{X}\right) \text{ or } X > \frac{C-M}{\alpha - \eta(q_1 + \bar{q}_2)} \\ -M\bar{q}_2 & \text{otherwise} \end{cases} \quad (1)$$

So for a given q_1 the follower will suspend if X falls below $\frac{C-M}{\alpha - \eta Q}$. For a given X the follower will suspend if the quantity of the leader q_1 is great than $\frac{1}{\eta}\left(\alpha - \frac{C-M}{X}\right)$. So the leader can squeeze the follower out of the market by setting its output quantity to a level $q_1 \geq \frac{1}{\eta}\left(\alpha - \frac{C-M}{X}\right)$. $\underline{q}_{1,s} = \frac{1}{\eta}\left(\alpha - \frac{C-M}{X}\right)$ is the lowest level for a given X that squeezes the follower out of the market.

Now we look at the leader: Note that $q_1 < \bar{q}_1 = \frac{\alpha}{\eta}$ in order for the leader to get a positive monopoly profit.

$$\pi_L(X) = \begin{cases} X(\alpha - \eta q_1)q_1 & \text{if the follower is in suspension} \\ X(\alpha - \eta(q_1 + \bar{q}_2))q_1 & \text{if the follower is producing} \end{cases} \quad (2)$$

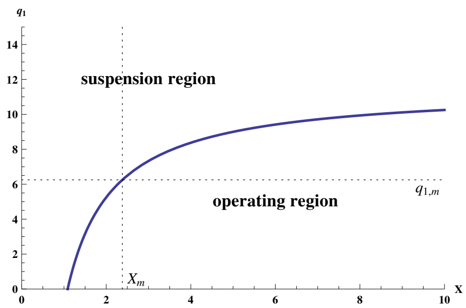

Figure 1: Illustration of the production and suspension regions of the follower as functions of q_1 and X.

Given that the follower is in suspension independent of the leader's output quantity the optimal output quantity of the leader is equal to $q_{1,m} = \frac{\alpha}{2\eta}$. The leader can keep the follower in suspension producing its optimal "monopoly-quantity" $q_{1,m}$ as long as $q_{1,m} \geq \underline{q}_{1,s}$ or $X \leq \frac{2(C-M)}{\alpha}$. Let's define this boundary by $\bar{X}_{s,m} = \frac{2(C-M)}{\alpha}$. The optimal output quantity in order to apply the squeeze strategy is equal to $q_{1,s} = max[\underline{q}_{1,s}, q_{1,m}]$. The leader accommodates the follower by producing the quantity $q_{1,a} = \frac{\alpha - \eta \bar{q}_2}{2\eta}$. The output quantities and corresponding profit function for the leader playing the different strategies is equal to

$$q_1^* = \begin{cases} q_{1,m} = \frac{\alpha}{2\eta} & \text{if leader can keep the follower out of the market "for free"} \\ \underline{q}_{1,s} = \frac{1}{\eta}\left(\alpha - \frac{C-M}{X}\right) & \text{if the leader applies the squeeze strategy by overproducing} \\ q_{1,a} = \frac{\alpha - \eta\bar{q}_2}{2\eta} & \text{if the leader applies the accommodate strategy} \end{cases} \tag{3}$$

$$\pi_L(X) = \begin{cases} X\frac{\alpha^2}{4\eta} & \text{if leader can squeeze the follower out of the market by producing } q_{1,m} \\ \frac{C-M}{\eta}\left(\alpha - \frac{C-M}{X}\right) & \text{if the leader applies the squeeze strategy producing } \underline{q}_{1,s} \\ X\frac{\left[\alpha^2 - \eta^2\bar{q}_2^2\right]}{4\eta} & \text{if the leader applies the accommodate strategy producing } q_{1,a} \end{cases} \tag{4}$$

We denote the profit of the leader using the accommodation strategy by $\pi_{L,a}(X) = X\frac{\left[\alpha^2 - \eta^2\bar{q}_2^2\right]}{4\eta}$. The profit of the leader using the squeeze strategy is equal to

$$\pi_{L,s}(X) = \begin{cases} X\frac{\alpha^2}{4\eta} & X \leq \bar{X}_{s,m} \\ \frac{C-M}{\eta}\left(\alpha - \frac{C-M}{X}\right) & X > \bar{X}_{s,m} \end{cases} \tag{5}$$

It is optimal for the leader to play the squeeze strategy if X so that $\pi_{L,s}(X) > \pi_{L,a}(X)$. The derivations in show that it is optimal for the leader to squeeze

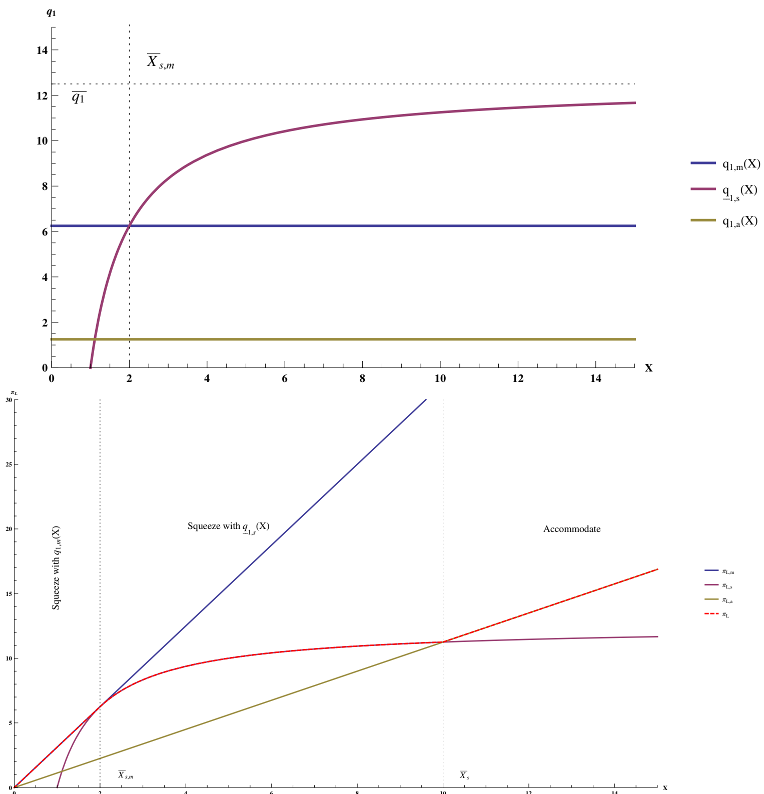

Figure 2: The different production quantities (upper plot) and profit functions (lower plot) of the leader as a function of X.

the follower out of the market for X so that $\underline{X}_s < X < \bar{X}_s$. If $X \leq \bar{X}_{s,m}$ the leader can keep the follower out of the market by producing $q_{1,m}$. If $X > \bar{X}_{s,m}$ the leader is overproducing with an amount equal to $\underline{q}_{1,s}$ in order to keep the follower out of the market.

Therefore the optimal production quantity and profit of the leader as a function of X is equal to

$$q_1^* = \begin{cases} \frac{\alpha}{2\eta} & X \leq \bar{X}_{s,m} \\ \frac{1}{\eta}\left(\alpha - \frac{C-M}{X}\right) & \bar{X}_{s,m} < X \leq \bar{X}_s \\ \frac{\alpha - \eta\bar{q}_2}{2\eta} & X > \bar{X}_s \end{cases} \tag{6}$$

$$\pi_L(X) = \begin{cases} X\frac{\alpha^2}{4\eta} & X \leq \bar{X}_{s,m} \\ \frac{C-M}{\eta}\left(\alpha - \frac{C-M}{X}\right) & \bar{X}_{s,m} < X \leq \bar{X}_s \\ X\frac{\left[\alpha^2 - \eta^2\bar{q}_2^2\right]}{4\eta} & X > \bar{X}_s \end{cases} \tag{7}$$

This results in the following profit flow of the follower

$$\pi_F(X) = \begin{cases} \frac{X}{2}(\alpha - \eta\bar{q}_2)\bar{q}_2 - C\bar{q}_2 & X > \bar{X}_s \\ -M\bar{q}_2 & X \leq \bar{X}_s \end{cases} \tag{8}$$

Proposition 3

- *The upper boundary of the squeeze region $\bar{X}_s$ is increasing in C, η and $\bar{q}_2$, and decreasing in α and M.*
- *The squeeze region where the leader produces $\underline{q}_{1,s}$ is always positive. Because $\bar{X}_s - \bar{X}_{s,m} = \frac{2C\eta\bar{q}_2}{\alpha(\alpha-\eta\bar{q}_2}$.*
- *If $\bar{q}_2 < \frac{2\alpha}{\eta}$ then $\bar{X}_s - \bar{X}_{s,m}$ is decreasing in α, i.e.* $\frac{\partial\left(\bar{X}_s - \bar{X}_{s,m}\right)}{\partial\alpha} < 0$.
- *$\bar{X}_s - \bar{X}_{s,m}$ is increasing in $\bar{q}_2$, i.e.* $\frac{\partial\left(\bar{X}_s - \bar{X}_{s,m}\right)}{\partial\bar{q}_2} > 0$.
- *$\bar{X}_s - \bar{X}_{s,m}$ is increasing in η, i.e.* $\frac{\partial\left(\bar{X}_s - \bar{X}_{s,m}\right)}{\partial\eta} > 0$.

3.2 Model derivations - Investment Decisions

In order to derive the optimal investment decisions of the leader and the follower, we solve the problem backwards, starting with the investment decision of the follower.

3.2.1 Follower's Optimal Investment Decision

Given that the leader has already invested, the follower cannot influence the investment decision of its competitor anymore. This means that the follower decision involves no strategic aspects. The follower has to determine the investment timing, which is similar to fixing a threshold level X_F.

The leader has already invested and is producing

$$q_1^* = \frac{\alpha}{2\eta} \quad \text{for } X \leq \bar{X}_{s,m} \tag{9}$$

as long as it is the only firm in the market, and

$$q_1^* = \begin{cases} \frac{\alpha}{2\eta} & X \leq \bar{X}_{s,m} \\ \frac{1}{\eta}\left(\alpha - \frac{C-M}{X}\right) & \bar{X}_{s,m} < X \leq \bar{X}_s \\ \frac{\alpha - \eta \bar{q}_2}{2\eta} & X > \bar{X}_s \end{cases} \tag{10}$$

after the follower enters the market.

Therefore, the profit flow of the follower is equal to

$$\pi_F(X) = \begin{cases} \frac{X}{2}(\alpha - \eta \bar{q}_2)\bar{q}_2 - C\bar{q}_2 & X > \bar{X}_s \\ -M\bar{q}_2 & X \leq \bar{X}_s \end{cases} \tag{11}$$

We denote the idle, operating and mothballing states by the labels i, o and m, respectively. We find the value of the firm in each state as the appropriate combinations of the expected profit or cost streams and the options to switch. After investment the firm must decide whether and when to mothball operation. It will mothball the operating project if the price falls to a threshold X_M. Given the project is in mothballing state, the firm will reactivate it if the price rises to a threshold X_R.

We find the value of the firm in each state as the appropriate combinations of the expected profits or cost streams and the options to switch. The firm is in an idle state over the interval $(0, X_F)$. The value in this state is given by the following equation:

$$V_i(X) = \mathcal{A}X^{\beta_1}, \tag{12}$$

where $\mathcal{A}$ is a constant to be determined. This is the value of the option to invest. The operating state prevails over the interval $(\bar{X}_s, \infty)$, where the value of the firm is given by the following equation:

$$V_o(X) = \mathcal{B}X^{\beta_2} + \frac{X(\alpha - \eta\bar{q}_2)\bar{q}_2}{2(r-\mu)} - \frac{C\bar{q}_2}{r}, \tag{13}$$

where constant $\mathcal{B}$ remains to be determined. The mothballed state can continue of the range of $(0, \bar{X}_s)$. The value of the mothballed project is given by

$$V_m(X) = \mathcal{D}X^{\beta_1} - \frac{M\bar{q}_2}{r}, \tag{14}$$

where constant $\mathcal{D}$ remains to be determined. We can derive the expressions for $\mathcal{B}$ and $\mathcal{D}$ by value matching and smooth-pasting at the threshold $\bar{X}_s$, where the follower is indifferent between mothballing and operating. Solving

$$V_o(\bar{X}_s) = V_m(\bar{X}_s) \tag{15}$$

$$V_o'(\bar{X}_s) = V_m'(\bar{X}_s). \tag{16}$$

leads to

$$\mathcal{B} = \bar{X}_s^{-\beta_2}\left(\frac{\beta_1}{\beta_1-\beta_2}\right)\left[\frac{X(\alpha-\eta\bar{q}_2)\bar{q}_2}{2(r-\mu)}\left(\frac{1-\beta_1}{\beta_1}\right)+\frac{C\bar{q}_2}{r}\right] \tag{17}$$

$$\mathcal{D} = \bar{X}_s^{-\beta_1}\left(\frac{\beta_2}{\beta_1-\beta_2}\right)\left[\frac{X(\alpha-\eta\bar{q}_2)\bar{q}_2}{2(r-\mu)}\left(\frac{1-\beta_2}{\beta_2}\right)+\frac{C\bar{q}_2}{r}\right] \tag{18}$$

At the investment threshold X_F, X_M and X_R, the following value matching and smooth-pasting conditions hold:

$$V_i(X_F) = V_o(X_F) - I_2, \tag{19}$$

$$V_i'(X_F) = V_o'(X_F). \tag{20}$$

Then the optimal investment threshold X_F of the follower is implicitly given by the solution of the following equation:

$$\mathcal{B}X_F^{\beta_2}\left(\frac{\beta_1-\beta_2}{\beta_1}\right)+\frac{X_F(\alpha-\eta\bar{q}_2)\bar{q}_2}{2(r-\mu)}\left(\frac{\beta_1-1}{\beta_1}\right)-\frac{C\bar{q}_2}{r}-I_2=0, \tag{21}$$

and the constant $\mathcal{A}$ is equal to

$$\mathcal{A} = X_F^{-\beta_1}\left[\mathcal{B}X_F^{\beta_2}+\frac{X_F(\alpha-\eta\bar{q}_2)\bar{q}_2}{2(r-\mu)}-\frac{C\bar{q}_2}{r}-I_2\right] \tag{22}$$

3.2.2 Leader's Optimal Investment Decision

In the next step we determine the investment decision of the leader, where the leader takes the strategy of the follower into account.

To derive the leader's value function, we first consider the leader's profit function for a given GBM level X when both firms are active in the market which is given by

$$\pi_L(X) = \begin{cases} X\frac{\alpha^2}{4\eta} & X \leq \bar{X}_{s,m} \\ \frac{C-M}{\eta}\left(\alpha - \frac{C-M}{X}\right) & \bar{X}_{s,m} < X \leq \bar{X}_s \\ X\frac{\left[\alpha^2-\eta^2\bar{q}_2^2\right]}{4\eta} & X > \bar{X}_s \end{cases} \tag{23}$$

The leader's value $V_L(X)$ and profit $\pi_L(X)$ given that both firms have invested has to satisfy the following differential equation

$$\frac{1}{2}\sigma^2X^2\frac{\partial^2 V_L}{\partial X^2}+\mu X\frac{\partial V_L}{\partial X}-rV_L+\pi_L=0 \tag{24}$$

Substituting $\pi_L(X)$ into this equation and applying value matching and smooth pasting at $X=\bar{X}_{s,m}$ and $X=\bar{X}_s$ leads to the following value function for the leader

$$V_L(X) = \begin{cases} \mathcal{L}X^{\beta_1} + \frac{X}{r-\mu}\frac{\alpha^2}{4\eta} & X \leq \bar{X}_{s,m} \\ \mathcal{M}_1 X^{\beta_1} + \mathcal{M}_2 X^{\beta_2} + \frac{(C-M)\alpha}{r\eta} - \frac{(C-M)^2}{\eta X(r+\mu-\sigma^2)} & \bar{X}_{s,m} < X \leq \bar{X}_s \\ \mathcal{N}X^{\beta_2} + \frac{X}{r-\mu}\frac{\left[\alpha^2-\eta^2\bar{q}_2^2\right]}{4\eta} & X > \bar{X}_s \end{cases}$$

where

$$\begin{aligned} \mathcal{L} &= \bar{X}_{s,m}^{-\beta_1}\left(\frac{1}{\beta_1-\beta_2}\right)\left[\frac{\bar{X}_{s,m}}{r-\mu}\frac{\alpha^2}{4\eta}(\beta_2-1) - \mathcal{M}_1\bar{X}_{s,m}^{\beta_1}(\beta_2-\beta_1) - \frac{(C-M)\alpha}{r\eta}\beta_2 + \frac{(C-M)^2}{\eta\bar{X}_{s,m}(r+\mu-\sigma^2)}(\beta_2 + \right. \\ \mathcal{M}_1 &= \bar{X}_s^{-\beta_1}\left(\frac{-1}{\beta_1-\beta_2}\right)\left[\frac{\bar{X}_s}{r-\mu}\frac{\left[\alpha^2-\eta^2\bar{q}_2^2\right]}{4\eta}(\beta_2-1) - \frac{(C-M)\alpha}{r\eta}\beta_2 + \frac{(C-M)^2}{\eta\bar{X}_s(r+\mu-\sigma^2)}(\beta_2+1)\right] \\ \mathcal{M}_2 &= \bar{X}_{s,m}^{-\beta_2}\left(\frac{1}{\beta_1-\beta_2}\right)\left[\frac{\bar{X}_{s,m}}{r-\mu}\frac{\alpha^2}{4\eta}(\beta_1-1) - \frac{(C-M)\alpha}{r\eta}\beta_1 + \frac{(C-M)^2}{\eta\bar{X}_{s,m}(r+\mu-\sigma^2)}(\beta_1+1)\right] \\ \mathcal{N} &= \bar{X}_s^{-\beta_2}\left(\frac{1}{\beta_1-\beta_2}\right)\left[\mathcal{M}_2\bar{X}_s^{-\beta_2}(\beta_1-\beta_2) - \frac{\bar{X}_s}{r-\mu}\frac{\left[\alpha^2-\eta^2\bar{q}_2^2\right]}{4\eta}(\beta_1-1) + \frac{(C-M)\alpha}{r\eta}\beta_1 - \frac{(C-M)^2}{\eta\bar{X}_s(r+\mu-\sigma\right. \end{aligned}$$

As $\beta_1 > 0$ and $\beta_2 < 0$ it is straightforward to conclude that $\mathcal{L} < 0$ and $\mathcal{N} > 0$. The value function in equation (125) is split into three regions. In the demand region $X \leq X_M$ demand is so low that the follower is mothballing its operation. This leaves the leader as the only active producer in the market earning a high profit. The second term of the value function represents the expected total discounted revenue the leader obtains when it is in the market as the only producer forever. The first term of the value function represents a negative term that corrects for the fact that demand might eventually increase so that it is optimal for the follower to resume production again, i.e. $X > X_M$, which would decrease the leader's profit. For demand intercept regions $X > X_M$ the demand is so high that it is optimal for the follower to be active and produce. The second term in the leader's value function in this region stands for the expected total discounted revenue the leader obtains when both firms are active forever. The first term represents the option value that the demand might fall below the mothballing threshold of the follower which would leave the leader in a monopoly situation and it would earn a higher profit.

Before the follower's market entry, the leader's value function is equal to

$$V_L(X) = \mathcal{O}X^{\beta_1} + \frac{X}{r-\mu}\frac{\alpha^2}{4\eta} \tag{29}$$

The leader's value function before and after the follower's entry is equal to

$$V_L(X) = \begin{cases} \mathcal{O}X^{\beta_1} + \frac{X}{r-\mu}\frac{\alpha^2}{4\eta} & \text{before} \\ \mathcal{N}X^{\beta_2} + \frac{X}{r-\mu}\frac{1}{4\eta}\left[\alpha^2 - \eta^2\bar{q}_2^2\right] & \text{after} \end{cases} \tag{30}$$

with

$$\mathcal{O} = X_F^{-\beta_1}\left[NX_F^{\beta_2} - \frac{X_F}{r-\mu}\frac{\eta\bar{q}_2^2}{4}\right] \tag{31}$$

according to value matching at the follower's investment threshold X_F. Intuitively, $\mathcal{O}$ is negative as $\mathcal{O}X^{\beta_1}$ corrects for the fact that when $X(t)$ reaches X_F, the follower enters the market which ends the leader's monopolistic privilege. Note that the follower would never invest just to enter into a mothballing state.

The investment threshold of the leader can be derived by value matching and smooth pasting the value of the leader before investment, i.e. $\mathcal{P}X^{\beta_1}$ with the value function after investment

$$V_L(X) = \begin{cases} \mathcal{P}X^{\beta_1} & X \leq X_L \\ \mathcal{O}X^{\beta_1} + \frac{X}{r-\mu}\frac{\alpha^2}{4\eta} - I_1 & X > X_L \end{cases} \tag{32}$$

which leads to

$$\mathcal{P} = X_L^{-\beta_1}\left[\mathcal{O}X_L^{\beta_1} + \frac{X_L}{r-\mu}\frac{\alpha^2}{4\eta} - I_1\right] \tag{33}$$

$$X_L = \left(\frac{\beta_1}{\beta_1 - 1}\right)\frac{I_1(r-\mu)4\eta}{\alpha^2} \tag{34}$$

4 Model Derivation- Stackelberg competition after investment

Here we assume that the leader cannot influence the mothballing threshold of the follower.

4.0.1 Follower's Optimal Investment Decision

We assume in what follows that firm 1 is the leader and firm 2 is the follower. Given that the leader has already invested, the follower cannot influence the investment decision of its competitor anymore. This means that the follower decision involves no strategic aspects. The follower has to determine the investment timing, which is similar to fixing a threshold level X_F.

As the leader has already invested and is producing a quantity $q_1(\bar{q}_2) = \frac{\alpha}{2\eta} - \frac{\bar{q}_2}{2}$, the profit flow of the follower is equal to

$$\pi_F(X) = \begin{cases} X(\alpha - \eta\bar{q}_2)\frac{\bar{q}_2}{2} - C\bar{q}_2 & X > X_M \\ -M\bar{q}_2 & X \leq X_M \end{cases} \tag{35}$$

We denote the idle, operating and mothballing states by the labels i, o and m, respectively. We find the value of the firm in each state as the appropriate combinations of the expected profit or cost streams and the options to switch. After investment the firm must decide whether and when to mothball operation. It will mothball the operating project if the price falls to a threshold X_M. Given the project is in mothballing state, the firm will reactivate it if the price rises to a threshold X_R.

We find the value of the firm in each state as the appropriate combinations of the expected profits or cost streams and the options to switch. The firm is in an idle state over the interval $(0, X_F)$. The value in this state is given by the following equation:

$$V_i(X) = AX^{\beta_1}, \tag{36}$$

where A is a constant to be determined. This is the value of the option to invest. The operating state prevails over the interval (X_M, ∞), where the value of the firm is given by the following equation:

$$V_o(X) = BX^{\beta_2} + \frac{X(\alpha - \eta\bar{q}_2)\bar{q}_2}{2(r-\mu)} - \frac{C\bar{q}_2}{r}, \tag{37}$$

where constant B remains to be determined. The mothballed state can continue of the range of $(0, X_R)$. The value of the mothballed project is given by

$$V_m(X) = DX^{\beta_1} - \frac{M\bar{q}_2}{r}, \tag{38}$$

where constant D remains to be determined. At the switching points X_F, X_M and X_R, we have appropriate value matching and smooth-pasting conditions. For the original investment, the conditions are

$$V_i(X_F) = V_o(X_F) - I_2, \tag{39}$$
$$V_i'(X_F) = V_o'(X_F). \tag{40}$$

For mothballing, the conditions are

$$V_o(X_M) = V_m(X_M) - E_M, \tag{41}$$
$$V_o'(X_M) = V_m'(X_M). \tag{42}$$

For reactivation, the conditions are

$$V_m(X_R) = V_o(X_R) - R, \tag{43}$$
$$V_m'(X_R) = V_o'(X_R). \tag{44}$$

This system of six equations determines the three thresholds X_F, X_M and X_R and the three constants A, B and D.

$$AX_F^{\beta_1} = BX_F^{\beta_2} + \frac{X_F(\alpha - \eta\bar{q}_2)\bar{q}_2}{2(r-\mu)} - \frac{C\bar{q}_2}{r} - I_2 \tag{45}$$
$$\beta_1 AX_F^{\beta_1 - 1} = \beta_2 BX_F^{\beta_2 - 1} + \frac{(\alpha - \eta\bar{q}_2)\bar{q}_2}{2(r-\mu)} \tag{46}$$
$$BX_M^{\beta_2} + \frac{X_M(\alpha - \eta\bar{q}_2)\bar{q}_2}{2(r-\mu)} - \frac{C\bar{q}_2}{r} = DX_M^{\beta_1} - \frac{M\bar{q}_2}{r} - E_M \tag{47}$$
$$\beta_2 BX_M^{\beta_2 - 1} + \frac{(\alpha - \eta\bar{q}_2)\bar{q}_2}{2(r-\mu)} = \beta_1 DX_M^{\beta_1 - 1} \tag{48}$$
$$DX_R^{\beta_1} - \frac{M\bar{q}_2}{r} = BX_R^{\beta_2} + \frac{X_R(\alpha - \eta\bar{q}_2)\bar{q}_2}{2(r-\mu)} - \frac{C\bar{q}_2}{r} - R \tag{49}$$
$$\beta_1 DX_R^{\beta_1 - 1} = \beta_2 BX_R^{\beta_2 - 1} + \frac{(\alpha - \eta\bar{q}_2)\bar{q}_2}{2(r-\mu)} \tag{50}$$

σ	B	D	A	X_M	X_R	X_F
0.075	36.56	0.0466	5.474	1.88	2.58	30.297
0.1	22.15	0.146	0.0002	1.824	2.66	32.76
0.15	15.12	0.506	0.008	1.73	2.83	38.25
0.2	13.56	1.008	0.052	1.65	2.98	44.51

Table 1: Parameter values: $r = 0.05$; $\sigma = 0.2$; $\mu = 0$;α =1; C=1; E_M=0; η =0.08; I_1=100; I_2=200; M=0.01; R=0.8

This can be rewritten to

$$BX_M^{\beta_2} + \frac{X_M(\alpha - \eta\bar{q}_2)\bar{q}_2}{2(r-\mu)} - \frac{C\bar{q}_2}{r} - DX_M^{\beta_1} + \frac{M\bar{q}_2}{r} = -E_M \tag{51}$$

$$\beta_2 BX_M^{\beta_2-1} + \frac{(\alpha - \eta\bar{q}_2)\bar{q}_2}{2(r-\mu)} - \beta_1 DX_M^{\beta_1-1} = 0 \tag{52}$$

$$BX_R^{\beta_2} + \frac{X_R(\alpha - \eta\bar{q}_2)\bar{q}_2}{2(r-\mu)} - \frac{C\bar{q}_2}{r} - DX_R^{\beta_1} + \frac{M\bar{q}_2}{r} = R \tag{53}$$

$$\beta_2 BX_R^{\beta_2-1} + \frac{(\alpha - \eta\bar{q}_2)\bar{q}_2}{2(r-\mu)} - \beta_1 DX_R^{\beta_1-1} = 0 \tag{54}$$

$$BX_F^{\beta_2} + \frac{X_F(\alpha - \eta\bar{q}_2)\bar{q}_2}{2(r-\mu)} - \frac{C\bar{q}_2}{r} - AX_F^{\beta_1} = I_2 \tag{55}$$

$$\beta_2 BX_F^{\beta_2-1} + \frac{(\alpha - \eta\bar{q}_2)\bar{q}_2}{2(r-\mu)} - \beta_1 AX_F^{\beta_1-1} = 0 \tag{56}$$

4.0.2 Leader's Optimal Investment Decision

In the next step we determine the investment decision of the leader, where the leader takes the strategy of the follower into account. The follower has two possibilities: investing at the same time as the leader or investing later. Given the current level of X, the leader knows that the follower will invest later if it... To derive the leader's value function, we first determine the leader's profit function for a given GBM level X when both firms are active in the market. Due to the flexibility to mothball the production, the follower might not produce for low levels of X but remain in a mothballing state. For these two cases, the leader's profit flow is qual to

$$\pi_L(X) = \begin{cases} X\frac{\alpha^2}{4\eta} & X \leq X_M \\ X\frac{1}{4\eta}\left[\alpha^2 - \eta^2\bar{q}_2^2\right] & X > X_M \end{cases} \tag{57}$$

The leader's value $V_F(X)$ and profit $\pi_L(X)$ given that both firms have invested has to satisfy the following differential equation

$$\frac{1}{2}\sigma^2 X^2 \frac{\partial^2 V_L}{\partial X^2} + \mu X \frac{\partial V_L}{\partial X} - rV_L + \pi_L = 0 \tag{58}$$

Substituting $\pi_L(X)$ into this equation and applying value matching and smooth pasting at $X = X_M$ leads to the following value function for the leader

$$V_L(X) = \begin{cases} \mathcal{L}X^{\beta_1} + \frac{X}{r-\mu}\frac{\alpha^2}{4\eta} & X \leq X_M \\ \mathcal{N}X^{\beta_2} + \frac{X}{r-\mu}\frac{1}{4\eta}\left[\alpha^2 - \eta^2\bar{q}_2^2\right] & X > X_M \end{cases}$$

where

$$\mathcal{L} = X_M^{-\beta_1}\frac{\beta_2 - 1}{\beta_1 - \beta_2}\frac{X_M}{r-\mu}\frac{\eta\bar{q}_2^2}{4} \tag{59}$$

$$\mathcal{N} = X_M^{-\beta_2}\frac{\beta_1 - 1}{\beta_1 - \beta_2}\frac{X_M}{r-\mu}\frac{\eta\bar{q}_2^2}{4} \tag{60}$$

As $\beta_1 > 0$ and $\beta_2 < 0$ it is straightforward to conclude that $\mathcal{L} < 0$ and $\mathcal{N} > 0$. The value function in equation (125) is split into two regions. In the demand region $X \leq X_M$ demand is so low that the follower is mothballing its operation. This leaves the leader as the only active producer in the market earning a high profit. The second term of the value function represents the expected total discounted revenue the leader obtains when it is in the market as the only producer forever. The first term of the value function represents a negative term that corrects for the fact that demand might eventually increase so that it is optimal for the follower to resume production again, i.e. $X > X_M$, which would decrease the leader's profit. For demand intercept regions $X > X_M$ the demand is so high that it is optimal for the follower to be active and produce. The second term in the leader's value function in this region stands for the expected total discounted revenue the leader obtains when both firms are active forever. The first term represents the option value that the demand might fall below the mothballing threshold of the follower which would leave the leader in a monopoly situation and it would earn a higher profit.

Before the follower's market entry, the leader's value function is equal to

$$V_L(X) = \mathcal{M}X^{\beta_1} + \frac{X}{r-\mu}\frac{\alpha^2}{4\eta} \tag{61}$$

In the following we analyze two strategies of the leader, entry deterrence and entry accommodation. The leader's value function before and after the follower's entry is equal to

$$V_L(X) = \begin{cases} \mathcal{M}X^{\beta_1} + \frac{X}{r-\mu}\frac{\alpha^2}{4\eta}, & \text{before} \\ \mathcal{N}X^{\beta_2} + \frac{X}{r-\mu}\frac{1}{4\eta}\left[\alpha^2 - \eta^2\bar{q}_2^2\right], & \text{after} \end{cases} \tag{62}$$

with

$$\mathcal{M} = X_F^{-\beta_1}\left[\mathcal{N}X_F^{\beta_2} - \frac{X_F}{r-\mu}\frac{\eta\bar{q}_2^2}{4}\right] \tag{63}$$

according to value matching at the follower's investment threshold X_F. Intuitively, $\mathcal{M}$ is negative as $\mathcal{M}X^{\beta_2}$ corrects for the fact that when $X(t)$ reaches X_F, the follower enters the market which ends the leader's monopolistic privilege. Note that the follower would never invest just to enter into a mothballing state.

Assuming that the leader cannot influence the mothballing decision of the follower we derive the optimal investment decision of the leader. The value of teh leateru before and after its invest is equal to:

$$V_L(X) = \begin{cases} \mathcal{O}X^{\beta_1}, & X < X_L \\ \mathcal{M}(X_F)X^{\beta_1} + \frac{X}{r-\mu}\frac{\alpha^2}{4\eta}, & X \geq X_L \end{cases} \tag{64}$$

Value matching and smooth pasting at the leader's investment threshold leads to:

$$X_L = \left(\frac{\beta_1}{\beta_1 - 1}\right)\frac{(r-\mu)I_1 4\eta}{\alpha^2}. \tag{65}$$

5 Output games

5.1 Dynamic output game - accommodate and squeeze strategy of the leader

We will formulate the dynamic output game similar to Behar and Ritz (2017) along the lines that the leader can choose between two strategies referred to as *accommodate* and *squeeze* strategy, respectively.

1. *Accommodate*: The leader maximizes its profits taking as given that the follower produces up to $\bar{q}_2$ for all $X > X_M$.

2. *Squeeze*: The leader increases its output quantity in order to force the follower into mothballing for a given level of X. Note that in Behar and Ritz (2017) the leader lowers its market price to a certain level in order to squeeze the player out of the market[4]. In Behar and Ritz (2017) it is assumed that the leader has market power.

First we look at the follower: As a first step we **assume that suspension and resumption of operation is costless for the follower**. **Furthermore we set the variable costs in suspension equal to zero**. In that case the follower will produce as long as $\pi_F(X, q_1) > 0$ and suspend otherwise. Therefore,

$$\pi_F(X, q_1) = \begin{cases} X(\alpha - \eta(q_1 + \bar{q}_2))\bar{q}_2 - C\bar{q}_2 & \text{if } q_1 < \frac{1}{\eta}\left(\alpha - \eta\bar{q}_2 - \frac{C}{X}\right) \text{ or } X > \frac{C}{\alpha - \eta(q_1+\bar{q}_2)} \\ 0 & \text{otherwise} \end{cases} \tag{66}$$

[4]Behar and Ritz (2017) state that "since OPEX is the only strategy player" in their model "it can equivalently choose price or its output level to maximize its profits." Since their model "features a dominant player with a competitive fringe, rather than oligopolistic interaction, it is not sensitive to whether the choice variable is price (Bertrand) or quantity (Cournot)".

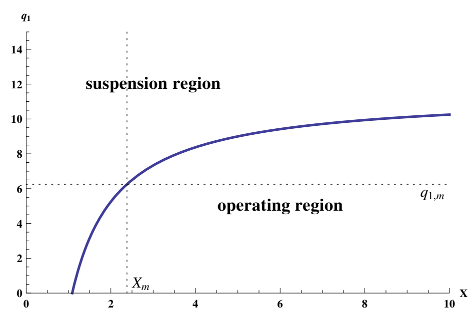

Figure 3: Illustration of the production and suspension regions of the follower as functions of q_1 and X.

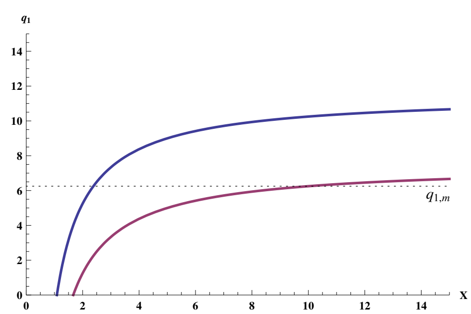

Figure 4: Effect of increasing $\bar{q}_2$ on suspension boundary of the follower. Blue line: $\bar{q}_2 = 1$ and pink line: $\bar{q}_2 = 5$

So for a given q_1 the follower will suspend if X falls below $\frac{C}{\alpha - \eta Q}$. For a given X the follower will suspend if the quantity of the leader q_1 is great than $\frac{1}{\eta}\left(\alpha - \frac{C}{X}\right)$. So the leader can squeeze the follower out of the market by setting its output quantity to a level $q_1 \geq \frac{1}{\eta}\left(\alpha - \frac{C}{X}\right)$. So $\underline{q}_{1,s} = \frac{1}{\eta}\left(\alpha - \frac{C}{X}\right)$ is the lowest level for a given X that squeezes the follower out of the market.

Now we look at the leader: Note that $q_1 < \bar{q}_1 = \frac{\alpha}{\eta}$ in order for the leader to get a positive monopoly profit.

$$\pi_L(X) = \begin{cases} X(\alpha - \eta q_1)q_1 & \text{if the follower is in suspension} \\ X(\alpha - \eta(q_1 + \bar{q}_2))q_1 & \text{if the follower is producing} \end{cases} \tag{67}$$

Given that the follower is in suspension independent of the leader's output quantity the optimal output quantity of the leader is equal to $q_{1,m} = \frac{\alpha}{2\eta}$. The leader can keep the follower in suspension producing its optimal "monopoly-quantity" $q_{1,m}$ as long as $q_{1,m} \geq \underline{q}_{1,s}$ or $X \leq \frac{2C}{\alpha}$. Let's define this boundary by $\bar{X}_{s,m} = \frac{2C}{\alpha}$. The optimal output quantity in order to apply the squeeze strategy

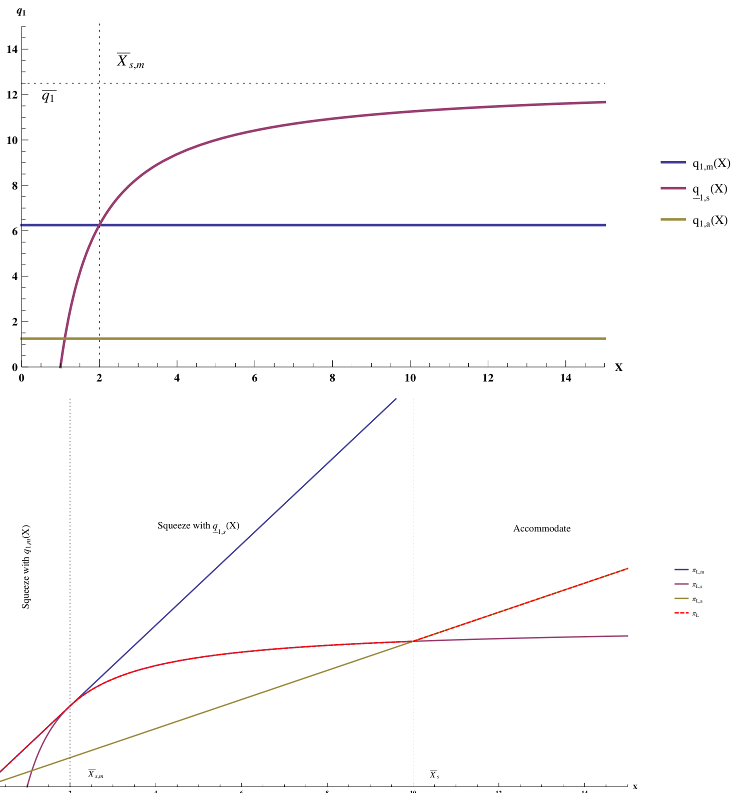

Figure 5: The different production quantities (upper plot) and profit functions (lower plot) of the leader as a function of X.

is equal to $q_{1,s} = max[\underline{q}_{1,s}, q_{1,m}]$. The leader accommodates the follower by producing the quantity $q_{1,a} = \frac{\alpha - \eta \bar{q}_2}{2\eta}$. The output quantities and corresponding profit function for the leader playing the different strategies is equal to

$$q_1^* = \begin{cases} q_{1,m} = \frac{\alpha}{2\eta} & \text{if leader can keep the follower out of the market "for free"} \\ \underline{q}_{1,s} = \frac{1}{\eta}\left(\alpha - \frac{C}{X}\right) & \text{if the leader applies the squeeze strategy by overproducing} \\ q_{1,a} = \frac{\alpha - \eta \bar{q}_2}{2\eta} & \text{if the leader applies the accommodate strategy} \end{cases} \tag{68}$$

$$\pi_L(X) = \begin{cases} X\frac{\alpha^2}{4\eta} & \text{if leader can squeeze the follower out of the market by producing } q_{1,m} \\ \frac{C}{\eta}\left(\alpha - \frac{C}{X}\right) & \text{if the leader applies the squeeze strategy producing } \underline{q}_{1,s} \\ X\frac{\left[\alpha^2 - \eta^2 \bar{q}_2^2\right]}{4\eta} & \text{if the leader applies the accommodate strategy producing } q_{1,a} \end{cases} \tag{69}$$

We denote the profit of the leader using the accommodation strategy by

$\pi_{L,a}(X) = X\frac{\left[\alpha^2-\eta^2\bar{q}_2^2\right]}{4\eta}$. The profit of the leader using the squeeze strategy is equal to

$$\pi_{L,s}(X) = \begin{cases} X\frac{\alpha^2}{4\eta} & X \leq \bar{X}_{s,m} \\ \frac{C}{\eta}\left(\alpha - \frac{C}{X}\right) & X > \bar{X}_{s,m} \end{cases} \tag{70}$$

Now we analyze in which cases the leader might want to play the squeeze strategy, i.e. for X so that $\pi_{L,s}(X) > \pi_{L,a}(X)$. This condition is equivalent to

$$P(X) = X^2\left(\eta^2\bar{q}_2^2 - \alpha^2\right) + 4XC\alpha - 4C^2 > 0 \tag{71}$$

$P(.)$ is a downward pointing parabola. We want to know for which values of X this function is positive. The maximum of this function lays at $X_{max} = \frac{2\alpha C}{(\eta\bar{q}_2+\alpha)(\eta\bar{q}_2\alpha)}$. As long as $\bar{q}_2 < \frac{\alpha}{\eta}$ it holds that $\pi_{L,s}(X) > \pi_{L,a}(X)$ for $X \in (\frac{2C}{\alpha+\eta q_2}, \frac{2C}{\alpha-\eta q_2})$ where we denote: $\underline{X}_s = \frac{2C}{\alpha+\eta q_2}$ and $\bar{X}_s = \frac{2C}{\alpha-\eta q_2}$. Note that $\underline{X}_s < \bar{X}_{s,m} < \bar{X}_s$ and $\bar{X}_{s,m} > 0$. Note that, of course, $\pi_{L,m}(X) > \pi_{L,s}(X)$.

Therefore the optimal production quantity of the leader as a function of X is equal to

$$q_1^* = \begin{cases} \frac{\alpha}{2\eta} & X \leq \bar{X}_{s,m} \\ \frac{1}{\eta}\left(\alpha - \frac{C}{X}\right) & \bar{X}_{s,m} < X \leq \bar{X}_s \\ \frac{\alpha-\eta\bar{q}_2}{2\eta} & X > \bar{X}_s \end{cases} \tag{72}$$

- The upper boundary of the squeeze region $\bar{X}_s$ is increasing in C, η and $\bar{q}_2$, and decreasing in α.
- The squeeze region where the leader produces $\underline{q}_{1,s}$ is always positive. Because $\bar{X}_s - \bar{X}_{s,m} = \frac{2C\eta\bar{q}_2}{\alpha(\alpha-\eta\bar{q}_2}$.
- If $\bar{q}_2 < \frac{2\alpha}{\eta}$ then $\bar{X}_s - \bar{X}_{s,m}$ is decreasing in α, i.e. $\frac{\partial\left(\bar{X}_s-\bar{X}_{s,m}\right)}{\partial\alpha} < 0$.
- $\bar{X}_s - \bar{X}_{s,m}$ is increasing in $\bar{q}_2$, i.e. $\frac{\partial\left(\bar{X}_s-\bar{X}_{s,m}\right)}{\partial\bar{q}_2} > 0$.
- $\bar{X}_s - \bar{X}_{s,m}$ is increasing in η, i.e. $\frac{\partial\left(\bar{X}_s-\bar{X}_{s,m}\right)}{\partial\eta} > 0$.

5.1.1 Positive variable mothballing costs

If we assume that there are positive variable costs related to staying in the mothballing state given by M. Then

$$\pi_F(X, q_1) = \begin{cases} X(\alpha - \eta(q_1 + \bar{q}_2))\bar{q}_2 - C\bar{q}_2 & \text{if } q_1 < \frac{1}{\eta}\left(\alpha - \eta\bar{q}_2 - \frac{C-M}{X}\right) \text{ or } X > \frac{C-M}{\alpha-\eta(q_1+\bar{q}_2)} \\ -M\bar{q}_2 & \text{otherwise} \end{cases} \tag{73}$$

So for a given q_1 the follower will suspend if X falls below $\frac{C-M}{\alpha-\eta Q}$. For a given X the follower will suspend if the quantity of the leader q_1 is great than

$\frac{1}{\eta}\left(\alpha - \frac{C-M}{X}\right)$. So the leader can squeeze the follower out of the market by setting its output quantity to a level $q_1 \geq \frac{1}{\eta}\left(\alpha - \frac{C-M}{X}\right)$. So $\underline{q}_{1,s} = \frac{1}{\eta}\left(\alpha - \frac{C-M}{X}\right)$ is the lowest level for a given X that squeezes the follower out of the market. By simply replacing C by C_M in all the expressions above, we take this cost in account.

5.2 Output game after both firms have invested - Stackelberg competition

We assume that after investment the leader and follower play a Stackelberg production game. As usual we solve the model by backward induction. Given that the follower is not flexible and assuming that the mothballing threshold is fixed and independent of q_1, its output is equal to:

$$q_2^*(q_1, X) = \begin{cases} \bar{q}_2 & \text{if } X > X_M \\ 0 & \text{if } X \leq X_M \end{cases} \tag{74}$$

The best response of the leader given $q_2^*(q_1, X)$ is then equal to

$$q_1^*(X) = \begin{cases} \frac{\alpha - \eta\bar{q}_2}{2\eta} & \text{if } X > X_M \\ \frac{\alpha}{2\eta} & \text{if } X \leq X_M \end{cases} \tag{75}$$

and the profit function equal to

$$\pi_L(X) = \begin{cases} X\frac{\alpha^2}{4\eta} & X \leq X_M \\ X\frac{1}{4\eta}\left[\alpha^2 - \eta^2\bar{q}_2^2\right] & X > X_M \end{cases} \tag{76}$$

This leads to the following profit function of the follower:

$$\pi_F(X) = \begin{cases} X\left(\alpha - \eta\bar{q}_2\right)\frac{\bar{q}_2}{2} & X > X_M \\ -M & X \leq X_M \end{cases} \tag{77}$$

5.3 Output game after both firms have invested

We assume that after investment the leader and follower play a Stackelberg production game. As usual we solve the model by backward induction. Given that the follower is not flexible and assuming that the mothballing threshold is fixed and independent of q_1, its output is equal to:

$$q_2^*(q_1, X) = \begin{cases} \bar{q}_2 & \text{if } X > X_M \\ 0 & \text{if } X \leq X_M \end{cases} \tag{78}$$

The profit function of the follower in the operating state is equal to

$$\pi_F o(X) = X\left(\alpha - \eta(q_1 + \bar{q}_2)\right)\bar{q}_2 - C\bar{q}_2 \tag{79}$$

, which is negative if $q_1 > \frac{1}{\eta}\left(\alpha - \frac{C}{X}\right) - \bar{q}_2$ or $X < \frac{C}{\alpha - \eta(q_1 + \bar{q}_2)}$

The best response of the leader given $q_2^*(q_1, X)$ is then equal to

$$q_1^*(X) = \begin{cases} \frac{\alpha - \eta\bar{q}_2}{2\eta} & \text{if } X > X_M \\ \frac{\alpha}{2\eta} & \text{if } X \leq X_M \end{cases} \tag{80}$$

and the profit function equal to

$$\pi_L(X) = \begin{cases} X\frac{\alpha^2}{4\eta} & X \leq X_M \\ X\frac{1}{4\eta}\left[\alpha^2 - \eta^2\bar{q}_2^2\right] & X > X_M \end{cases} \tag{81}$$

This leads to the following profit function of the follower:

$$\pi_F(X) = \begin{cases} X\left(\alpha - \eta\bar{q}_2\right)\frac{\bar{q}_2}{2} & X > X_M \\ -M & X \leq X_M \end{cases} \tag{82}$$

5.4 Output game after both firms have invested and are flexible- Stackelberg competition

As usual we solve the model by backward induction. Given the output of the leader q_1 and the mothballing threshold X_M the output that maximizes the follower's profit is equal to

$$q_2^*(q_1, X) = \begin{cases} \frac{1}{2\eta}\left[\alpha - \frac{C}{X}\right] - \frac{q_1}{2} & \text{if } X > X_M \\ 0 & \text{if } X \leq X_M \end{cases} \tag{83}$$

The best response of the leader given $q_2^*(q_1, X)$ is then equal to

$$q_1^*(X) = \begin{cases} \frac{1}{2\eta}\left[\alpha + \frac{C}{X}\right] & \text{if } X > X_M \\ \frac{\alpha}{2\eta} & \text{if } X \leq X_M \end{cases} \tag{84}$$

and the profit function equal to

$$\pi_L(X) = \begin{cases} (X(\alpha - \eta(q_1 + \bar{q}_2)) - C)\bar{q}_2 & X > X_M \\ -M\bar{q}_2 & X \leq X_M \end{cases} \tag{85}$$

This leads to the following output of the follower:

$$q_2^*(q_1, X) = \begin{cases} \frac{1}{4\eta}\left[\alpha - \frac{3C}{X}\right] & \text{if } X > X_M \\ 0 & \text{if } X \leq X_M \end{cases} \tag{86}$$

The profit functions of the follower and leader, respectively are then equal to

$$\pi_F(X) = \begin{cases} \frac{X}{16\eta}\left(\alpha - 3\frac{C}{X}\right)^2 & X > X_M \\ -M & X \leq X_M \end{cases} \tag{87}$$

$$\pi_L(X) = \begin{cases} \frac{X}{8\eta}\left(\alpha + \frac{C}{X}\right)^2 & X > X_M \\ ... & X \leq X_M \end{cases} \tag{88}$$

6 Further cases

These are the derivations assuming that the leader does not consider the squeeze strategy.

6.0.1 Follower's Optimal Investment Decision

We assume in what follows that firm 1 is the leader and firm 2 is the follower. Given that the leader has already invested, the follower cannot influence the investment decision of its competitor anymore. This means that the follower decision involves no strategic aspects. The follower has to determine the investment timing, which is similar to fixing a threshold level X_F.

As the leader has already invested and is producing a quantity $q_1(\bar{q}_2) = \frac{\alpha}{2\eta} - \frac{\bar{q}_2}{2}$, the profit flow of the follower is equal to

$$\pi_F(X, q_1) = \begin{cases} X(\alpha - \eta(q_1 + \bar{q}_2))\bar{q}_2 - C\bar{q}_2 & X > X_M \\ -M\bar{q}_2 & X \leq X_M \end{cases} \tag{89}$$

We denote the idle, operating and mothballing states by the labels i, o and m, respectively. We find the value of the firm in each state as the appropriate combinations of the expected profit or cost streams and the options to switch. After investment the firm must decide whether and when to mothball operation. It will mothball the operating project if the price falls to a threshold X_M. Given the project is in mothballing state, the firm will reactivate it if the price rises to a threshold X_R.

We find the value of the firm in each state as the appropriate combinations of the expected profits or cost streams and the options to switch. The firm is in an idle state over the interval $(0, X_F)$. The value in this state is given by the following equation:

$$V_i(X) = AX^{\beta_1}, \tag{90}$$

where A is a constant to be determined. This is the value of the option to invest. The operating state prevails over the interval (X_M, ∞), where the value of the firm is given by the following equation:

$$V_o(X) = BX^{\beta_2} + \frac{X(\alpha - \eta(q_1 + \bar{q}_2))\bar{q}_2}{r - \mu} - \frac{C\bar{q}_2}{r}, \tag{91}$$

where constant B remains to be determined. The mothballed state can continue of the range of $(0, X_R)$. The value of the mothballed project is given by

$$V_m(X) = DX^{\beta_1} - \frac{M\bar{q}_2}{r}, \tag{92}$$

where constant D remains to be determined. At the switching points X_F, X_M and X_R, we have appropriate value matching and smooth-pasting conditions. For the original investment, the conditions are

$$V_i(X_F) = V_o(X_F) - I_2, \tag{93}$$
$$V_i'(X_F) = V_o'(X_F). \tag{94}$$

For mothballing, the conditions are

$$V_o(X_M) = V_m(X_M) - E_M, \tag{95}$$
$$V_o'(X_M) = V_m'(X_M). \tag{96}$$

For reactivation, the conditions are

$$V_m(X_R) = V_o(X_R) - R, \tag{97}$$
$$V_m'(X_R) = V_o'(X_R). \tag{98}$$

This system of six equations determines the three thresholds X_F, X_M and X_R and the three constants A, B and D.

$$AX_F^{\beta_1} = BX_F^{\beta_2} + \frac{X_F(\alpha - \eta(q_1 + \bar{q}_2))\bar{q}_2}{r - \mu} - \frac{C\bar{q}_2}{r} \tag{99}$$
$$\beta_1 AX_F^{\beta_1 - 1} = \beta_2 BX_F^{\beta_2 - 1} + \frac{(\alpha - \eta(q_1 + \bar{q}_2))\bar{q}_2}{r - \mu} \tag{100}$$
$$BX_M^{\beta_2} + \frac{X_M(\alpha - \eta(q_1 + \bar{q}_2))\bar{q}_2}{r - \mu} - \frac{C\bar{q}_2}{r} = DX_M^{\beta_1} - \frac{M\bar{q}_2}{r} - E_M \tag{101}$$
$$\beta_2 BX_M^{\beta_2 - 1} + \frac{(\alpha - \eta(q_1 + \bar{q}_2))\bar{q}_2}{r - \mu} = \beta_1 DX_M^{\beta_1 - 1} \tag{102}$$
$$DX_R^{\beta_1} - \frac{M\bar{q}_2}{r} = BX_R^{\beta_2} + \frac{X_R(\alpha - \eta(q_1 + \bar{q}_2))\bar{q}_2}{r - \mu} - \frac{C\bar{q}_2}{r} \tag{103}$$
$$\beta_1 DX_R^{\beta_1 - 1} = \beta_2 BX_R^{\beta_2 - 1} + \frac{(\alpha - \eta(q_1 + \bar{q}_2))\bar{q}_2}{r - \mu} \tag{104}$$

This is equivalent to the following system of equations[5]:

$$\begin{aligned}
BX_M^{\beta_2} + \frac{X_M(\alpha - \eta(q_1 + \bar{q}_2))\bar{q}_2}{r-\mu} - \frac{C\bar{q}_2}{r} - DX_M^{\beta_1} + \frac{M\bar{q}_2}{r} &= -E_M && (109)\\
\beta_2 BX_M^{\beta_2-1} + \frac{(\alpha - \eta(q_1 + \bar{q}_2))\bar{q}_2}{r-\mu} - \beta_1 DX_M^{\beta_1-1} &= 0 && (110)\\
BX_R^{\beta_2} + \frac{X_R(\alpha - \eta(q_1 + \bar{q}_2))\bar{q}_2}{r-\mu} - \frac{C\bar{q}_2}{r} - DX_R^{\beta_1} + \frac{M\bar{q}_2}{r} &= R && (111)\\
\beta_2 BX_R^{\beta_2-1} + \frac{(\alpha - \eta(q_1 + \bar{q}_2))\bar{q}_2}{r-\mu} - \beta_1 DX_R^{\beta_1-1} &= 0 && (112)\\
BX_F^{\beta_2} + \frac{X_F(\alpha - \eta(q_1 + \bar{q}_2))\bar{q}_2}{r-\mu} - \frac{C\bar{q}_2}{r} - AX_F^{\beta_1} &= I_2 && (113)\\
\beta_2 BX_F^{\beta_2-1} + \frac{(\alpha - \eta(q_1 + \bar{q}_2))\bar{q}_2}{r-\mu} - \beta_1 AX_F^{\beta_1-1} &= 0 && (114)
\end{aligned}$$

Now we analyse how the leader can influence the production decision of the leader by setting it's output quantity. Note that the leader is flexible and therefore, can adapt its output quantity freely. Therefore, we study the interaction between mothballing and resumption of the follower and the influence of the leader's production quantity q_1 on it. This means we study the first four equations of the system:

$$\begin{aligned}
BX_M^{\beta_2} + \frac{X_M(\alpha - \eta(q_1 + \bar{q}_2))\bar{q}_2}{r-\mu} - \frac{C\bar{q}_2}{r} - DX_M^{\beta_1} + \frac{M\bar{q}_2}{r} &= -E_M && (115)\\
\beta_2 BX_M^{\beta_2-1} + \frac{(\alpha - \eta(q_1 + \bar{q}_2))\bar{q}_2}{r-\mu} - \beta_1 DX_M^{\beta_1-1} &= 0 && (116)\\
BX_R^{\beta_2} + \frac{X_R(\alpha - \eta(q_1 + \bar{q}_2))\bar{q}_2}{r-\mu} - \frac{C\bar{q}_2}{r} - DX_R^{\beta_1} + \frac{M\bar{q}_2}{r} &= R && (117)\\
\beta_2 BX_R^{\beta_2-1} + \frac{(\alpha - \eta(q_1 + \bar{q}_2))\bar{q}_2}{r-\mu} - \beta_1 DX_R^{\beta_1-1} &= 0 && (118)
\end{aligned}$$

We can regard this system of four equations in the four unknowns B, D, X_M and X_R, and solve it on its own. This system of equations is similar to the one presented in Chapter 7, Subsection 1.A in **?**[6].

[5]Defining the function $G(X) = BX^{\beta_2} + \frac{X(\alpha-\eta(q_1+\bar{q}_2))\bar{q}_2}{r-\mu} - \frac{C\bar{q}_2}{r} - DX^{\beta_1} + \frac{M\bar{q}_2}{r}$ we can rewrite the the first four equations of this system into

$$\begin{aligned}
G(X_M) &= -E_M && (105)\\
G_X(X_M) &= 0 && (106)\\
G(X_R) &= R && (107)\\
G_X(X_R) &= 0 && (108)
\end{aligned}$$

[6]As **?** it can be shown that a solution to the system exists, is unique, and has economically intuitive basic properties. **?** refer to (**?**, Appendix A)

We combine the first two equations to derive the unknown D in terms of X_M:

$$D = X_M^{-\beta_1}\left(\frac{\beta_2}{\beta_2-\beta_1}\right)\left[\frac{X_M(\alpha-\eta(q_1+\bar{q}_2))\bar{q}_2}{r-\mu}\left(\frac{\beta_2-1}{\beta_2}\right)-\frac{Cq_2}{r}+\frac{Mq_2}{r}+E_M\right] \quad (119)$$

Combining the last two equations to derive the unknown B in terms of X_R:

$$B = X_R^{-\beta_2}\left(\frac{\beta_1}{\beta_1-\beta_2}\right)\left[\frac{X_R(\alpha-\eta(q_1+\bar{q}_2))\bar{q}_2}{r-\mu}\left(\frac{1-\beta_1}{\beta_1}\right)+\frac{Cq_2}{r}-\frac{Mq_2}{r}+R\right] \quad (120)$$

With that we can implicitly represent the two thresholds X_M and X_R as the solutions to the two equations:

$$B(X_R,q_1)X_M^{\beta_2}+\left(\frac{\beta_1-1}{\beta_1-\beta_2}\right)\left[\frac{X_M(\alpha-\eta(q_1+\bar{q}_2))\bar{q}_2}{r-\mu}\right]+\left[\frac{Cq_2}{r}-\frac{Mq_2}{r}-E_M\right]\left(\frac{\beta_1}{\beta_2-\beta_1}\right) \quad (121)$$

$$\left(\frac{1-\beta_2}{\beta_1-\beta_2}\right)\left[\frac{X_R(\alpha-\eta(q_1+\bar{q}_2))\bar{q}_2}{r-\mu}\right]+\left[\frac{Cq_2}{r}-\frac{Mq_2}{r}+R\right]\left(\frac{\beta_2}{\beta_1-\beta_2}\right)-D(X_M,q_1)X_R^{\beta_1} \quad (122)$$

6.0.2 Leader's Optimal Investment Decision

In the next step we determine the investment decision of the leader, where the leader takes the strategy of the follower into account. The follower has two possibilities: investing at the same time as the leader or investing later. Given the current level of X, the leader knows that the follower will invest later if it... To derive the leader's value function, we first determine the leader's profit function for a given GBM level X when both firms are active in the market. Due to the flexibility to mothball the production, the follower might not produce for low levels of X but remain in a mothballing state. For these two cases, the leader's profit flow is qual to

$$\pi_L(X) = \begin{cases} X\frac{\alpha^2}{4\eta} & X \le X_M \\ X\frac{1}{4\eta}\left[\alpha^2-\eta^2\bar{q}_2^2\right] & X > X_M \end{cases} \quad (123)$$

The leader's value $V_F(X)$ and profit $\pi_L(X)$ given that both firms have invested has to satisfy the following differential equation

$$\frac{1}{2}\sigma^2X^2\frac{\partial^2 V_L}{\partial X^2}+\mu X\frac{\partial V_L}{\partial X}-rV_L+\pi_L=0 \quad (124)$$

Substituting $\pi_L(X)$ into this equation and applying value matching and smooth pasting at $X=X_M$ leads to the following value function for the leader

$$V_L(X) = \begin{cases} \mathcal{L}X^{\beta_1}+\frac{X}{r-\mu}\frac{\alpha^2}{4\eta} & X \le X_M \\ \mathcal{N}X^{\beta_2}+\frac{X}{r-\mu}\frac{1}{4\eta}\left[\alpha^2-\eta^2\bar{q}_2^2\right] & X > X_M \end{cases}$$

where

$$\mathcal{L} = X_M^{-\beta_1} \frac{\beta_2 - 1}{\beta_1 - \beta_2} \frac{X_M}{r - \mu} \frac{\eta \bar{q}_2^2}{4} \tag{125}$$

$$\mathcal{N} = X_M^{-\beta_2} \frac{\beta_1 - 1}{\beta_1 - \beta_2} \frac{X_M}{r - \mu} \frac{\eta \bar{q}_2^2}{4} \tag{126}$$

As $\beta_1 > 0$ and $\beta_2 < 0$ it is straightforward to conclude that $\mathcal{L} < 0$ and $\mathcal{N} > 0$. The value function in equation (125) is split into two regions. In the demand region $X \leq X_M$ demand is so low that the follower is mothballing its operation. This leaves the leader as the only active producer in the market earning a high profit. The second term of the value function represents the expected total discounted revenue the leader obtains when it is in the market as the only producer forever. The first term of the value function represents a negative term that corrects for the fact that demand might eventually increase so that it is optimal for the follower to resume production again, i.e. $X > X_M$, which would decrease the leader's profit. For demand intercept regions $X > X_M$ the demand is so high that it is optimal for the follower to be active and produce. The second term in the leader's value function in this region stands for the expected total discounted revenue the leader obtains when both firms are active forever. The first term represents the option value that the demand might fall below the mothballing threshold of the follower which would leave the leader in a monopoly situation and it would earn a higher profit.

Before the follower's market entry, the leader's value function is equal to

$$V_L(X) = \mathcal{M} X^{\beta_1} + \frac{X}{r - \mu} \frac{\alpha^2}{4\eta} \tag{127}$$

In the following we analyze two strategies of the leader, entry deterrence and entry accommodation. The leader's value function before and after the follower's entry is equal to

$$V_L(X) = \begin{cases} \mathcal{M} X^{\beta_1} + \frac{X}{r-\mu} \frac{\alpha^2}{4\eta} & \text{before} \\ \mathcal{N} X^{\beta_2} + \frac{X}{r-\mu} \frac{1}{4\eta} \left[\alpha^2 - \eta^2 \bar{q}_2^2\right] & \text{after} \end{cases} \tag{128}$$

with

$$\mathcal{M} = X_F^{-\beta_1} \left[\mathcal{N} X_F^{\beta_2} - \frac{X_F}{r - \mu} \frac{\eta \bar{q}_2^2}{4} \right] \tag{129}$$

according to value matching at the follower's investment threshold X_F. Intuitively, $\mathcal{M}$ is negative as $\mathcal{M} X^{\beta_2}$ corrects for the fact that when $X(t)$ reaches X_F, the follower enters the market which ends the leader's monopolistic privilege. Note that the follower would never invest just to enter into a mothballing state.

References

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