Trade and Vertical Differentiation*

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Abstract

This paper discusses a trade model with many countries, many goods produced in multiple quality versions, and non-homothetic preferences. It embeds in the same model a series of results that have been empirically confirmed: high-income countries specialize in the production of high-quality goods and trade more of those. Richer countries purchase more high-quality varieties. They import more high-quality products from the most productive exporters. The paper then studies the impact of productivity and population changes on the quality composition of exports.

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1 Introduction

In the last decades, the patterns of international trade have shown large discrepancies in the quality of traded goods.[1] The role of quality in explaining trade has stimulated recent developments in economic theory. In this field, product quality has been modelled as a demand shifter of a horizontally differentiated product, and consumers are identical.[2] This implies that all consumers purchase all available goods, and only one quality of each good is present in the markets. This perspective strongly differs from the standard vertical differentiation approach, where consumers may differ with respect to purchasing power (income) and more than one quality version of each good can be bought.[3]

The objective of this paper is to understand the trade patterns in the context of many countries and many vertically differentiated goods. In particular, we study how the quality range of exported and imported goods changes with country productivity, population and income. The paper presents an easy and tractable model of vertical differentiation with many goods and countries. Each country produces a set of varieties with high and low quality versions while consumers have non-homothetic preferences and purchase a single version of every variety from every country. While higher quality versions yield higher utility, they are more costly to produce. For each variety, consumers then compare the prices of each quality version with their marginal utility. The innovation of the paper is to introduce a class of quality and cost profiles that makes consumer expenditures linear in the consumer's marginal utility. As a result, the trade equilibrium is described by a set of linear equations that can readily be solved and discussed.

The paper embeds in the same model a series of empirically founded properties on product

[1] See Fieler 2011, Hallak 2010, Choi *et al.*, 2009, Dalgin *et al.* 2008, Hallak 2006, Hummels and Klenow 2005 and Schott 2004, *inter alia*.

[2] See Jaimovic and Merella 2015, 2012, Comite *et al.* 2014, Picard 2014, Baldwin and Harrigan 2012, Fajgelbaum *et al.* 2011, and Verhoogen 2008, *inter alia*.

[3] The seminal papers are Mussa and Rosen 1978; Gabszevicz and Thisse 1979; Shaked and Sutton 1982.

quality and trade. We indeed show that richer countries import more high-quality varieties. Also, a country imports more high quality goods from the exporter with higher productivity. In line with the Linder hypothesis (1961), high-income countries specialize in the production of high-quality goods and trade more of those.

We then examine the effects of changes in a country's productivity and population on the quality composition of trade goods . We show that a rise in a country's productivity entices this country to specialize in high quality goods and makes it consumes a wider range of local and imported high quality varieties. Other countries then begin to import a wider set of high quality varieties from this country but a narrower set from each other. Finally, we show that a rise in a country's population leads to wider consumption of local high-quality goods, and under certain conditions, a purchase of a narrower range of high quality imports.

The present analysis is linked to two strands on the theoretical literature of international trade and vertical differentiation. The first strand includes the seminal studies of general equilibrium models with trade and vertical differentiation (Flam and Helpman, 1987; Stokey, 1991; Matsuyama, 2000; and others). First, this strand usually considers endogenous spectrum of quality. As a case in point, Flam and Helpman (1987) consider the continuous versioning of a unique good while we consider discrete quality versions. Since only one good is produced, those models are unable to highlight intra-industry trade. we expands the analysis to sets of horizontally differentiated goods. Second, we examine trade partners that exhibits comparable conditions. This contrasts with the standard North-South setting where an exogenous asymmetry is imposed (Matsuyama, 2000). Finally, while this literature assumes endogenous location of production, we consider that all versions of a same variety are produced in the same country. This allows us to analyze less extreme trade patterns where exports include mixes of high and low quality goods in all trade directions.

In the recent years, there has been a renewed interest in the analysis of quality in trade, with the aim to explain product quality in micro-trade data (Jaimovic and Merella, 2015 and 2012, and Fieler, 2012). Unlike our approach, these studies discuss relative prices of traded varieties of various quality levels, the import penetration and the export compositions. In contrast to this paper, Jaimovich and Merella (2012, 2015) assume the divisible rather than indivisible goods - as in the usual traditional differentiation literature with unit purchases. Consistent with our findings, Jaimovich and Merella (2012) show that a higher global productivity can increase the production and consumption of high quality goods across borders (Linder hypothesis). Finally, Fieler (2012) and Jaimovich and Merella (2012, 2015) assume infinitely small countries, so that a change in national economic conditions does not affect global demand and supply. On the contrary, we are able to investigate the effects on countries with finite population size.

The remainder of the paper is organized as follows. Section 2 describes the model. The trade equilibrium and its properties is examined in Section 3. Section 7 concludes.

2 Model

We consider an economy with N trading countries $i \in \{1, ..., N\}$ populated by a mass M_i of individuals who are each endowed with s_i labor units (skill). The share of country i's population in the world is denoted as $m_i = M_i/M$ where $M = \sum_i M_i$. Each country i is endowed with an idiosyncratic set of differentiated varieties $z \in \mathcal{Z}_i$. We denote by n the mass of varieties produced in each country. Each variety can be versioned with high or low quality, denoted by $k \in \{H, L\}$. The world set of varieties is defined as $\mathcal{Z} = \cup_i \mathcal{Z}_i$.

Production Follow Armington (1961), each country i produces a set of varieties $z \in \mathcal{Z}_i$ requires $a_H(z)$ and $a_L(z)$ labor units for the high and low quality versions of the variety. Varieties in i cannot be produced abroad and *vice versa*. Under perfect competition and absence of trade cost, the price of variety z is equal to its unit cost:

$$p_k\left(z\right) = a_k(z)w_i,\; k \in \{H, L\},\; z \in \mathcal{Z}_i, \tag{1}$$

where w_i is the wage (per labor unit) in country i. We assume that quality upgrades are more difficult to obtain for more costly varieties. Input functions a_H and $a_L : [1-n, 1) \to \mathbb{R}^+$, $n \in (0, 1]$, follow the profiles $a_L\left(z\right) = 1$ and $a_H\left(z\right) = 1 + 1/\left(1-z\right)^2$. Ceteris paribus, the cost of high quality varieties z increases with their index z and becomes prohibitive when $z \to 1$ as $a_H\left(z\right) \to \infty$.[4]

Although varieties are perfectly differentiated within and between countries, their production functions and quality profiles are the same in every country for the sake of simplicity. With a unit mass of idiosyncratic varieties in each country, the total number of varieties is equal to Nn.

Demands A variety z yields to the consumer a utility level $b_H\left(z\right) > 0$ for its high quality version and $b_L\left(z\right) > 0$ for its low quality version. For conciseness, we shall call $b_i\left(z\right)$ also product quality. The quality profiles are given by $b_L\left(z\right) = 1$ and $b_H(z) = 1 + 1/\left(1-z\right)$. Hence, high quality goods with higher index z are associated with higher utility increases.

Every individual consume a unit of every variety $z \in \mathcal{Z}_j$ produced in every country j. An individual in country i maximizes her utility

$$U_i = \sum_{j=1}^{N} \sum_{k=H,L} \int_{\mathcal{Z}_j} b_k\left(z\right) x_k\left(z\right) \mathrm{d}z,$$

[4] This is an instance of Inada condition.

subject to her budget constraint

$$\sum_{j=1}^{N} \sum_{k=H,L} \int_{\mathcal{Z}_j} p_k(z)\, x_k(z)\, \mathrm{d}z = w_i s_i,$$

where $p_k(z) > 0$ is the (destination) consumer prices and $x_k(z) \in \{0,1\}$ the unitary consumption decision of variety z ($x_H(z) + x_L(z) = 1$). In country i, each individual earns the income $w_i s_i$ by offering her s_i labor units. Replacing the prices by their values in (1), there exits a positive scalar μ_i such that the individual i buys the high quality version H of a variety $z \in \mathcal{Z}_j$ if

$$b_H(z) - \frac{1}{\mu_i} a_H(z) w_j \geq b_L(z) - \frac{1}{\mu_i} a_L(z) w_j, \tag{2}$$

and the low quality L otherwise. The scalar μ_i measures the inverse of the marginal utility of income and is equal to the inverse of the Lagrange multiplier of the budget constraint.

By (2), the set of high-quality varieties produced in country j consumed in country i is given by

$$\mathcal{H}\left(\frac{\mu_i}{w_j}\right) \equiv \left\{ z : \frac{\mu_i}{w_j} \geq \ell(z), z \in \mathcal{Z}_j \right\}, \tag{3}$$

where

$$\ell(z) \equiv \frac{a_H(z) - a_L(z)}{b_H(z) - b_L(z)} = \frac{1}{1-z},$$

denotes the per-quality-unit labor input of upgrading variety z. For the sake of brevity, we shall call this the "per-quality input". Per-quality input monotonically rises from $1/n$ to infinity as the variety index z increases from $1-n$ to 1. Per-quality input has inverse function $\ell^{-1}(y) = 1 - 1/y$. The sets of the purchased low quality varieties is defined as $\mathcal{L}(\mu_i/w_j) = \mathcal{Z}_i \backslash \mathcal{H}(\mu_i/w_j)$. From the above definition, it is apparent that μ_i/w_i is a sufficient statistics for the mass of consumers' purchases of local high-quality varieties $\mathcal{H}(\mu_i/w_i)$ and μ_i/w_j for their consumption of high quality imports $\mathcal{H}(\mu_i/w_j)$.

To be valid, the above demands require the two following restrictions. First, every individual must buy a mix of high and low qualities. For this, it should be that $\mu_i/w_j \in [\ell(1-n), \ell(1)) = [1/n, \infty)$, $\forall i, j$, Second, individuals must be productive (rich) enough to buy all varieties. This corresponds to "full market coverage" condition, which is standard in vertical differentiation models. A first condition is that a consumer who prefers a high quality over low quality good also chooses to purchase this high quality good. This implies that the per-quality input schedule ℓ lies above the schedule a_H/b_H, a condition that is always satisfied under the above primitives. A second condition must ensure that low quality goods are always purchased: that is, μ_i/w_j lies above than the schedule a_L/b_L. This conditions is satisfied under our primitives if $\frac{\mu_i}{w_j} \geq 1$, $\forall i, j$. Since $n < 1$, this implies the following simple restriction:

$$\frac{\mu_i}{w_j} \geq \frac{1}{n} \tag{4}$$

As μ_i/w_j will be shown to be positively related to income, this condition expresses that consumers should have a high enough income to purchase all low quality varieties.

Figure 1 represents the per-quality input of varieties $z \in \mathcal{Z}_j$ produced in any country j. Consumptions of high and low quality varieties can readily be inferred for a consumer in country i. The latter has an inverse marginal utility μ_i and consumes the sets of high and low-quality varieties from j, $\mathcal{H}(\mu_i/w_j)$ and $\mathcal{L}(\mu_i/w_j)$. The first assumption imposes the equilibrium to lie below the highest value of $\ell(n)$ while the second one constrains the equilibrium to lie above the highest curve $a_H(z)/b_H(z)$ and $a_L(z)/b_L(z)$.

We denote the labor content of the set of varieties produced in country j and consumed by

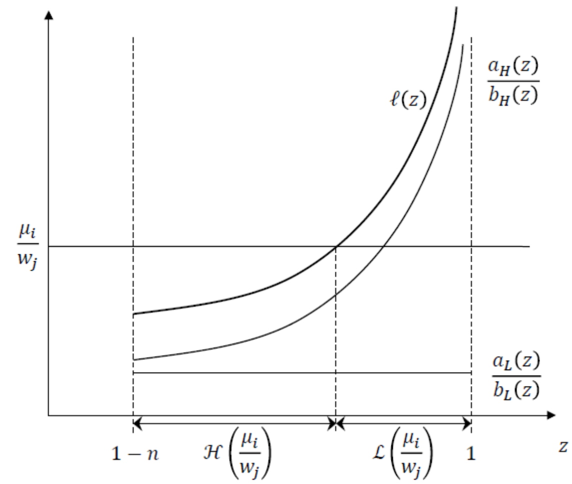

Figure 1: Country i' individual demand for high- and low-quality varieties from country j.

an individual in country i as

$$E\left(\frac{\mu_i}{w_j}\right) \equiv \int_{\mathcal{H}\left(\frac{\mu_i}{w_j}\right)} a_H(z)\mathrm{d}z + \int_{\mathcal{L}\left(\frac{\mu_i}{w_j}\right)} a_L(z)\mathrm{d}z.$$

This represents her *expenditure* on varieties imported from j in terms of importing country's wage. Using the above setting the function E successively reduces to

$$\begin{aligned} E(y) &= \int_{\mathcal{H}(y)} [a_H(z) - a_L(z)]\,\mathrm{d}z + \int_{\mathcal{Z}} a_L(z)\mathrm{d}z \\ &= \int_{1-n}^{\ell^{-1}(y)} [a_H(z) - a_L(z)]\,\mathrm{d}z + \int_{1-n}^{1} a_L(z)\mathrm{d}z \\ &= \int_{1-n}^{1-1/y} (1-z)^{-2}\,\mathrm{d}z + \int_{1-n}^{1} \mathrm{d}z \\ &= y - \frac{1-n^2}{n} \end{aligned}$$

Hence,

$$E(y) = y - r$$

where r is the constant

$$r = \frac{1-n^2}{n} > 0.$$

The total expenditure of an individual in country i simplifies to

$$E_i = \sum_{j=1}^{N} w_j E\left(\frac{\mu_i}{w_j}\right) = N\mu_i - r\left(\sum_{j=1}^{N} w_j\right). \tag{5}$$

Importantly, our model reduces to a set-up where wages w_j and inverse marginal utility of income μ_i appear in a linear way. Finally, to balance budget, expenditure E_i should equal to incomes $s_i w_i$. Using this in the above identity for real expenditure, we have

$$\mu_i = \frac{s_i w_i}{N} + \frac{r}{N}\sum_{l=1}^{N} w_l, \quad i \in \{1, ..., N\}. \tag{6}$$

The inverse marginal utility of income μ_i reflects the consumer's incentive to purchase an upgraded quality version of the good amongst her basket of low quality goods. Note that multiplying all prices by any constant scalar leads to multiply the value of μ_i by the same scalar. As a result μ_i/w_i and $\mathcal{H}(\mu_i/w_i)$ are invariant to global price increases. Demands for high- and low-quality goods are homogenous of degree zero.

To close the model, we express the trade balance condition for each country i, which equates the values of its imports and exports:

$$\sum_{l \neq i} m_i w_l E\left(\frac{\mu_i}{w_l}\right) = \sum_{l \neq i} m_l w_i E\left(\frac{\mu_l}{w_i}\right).$$

Adding $m_i w_i E\left(\mu_i/w_i\right)$ on both sides and substituting for E yields

$$\sum_{l=1}^{N} m_i\left(\mu_i - r w_l\right) = \sum_{l=1}^{N} m_l\left(\mu_l - r w_i\right), \quad i \in \{1, ..., N\}. \tag{7}$$

To sum up, our model is characterized by two sets of equations (6) and (7) that are linear in w_i and μ_i, $i \in \{1, ..., N\}$.

Finally, we establish three measures of interest for the sequel discussion. First, the average price of imports is given by

$$\overline{p}_{ij} \equiv \frac{1}{n}\left(\int_{\mathcal{H}(\mu_i/w_j)} w_j a_H(z)\mathrm{d}z + \int_{\mathcal{L}(\mu_i/w_j)} w_j a_L(z)\mathrm{d}z\right) = \frac{1}{n} w_j E\left(\frac{\mu_i}{w_j}\right). \tag{8}$$

Second, the share of high quality purchases in imported goods is equal to

$$\frac{\int_{\mathcal{H}(\mu_i/w_j)} \mathrm{d}z}{\int_{\mathcal{Z}_j} \mathrm{d}z} = \frac{\int_{1-n}^{\ell^{-1}\left(\frac{\mu_i}{w_j}\right)} \mathrm{d}z}{\int_{1-n}^{1} \mathrm{d}z} = 1 - \frac{1}{n}\frac{w_j}{\mu_i}$$

So, the ratio μ_i/w_j is a sufficient statistics for this share. Finally, the indirect utility simplifies to

$$\begin{aligned} V_i &= \int_{\mathcal{H}\left(\frac{\mu_i}{w_j}\right)} b_H(z)\mathrm{d}z + \int_{\mathcal{L}\left(\frac{\mu_i}{w_j}\right)} b_L(z)\mathrm{d}z \\ &= \sum_{j=1}^{N}\left[\int_{1-n}^{\ell^{-1}(\mu_i/w_j)} \left[b_H(z) - b_L(z)\right]\mathrm{d}z + \int_{1-n}^{1} b_L(z)\,\mathrm{d}z\right] \\ &= \sum_{j=1}^{N}\left[\int_{1-n}^{\ell^{-1}(\mu_i/w_j)} (1-z)^{-1}\,\mathrm{d}z + n\right] \end{aligned}$$

Hence, after simplifications,

$$V_i = \sum_{j=1}^{N} \ln\left(\frac{\mu_i}{w_j}\right) + N\left(n + \ln n\right).$$

As a result, the ratios μ_i/w_j are also sufficient statistics for utility. This expression also sheds new light on the interpretation of the utility parameter β. As it augments consumer's utility of the n local varieties, it is a measure of the love for product diversity. Welfare increases with the (exogenous) global mass of varieties nN, because it separately increases with both the countries's mass of varieties n and the number of countries N.

3 Equilibrium

A trade equilibrium is defined by the profiles of prices $p_H(z)$ and $p_L(z)$, $z \in \mathcal{Z}_j$, that make firms break even (condition (1)) in every country $j \in \{1, ..., N\}$, the vector of inverse marginal utility of income $\mu = (\mu_1,..., \mu_N)$ that matches individuals' optimal consumption choices at given prices (condition (6)), the vector of wages $w = (w_1, ..., w_N)$ that balances trade conditions (7). Finally, under condition (4), consumers buy all varieties and a mix of qualities at the equilibrium.

Since prices are directly derived from wages, it is sufficient to check the $2N$ conditions (7) and (6), which are linear in μ and w. Given demand homogeneity of degree zero and Walras law, the equilibrium is the solution of $2N - 1$ equations and $2N - 1$ values of w and μ. In the sequel we concentrate on the relative wage and marginal utility of income w_i/w_j and μ_i/w_j. Conditions (7) and (6) gives the following unique solution for relative wages

$$\frac{w_i}{w_j} = \frac{m_j s_j + r}{m_i s_i + r}. \tag{9}$$

The above first identity is remarkable because it is mainly expressed in terms of the countries's labor supply, $m_j s_j$. Relative wages between two countries w_i/w_j are inversely related to the ratio of their labor supplies. Very intuitively, larger labor supplies push the price of labor down.

Given the above, one gets the relative inverse marginal utility of income

$$\frac{\mu_i}{w_j} = \frac{1}{N}\left(\frac{w_i}{w_j} s_i + r\sum_{l=1}^{N}\frac{w_l}{w_j}\right). \tag{10}$$

Thus, the incentive to purchase high quality goods in country i from j, μ_i/w_j, increases with the individual's productivity s_i and relative wages w_i/w_j between countries i and j. The last identity can be written as function of the exogenous variables as

$$\frac{\mu_i}{w_j} = \frac{1}{N}\left(\frac{m_j s_j + r}{m_i s_i + r} s_i + r \sum_{l=1}^{N} \frac{m_j s_j + r}{m_l s_l + r}\right). \tag{11}$$

Hence, if it exists, the equilibrium is unique. The only restrictions for the existence is condition (4). For readability we focus on the existence of a trade equilibrium with symmetric countries where $m_i = m$ and $s_i = s$.

Proposition 1 *A symmetric country trade equilibrium exists and is unique for* $s \geq nN$*.*

Proof. At the symmetric equilibrium $\mu_i/w_j = \mu^0 = s/N + r$. Condition (4) impose $\mu_i/w_j \geq 1/n$; that is, $s/N \geq 1/n - r = n$. ■

The symmetric country trade equilibrium exists for a large range of productivity levels. However, individual's productivity and therefore income must rise with the number of countries because, in this Armington model, consumers are required to purchase all varieties from each country. This contrasts to usual models with divisible goods. Finally, by continuity, trade equilibria exist for not too asymmetric country productivities.

In the sequel we assume a set of parameters such that a trade equilibrium exists. We now turn to the discussion of the properties of trade equilibria.

4 Properties

In this section, we discuss the equilibrium properties with respect to the countries' productivity and population sizes. We first consider the trade properties between country pairs because of their application in empirical studies.

4.1 Country pair properties

We discuss the effect of a third country on the trade patterns of two similar or different countries. First note that, by (9), *a higher productivity s_i in country i reduces its wage relative to any other country.* This is because its labor supply rises while the mass of local variety does not change.

Exports from the same origin Take two countries i and j importing from the same exporting country l ($l \neq i \neq j$). Then, by (10), we can write

$$\frac{\mu_i}{w_l} - \frac{\mu_j}{w_l} = \frac{1}{N} \frac{w_j s_j}{w_l} \left(\frac{w_i s_i}{w_j s_j} - 1 \right), \tag{12}$$

so that

$$\frac{\mu_i}{w_l} \geq \frac{\mu_j}{w_l} \iff \frac{w_i s_i}{w_j s_j} \geq 1.$$

Therefore, given that μ_i / w_l is a sufficient statistic for the larger share of high-quality varieties and its associated utility, the last condition states that *a country with larger per capita income imports a larger share of high-quality varieties from a same country l and gets a larger utility from its imports from country l.* By (8), it can further be shown that average import prices rank such as

$$\overline{p}_{il} \geq \overline{p}_{jl} \iff \frac{\mu_i}{w_l} \geq \frac{\mu_j}{w_l}.$$

Therefore, *the average import price is larger to the country with larger per capita income.* Empirically, one should finds a positive correlation between import prices and importer income per capita. Finally, by (9), the ratio of income per capita can be related to exogenous productivity parameters as

$$\frac{w_i s_i}{w_j s_j} = \frac{s_i / \left(m_i s_i + r \right)}{s_j / \left(m_j s_j + r \right)}$$

This implies that more productive countries import a larger share of high quality goods and have higher average import prices.

Imports from different origins

Take a country l that imports from two different exporting countries i and j ($l \neq i \neq j$). Then, by (10),

$$\frac{\mu_l}{w_i} - \frac{\mu_l}{w_j} = \frac{1}{N}\left(\frac{1}{w_i} - \frac{1}{w_j}\right)\left(w_l s_l + r\sum_{k=1}^{N} w_k\right).$$

So, we have

$$\frac{\mu_l}{w_i} \geq \frac{\mu_l}{w_j} \iff \frac{w_i}{w_j} \leq 1 \iff \frac{m_i s_i}{m_j s_j} \geq 1.$$

Therefore, country l imports a larger share of high-quality products from the country with higher labor supply. Controlling for exporter sizes, *country l imports a larger share of high quality varieties and thus have higher expenditures for the varieties manufactured by the more productive exporters.*

Using (8), one shows that average import prices rank such as

$$\overline{p}_{li} \geq \overline{p}_{lj} \iff w_i \leq w_j$$

Therefore, *the average import price to country l is larger for the goods shipped from more productive exporters.* Empirically, this should lead to an positive correlation between exporter income per capita and unit price.

Linder hypothesis According to the Linder's (1961) hypothesis, rich countries trade more numerous high-quality goods with each other than poorer countries. To show this in the present model, consider three countries (i, j, l) with same size $(m_i = m_j = m_l)$ such that countries i and j have the same high productivity while country l is less productive

$(s_i = s_j > s_l)$. Then, wages become

$$\frac{w_i}{w_j} = 1 > \frac{w_i}{w_l}.$$

The wage is lower in the more productive country because of its more abundant labor supply. This gives $w_i = w_j < w_l$. At the same time, from (10), the incentives to purchase high quality goods compare as follows:

$$\frac{\mu_i/w_j}{\mu_j/w_i} = 1 \quad \text{and} \quad \frac{\mu_i/w_j}{\mu_i/w_l} = \frac{w_l}{w_j} > 1.$$

From the first identity, we observe that the two more productive countries import the same range of high quality goods. From the second inequality, country i imports more numerous high-quality goods from the more productive country than from the lower productivity one. By symmetry, country j does the same. Hence, controlling for population sizes, *two high income countries specialize in the production of higher quality goods and trade more of those*, which confirms Linder (1961).

We now turn to the study of the effects of productivity and population size on the consumptions of high quality varieties.

4.2 Productivity changes

Consider an increase in the productivity s_i of country i. Then, its labor supply $m_i s_i$ rises and its wage falls relative to other countries as we compute

$$\frac{\mathrm{d}\left(w_i/w_j\right)}{\mathrm{d}s_i} = -\frac{m_i\left(m_j s_j + r\right)}{\left(m_i s_i + r\right)^2} < 0. \tag{13}$$

This depresses its relative prices and makes the country more competitive in international markets. As a result, every other country $j \neq i$ imports more numerous high-quality goods from country i, substituting for the trade of high quality goods with third countries $l \neq j \neq i$. Indeed, one can compute the changes in high quality imports into country j from countries i and $l \neq i$ as

$$\frac{\mathrm{d}\left(\mu_j/w_i\right)}{\mathrm{d}s_i} = m_i \frac{s_j + r\sum_{l=1,l\neq i}^{N} \frac{r+m_j s_j}{m_l s_l + r}}{N\left(r+m_j s_j\right)} > 0 \quad \text{and} \quad \frac{\mathrm{d}\mu_j/w_l}{\mathrm{d}s_i} = -\frac{r\left(m_l s_l + r\right)}{N\left(m_i s_i + r\right)^2} < 0.$$

At a given wage, country i's workers benefit from larger incomes and from cheaper production of local high-quality goods. But, although their relative wage falls and import prices become higher relative to their incomes, they import a wider range of high quality goods as indeed,

$$\frac{\mathrm{d}\left(\mu_i/w_j\right)}{\mathrm{d}s_i} = r\frac{\left(1-m_i\right)\left(r+m_j s_j\right)}{N\left(r+m_i s_i\right)^2} > 0.$$

They however purchase a larger range of local high variety goods as

$$\frac{\mathrm{d}\left(\mu_i/w_i\right)}{\mathrm{d}s_i} = \frac{1}{N}\left(1 + r\sum_{l=1,l\neq i}^{N} \frac{m_i s_i + r}{m_l s_l + r}\right) > 0.$$

Proposition 2 *In the equilibrium of trade network with N countries, a rise in productivity of country i entices this country to specialize in high quality goods. Country i consumes a wider range of local and imported high quality varieties. Other countries import more high quality varieties from country i and less from each other.*

One consequence of the proposition is that the average quality of home imports increases when the home productivity rises. The result supports Jaimovic and Merella's (2012) study.

4.3 Population changes

Consider an infinitesimal increase in country i's population size, $\mathrm{d}M_i$. Keeping constant other countries' populations, this impacts the population ratios of all countries as follows:

$$\mathrm{d}m_i = \frac{M_i + \mathrm{d}M_i}{M + \mathrm{d}M_i} - \frac{M_i}{M} \simeq (1 - m_i)\,\frac{\mathrm{d}M_i}{M},$$
$$\mathrm{d}m_j = \frac{M_j}{M + \mathrm{d}M_i} - \frac{M_j}{M} \simeq -m_j\frac{\mathrm{d}M_i}{M}.$$

It increases country i's population ratio m_i and decreases other countries' m_j, $j \neq i$, in proportion to global population changes $\mathrm{d}M_i/M$ and initial population distributions. Combining this with the effects of population ratios on μ_i/w_j we can establish the following comparative statics properties. First, there is a decrease in wage for country i relative to other countries $j \neq i$. Indeed, we show in the Appendix that $\mathrm{d}\left(w_i/w_j\right)/\mathrm{d}M_i < 0$. This is because country i's population growth raises labor supply and decreases local production cost and product prices. As their local prices fall and import prices rise, individuals in country i have incentive to augment their consumption of local high-quality varieties. We indeed show that $\mathrm{d}\left(\mu_i/w_i\right)/\mathrm{d}M_i > 0$ while $\mathrm{d}\left(\mu_i/w_j\right)/\mathrm{d}M_i < 0$ if countries' labor supplies are close to symmetry $(s_l m_l \simeq s_j m_j)$.

Proposition 3 *Consider a rise in the population of country i in a trade network with N country. This implies:*

- *a decrease in wage for country i relative to other countries $j \neq i$;*
- *a rise in country l's wage relative to country j's if l has a larger effective labor supply than j $(m_l s_l > m_j s_j)$;*
- *a rise in country i's consumption of its local high-quality goods;*
- *a decrease in the range of high-quality imports consumed by country i's consumers, if countries are sufficiently symmetric.*

The first line of Proposition 3 is intuitive. A larger domestic population increases labor supply in country i and reduces local wages. Therefore, the growing country incurs a fall in its wage with respect to each other trade partner. By the same token, other countries have a rise in their wages relative to country i.

The terms of trade between each other countries also change: a country l has a rise in its wage compared to country j if it has a larger effective labor supply $m_j s_j > m_l s_l$. Moreover, the fall in wages negatively affect domestic consumers' purchasing power so that they buy fewer high-quality local goods.

The effects of a rise in country i population on high quality imports is unclear. The first part of (**??**) in the appendix, is always negative, reflecting the fall in wage due to the increase in supply in country i. The second effect in the second part of the equation is ambiguous, and it is determined by the differences in effective labor supplies of other countries, which affect the interplays of wages among countries. Suppose, for instance, that country j has the highest effective labor supply of the whole economy. Then, purchasing goods from country j becomes more expensive for country i consumers, who reduce the number of high-quality goods imported from j. If conversely, country j has a very low effective labor supply, the effect due by the difference in productivity of other countries might be positive for high quality import of country i and might also compensate the fall in wage.

Finally, if countries are symmetric, the increase in population depresses the range of high quality goods purchased by country i. In this case the effect of differences in productivity is nil, leaving the fall in purchasing power driven by the decrease in country i wages.

5 Ad-valorem trade costs

We consider the presence of symmetric ad valorem (iceberg) trade costs $\tau_{ij} \geq 1$ where a share $1/\tau_{ij}$ of each good arrives at destination i after shipment from country j. Trade cost are symmetric across countries and nil within countries: $\tau_{ji} = \tau_{ij}$ and $\tau_{ii} = 1$. Accordingly, the (destination) consumption price of an unit $z \in \mathcal{Z}_j$ imported from country j to country i is given by $p_{ijk}(z) = \tau_{ij} w_j a_k(z)$, $k = H, L$, and an individual in country i with inverse marginal utility μ_i will purchase all high-quality imports z if $\mu_i/(\tau_{ij} w_j) \geq \ell(z)$. Incentives to purchase high quality goods are then given by the statistics $\mu_i/(\tau_{ij} w_j)$: the higher this is, the wider the range of consumed high-quality imports. Hence, ceteris paribus, a higher τ_{ij} entices consumers to reduce their range of high quality goods. Using the same argument for (4), it can be shown that the requirement $\mu_i/(\tau_{ij} w_j) \geq 1/n$ entices every consumer to buy a mix of all goods with high and low quality versions.

Following the previous procedure and using the above definition of E, the expenditure of an individual in country i for goods produced in j is successively given by

$$\begin{aligned} E_{ij} &\equiv \int_{\mathcal{H}\left(\frac{\mu_i}{\tau_{ij} w_j}\right)} \tau_{ij} w_j a_H(z) dz + \int_{\mathcal{L}\left(\frac{\mu_i}{\tau_{ij} w_j}\right)} \tau_{ij} w_j a_L(z) dz \\ &= \tau_{ij} w_j E\left(\frac{\mu_i}{\tau_{ij} w_j}\right) \\ &= \tau_{ij} w_j \left(\frac{\mu_i}{\tau_{ij} w_j} - r\right) \\ &= \mu_i - r \tau_{ij} w_j. \end{aligned}$$

Her income is equal to her total expenditure: $w_i s_i = E_i \equiv \sum_{j=1}^{N} E_{ij}$. That is,

$$w_i s_i = N\mu_i - r \sum_{j=1}^{N} \tau_{ij} w_j.$$

This gives the incentives to purchase as a function of relative factor prices and trade costs:

$$\frac{\mu_i}{\tau_{ij}w_j} = \frac{1}{N}\frac{s_i}{\tau_{ij}}\frac{w_i}{w_j} + \frac{r}{N}\sum_{l=1}^{N}\frac{\tau_{il}}{\tau_{ij}}\frac{w_l}{w_j}. \tag{14}$$

In country i trade balances the value of imports and exports as

$$\sum_{j\neq i}^{N} m_i \tau_{ij} w_j E\left(\frac{\mu_i}{\tau_{ij}w_j}\right) = \sum_{j\neq i}^{N} m_j \tau_{ji} w_i E\left(\frac{\mu_j}{\tau_{ji}w_i}\right),$$

Given the linear expenditure function, the balanced trade condition simplifies to

$$\sum_{j=1}^{N} m_i \left(\mu_i - r\tau_{ij}w_j\right) = \sum_{j=1}^{N} m_j \left(\mu_j - r\tau_{ji}w_i\right).$$

It is useful to denote the country i's average ad-valorem trade cost $\overline{\tau}_i \equiv 1+\sum_{j=1}^{N} m_j \left(\tau_{ij}-1\right)$ where the second term measures the average trade cost of country i's exports weighted by the export destination populations. Hence, the relative factor prices and incentives to purchase high-quality goods simplify to

$$\frac{w_i}{w_j} = \frac{m_j s_j + r\overline{\tau}_j}{m_i s_i + r\overline{\tau}_i}, \tag{15}$$

$$\frac{\mu_i}{\tau_{ij}w_j} = \frac{1}{N\tau_{ij}}\left(\frac{m_j s_j + r\overline{\tau}_j}{m_i s_i + r\overline{\tau}_i} s_i + r\sum_{l=1}^{N}\tau_{il}\frac{m_j s_j + r\overline{\tau}_j}{m_l s_l + r\overline{\tau}_l}\right). \tag{16}$$

Those expressions compare to the ones without trade costs.

Finally, we recall our three measures of interest. The share of high quality purchases in imported goods is given by

$$\frac{\int_{\mathcal{H}(\mu_i/\tau_{ij}w_j)} \mathrm{d}z}{\int_{\mathcal{Z}_j} \mathrm{d}z} = 1 - \frac{1}{n}\frac{\tau_{ij}w_j}{\mu_i}.$$

The indirect utility in country i simplifies to

$$V_i = \sum_{j=1}^{N} \ln\left(\frac{\mu_i}{\tau_{ij} w_j}\right) + N\left(n + \ln n\right).$$

As a result, the ratios $\mu_i/\tau_{ij}w_j$ are also sufficient statistics for the share of high quality goods and the utility from imports. Because of trade costs, the average import prices must be distinguished by whether they are evaluated at orign or destination. Following international trade terminology, freight on board (fob) prices do not include trade costs while cost, insurance & freigh (cif) prices include them. Exports are most generally reported in fob values at the borders of exporting countries and imports are denominated in cif prices at the gates of importing countries. As a result, we extend our earlier definition of average prices as

$$\overline{p}_{ij}^{\text{fob}} = \frac{1}{n} w_j E\left(\frac{\mu_i}{\tau_{ij} w_j}\right) \quad \text{and} \quad \overline{p}_{ij}^{\text{cif}} = \frac{1}{n} \tau_{ij} w_j E\left(\frac{\mu_i}{\tau_{ij} w_j}\right). \tag{17}$$

Trade equilibrium with symmetric countries To make thing clear, we firstly consider the case of symmetric countries and trade costs ($s_i = s$, $m_i = 1/N$, $\tau_{ij} = \tau$, $i \neq j$ while $\overline{\tau}_i \equiv \overline{\tau} = 1 + (\tau - 1)(N-1)/N$). Equilibrium conditions simplify as

$$\frac{w_i}{w_j} = 1, \quad \frac{\mu_i}{w_i} = \frac{1}{N}\left[s + r + r\tau(N-1)\right] \quad \text{and} \quad \frac{\mu_i}{\tau_{ij} w_j} = \frac{1}{N}\left[\frac{s+r}{\tau} + r(N-1)\right].$$

Hence, a global fall in ad-valorem trade cost (lower τ) entices workers to consume fewer local high-quality goods (μ_i/w_i falls) and a larger share of high-quality imports ($\mu_i/(\tau_{ij}w_j)$ rises). The trade equilibrium exists if $\mu_i/(\tau_{ij}w_j) \geq 1/n$; that is, after simplifications, if $s \geq Nn + (\tau - 1)\left[(N-1)n + 1/n\right]$. As trade costs rises, individuals' productivity s must be increased as to sustain consumption of all goods.

Denoting wages to $w_i = w_j = w$, the average fob and cif prices compute as

$$\overline{p}_{ij}^{\text{fob}} = \frac{w}{nN}\left(\frac{s+r}{\tau} - r\right) \quad \text{and} \quad \overline{p}_{ij}^{\text{cif}} = \tau\overline{p}_{ij}^{\text{fob}} = \frac{w}{nN}\left(s + r - r\tau\right).$$

So, both average prices rise with the fall in trade cost. Lower trade costs indeed entice consumers to import a larger share of high-quality goods, which pushes up the average fob price. Interestingly, the average cif price rises. Consumers increase more their expenditure on import than what they save on trade cost. This is because they reduce their purchases of local high-quality goods. This can be expressed in the country utility, which successively computes as

$$\begin{aligned} V_i &= N \ln \frac{\mu_i}{w_i} - (N-1)\ln\tau + \text{constant}, \\ &= N \ln\left[s + r + r\tau\left(N-1\right)\right] - \left(N-1\right)\ln\tau + \text{constant} \end{aligned}$$

The first and second terms express the impact of local consumption and the effect of trade cost on imports. It can be shown that the utility falls with τ under the above trade equilibrium existence condition.

By a continuity argument, the same properties apply for not too dissimilar countries.

Proposition 4 *A fall in a global symmetric ad-valorem trade cost entices each country to consume fewer high-quality goods from home and more from abroad. It boosts exports of high quality goods, increases both average* fob *and* cif *prices and finally raises utility everywhere.*

This proposition highlights the tradeoff between quality and trade cost for fixed number and quantiy of goods consumed. It therefore complements the trade literature about the tradeoffs between trade costs, intensive and extentive margins of trade.

We can again study country pairs as in Subsection 4.1 but in the presence of trade costs.

Exports from the same origin Take two countries i and j importing from the same exporter l ($l \neq i \neq j$). We know that high-quality import shares and utility from those imports depend on the incentives to buy high-quality goods $\mu_i / (\tau_{il} w_l)$ and $\mu_j / (\tau_{jl} w_l)$. Interestingly, the comparison of average fob import prices also depend on those ratios since, using (17), one gets

$$\overline{p}_{il}^{\text{fob}} \geq \overline{p}_{jl}^{\text{fob}} \iff \frac{\mu_i}{\tau_{il} w_l} \geq \frac{\mu_j}{\tau_{jl} w_l}.$$

Then, cross-country comparisons between high-quality import shares, utility and average fob import prices can be studied with the differences in incentives to buy high-quality goods. By (14), the latter compute as

$$\frac{\mu_i}{\tau_{il} w_l} - \frac{\mu_j}{\tau_{jl} w_l} = \frac{1}{N w_l} \left(\frac{s_i w_i + r \sum_{h=1}^{N} w_h + r \sum_{h=1}^{N} (\tau_{ih} - 1) w_h}{\tau_{il}} - \frac{s_j w_j + r \sum_{h=1}^{N} w_h + r \sum_{h=1}^{N} (\tau_{jh} - 1) w_h}{\tau_{jl}} \right), \tag{18}$$

which reduces to (12) in the absence of trade cost. The difference in high-quality import shares and average fob import prices depend on the difference between each term in the parentheses. Ceteris paribus, high-quality import share and average fob import prices in country i is larger when the first term becomes larger. This occurs if country i has higher per-capita income $s_i w_i$, lower bilateral trade cost τ_{li} and higher remoteness measured by the average trade cost $\sum_{h=1}^{N} (\tau_{ih} - 1) w_h$. The same hold for utility of imports.

To express the above condition as a function of exogenous parameters, it is convenient to define the following *average relative price:*

$$\omega_i \equiv \frac{w_i}{\frac{1}{N} \sum_j w_j} = \frac{(m_i s_i + r \overline{\tau}_i)^{-1}}{\frac{1}{N} \sum_j (m_j s_j + r \overline{\tau}_j)^{-1}}.$$

It is smaller in a country i that has higher supply of labor units $m_i s_i$ relative to other countries.

This translates an average deterioration of its terms of trade. Ceteris paribus, it is also smaller for a relatively more remote country i (higher $\overline{\tau}_i$), which reflects a deterioration of terms of trade caused by a lower international demand for its exports. Substituting for μ_i and μ_j, we obtain

$$\frac{\mu_i}{\tau_{il} w_l} - \frac{\mu_j}{\tau_{jl} w_l} = \frac{1}{N\omega_l}\left[\frac{1}{\tau_{il}}\left(s_i\omega_i + rN + r\sum_{h=1}^{N}(\tau_{ih}-1)\,\omega_h\right) - \frac{1}{\tau_{jl}}\left(s_j\omega_j + rN + r\sum_{h=1}^{N}(\tau_{jh}-1)\,\omega_h\right)\right]$$

The structure of this condition is the same as (18) after substitution of w_i by ω_i. So, ceteris paribus, the high-quality import share and average fob import price $\overline{p}_{il}$ in country i are larger when the latter country has higher productivity s_i, average relative price ω_i, lower bilateral trade cost τ_{li} and higher remoteness measured by the average 'relative' trade cost $\sum_{h=1}^{N}(\tau_{ih}-1)\,\omega_h$. However, average relative price ω_i also falls with remoteness, as measured by $\overline{\tau}_i$. So, the impact of remoteness is a priori unclear.

Such effects of distance and remoteness have been empirically verified by Zhang and Manova (2012), Crozet et al. (2012) and others.

Imports from different origins Now, consider a country l that imports from two different exporters i and j ($l \neq i \neq j$). Using (17), we obtain the following conditions on the ranking of average cif price:

$$\overline{p}_{li}^{cif} \geq \overline{p}_{lj}^{cif} \iff \frac{w_i}{w_j} \leq \frac{\tau_{lj}}{\tau_{li}} \iff 1 \leq \frac{(m_i s_i + r\overline{\tau}_i)/\tau_{li}}{(m_j s_j + r\overline{\tau}_j)/\tau_{lj}}$$

Therefore, after controling for productivity, population and remoteness ($m_i s_i = m_j s_j$ and $\overline{\tau}_i = \overline{\tau}_j$), the average cif price is higher in the importing country with lower bilateral trade barriers. Similarly, after controling for productivity, population and bilateral trade barriers

($m_i s_i = m_j s_j$ and $\tau_{li} = \tau_{lj}$), the average cif price is higher in the importing country i facing a larger remoteness, defined as average trade cost $\overline{\tau}_i$.

The shares of high quality imports and their contribution to utility increase with the ratios $\mu_l / (\tau_{il} w_i)$ and $\mu_l / (\tau_{jl} w_j)$. To compare high-quality shares and utility contributions of imports from various countries, we simply study the difference

$$\frac{\mu_l}{\tau_{li} w_i} - \frac{\mu_l}{\tau_{lj} w_j} = \frac{1}{N}\left(\frac{1}{\tau_{li} w_i} - \frac{1}{\tau_{lj} w_j}\right)\left(s_l w_l + r \sum_{h=1}^{N} w_h\right)$$
$$+ r\left(\frac{1}{\tau_{li} w_i} - \frac{1}{\tau_{li} w_j}\right)\frac{1}{N}\sum_{h=1}^{N}(\tau_{hl} - 1)\, w_h.$$

When this expression is positive, exporter i ships a higher share of high-quality goods to importer l than exporter j. The first term in the RHS measures the direct effect of trade barrier and is equivalent to the expression obtained in the absence of trade costs. Accordingly, a higher bilateral trade barrier between i and l, relatively to that between j and l, entices exporter i to ship a smaller share of high-quality goods to the importing country l than what exporter j does. This in turn implies that country l gets a higher utility out of its imports from country i. The second term in the RHS adds the effect of remoteness of importing country l, measured by its average trade cost $\frac{1}{N}\sum_{h=1}^{N}(\tau_{hl} - 1)\, w_k$. It can then be seen that higher remoteness amplifies the effects of bilateral trade costs on high-quality import shares and utility from imports. Again, such effects of distance and remoteness have been empirically verified by Zhang and Manova (2012), Crozet et al. (2012) and others.

Gravity [unfinished] We end up with the discussion of gravity equation. Country j's export to country i is captured by the expenditure and number of high quality variety, which increases with the statistics $\mu_i / (\tau_{ji} w_j)$. The expenditure on import from j to i is given by

$$E_{ij} = w_j E\left(\frac{\mu_i}{\tau_{ij} w_j}\right) = \frac{1}{N}\frac{1}{\tau_{ij}}\left(s_i w_i + \frac{r}{N}\sum_{l=1}^{N}\tau_{il} w_l\right) - r w_j.$$

Noting that $Y_j = \sum_{i=1}^{N} m_i E_{ij}$ we have

$$Y_j = \frac{1}{N}\left(\sum_{h=1}^{N}\frac{1}{\tau_{jh}} m_h s_h w_h + \frac{r}{N}\sum_{h=1}^{N}\sum_{l=1}^{N}\frac{\tau_{hl}}{\tau_{jh}} m_h w_l\right) - r w_j$$

$$\iff$$

$$r w_j = -Y_j + \left(\sum_{h=1}^{N}\frac{1}{\tau_{jh}} m_h s_h w_h + r\sum_{h=1}^{N}\sum_{l=1}^{N}\frac{\tau_{hl}}{\tau_{jh}} m_h w_l\right)$$

Therefore the gravity equation becomes

$$E_{ij} = \frac{1}{N}\frac{1}{\tau_{ij}}\left(s_i w_i + \frac{r}{N}\sum_{l=1}^{N}\tau_{il} w_l\right) + Y_j - \left(\sum_{h=1}^{N}\frac{m_h s_h w_h}{\tau_{hj}} + r\sum_{h=1}^{N}\sum_{l=1}^{N}\frac{\tau_{lh} m_h w_l}{\tau_{hj}}\right)$$

For $r \to 0$, the gravity equation simplifies to

$$E_{ij} = \frac{1}{N}\frac{1}{\tau_{ij}}(s_i w_i) + N Y_j - \left(\sum_{h=1}^{N}\frac{m_h s_h w_h}{\tau_{hj}}\right)$$

Similarly,

$$Y_i = \sum_{j=1}^{N} m_j E_{ij} = s_i w_i \frac{1}{N}\sum_{j=1}^{N} m_j \frac{1}{\tau_{ij}} + \frac{r}{N^2}\sum_{j=1}^{N}\sum_{l=1}^{N} m_j \frac{\tau_{il}}{\tau_{ij}} w_l - r\sum_{j=1}^{N} m_j w_j.$$

$$\iff$$

$$s_i w_i = \frac{Y_i + r\sum_{j=1}^{N} w_j - \frac{r}{N^2}\sum_{j=1}^{N}\sum_{l=1}^{N} m_j \frac{\tau_{il}}{\tau_{ij}} w_l}{\frac{1}{N}\sum_{j=1}^{N}\frac{m_j}{\tau_{ij}}}$$

6 Linear trade costs

In this section, we consider the presence of linear trade costs. Alchian and Allen's (1964) postulate that a per unit transactions cost lowers the relative price of high quality goods and raises the relative demand for them. Hummels and Skiba (2003) confirm this hypothesis by showing that exporters charge destination prices that vary positively with per unit linear shipping costs and negatively with ad valorem tariffs. We therefore start with linear trade cost.

For the sake of simplicity, we consider trade cost $t_{ji}(z)$ is incurred in the country of destination and depends on the nature of each good but not its quality. For instance, transport costs and tariffs depend mainly on quantity rather than quality of watches, cars, etc... Therefore, the total price of an imported unit $z \in \mathcal{Z}_j$ of quality $k = H, L$, from country j to country i amounts to the sum of the mill price $w_j a_k(z)$ and trade cost $w_i t_{ji}(z)$. There is no trade cost within a same country: $t_{jj}(z) = 0$, $z \in \mathcal{Z}_j$. Since trade costs are the same for high and low qualities, the per-quality input $\ell(z)$ is independent of trade costs. The consumer makes the same choice between high and low quality if she faced the same inverse marginal utility μ_i and wages w_i as without trade costs. The point is that the inverse marginal utility and wages and therefore the product portfolio will change because of higher prices.

The expenditure is given by

$$E_i = \sum_{l=1}^{N} \left(\int_{\mathcal{H}\left(\frac{\mu_i}{w_l}\right)} w_l a_H(z) dz + \int_{\mathcal{L}\left(\frac{\mu_i}{w_l}\right)} w_l a_L(z) dz + w_i \int_0^n t_{li}(z) dz \right),$$

where w is the vector of wages $(w_1, ..., w_n)$. This gives

$$E_i = w_i \left[t_i + \sum_{l=1}^{n} \frac{w_l}{w_i} E\left(\frac{\mu_i}{w_l}\right) \right],$$

where $t_i = \sum_{l=1}^{N} \int_0^n t_{li}(z) dz$ is country i's total trade import cost and where $E(y)$ is defined as before. Balanced trade imposes that the values of exports and imports equate at the mill, "before" payment of trade costs (those are taken in charge by the consumers at destination). That is,

$$\sum_{l\neq i}^{n} \frac{w_l}{w_i} E\left(\frac{\mu_i}{w_l}\right) = \sum_{l\neq i}^{n} \frac{w_i}{w_l} E\left(\frac{\mu_l}{w_i}\right),$$

which is also the same as before. In the equilibrium the balance trade is satisfied as well as the budget balance $E_i = w_i s_i$. The equilibrium is then the same as without trade cost, except that s_i should be replaced by $s_i - t_i$. Therefore, in this framework, *a lower import linear cost is equivalent to a rise in productivity,* s_i. If one interprets s_i as a country fixed 'work time', then t_i is simply the number of hours spent in transporting the goods to home. A lower t_i allows workers to supply more time for production. We can apply Proposition 3 as it follows:

Proposition 5 *Consider the reduction of a unilateral, linear import trade cost* t_i *in country. Then, this country faces a fall in its wage compared to other countries, a specialization in high quality exports and a fall in the range of high quality imports. It increases its consumption of local high quality goods when it is not too large compared to other countries.*

The relative wage falls because workers supply more resources ($s_i - t_i$) for production. This makes the country more competitive for exports but less attractive for imports. When the country is not too large, the terms of trade do not vary dramatically. Local consumers then substitute foreign high quality goods for local ones. The removal of a such unilateral trade cost should be beneficial to the country.

Alchian and Allen's (1964) hypothesis is formulated in terms of export trade cost. Suppose that country i has an rise in its export trade cost. This means that all other countries have a rise in their total trade import cost t_j, $j \neq i$, which is equivalent to a drop in their productivity s_j. Then, according to Proposition 2, other countries j consume a narrower range of local and imported high-quality varieties. Country j substitutes the highest-priced high-quality imports with the highest-priced low-quality imports. The average price of high quality imports drops while the average price of low quality imports fall. On average, the price of high-quality relative to low-quality goods drop, which is consistent with Alchian and Allen's (1964) hypothesis.

Although the above argument is simple, changes in trade costs are generally not unilateral. Longer shipping distances between two countries lead to higher transportation costs in both directions. Improvement in transport technologies affect all directions at the same time. Most import tariff levels between countries are bilateral as the result of trade negotiation and trade policy harmonization. For those reasons, we need to investigate the effect of bilateral trade costs.

We examine the effects of a global fall in linear trade costs so that $t_i = t$ for all country i. Since the reduction of trade cost t for importing to country i corresponds to an increase in $s_i - t$, a fall in t is equivalent to an increase of each productivity s_i such that $\mathrm{d}s_i/\mathrm{d}t = -1\ \forall i$. The changes in a variable X are then given by

$$\frac{\mathrm{d}X_j}{\mathrm{d}t} = \sum_i \frac{\partial X_j}{\partial s_i}\frac{\mathrm{d}s_i}{\mathrm{d}t} = -\sum_i \frac{\partial X_j}{\partial s_i}.$$

Using (9), the changes in relative wages and incentives to purchase high quality goods therefore obtain as

$$\begin{aligned}
\frac{\mathrm{d}\left(w_i/w_j\right)}{\mathrm{d}t} &= \frac{m_j s_j - m_i s_i}{\left(m_i s_i + r\right)^2}, \\
\frac{\mathrm{d}\left(\mu_i/w_i\right)}{\mathrm{d}t} &= \frac{1}{N}\left(-1 + r\sum_{l=1}^{N}\frac{m_i s_i - m_l s_l}{\left(m_l s_l + r\right)^2}\right), \\
\frac{\mathrm{d}\left(\mu_i/w_j\right)}{\mathrm{d}t} &= \frac{1}{N}\left(s_i\frac{m_j s_j - m_i s_i}{\left(m_i s_i + r\right)^2} - \frac{m_j s_j + r}{m_i s_i + r} + r\sum_{l=1}^{N}\frac{m_j s_j - m_l s_l}{\left(m_l s_l + r\right)^2}\right).
\end{aligned}$$

With fully symmetric countries $m_i = 1/N$ and $s_i = s$ we get

$$\frac{\mathrm{d}\left(w_i/w_j\right)}{\mathrm{d}t} = 0 > \frac{\mathrm{d}\left(\mu_i/w_i\right)}{\mathrm{d}t} = \frac{\mathrm{d}\left(\mu_i/w_j\right)}{\mathrm{d}t} = -\frac{1}{N}.$$

So, a fall in t increases both μ_i/w_i and μ_i/w_j, which implies higher consumption of high-quality goods from home and abroad. All countries produce more numerous high quality goods. It is clear that those relationships hold when countries are not too dissimilar $\left(m_j s_j\right)/\left(m_i s_i\right) \to 1$.

Corollary 1 *Consider a fall in a global import trade cost t in a trade network with not too dissimilar countries. Then, each country produces more numerous high quality goods and consume more of them from home and abroad.*

This corollary supports Alchian and Allen's (1964) conjecture that a linear transactions cost raise the relative demand of high quality goods. Hence, linear and ad-valorem trade costs differ in their effects on local consumption: a fall of the former raises the consumption of local high-quality goods while a reduction of the latter diminishes it. This is a possible empirical prediction to test.

A symmetric argument holds when linear trade cost accrues to the exporter. A fall in one country's export trade cost is then equivalent to a rise exporters' productivity. That raise its labor supply and pushes its wage down compared to other countries. Specialization in high quality exports take place and the country reduces its range of high quality imports. It increases its consumption of local high quality goods when it is not too large. When the fall in export trade cost is global and countries are similar, then they also produce more numerous high quality goods and consume more of them from home and abroad.

7 Concluding remarks

In this paper we have analyzed a trade model where preferences are non-homothetic, each product is versioned in two different qualities and where a many countries exhibit different size and productivity. Once we derived the equilibrium, we have first examined the effects of differences in productivity among countries. We have shown that a rise in the productivity of one country implies a fall in domestic wage relative to other countries. Richest countries demand more high-quality varieties from abroad. Between two countries of same size, the more productive specializes in exporting goods of higher quality. Finally, high-income countries specialize in the production of high-quality goods and trade more of those, as suggested by the Linder hypothesis (1961).

We have then investigated the effects of changes in population and productivity in one country. An increase in population induces a decrease in relative prices and, subsequently, in the consumption of high quality goods. An rise in productivity favors the consumption of local high-quality goods only if the relative size of the country is sufficiently small, while high quality exports decrease.

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Appendix

Population changes

Consider an absolute increase in the population size M_i of country i by $\mathrm{d}M_i$. This implies the simultaneous first order changes in relative population sizes

$$\mathrm{d}m_i = \frac{M_i + \mathrm{d}M_i}{M + \mathrm{d}M_i} - \frac{M_i}{M} \simeq (1 - m_i)\frac{\mathrm{d}M_i}{M},$$
$$\mathrm{d}m_j = \frac{M_j}{M + \mathrm{d}M_i} - \frac{M_j}{M} \simeq -m_j\frac{\mathrm{d}M_i}{M}.$$

Hence, for any variable X, an increase in the population size M_i implies

$$\frac{\mathrm{d}X}{\mathrm{d}M_i} = \frac{\partial X}{\partial m_i}\frac{\mathrm{d}m_i}{\mathrm{d}M_i} + \sum_{k \neq i}\frac{\partial X}{\partial m_k}\frac{\mathrm{d}m_k}{\mathrm{d}M_i} = \frac{1}{M}\left[(1 - m_i)\frac{\partial X}{\partial m_i} - \sum_{k \neq i} m_k\frac{\partial X}{\partial m_k}\right].$$

Relative factor prices For $i \neq j \neq l$,

$$\frac{\partial w_i/w_j}{\partial m_i} = -\frac{s_i\left(m_j s_j + r\right)}{\left(m_i s_i + r\right)^2} < 0, \quad \frac{\partial w_j/w_i}{\partial m_i} = \frac{s_i}{m_j s_j + r} > 0 \quad \text{and} \quad \frac{\partial w_l/w_j}{\partial m_i} = 0.$$

Hence, we have

$$\begin{aligned}\frac{\mathrm{d}w_i/w_j}{\mathrm{d}M_i} &= \frac{1}{M}\left[(1 - m_i)\frac{\partial w_i/w_j}{\partial m_i} - \sum_{k \neq i} m_k\frac{\partial w_i/w_j}{\partial m_k}\right] \\ &= \frac{1}{M}\left[(1 - m_i)\frac{\partial w_i/w_j}{\partial m_i} - m_j\frac{\partial w_i/w_j}{\partial m_j}\right] \\ &= -\frac{1}{M}\frac{m_j s_j + r}{m_i s_i + r}\left[\frac{(1 - m_i)\, s_i}{(m_i s_i + r)} + \frac{m_j s_j}{(m_j s_j + r)}\right] < 0. \qquad (19)\end{aligned}$$

So, the more populated country incurs a fall in its wage with respect to each other trade partner.

Also,

$$
\begin{aligned}
\frac{\mathrm{d}w_j/w_i}{\mathrm{d}M_i} &= \frac{1}{M}\left[(1-m_i)\frac{\partial w_j/w_i}{\partial m_i} - m_j\frac{\partial w_j/w_i}{\partial m_j} - \sum_{k\neq i\neq j} m_k \frac{\partial w_j/w_i}{\partial m_k}\right], \\
&= \frac{1}{M}\left[(1-m_i)\frac{\partial w_j/w_i}{\partial m_i} - m_j\frac{\partial w_j/w_i}{\partial m_j}\right], \\
&= \frac{(m_i s_i + r)}{M\,(m_j s_j + r)}\left[\frac{(1-m_i)\,s_i}{m_i s_i + r} + \frac{m_j s_j}{m_j s_j + r}\right] > 0. \qquad (20)
\end{aligned}
$$

So, the other countries have a rise in their wages with respect to the more populated country. Finally,

$$
\begin{aligned}
\frac{\mathrm{d}w_l/w_j}{\mathrm{d}M_i} &= \frac{1}{M}\left((1-m_i)\frac{\partial w_l/w_j}{\partial m_i} - \sum_{k\neq i} m_k \frac{\partial w_l/w_j}{\partial m_k}\right), \\
&= -\frac{1}{M}\left(m_l\frac{\partial w_l/w_j}{\partial m_l} + m_j\frac{\partial w_l/w_j}{\partial m_j}\right), \\
&= \frac{1}{M}\frac{r\,(m_l s_l - m_j s_j)}{(m_l s_l + r)^2}. \qquad (21)
\end{aligned}
$$

This is positive for $m_l s_l > m_j s_j$. A country l has a rise in its wage compared to country j if it has a larger effective labor supply. In turn

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}M_i}\left(\sum_{l=1}^{N} w_l/w_j\right) &= \sum_{l=1}^{N}\frac{\mathrm{d}w_l/w_j}{\mathrm{d}M_i}, \\
&= \frac{\mathrm{d}w_i/w_j}{\mathrm{d}M_i} + \sum_{l\neq i}^{N}\frac{\mathrm{d}w_l/w_j}{\mathrm{d}M_i}.
\end{aligned}
$$

By (19) and (21), this is

$$\frac{\mathrm{d}}{\mathrm{d}M_i}\left(\sum_{l=1}^{N} w_l/w_j\right) = -\frac{1}{M}\frac{m_j s_j + r}{m_i s_i + r}\left[\frac{(1-m_i)\,s_i}{(m_i s_i + r)} + \frac{m_j s_j}{(m_j s_j + r)}\right] \tag{22}$$

$$+\frac{1}{M}\sum_{l\neq i}^{N}\frac{(m_j s_j + r)\,m_l s_l - (m_l s_l + r)\,m_j s_j}{(m_l s_l + r)^2}$$

$$= -s_i\frac{m_j s_j + r}{M\,(m_i s_i + r)^2} + \frac{1}{M}\sum_{l}^{N}\frac{r\,(m_l s_l - m_j s_j)}{(m_l s_l + r)^2}$$

The first part is negative. A sufficient condition of negativity of the second part is $m_j s_j < m_l s_l$ for all $l \neq j$. The expression is also negative if countries' labor supply are close to symmetry $m_l s_l \to m_j s_j$.

Country i local consumption By (10), the incentives to consume local high quality goods are given by

$$\frac{\mathrm{d}\mu_i/w_i}{\mathrm{d}M_i} = \frac{r}{N}\left(\sum_{l=1}^{N}\frac{\mathrm{d}w_l/w_i}{\mathrm{d}M_i}\right) = \frac{r}{N}\left(\sum_{l\neq i}^{N}\frac{\mathrm{d}w_l/w_i}{\mathrm{d}M_i}\right),$$

which is positive by (20).

Country i imports from country j Differentiating μ_i/w_j in (10) with respect to M_i yields:

$$\frac{\mathrm{d}\mu_i/w_j}{\mathrm{d}M_i} = \frac{1}{N}\left(s_i\frac{\mathrm{d}w_i/w_j}{\mathrm{d}M_i} + r\frac{\mathrm{d}}{\mathrm{d}M_i}\left(\sum_{l=1}^{N} w_l/w_j\right)\right).$$

By (19) and (22), the first term is negative while the second is negative if $m_j s_j < m_l s_l$ for all $l \neq j$ or if countries' labor supply are close to symmetry $m_l s_l \to m_j s_j$.

After some simplifications we get

$$\frac{\mathrm{d}\mu_i/w_j}{\mathrm{d}M_i} - \frac{\mathrm{d}\mu_i/w_k}{\mathrm{d}M_i} = \frac{1}{N}s_i\left(\frac{\mathrm{d}w_i/w_j}{\mathrm{d}M_i} - \frac{\mathrm{d}w_i/w_k}{\mathrm{d}M_i}\right) + \frac{r}{N}\left(\frac{\mathrm{d}}{\mathrm{d}M_i}\left(\sum_{l=1}^{N} w_l/w_j\right) - \frac{\mathrm{d}}{\mathrm{d}M_i}\left(\sum_{l=1}^{N} w_l/w_k\right)\right)$$
$$= -\frac{1}{MN}\left(m_j s_j - m_k s_k\right)\left[s_i\frac{2r+s_i}{\left(r+m_i s_i\right)^2} + \sum_{l}^{N}\frac{r^2}{\left(m_l s_l + r\right)^2}\right]$$

Therefore, a rise in country i's population entices this country to replace its high quality imports from high labor supply countries by high quality imports from low labor supply countries ($\frac{\mathrm{d}\mu_i/w_j}{\mathrm{d}M_i} - \frac{\mathrm{d}\mu_i/w_k}{\mathrm{d}M_i} > 0 \iff m_j s_j < m_k s_k$).

Country j imports from country l Differentiating μ_j/w_l in (10) with respect to M_i yields:

$$\frac{\mathrm{d}\mu_l/w_j}{\mathrm{d}M_i} = \frac{1}{N}\left(s_l\frac{\mathrm{d}}{\mathrm{d}M_i}\frac{w_l}{w_j} + r\frac{\mathrm{d}}{\mathrm{d}M_i}\sum_{k=1\neq i\neq j}^{N}\frac{w_k}{w_j} + r\frac{\mathrm{d}}{\mathrm{d}M_i}\frac{w_i}{w_j}\right)$$

The last term is always negative. The first and second terms are negative if $m_j s_j < m_l s_l$ for all $l \neq j$ or $m_l s_l \to m_j s_j$. So, under the latter condition, the expression is negative.