Fee-shifting with Two-sided Asymmetric Information*

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Abstract

We introduce a tractable model of fee-shifting where both parties to a legal dispute hold private information but one party might be better informed than the other. Contrary to the existing literature, we show that when the informational advantage of one party over the other is not too great, fee-shifting does not affect the probability of litigation. This result holds both with uncertainty about the amount of the award and with uncertainty about the probability of victory. Next to the traditional American and English rules, we examine endogenous fee-shifting rules where fee-shifting depends on the precision of the evidence or on the margin of victory. We offer testable implications concerning the settlement rate, the filing rate, case selection, the accuracy of judicial decisions, and the endogenous determination of litigation expenditures.

Keywords: settlement, fee-shifting, two-sided asymmetric information, margin of victory.

1 Introduction

A widely-used method to discourage litigation consists of shifting part of its costs to the losing party under the so-called English rule. (Under the American rule, each party pays his or her own fees.) The White House Task Force on High-Tech Patent Issues recommended that legislation should "[p]ermit more discretion in awarding fees to prevailing parties in patent cases, providing district courts with more discretion to award attorney's fees [...] as a sanction for abusive court filings".[1] Along the same lines, in sponsoring the Innovation Act,[2] congressman Bob Goodlatte stressed that fee-shifting along with other provisions "will eliminate the abuses of [the] patent system by discouraging frivolous patent litigation."[3] These initiatives build on the idea that fee-shifting discourages filings with low probability of success.[4] This effect is reinforced by the

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[1] *Fact Sheet: White House Task Force on High-Tech Patent Issues* of June 04, 2014 available at <http://www.whitehouse.gov/the-press-office/2013/06/04/fact-sheet-white-house-task-force-high-tech-patent-issues> last accessed on October 24, 2014.

[2] H.R.3309 Innovation Act.

[3] Ad available at <https://www.youtube.com/watch?v=uSEH7nYTRh4> last accessed on October 24, 2014.

[4] Shavell (1982); Rosenberg and Shavell (1985); Farmer and Pecorino (1998); see also Katz (1990). See most recently Liang and Berliner (2013).

fact that fee-shifting encourages spending at trial, which makes litigation more expensive and hence less appealing.[5] However, the literature also finds that, holding filing and expenditures constant, fee-shifting may discourage settlement, hence offsetting the positive effects illustrated above, when there is uncertainty about the probability of victory but not if uncertainty is about the magnitude of the award.

We present a tractable model of two-sided asymmetric information and show that there is no difference in settlement rates between the American and the English rule, irrespective of whether uncertainty concerns the probability of victory or the amount of the award. In our model, both parties hold private information, but one party might be better informed than the other. When the asymmetry of information is balanced, each party anticipates the strategic behavior of the other party and adjusts his or her settlement bid. As a result, the settlement amount changes in response to fee-shifting but the probability of settlement remains the same.[6] It is instructive to contrast our results with those derived from existing models.

In divergent-prior models (Shavell, 1982), litigation arises between two sufficiently optimistic parties—that is, with uncertainty about the probability of winning, both parties think to have good chances of winning. To optimistic parties, fee-shifting makes litigation more desirable at the margin because it increases the wedge between winning and losing at trial. In contrast, in our model, parties are not myopically optimistic: a party takes into account that the other party also has relevant private information and adjusts his or her strategy, dissipating the negative effects of fee-shifting.

In models of one-sided asymmetric information (screening: Bebchuk, 1984; signaling: Reinganum and Wilde, 1986), litigation arises because the informed party can exploit his or her informational advantage to obtain a favorable settlement at the expense of going to trial some of the time. With uncertainty about the probability of winning, under the English rule the informed party has private information not only on the probability of winning but also on the probability of paying the entire court fee; hence his or her posture at the settlement stage will be more aggressive and, in equilibrium, there will be more litigation. In contrast, in our model, the degree of the parties' asymmetric information is balanced (though not necessarily identical) and hence neither party can exploit his or her private information at the expense of the other party, dissipating again the negative effects of fee-shifting.

Extant models of two-sided asymmetric information are of two types. In Spier (1994), the parties are asymmetrically informed about the same aspect of the dispute, as in our model.[7] She characterizes the optimal direct mechanism and shows that fee-shifting encourages settlement if it is conditioned on the trial outcome and on the parties settlement bids. A crucial difference with our framework is that in our model the parties' bids are not verifiable and hence the court cannot condition fee-shifting on them.[8] In Chopard et al. (2010), differently from us, the parties are asymmetrically informed on two different variables:[9] each party knows his or her own litigation costs. The private information held by one party does not balance but rather adds to the private information held by the other party and the effect of fee-shifting is ambiguous.

In our model, two litigants have common information about the merits of the case—which can initially be in favor of either party—but hold private information about the evidence that each of them has. Bargaining

[5] Braeutigam et al. (1984); Katz (1987); Plott (1987); Cooter and Rubinfeld (1989); Hause (1989); Hyde and Williams (2002); Choi and Sanchirico (2004).

[6] This result was conjectured in Gong and Mcafee (1994). In a model with two-sided asymmetric information and binary signals they find no effect of fee-shifting rules but remark that the model is not adequate to answer questions about the likelihood of settlement given the coarse signal structure. In our model, signals are drawn from a continuum.

[7] Schweizer (1989) considers discrete information (good versus bad cases), while in our model information is on a continuum (probability of victory or amount of the award), and only analyzes the English rule without comparing it to the American rule.

[8] In addition, in her model the court observes a signal whose distribution depends on the parties' private information, while in our model the court observes the parties' signals (the evidence they bring) directly. Note also that our framework leaves out the possibility that the parties keep bargaining until the trial date (Spier, 1992).

[9] In Sobel (1989) and Daughety and Reinganum (1994), the defendant knows the probability of liability and the plaintiff knows the amount of damages. These papers, however, do not study fee-shifting.

during the settlement phase is modeled as a one-shot simultaneous-bid process (think of parties communicating their bids to a mediator; Chatterjee and Samuelson, 1983). If the plaintiff's demand is lower than the defendant's offer, the parties settle for an amount halfway between demand and offer; otherwise they litigate. If there is a trial, the court adjudicates the case based on the evidence that the parties submit. Based on the same evidence, the court decides how to allocate the litigation costs. The closest article to ours is Friedman and Wittman (2006). A slightly adapted version of their model is derived as a special case of our model and their results are replicated. We generalize their setting in three ways. First, we allow the merits of the case to be in favor of either party, while in their model the parties have equal merits. This allows us to measure how close the court decision (based on the evidence submitted by the parties) is to the merits of the case (which the court cannot observe). Second, we vary the degree of asymmetric information, allowing one party to be better informed than the other, while in their model the parties are privately informed to the same extent. Third, we consider various fee-shifting regimes while they only focus on the American rule.

Next to demonstrating our result with reference to the American and the English rule, where fee-shifting only depends on who wins the case, our model allows an analysis of ways in which fee-shifting can be endogenously determined at trial. Two recent unanimous decisions by the Supreme Court in the *Octane* and *Highmark* cases[10] have given courts more discretion in determining fee-shifting. Also private parties seem to attach value to judicial discretion: 4.3% of the contracts in a sample of large corporations' public securities filings explicitly provide for discretion in the application of fee-shifting rules (Eisenberg and Miller, 2012).[11] Indeed, in general, fee-shifting does not follow automatically from the outcome of adjudication. In most countries, fee-shifting results from a case-by-case determination ex post, so that similar cases might end with the same judgment on the merits but different fee-shifting outcomes. When deciding on the allocation of the litigation costs, the court does not only consider whether a party lost but also how clear the parties' merits appear to be. Already in ancient Athens, the losing party was subjected to penalties for frivolous litigation only if he failed to secure a minimum number of votes in his favor by the jury, suggesting that the case was patently meritless (Thuer, 2012). Modern courts take a similar approach: the losing party is obliged to pay the other party's costs only when the evidence suggests that he or she went to trial with a particularly weak case (Reimann, 2012).[12]

We endogenize the determination of fee-shifting by allowing the court to allocate the litigation costs based not only on the outcome of adjudication (who wins) but also on the precision of the evidence independently submitted by the parties (how confident the court is about who should win). Therefore, two cases might end with the same judgment on the merits but different fee-shifting arrangements. Different fee-shifting rules can be characterized by the sensitivity of the fee-shifting decision to the precision of the evidence on a continuum ranging from the American rule (infinite sensitivity—there is never fee-shifting) to the English rule (no sensitivity—there is always fee-shifting). We show that our main result, the irrelevance of fee-shifting for the settlement decision, continues to hold in our model of endogenous fee-shifting based on the precision of the evidence as well as in an alternative model of endogenous fee-shifting based on the margin of victory, which has already been studied in the literature. In Spier (1994), the optimal direct mechanism prescribes that fees be shifted to the plaintiff if the award is below a threshold and to the defendant if the award is

[10] *Octane Fitness v. ICON Health & Fitness* 134 S. Ct. 1749 (April 29, 2014) and *Highmark v. Allcare Health Management System* 134 S. Ct. 1744 (2014).

[11] Eisenberg et al. (2013) show that courts in Israel use their discretion also to implement one-way fee-shifting.

[12] While a large and important literature has studied the litigation process under different fee-shifting rules, the role of the court has not been examined. The main focus has been on exogenous fee-shifting rules that determine ex ante who pays the litigation costs: for instance, Shavell (1982); Reinganum and Wilde (1986); Kaplow (1993); Gravelle (1993); Bebchuk and Chang (1996). Another strand of the literature studies the effects of conditioning the allocation of the litigation costs to the parties' pretrial announcements, which we do not study here; see Miller (1986); Spier (1994); Chung (1996). For a recent survey of the literature on fee-shifting see Katz and Sanchirico (2012).

above that threshold; in turn, the threshold is endogenously determined by the parties' settlement bids. In Bebchuk and Chang (1996) there are two different thresholds so that in the intermediate region the court fees are shared. Since in our model the settlement bids are not verifiable, we implement the latter approach to the margin of victory.

Our model also yields additional testable implications. Fee-shifting does not affect case selection for trial, which instead only depends on the merits of the case and the court fees. Fee-shifting, however, dramatically affects the final outcome of the case and hence has profound distributional implications. Yet, our results do not support the view that a more permissive fee-shifting policy enhances the accuracy of judicial decisions (and of the corresponding settlements). Rather, whether more or less fee-shifting is desirable for accuracy reasons depends on other factors and, principally, the court fees. With low court fees, the English rule performs better than the American rule and vice versa. Moreover, the optimal fee-shifting rule is not necessarily a corner solution. Since accuracy is operationalized as the difference between the outcome of the case and its merits, these considerations can also be used to study the effects of fee-shifting on primary behavior. As a first approximation, if the merits of the case reflect the true level of damages, incentives for primary behavior improve if the expected outcome of a case is close to its "true" outcome.

While some costs can be shifted, virtually no legal system implements a fee-shifting rule that covers all litigation costs. Most commonly, only the court fees can be shifted but some countries allow shifting the court fees plus some reasonable, predetermined or capped portion of the lawyers' fees. The latter costs are typically not under the control of the parties and concern the lawyer's side of the regular costs of court proceedings (Reimann, 2012). To capture this fact, we distinguish between court fees, which can be shifted and are predetermined, and lawyers' fees, which cannot be shifted and are essentially determined by the parties through their choice of lawyers. In an augmented model we study the parties' choice of lawyers and allow for the endogenous determination of litigation costs. We find that parties tend to hire a more expensive lawyer if the case is more uncertain, if the court fee is higher and if there is a higher degree of fee-shifting. The latter confirms the results obtained by a long legacy of studies pointing out that the English rule yields larger litigation expenditures than the American rule.[13]

This paper is organized as follows. Section 2 presents the model setup. The basic model focuses on litigation about dividing an asset of known value (in this model parties are uncertain about the amount of the award),[14] and allows fee-shifting based on the precision of the evidence (the American and English rules are derived as special cases). We develop this model fully and derive the parties' equilibrium bids at the settlement stage in Section 3. Section 3.5 illustrates the equilibrium with the American and the English rules and compares our model to the common divergent-prior model. In Section 4 we present our central result: fee-shifting does not affect the settlement rate. In Section 5 we study the characteristics of litigated cases: case selection, accuracy and decisions to file and defend against lawsuits. In Section 6 we augment the model and study endogenous legal expenditures. In Section 7 we show that our main results remain valid in a model with endogenous fee-shifting based on the margin of victory and in a model of suits about the determination of liability, where parties are uncertain about the probability of victory. Section 8 concludes. The Appendix contains all proofs.

[13] This is so even if our model lets the parties select a lawyer (and hence determine legal expenditures) before the bargaining phase, while in general in the literature bargaining occurs before selecting legal expenditures. See footnote 5 for references.

[14] Note, however, that we allow the court to condition fee-shifting on the decision on the merits as in Spier (1994). This is not generally allowed in models of uncertainty about the amount of the award, which find no effect of fee-shifting in this case (Reinganum and Wilde, 1986). We do so to stack the deck against our main claim.

2 Model

We analyze the behavior of two risk-neutral parties: the plaintiff Π files a lawsuit against the defendant Δ to seek a judgment—such as a damages award or a share of an undivided asset—whose true value is $q \in (0, 1)$. The quality q of the plaintiff's case is known to the parties but is not verifiable in court. Therefore, to make his or her case in court, each party must collect a piece of hard evidence, such as an expert testimony. Prior to trial, the parties try to settle the case. The game unfolds as follows:

Time 1: Evidence collection. Both parties jointly observe the quality of the plaintiff's case q and the distribution of the evidence (Figure 1). The plaintiff draws a signal $\theta_\Pi \sim U[0, q]$, that is, a piece of positive evidence proving that $q \geq \theta_\Pi$; simultaneously, the defendant draws a signal $\theta_\Delta \sim U[q, 1]$, that is, a piece of negative evidence proving that $q \leq \theta_\Delta$. A party's signal cannot be credibly conveyed to the other party prior to trial (there is two-sided asymmetric information) but is verifiable in court.

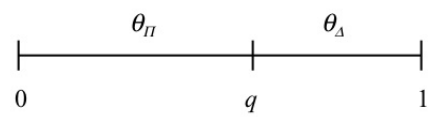

Fig. 1: Evidence signals

Time 2: Settlement negotiations. At the settlement stage, the parties make simultaneous bids. If the plaintiff's demand is weakly lower than the defendant's offer ($p \leq d$), they settle for $\frac{p+d}{2}$ and the game ends. Otherwise ($p > d$), they litigate.

Time 3: Adjudication and fee-shifting. At trial, the court verifies the evidence submitted by the parties—for instance, it hears the experts—and adjudicates the case by awarding $J = \frac{\theta_\Pi + \theta_\Delta}{2}$ to the plaintiff. The court also allocates the court fee to the losing party according to

$$\alpha_t(\theta_\Delta, \theta_\Pi) = \begin{cases} 0 & \text{if} \quad \theta_\Pi < 1 - \theta_\Delta \quad \text{and} \quad \theta_\Delta < t \\ \frac{1}{2} & \text{if} \quad \theta_\Pi = 1 - \theta_\Delta \quad \text{or} \quad (\theta_\Delta \geq t \text{ and } \theta_\Pi \leq 1 - t) \\ 1 & \text{if} \quad \theta_\Pi > 1 - \theta_\Delta \quad \text{and} \quad \theta_\Pi > 1 - t \end{cases} \tag{1}$$

where α is the share of the total court fee $c \geq 0$ paid by the defendant and t identifies the fee-shifting rule. Note that $t = 0$ describes the American rule (the court fee is shared) and $t = 1$ the English rule (the loser pays the court fee). For values of $t \in (0, 1)$ the corresponding fee-shifting rule gives some weight to the quality of the evidence submitted by the parties and shifts the court fee only if the evidence is sufficiently precise, that is, if the signals θ_Π and θ_Δ are close to each other and hence identify a narrow range for q.

In the following subsections, we expand on and provide the micro-foundations of the evidence collection process, the settlement-negotiation protocol and the adjudication and fee-shifting rules. The model is solved in Section 3, where we identify the plaintiff's settlement demand as a function of the plaintiff's signal and the defendant's settlement offer as a function of the defendant's signal at the equilibrium. These bid functions will depend on the two parameters of the game (the quality of the case, q, and the fee-shifting rule, t) and will determine the equilibrium rate of litigation and other characteristics of tried and settled cases, which we explore in the following sections.

2.1 Evidence collection

There is a population of experts, q of whom are pro-plaintiff—that is, they support the plaintiff's claim and show that damages are higher than a certain threshold—while the remaining $1-q$ are pro-defendant—that is, they can demonstrate that damages are below a certain threshold. Next to measuring the true value of the plaintiff's case, q also naturally captures its evidentiary quality; with a higher q there is more abundant evidence supporting the plaintiff, vice versa, with a low q the plaintiff's case is difficult to prove.[15]

Each expert has a piece of hard evidence (Bull and Watson, 2004, 2007; Gennaioli and Perotti, 2009), which varies in strength. A strong piece of evidence is very close to q: for the plaintiff, strong evidence is a large θ_Π—showing that damages are high—while for the defendant strong evidence is a low θ_Δ. While it is easy for the parties to identify experts in their favor—that is, parties know whether an expert belongs to $[0, q]$ or to $[q, 1]$—the strength of the evidence is revealed only after each party has been paired with an expert—that is, parties cannot observe the value of θ while choosing an expert.[16] This justifies the information structure (Figure 1).[17] Assuming a uniform distribution is standard and guarantees the tractability of the model.[18]

Note that this formulation allows us to vary the amount of asymmetric information that the parties have. If q is large, the plaintiff has a good case and is also better informed than the defendant, because the variance of the plaintiff's signal is larger than the variance of the defendant's signal; and vice versa if q is small.[19]

2.2 Settlement negotiations

The settlement-negotiation phase is modeled as in Chatterjee and Samuelson (1983) and Friedman and Wittman (2006): the parties submit simultaneous bids (to a mediator) and settle if the bids cross; they litigate otherwise. Since each party only observes his or her evidence signal, a party's bid will be a function of his or her signal and of the parameters q and t and will not depend on the signal observed by the other party. This framework does not require us to make assumptions on who makes a take-it-or-leave-it offer and preserves the symmetry of the game. In addition, the resulting settlement-negotiation game is tractable and can be extended in several ways.

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[15] Since q determines both the true value of the plaintiff's case and the share of pro-plaintiff's experts, our model has the very natural feature that it is easier to prove that damages are high if they are in fact high; this assumption could be relaxed without affecting the main message of the model but at the price of more cumbersome notation and formulas.

[16] An alternative way to describe the evidence collection process is to assume that experts have two characteristics $\{\phi, \theta\}$ with $\theta \sim U[0, 1]$ and

$$\phi = \begin{cases} 0 & \text{if} \quad \theta \geq q \\ 1 & \text{if} \quad \theta \leq q \end{cases}$$

(Note that the slight ambiguity that arises when $\theta = q$ does not cause problems as it has mass zero but is necessary to keep the model symmetric.) We assume that ϕ is directly observable to a party, while θ is revealed only after choosing an expert. A party may need some time and effort to understand if the expert has strong or weak evidence. Note that for the plaintiff, the worst positive signal, $\{1, 0\}$, is preferable to any negative signal, $\{0, \theta\}$ for any θ. This is because the former indicates that q can take any value (thus, it carries no information), while the latter gives an upper bound for q. Thus, it is a dominant strategy for the plaintiff to choose a pro-plaintiff expert rather than a pro-defendant expert; likewise, the defendant prefers a pro-defendant expert. Consequently, we can define $\theta_\Pi \in \{\theta \mid \phi = 0\}$ and $\theta_\Delta \in \{\theta \mid \phi = 1\}$ as the plaintiff's and the defendant's signal. Given that θ is uniformly distributed, we have $\theta_\Pi \sim U[0, q]$ and $\theta_\Delta \sim U[q, 1]$.

[17] Allowing the parties to collect multiple pieces of evidence would significantly complicate the analysis without affecting the main results as long as parties have symmetric access to evidence. In Section 6 we will allow parties to invest in lawyers of different abilities.

[18] Gong and Mcafee (1994); Friedman and Wittman (2006); Gennaioli and Perotti (2009).

[19] In reality, merits and asymmetric information might vary in different ways, while our framework uses q to vary both of them at the same time. However, our results are not qualitatively affected by this feature of the model. To see why this is the case, note that a model in which the parties' signals are independent and uniformly distributed on $\left[\tilde{q} - \frac{1}{2}, \tilde{q} + \frac{1}{2}\right]$ (where the variance of the signals does not change with $\tilde{q}$) is essentially equivalent to a special case of our model with equal merits ($q = \frac{1}{2}$) and hence our results continue to hold.

2.3 Adjudication and fee-shifting

At trial, the court cannot observe the quality q of the plaintiff's case and hence cannot generally set $J = q$. Due to numerous legal restrictions, courts are usually modeled as non-Bayesian actors (Daughety and Reinganum, 2000b; Gennaioli and Shleifer, 2007).[20] Daughety and Reinganum (2000a) identify a unique family of judgments that, among other properties, are (1) strictly monotonically increasing in each of the signals, (2) bounded by the minimum and maximum of the signals, and (3) symmetric with respect to the signals. The only member of this family of judgments that is based on a neutral interpretation of the law and preponderance of evidence is $J = \frac{\theta_\Pi + \theta_\Delta}{2}$.[21] As expected, J lies between θ_Π and θ_Δ, increases in both evidence signals, treats the parties symmetrically and is typically different from q.[22]

Next to adjudicating the case, the court also decides who pays the total court fee $c > 0$. The fee-shifting rule in (1) depends on two factors. The first factor is the relative strength of the parties' evidence signals, which determines who should pay c. The plaintiff's evidence is stronger than the defendant's evidence if $\theta_\Pi > 1 - \theta_\Delta$ and, vice versa, the defendant's evidence is stronger than the plaintiff's evidence if $\theta_\Pi < 1 - \theta_\Delta$. Quite naturally, the court considers shifting the court fee to the party submitting the weaker signal, never the opposite.

Whether the court fee is shifted depends on the second factor: the cumulative precision of the evidence submitted to the court. Precision is naturally captured by the distance between the parties' signals, $\theta_\Delta - \theta_\Pi$, which directly measures the length of the range of uncertainty for q, which in turn the court cannot observe. Intuitively, if the plaintiff's and the defendant's signals are far apart, the range in which the true merits can fall is large and hence J might be far from q. The fee-shifting rule in (1) is the simplest, one-parameter characterization of a family of rules that captures this intuition.[23] These fee-shifting rules allocate the court fee to the losing party only if the winner's signal is sufficiently strong, that is, if it is above a threshold if the plaintiff wins or below a threshold if the defendant wins; the threshold $t \in [0, 1]$ fully characterizes the rule.

A larger value of t implies that the precision of the signals is given more weight in determining fee-shifting. Figure 2 depicts different types of fee-shifting rules characterized by different values of t. If $t = 0$, fee-shifting is infinitely sensitive to the precision of the evidence so that fee-shifting never occurs and court fee is shared; this is the American rule, where $\alpha = \frac{1}{2}$ irrespective of the signals. If instead $t = 1$, fee-shifting is insensitive to the precision of the evidence and the loser always pays the court fee; this is the English rule, where $\alpha = 0$

[20] Most importantly, this formulation implies that the court does not infer anything from the fact that a party presents no evidence and treats it as simply an uninformative signal. Thus, no evidence submitted by the plaintiff is equivalent to $\theta_\Pi = 0$, while no evidence submitted by the defendant is equivalent to $\theta_\Delta = 1$. This in turn implies that we could dispense of our assumption that the parties can observe the type of evidence (positive or negative) before choosing an expert. We could simply allow a plaintiff who has accidentally chosen a pro-defendant expert to submit no evidence and likewise for the defendant. Doing so would change the distribution of evidence (it would put positive mass on $\theta_\Pi = 0$ and on $\theta_\Delta = 1$) therefore complicating the analysis, but would not affect the basic structure of the model.

[21] More precisely, the family of judgments identified by Daughety and Reinganum (2000a) is (in our own notation) $\tilde{J}(\theta_\Delta, \theta_\Pi) = \left(\left(\frac{\theta_\Delta^\xi + \theta_\Pi^\xi}{2}\right)^{\frac{1}{\xi}}, \gamma\right)$, where $\xi \neq 0$ is a metric of the court's interpretation of the law, and γ is the evidence threshold. A large ξ magnifies the effect of the winner's signal, while a low ξ magnifies the impact of the loser's signal. Our formulation of the judgment is obtained by setting $\xi = 1$, which can be thought of as a neutral interpretation of the law, where the winner's and the loser's signal have the same weight. The evidence threshold in our framework is set at $\gamma = \frac{1}{2}$, which is the preponderance of evidence threshold: if $J > \frac{1}{2}$, the plaintiff wins, otherwise the defendant wins.

[22] The expected value of J for a given q is $\mathrm{E}_{\theta_\Pi, \theta_\Delta}[J] = \frac{2q+1}{4} \in \left(\frac{1}{4}, \frac{3}{4}\right)$, which is biased towards $\frac{1}{2}$, that is, is greater than q for $q < \frac{1}{2}$ and less than q for $q > \frac{1}{2}$. The party with the greater merits wins more often in court but this advantage exhibits decreasing marginal returns. Assuming that q is symmetrically distributed around $\frac{1}{2}$ (which implies $\mathrm{E}[q] = \frac{1}{2}$ and includes the uniform distribution) and taking the expectation over q yields an unbiased expected judgment equal to $\mathrm{E}_q\left[\mathrm{E}_{\theta_\Pi, \theta_\Delta}[J]\right] = \frac{1}{2}$. The assumption of symmetry is consistent with our more general choice to consider litigation between (ex ante) symmetric parties.

[23] Note that the condition "($\theta_\Pi < 1 - \theta_\Delta$ and $\theta_\Delta \geq t$) or ($\theta_\Pi > 1 - \theta_\Delta$ and $\theta_\Pi \leq 1 - t$)" is equivalent to the simpler condition "$\theta_\Delta \geq t$ and $\theta_\Pi \leq 1 - t$".

if the defendant wins ($\theta_\Pi < 1 - \theta_\Delta$) and $\alpha = 1$ if the plaintiff wins ($\theta_\Pi > 1 - \theta_\Delta$). For intermediate values of t, fee-shifting occurs only if the evidence of the winning party is sufficiently strong or—which is the same—if the distance between the parties' signals $\theta_\Delta - \theta_\Pi$ is sufficiently small and hence evidence is sufficiently precise. In all cases, if neither party wins ($\theta_\Pi = 1 - \theta_\Delta$) or if the winner's evidence does not meet the threshold, each party pays his or her own costs.

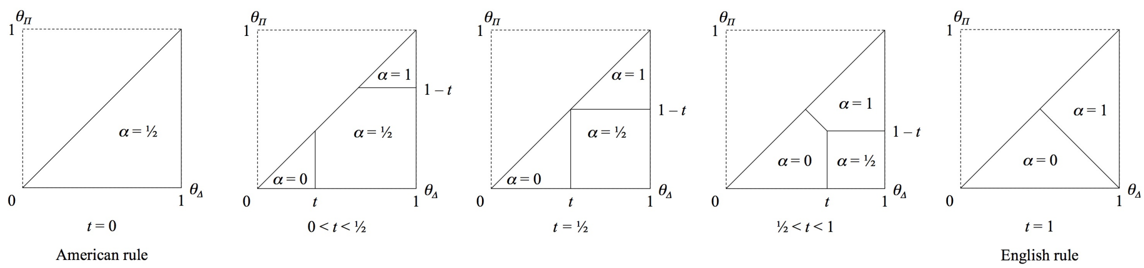


Fig. 2: Fee-shifting rules

This family of endogenous fee-shifting rules have the following three appealing properties:

- *Symmetry.* If the parties were to exchange their signals, the plaintiff would pay the defendant's share and vice versa, as is evident from Figure 2:

$$\alpha_t(\theta_\Pi, \theta_\Delta) = 1 - \alpha_t((1 - \theta_\Delta), (1 - \theta_\Pi))$$

- *Responsiveness to the strength of the evidence.* Fee-shifting occurs in a broader range of cases if the evidence in favor of the winning party becomes stronger; that is, α weakly increases in $\theta_\Pi + \theta_\Delta$. The solid lines in Figure 3 are "iso-strength" lines, along which the sum $\theta_\Pi + \theta_\Delta$ and, hence, the judgment J are constant. As we move north-east, strength increases and α may therefore also increase, as can be verified in Figure 2.

- *Responsiveness to the precision of the evidence.* Fee-shifting occurs in a broader range of cases if the evidence becomes more precise, that is, α weakly moves away from $\frac{1}{2}$ if $\theta_\Delta - \theta_\Pi$ decreases. The dashed lines in Figure 3 are "iso-precision" lines, along which the difference $\theta_\Delta - \theta_\Pi$ is constant. As we move north-west, precision increases and hence α may move away from $\frac{1}{2}$, as can be verified in Figure 2.

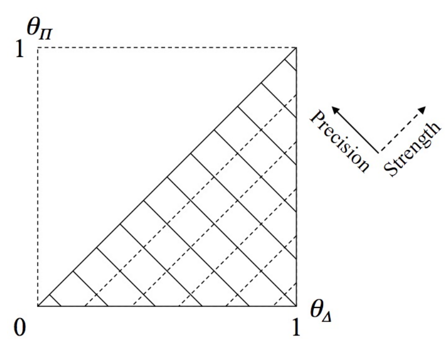


Fig. 3: Properties of the fee-shifting rules

The last two properties imply that two lawsuits could end with the same judgment, while yielding two different allocations of the court fee.[24] Assume that $t = \frac{2}{3}$ and consider two hypothetical lawsuits. In lawsuit A, the parties submit evidence $\theta_\Pi = \frac{1}{4}$ and $\theta_\Delta = 1$; accordingly, the court decides $J = \frac{\frac{1}{4}+1}{2} = \frac{5}{8}$ on the merits (the plaintiff wins) and shares the court fee since the evidence is not precise enough (the signals are far apart): $\alpha = \frac{1}{2}$ because, applying (1), $\theta_\Delta = 1 > \frac{2}{3} = t$ and $\theta_\Pi = \frac{1}{4} < \frac{1}{3} = 1 - t$. In contrast, in lawsuit B, the parties submit evidence $\theta_\Pi = \frac{1}{2}$ and $\theta_\Delta = \frac{3}{4}$. The court decides $J = \frac{\frac{1}{2}+\frac{3}{4}}{2} = \frac{5}{8}$ as in lawsuit A but now $\alpha = 1$ (due to $\theta_\Pi = \frac{1}{2} > \frac{1}{4} = 1 - \theta_\Delta$ and $\theta_\Pi = \frac{1}{2} > \frac{1}{3} = 1 - t$): the defendant pays the entire court fee since the evidence against him or her is more precise than in lawsuit A (the signals in lawsuit B are closer to each other). An increase in the fee-shifting parameter t allows the court to "punish" the loser more often; with $t > \frac{4}{5}$, even lawsuit A would result in fee-shifting to the defendant. In contrast, lower levels of t condition the fee-shifting decision to more precise evidence.

Figure 2 describes fee-shifting as applied by the court ex post. Since the court cannot observe q, the fee-shifting rule is defined on the whole interval $[0, 1]$ for both signals. The parties, however, observe q and hence their expectations about fee-shifting will reflect the fact that the plaintiff's signal is less than q while the defendant's signal is greater than q. To see why this is relevant, consider the following example: assume that $q = \frac{1}{2}$ and that $t = \frac{1}{3}$ (an instance of Case 1 in Figure 4). In this case, the defendant's signal is necessarily above $\frac{1}{2}$ and hence $\theta_\Delta < t = \frac{1}{3}$ can never be satisfied. Similarly, the plaintiff's signal is surely below $\frac{1}{2}$ and hence cannot satisfy the fee-shifting condition $\theta_\Pi > 1 - t = \frac{2}{3}$. The same is true for all values of t and q such that $t \leq q \leq 1 - t$. Therefore, in Case 1 the court fee is always shared.

Consider now a different case. Assume $q = \frac{1}{3}$ and $t = \frac{1}{2}$ (an instance of Case 2 in Figure 4). In this case, it is possible that the defendant's signal be above q and below t (for instance, $\theta_\Delta = \frac{5}{12}$) so that if the defendant wins (which happens if, for instance, $\theta_\Pi = \frac{3}{12} < \frac{7}{12} = 1 - \theta_\Delta$), then the court fee is shifted to the plaintiff. A winning plaintiff, in turn, can never satisfy the fee-shifting requirement $\theta_\Pi > \frac{1}{2} = 1 - t$. Hence, in this case, the only two possibilities are fee-shifting in favor of the defendant or no fee-shifting.

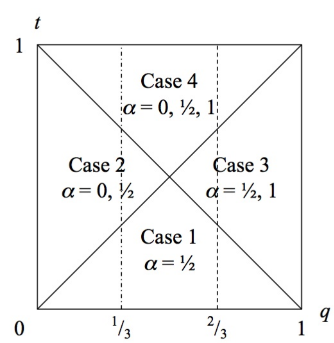


Fig. 4: Four cases: no fee-shifting (Case 1), one-way fee-shifting to the plaintiff (Case 2), one-way fee-shifting to the defendant (Case 3), and two-way fee-shifting (Case 4)

Depending on how q compares to t and to $1 - t$, we can distinguish among four possible cases illustrated in Figure 4. To illustrate, consider $q = \frac{1}{3}$: for $t \leq q$ (Case 1), there is no fee-shifting; for $q < t \leq 1 - q$ (Case 2), there is one-way fee-shifting to the plaintiff; finally, for $t > 1 - q$ (Case 4), there is fee-shifting in both directions. In the mirror case of $q = \frac{2}{3}$, we have that: for $t \leq 1 - q$ (Case 1), there is no fee-shifting; for $1 - q < t \leq q$ (Case 3), there is one-way fee-shifting to the defendant; finally, for $t > q$ (Case 4), there is fee-shifting in both directions. Table 1 formalizes these observations.

Since the parties know q, these four cases are relevant when determining their expectations about the

[24] See also Spier (1994). This is not the case when fee-shifting depends on the margin of victory as in Bebchuk and Chang (1996), which we study in Section 7.

Case 1	$t \leq q \leq 1-t$	$\alpha_t = \frac{1}{2}$
Case 2	$q < t < 1-q$	$\alpha_t\left(\theta_\Delta\right) = \begin{cases} 0 & \text{if} \quad \theta_\Delta < t \\ \frac{1}{2} & \text{if} \quad \theta_\Delta \geq t \end{cases}$
Case 3	$1-q < t < q$	$\alpha_t\left(\theta_\Pi\right) = \begin{cases} \frac{1}{2} & \text{if} \quad \theta_\Pi \leq 1-t \\ 1 & \text{if} \quad \theta_\Pi > 1-t \end{cases}$
Case 4	$1-t \leq q \leq t$	$\alpha_t\left(\theta_\Delta, \theta_\Pi\right) = \begin{cases} 0 & \text{if} \quad \theta_\Delta < 1-\theta_\Pi \quad \text{and} \quad \theta_\Delta < t \\ \frac{1}{2} & \text{if} \quad \theta_\Delta = 1-\theta_\Pi \quad \text{or} \quad (\theta_\Delta \geq t \text{ and } \theta_\Pi \leq \text{1-t}) \\ 1 & \text{if} \quad \theta_\Delta > 1-\theta_\Pi \quad \text{and} \quad \theta_\Pi > 1-t \end{cases}$

Tab. 1: Four cases of fee-shifting

allocation of the court fee. In particular, Case 2 and Case 3 are radically different with respect to what parties know. In Case 2, only the defendant's evidence signal can go over the threshold, and hence fee-shifting only depends on the evidence submitted by the defendant. Therefore, the defendant knows for sure whether the court will shift the court fee to the plaintiff ($\alpha = 0$) or share it ($\alpha = \frac{1}{2}$). In contrast, the plaintiff cannot observe the defendant's signal before trial and hence cannot predict the allocation of the court fee. For the plaintiff fee-shifting is uncertain: $\alpha \in \left\{0, \frac{1}{2}\right\}$. This asymmetry in information about fee-shifting adds to the two-sided asymmetry of information about evidence and will be important in determining the parties' settlement behavior. Case 3 is the mirror image of Case 2 and displays an informational advantage for the plaintiff. Case 1 and 4 are instead symmetric with respect to information about fee-shifting because either there is no fee-shifting (Case 1) or fee-shifting depends on the evidence submitted by both parties (Case 4).

3 Settlement behavior in equilibrium

We are now ready to characterize the equilibrium of the settlement game that the parties play at time 2. After observing θ_Π, the plaintiff chooses a settlement demand p so as to maximize the expected gain Π given the defendant's settlement offer d. The plaintiff's gain has two components: the expected outcome of settlement—the first term, which occurs when $p \leq d$—and the expected outcome of litigation—the second term, when $p > d$ and the parties fail to settle. For each of these two components, the plaintiff's gain is calculated on all possible defendant's signals.[25] Similarly, the defendant minimizes the expected cost Δ.

$$\begin{aligned} \Pi\left(p\right) &= \int_{\{p \leq d\}} \frac{p+d}{2} d\theta_\Delta + \int_{\{p > d\}} \left[J - (1-\alpha)c\right] d\theta_\Delta \\ \Delta\left(d\right) &= \int_{\{p \leq d\}} \frac{p+d}{2} d\theta_\Pi + \int_{\{p > d\}} \left[J + \alpha c\right] d\theta_\Pi \end{aligned}$$

Since the parties submit their bids simultaneously, the Nash equilibrium is a pair $\{p, d\}$ of the plaintiff's demand and the defendant's offer, conditional on the fact that each party observes his or her own signal but not the signal of the other party. Therefore, in equilibrium, the plaintiff's demand p must be a function of θ_Π, and the defendant's offer d must be a function of θ_Δ. Moreover, in equilibrium, the parties' bids will not be equal to their signals due to the fact that overstating one's position reduces the probability of settlement for both parties (a common cost) while improving the settlement amount (a private benefit). The following linear transformations of the parties' signals will allow us to more easily visualize and analyze the results:[26]

Plaintiff's normalized evidence signal: $z_\Pi = \frac{\theta_\Pi}{q} \sim U\left[0, 1\right]$;

Defendant's normalized evidence signal: $z_\Delta = \frac{\theta_\Delta - q}{1-q} \sim U\left[0, 1\right]$.

[25] The set $\{p \leq d\}$ is simply the set of defendant's signals that, given the plaintiff's signal, result in bids $p \leq d$ and hence in settlement, similarly for the other set. In the defendant's payoff function these sets are defined analogously.

[26] Since the parties know q these transformations are well defined.

The same transformations also apply to the judgment, so that $J(z_\Pi, z_\Delta) = \frac{z_\Delta(1-q)+q+qz_\Pi}{2}$, and to the fee-shifting rules. (With a slight abuse of notation we keep using J and α for the new functions, the arguments are enough to avoid confusion.)

Case 1	$t \le q \le 1-t$	$\alpha_t = \frac{1}{2}$
Case 2	$q < t < 1-q$	$\alpha_t(z_\Delta) = \begin{cases} 0 & \text{if} \quad z_\Delta < \frac{t-q}{1-q} \\ \frac{1}{2} & \text{if} \quad z_\Delta \ge \frac{t-q}{1-q} \end{cases}$
Case 3	$1-q < t < q$	$\alpha_t(z_\Pi) = \begin{cases} \frac{1}{2} & \text{if} \quad z_\Pi \le \frac{1-t}{q} \\ 1 & \text{if} \quad z_\Pi > \frac{1-t}{q} \end{cases}$
Case 4	$1-t \le q \le t$	$\alpha_t(z_\Delta, z_\Pi) = \begin{cases} 0 & \text{if} \quad z_\Delta < 1 - \frac{q}{1-q} z_\Pi \quad \text{and} \quad z_\Delta < \frac{t-q}{1-q} \\ \frac{1}{2} & \text{if} \quad z_\Delta = 1 - \frac{q}{1-q} z_\Pi \quad \text{or} \quad \left(z_\Delta \ge \frac{t-q}{1-q} \text{ and } z_\Pi \le \frac{1-t}{q}\right) \\ 1 & \text{if} \quad z_\Delta > 1 - \frac{q}{1-q} z_\Pi \quad \text{and} \quad z_\Pi > \frac{1-t}{q} \end{cases}$

Tab. 2: Four cases of fee-shifting with normalized signals

Let $p = P(z_\Pi)$ be the plaintiff's settlement demand as a function of his or her normalized signal and $d = D(z_\Delta)$ be the defendant's offer. We proceed in the usual way (Chatterjee and Samuelson, 1983; Friedman and Wittman, 2006) and assume that the parties' bid functions are linear and increasing in their signals. Given monotonicity, we can define inverse functions of the bids. Accordingly, $P^{-1}(d)$ is the value of the plaintiff's signal such that the plaintiff's demand p is equal to the defendant's offer d. For a given defendant's demand d, if the plaintiff's signal is $z_\Pi \le P^{-1}(d)$, then $P(z_\Pi) \le d$ and the parties settle; otherwise, the parties litigate. Similarly, $D^{-1}(p)$ is the value of the defendant's signal such that the defendant's demand d is equal to the plaintiff's offer p. For a given plaintiff's offer p, if the defendant's signal is $z_\Delta \ge D^{-1}(p)$, then $D(z_\Delta) \ge p$ and the parties settle; otherwise, the parties litigate. Using these observations, we can write the parties' optimization problem:

$$\begin{aligned} \max_p \Pi(p) &= \max_p \left[\int_{D^{-1}(p)}^{1} \frac{p+D(z_\Delta)}{2} dx + \int_{0}^{D^{-1}(p)} \left[\frac{qz_\Pi + z_\Delta(1-q)+q}{2} - (1-\alpha)c \right] dz_\Delta \right] \\ \min_d \Delta(d) &= \min_d \left[\int_{0}^{P^{-1}(d)} \frac{P(z_\Pi)+d}{2} dy + \int_{P^{-1}(d)}^{1} \left[\frac{qz_\Pi + z_\Delta(1-q)+q}{2} + \alpha c \right] dz_\Pi \right] \end{aligned} \tag{2}$$

To find the parties' equilibrium bid functions for each fee-shifting rule t, we adapt the method used in Friedman and Wittman (2006) to our framework. We use the assumption of linearity, that is, we impose that the bids have the form $p = e + fz_\Pi$ and $d = a + bz_\Delta$. From there, we can write the inverse bid functions explicitly and substitute them into (2). Depending on the case, we substitute the appropriate formulation of α from Table 2 into (2). We then calculate the first-order conditions of the expected payoffs for the plaintiff and the defendant; given the discontinuities in α, we do this piecewise (the second order conditions are satisfied). Finally, from the first-order conditions we derive the unique pure-strategy piecewise linear bid functions, that is, we find the value of the coefficients a, b, e and f. We repeat the same procedure in each of the four cases in Table 2 to provide constructive proofs of the propositions that follow. The proofs are in the Appendix; here we provide intuitions and results.

From now on we assume that $\frac{1}{3} \le q \le \frac{2}{3}$, that is, we restrict the analysis to situations with balanced two-sided asymmetric information, where the difference in information between the parties is not too wide and there exist pure-strategy Nash equilibria. As q takes extreme values, we continuously approach the one-sided asymmetric information framework—with $q = 0$ only the defendant is informed and with $q = 1$ only the plaintiff is informed—and our pure-strategy equilibria fail.

3.1 Case 1: $t \leq q \leq 1 - t$

In Case 1, both parties face the same court fee $\frac{c}{2}$ and there is no fee-shifting. This case includes the American rule ($t = 0$).

Proposition 1. *If $t \leq q \leq 1 - t$ (Case 1), the equilibrium bid functions at the settlement stage are:*

$$\begin{aligned} P_1(z_\Pi) &= \tfrac{1}{2} - 3\left(\tfrac{5}{6} - q\right)c + \tfrac{1}{3}z_\Pi \\ &\quad \textit{truncated above at } D_1(1) \textit{ or below at } D_1(0) \\ D_1(z_\Delta) &= \tfrac{1}{6} + 3\left(q - \tfrac{1}{6}\right)c + \tfrac{1}{3}z_\Delta \\ &\quad \textit{truncated above at } P_1(1) \textit{ or below at } P_1(0) \end{aligned}$$

The American rule ($t = 0$) belongs to this case.

As expected, the parties' bids are linearly increasing in their signals. Depending on c, we can distinguish three cases. If $c < \frac{1}{6}$, then $P(z) > D(z)$ and the truncations are inessential because they occur in the trial region and hence do not affect the outcome.[27] If $c = \frac{1}{6}$, the bid functions overlap and there are no truncations. Finally, if $c > \frac{1}{6}$, then $P(z) < D(z)$ and the defendant offer is truncated above at $P_1(1)$, since the defendant will not offer more than the maximum plaintiff's demand. Similarly, the plaintiff's demand is truncated below at $D_1(0)$, since the plaintiff will not demand less than the minimum defendant's offer. These truncations are important because they occur in the settlement region—when $P(z_\Pi) \leq D(z_\Delta)$—and hence affect the parties' payoffs.

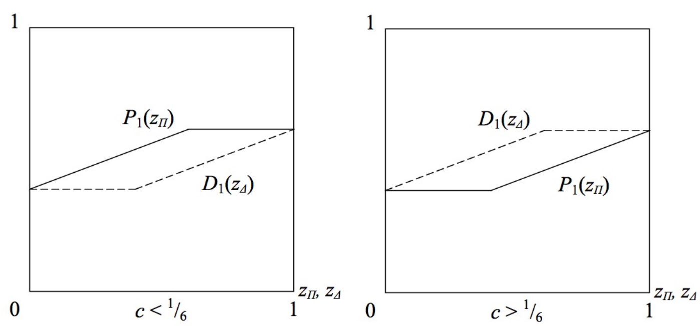


Fig. 5: Case 1 equilibrium bids

Suppose that $c > \frac{1}{6}$ and the defendant's signal is $z_\Delta = 0$; denote as z_Π^* the plaintiff's signal such that $P_1(z_\Pi^*) = D_1(0)$. If the plaintiff draws $z_\Pi \leq z_\Pi^*$, then the case settles. However, by demanding $P_1(z_\Pi^*) = D_1(0)$ instead of $P_1(z_\Pi) < D_1(0)$ for any signal less than z_Π^*, the plaintiff can improve the settlement amount without affecting the probability of litigation. In turn, it is a best response for the defendant to keep offering $D_1(0)$. The truncation of the defendant's bid is generated in a similar way.

This pure-strategy equilibrium breaks down if $q < \frac{1}{3}$ or $q > \frac{2}{3}$. A high value of q implies that the plaintiff is better informed than the defendant, since the variance of the plaintiff's signal is larger than the variance of the defendant's signal. For $q > \frac{2}{3}$, the defendant faces a lemons problem as the settlement he or she will be able to "buy" from the plaintiff will correspond to the low-value trials. These are lemons settlements that

[27] In this case, $P(z_\Pi)$ is truncated above in a region where litigation occurs for sure given that the defendant offers less than the plaintiff for any value of his or her signal. Therefore, in this region any plaintiff's demand above the defendant's offer is an equilibrium. Similarly, $D(z_\Delta)$ is truncated below, in the region where the plaintiff always demands more than what the defendant offers and hence the case goes to court. Yet, the truncations are irrelevant for the outcome since those cases go to trial and hence the bids are essentially unique.

the defendant will try to avoid by lowering his offer in response to the plaintiff's truncation. That is, offering $D(z_\Delta)$ is no longer a best response to the plaintiff demanding $P_1(z^*_\Pi)$ and the pure-strategy equilibrium collapses. When $q < \frac{1}{3}$, extreme asymmetric information affects the plaintiff's response to the defendant's truncations in a similar way.

The same conditions will be found in the other three cases and, in what follows, we will focus on the analysis of pure-strategy equilibria with $\frac{1}{3} \leq q \leq \frac{2}{3}$, when the parties's positions are not too unbalanced. This condition also guarantees that the bids respond in a straightforward way to an increase in the court fee: as c increases, the plaintiff reduces his or her demand to facilitate settlement and, likewise, the defendant increases his or her offer.

Figure 6 illustrates the parties settlement decision as a result of their equilibrium bids for $c < \frac{1}{6}$ (if $c > \frac{1}{6}$ the settlement line is above the diagonal). The parties litigate if $P_1(z_\Pi) > D_2(z_\Delta)$, that is, if the parties signals are above the settlement line $z_\Pi = z_\Delta + 6c - 1$. Relatively pessimistic parties (low z_Π and high z_Δ) settle, while relatively optimistic parties go to trial.

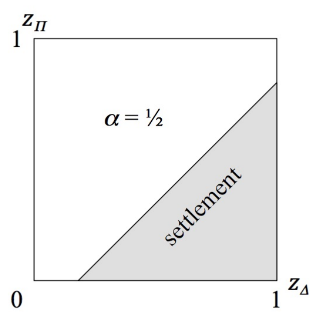


Fig. 6: Case 1: litigation and settlement ($c < \frac{1}{6}$)

3.2 Case 2: $q < t < 1 - q$

In Case 2, the values of t and q are such that there are two possible fee-shifting outcomes: $\alpha = 0$ (the plaintiff pays all costs) and $\alpha = \frac{1}{2}$ (each party pays half of the costs).

Proposition 2. *If $q < t < 1 - q$ (Case 2), the equilibrium bid functions at the settlement stage are:*

$$P_2(z_\Pi) = \begin{cases} \frac{1}{2} - 3(1-q)c + \frac{1}{3}z_\Pi \equiv \underline{P}_2(z_\Pi) & \text{if } z_\Pi < 6c - 1 + \frac{t-q}{1-q} \\ \frac{1}{2} - 3\left(\frac{5}{6} - q\right)c + \frac{1}{3}z_\Pi \equiv \overline{P}_2(z_\Pi) & \text{if } z_\Pi \geq 6c - 1 + \frac{t-q}{1-q} \end{cases}$$
truncated above at $D_2(1)$ or below at $D_2(0)$

$$D_2(z_\Delta) = \begin{cases} \frac{1}{6} + 3\left(q - \frac{1}{3}\right)c + \frac{1}{3}z_\Delta \equiv \underline{D}_2(z_\Delta) & \text{if } z_\Delta < \frac{t-q}{1-q} \\ \frac{1}{6} + 3\left(q - \frac{1}{6}\right)c + \frac{1}{3}z_\Delta \equiv \overline{D}_2(z_\Delta) & \text{if } z_\Delta \geq \frac{t-q}{1-q} \end{cases}$$
truncated above at $P_2(1)$ or below at $P_2(0)$

Since fee-shifting in Case 2 only depends on the defendant's signal, the defendant's offer shifts upwards at $z_\Delta = \frac{t-q}{1-q}$ (Figure 7), at the point where fee-shifting changes from $\alpha = 0$ to $\alpha = \frac{1}{2}$ and litigation becomes more expensive for the defendant (see Table 2), triggering a higher offer. In turn, the plaintiff cannot perfectly anticipate the fee-shifting decision by the court because the defendant's signal is private information prior to trial. However, the plaintiff knows than litigating against a defendant with a signal below the threshold $\frac{t-q}{1-q}$ is more costly than litigating against a defendant with a signal above that threshold.

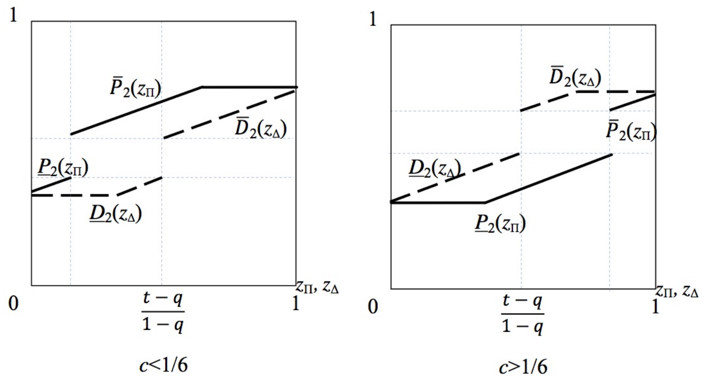


Fig. 7: Case 2 equilibrium bids

Anticipating the defendant's bidding strategy, the plaintiff's best response is to demand more in the latter case than in the former case. The lower segment of the plaintiff's demand, $\underline{P}_2(z_\Pi)$ in Figure 7, reflects the fact that trial ($p > d$) occurs only with defendants who have drawn a signal below the fee-shifting threshold $\frac{t-q}{1-q}$, which results in fee-shifting to the plaintiff ($\alpha = 0$). Therefore, the plaintiff's demand is relatively low. Instead, the upper segment of the plaintiff's demand, $\overline{P}_2(z_\Pi)$, reflects the fact that, at the margin, trial occurs with defendants who have drawn a signal above the threshold, which results in a shared court fee ($\alpha = \frac{1}{2}$). Trial is now cheaper for the plaintiff, which allows for more daring settlement demands.

Figure 8 illustrates the parties' settlement decisions in Case 2. The parties litigate if $P_2(z_\Pi) > D_2(z_\Delta)$, which gives the same settlement line as in Case 1: $z_\Pi = z_\Delta + 6c - 1$. If the parties go to trial, the court fee is allocated depending on the defendant's signal.

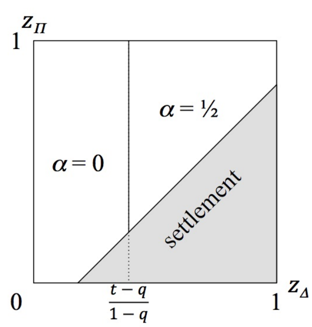


Fig. 8: Case 2: litigation and settlement ($c < \frac{1}{6}$)

3.3 Case 3: $1 - q < t < q$

Case 3 is the mirror image of Case 2. In Case 3, the feasible fee-shifting outcomes are $\alpha = \frac{1}{2}$ (each party pays half of the costs) and $\alpha = 1$ (the defendant pays all costs), and depend only on the plaintiff's signal.

Proposition 3. *If $1-q<t<q$ (Case 3), the equilibrium bid functions at the settlement stage are:*

$$P_3(z_\Pi) = \begin{cases} \frac{1}{2}-3\left(\frac{5}{6}-q\right)c+\frac{1}{3}z_\Pi & \equiv \underline{P}_3(z_\Pi) \quad \text{if} \quad z_\Pi \leq \frac{1-t}{q} \\ \frac{1}{2}-3\left(\frac{2}{3}-q\right)c+\frac{1}{3}z_\Pi & \equiv \overline{P}_3(z_\Pi) \quad \text{if} \quad z_\Pi > \frac{1-t}{q} \end{cases}$$
truncated above at $D_3(1)$ or below at $D_3(0)$

$$D_3(z_\Delta) = \begin{cases} \frac{1}{6}+3\left(q-\frac{1}{6}\right)c+\frac{1}{3}z_\Delta & \equiv \underline{D}_3(z_\Delta) \quad \text{if} \quad z_\Delta \leq 1-6c+\frac{1-t}{q} \\ \frac{1}{6}+3qc+\frac{1}{3}z_\Delta & \equiv \overline{D}_3(z_\Delta) \quad \text{if} \quad z_\Delta > 1-6c+\frac{1-t}{q} \end{cases}$$
truncated above at $P_3(1)$ or below at $P_3(0)$

The plaintiff's demand shifts upward at $z_\Pi=\frac{1-t}{q}$, at the point where fee-shifting changes in his or her favor; the defendant's offer shifts in response to the plaintiff's strategy, as illustrated in Figure 9.

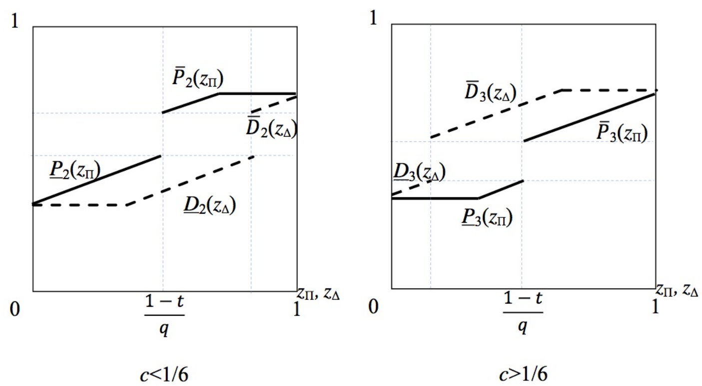


Fig. 9: Case 3 equilibrium bids

Figure 10 illustrates the parties settlement decision in Case 3. The parties litigate if $P_3(z_\Pi)>D_3(z_\Delta)$, which gives again the same settlement line as in Case 1: $z_\Pi=z_\Delta+6c-1$. If the parties go to trial, the court fee is allocated depending on the plaintiff's signal.

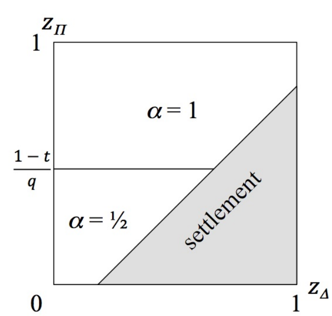


Fig. 10: Case 3: litigation and settlement ($c<\frac{1}{6}$)

3.4 Case 4: $1-t\leq q\leq t$

In Case 4, fee-shifting depends on both signals and hence the bids resemble a combination of the previous two cases. A party's bid shifts either in response to that party's own signal or in response to a shift in the other party's bid. Examining Case 4 yields two subcases, depending on the level of c.[28] Case 4A: if the court fee is smaller than a threshold ($c \leq \frac{1}{6}\frac{1-t}{q(1-q)}$), settlement occurs only in the south-east half of Figure 12, where the parties are relatively pessimistic about the trial outcome (the plaintiff has a low signal and the defendant has a high signal). In this case, the parties' bids have three parts, corresponding to the three possible fee-shifting outcomes.

Proposition 4. *If $1-t \leq q \leq t$ and $c \leq \frac{1}{6}\frac{1-t}{q(1-q)}$ (Case 4A), the equilibrium bid functions at the settlement stage are:*

$$P_{4A}(z_\Pi) = \begin{cases} \frac{1}{2} - 3(1-q)c + \frac{1}{3}z_\Pi \equiv \underline{P}_4(z_\Pi) & \text{if } z_\Pi < 6c - 1 + \frac{t-q}{1-q} \\ \frac{1}{2} - 3\left(\frac{5}{6} - q\right)c + \frac{1}{3}z_\Pi \equiv \dot{P}_4(z_\Pi) & \text{if } 6c - 1 + \frac{t-q}{1-q} \leq z_\Pi \leq \frac{1-t}{q} \\ \frac{1}{2} - 3\left(\frac{2}{3} - q\right)c + \frac{1}{3}z_\Pi \equiv \overline{P}_4(z_\Pi) & \text{if } z_\Pi > \frac{1-t}{q} \end{cases}$$

truncated above at $D_{4A}(1)$ or below at $D_{4A}(0)$

$$D_{4A}(z_\Delta) = \begin{cases} \frac{1}{6} + 3\left(q - \frac{1}{3}\right)c + \frac{1}{3}z_\Delta \equiv \underline{D}_4(z_\Delta) & \text{if } z_\Delta < \frac{t-q}{1-q} \\ \frac{1}{6} + 3\left(q - \frac{1}{6}\right)c + \frac{1}{3}z_\Delta \equiv \dot{D}_4(z_\Delta) & \text{if } \frac{t-q}{1-q} \leq z_\Delta \leq 1 - 6c + \frac{t-q}{1-q} \\ \frac{1}{6} + 3qc + \frac{1}{3}z_\Delta \equiv \overline{D}_4(z_\Delta) & \text{if } z_\Delta > 1 - 6c + \frac{t-q}{1-q} \end{cases}$$

truncated above at $P_{4A}(1)$ or below at $P_{4A}(0)$

Case 4B: as the court fee increases above the threshold ($c > \frac{1}{6}\frac{1-t}{q(1-q)}$), more cases settle and settlement occurs also in part of the north-west half of Figure 12. Given that c is high, only very optimistic parties litigate. Those cases are characterized by a high z_Π (optimistic plaintiff) and a low z_Δ (optimistic defendant), so that evidence is very precise ($z_\Delta - z_\Pi$ is low) and the court fee is never shared. Reflecting the two possible allocations of the court fee, $\alpha = 0$ and $\alpha = 1$, bids have now only two parts. (Note that with $t = 1$ the threshold condition becomes $c > \frac{1}{6}\frac{1-t}{q(1-q)} = 0$, so that the English rule falls in Case 4B.)

Proposition 5. *If $1-t \leq q \leq t$ and $c > \frac{1}{6}\frac{1-t}{q(1-q)}$ (Case 4B), the equilibrium bid functions at the settlement stage are:*

$$P_{4B}(z_\Pi) = \begin{cases} \frac{1}{2} - 3(1-q)c + \frac{1}{3}z_\Pi \equiv \underline{P}_4(z_\Pi) & \text{if } z_\Pi \leq 6c(1-q) \\ \frac{1}{2} - 3\left(\frac{2}{3} - q\right)c + \frac{1}{3}z_\Pi \equiv \overline{P}_4(z_\Pi) & \text{if } z_\Pi > 6c(1-q) \end{cases}$$

truncated above at $D_{4B}(1)$ or below at $D_{4B}(0)$

$$D_{4B}(z_\Delta) = \begin{cases} \frac{1}{6} + 3\left(q - \frac{1}{3}\right)c + \frac{1}{3}z_\Delta \equiv \underline{D}_4(z_\Delta) & \text{if } z_\Delta \leq 1 - 6cq \\ \frac{1}{6} + 3qc + \frac{1}{3}z_\Delta \equiv \overline{D}_4(z_\Delta) & \text{if } z_\Delta > 1 - 6cq \end{cases}$$

truncated above at $P_{4B}(1)$ or below at $P_{4B}(0)$

The English rule ($t = 1$) belongs to this case.

While the two components of the bids are the same as the two external components of the bids in Case 4A (the middle segment of the bids disappears), the thresholds are different.

Figure 11 illustrates these results. Figure 12 illustrates the parties' settlement decision in Case 4.[29] The parties litigate if $P_{4A}(z_\Pi) > D_{4A}(z_\Delta)$ or $P_{4B}(z_\Pi) > D_{4B}(z_\Delta)$, depending on whether c is below or above the threshold. Both cases give the same settlement line as in Case 1: $z_\Pi = z_\Delta + 6c - 1$.

[28] Note that these subcases depend on c and do not correspond to the two subcases drawn in Figure 4, which instead depend on q. The latter need to be distinguished in the analysis but yield the same bid functions and hence do not show up in the results presented here.

[29] Note further that Figure 12 is drawn for $q > \frac{1}{2}$; with $q < \frac{1}{2}$ the negative-slope line dividing $\alpha = 0$ from $\alpha = 1$ would start from above the north-west vertex. Note further that the settlement line lies above the diagonal if $c > \frac{1}{6}$.

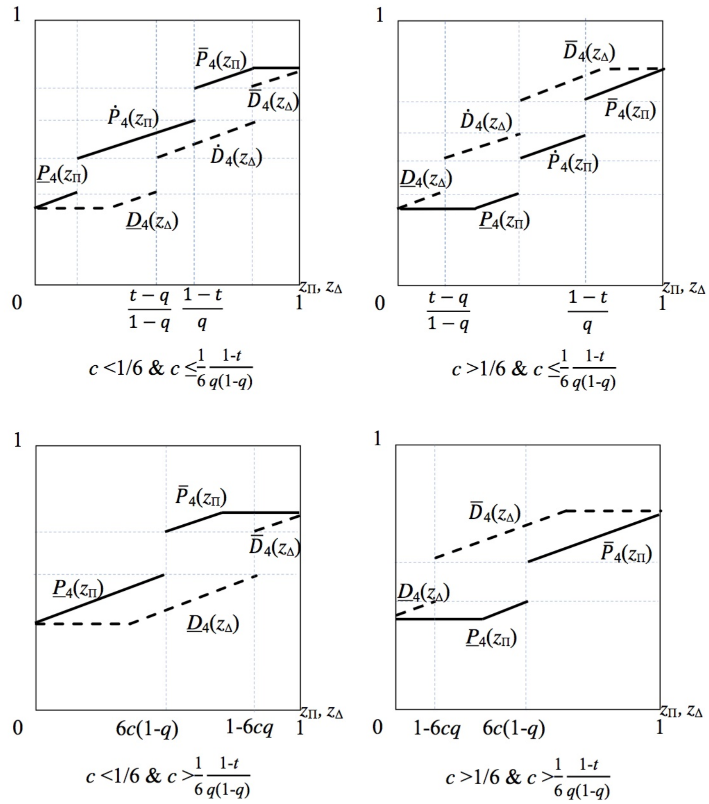
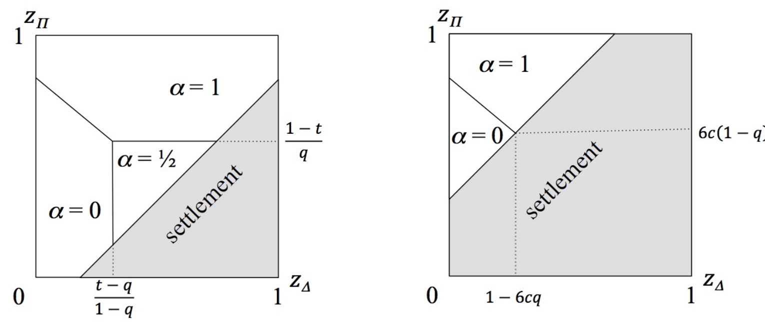


Fig. 11: Case 4 equilibrium bids: Case 4A (figures above) and Case 4B (figures below)

3.5 An illustration: American rule versus English rule in our model and in the divergent-prior model

To illustrate our results, it is instructive to compare the parties' settlement behavior under the American and the English rules. We can obtain the equilibrium bids for the American rule by setting $t = 0$ in the bids of Proposition 1; similarly, we obtain the equilibrium bids for the English rule by setting $t = 1$ in the bids of Proposition 5. For simplicity we focus in both cases on $q = \frac{1}{2}$, when the parties' have equal merits and the asymmetry of information between them is perfectly balanced.



Case 4A ($c \leq \frac{1}{6}\frac{1-t}{q(1-q)}$) Case 4B ($c > \frac{1}{6}\frac{1-t}{q(1-q)}$)

Fig. 12: Case 4: litigation and settlement with $q > \frac{1}{2}$

American rule with equal merits (Case 1, with $t = 0$ and $q = \frac{1}{2}$):

$$\begin{aligned} P_A(z_\Pi) &= \tfrac{1}{2} - c + \tfrac{1}{3} z_\Pi \\ &\quad \text{truncated above at } D_A(1) \text{ or below at } D_A(0) \\ D_A(z_\Delta) &= \tfrac{1}{6} + c + \tfrac{1}{3} z_\Delta \\ &\quad \text{truncated above at } P_A(1) \text{ or below at } P_A(0) \end{aligned}$$

English rule with equal merits (Case 4B, with $t = 1$ and $q = \frac{1}{2}$):

$$\begin{aligned} P_E(z_\Pi) &= \begin{cases} \frac{1}{2} - \frac{3}{2}c + \frac{1}{3} z_\Pi & \text{if} \quad z_\Pi \leq 3c \\ \frac{1}{2} - \frac{1}{2}c + \frac{1}{3} z_\Pi & \text{if} \quad z_\Pi > 3c \end{cases} \\ &\quad \text{truncated above at } D_E(1) \text{ or below at } D_E(0) \\ D_E(z_\Delta) &= \begin{cases} \frac{1}{6} + \frac{1}{2}c + \frac{1}{3} z_\Delta & \text{if} \quad z_\Delta \leq 1 - 3c \\ \frac{1}{6} + \frac{3}{2}c + \frac{1}{3} z_\Delta & \text{if} \quad z_\Delta > 1 - 3c \end{cases} \\ &\quad \text{truncated above at } P_E(1) \text{ or below at } P_E(0) \end{aligned}$$

Note that the results obtained under the American rule replicate those in Friedman and Wittman (2006).[30] Figure 13 shows what happens to the parties' bid functions when we move from the American rule to the English rule. Under the American rule both parties' (black) bids are continuous in their evidence signals. Instead, under the English rule, both the plaintiff's demand and the defendant's offer are discontinuous at the point where each party expects the court fee to be shifted from the plaintiff to the defendant. The left-hand portion of the parties' (grey) bids is lower than under the American rule: this is the region in which both parties expect the plaintiff to bear the entire court fee if the case went to trial. As a result both parties bid less than under the American rule. Similarly, the right-hand portion of the parties' bids is above the American-rule bids, since now the defendant is expected to bear the court fee. In both cases, the shift up- or downwards is equal to $\frac{c}{2}$, which is the difference between litigating under the American rule and litigating under the English rule.

What is important is that these shifts do not change the horizontal distance between the parties' bids, which remains equal to $6c - 1$. This distance is the crucial determinant of litigation. Parties litigate if the plaintiff's demand is above the defendant's offer, $P(z_\Pi) > D(z_\Delta)$, which occurs if the plaintiff's signal z_Π is at least at a distance $6c - 1$ to the right of the defendant's signal z_Δ. Since the parties adjust their

[30] To reproduce the results in Friedman and Wittman (2006) multiply c, z_Π and z_Δ by 2. This is necessary because we use c to denote the total court fee, while they use c to denote the individual court fee, so that the total is $2c$ in their framework. Moreover, with $q = \frac{1}{2}$, the pre-normalization signal space is of length $\frac{1}{2}$ in our model, while it is of length 1 in their model so that signals need to be scaled.

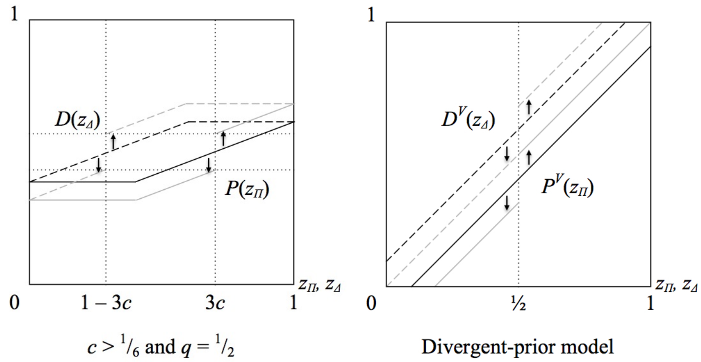


Fig. 13: American rule (black) and English rule (grey): our model (left) compared to the divergent-prior model (right)

bidding strategies to the fee-shifting rule, the American rule and the English rule yield the same probability of litigation.

To appreciate the implications of this result, compare it with the outcome that can be obtained with the naive bid functions used in the divergent-prior model (Landes, 1971; Gould, 1973; Posner, 1973; Shavell, 1982). In this model a party fails to consider that the other party might have received a different signal, and believes that his or her signal perfectly identifies the trial outcome; that is, the plaintiff believes that $J^V = z_\Pi$ while the defendant believes that $J^V = z_\Delta$.[31] A party's settlement bid is equal to his or her expected trial outcome net of the court fee.

American rule in the devergent-prior model:

$$\begin{aligned} P_A^V(z_\Pi) &= z_\Pi - \tfrac{c}{2} \\ D_A^V(z_\Delta) &= z_\Delta + \tfrac{c}{2} \end{aligned}$$

The plaintiff's demand is above the defendant's offer if the plaintiff's signal z_Π is at least at a distance c to the right of the defendant's signal, according to the familiar mutual-optimism condition for litigation with divergent priors $z_\Pi - z_\Delta > c$.[32] Although the magnitude is different, this result is not qualitatively different from ours: if the court fee increases, the parties will litigate less often. In contrast, the English rule yields radically different results. Now the plaintiff expects to pay nothing in case of victory at trial—that is,

[31] Note that these believes might also derive from the conviction that the other party must be bringing the same evidence—that is, that $z_\Pi = z_\Pi$—but that he or she makes mistakes in interpreting it (each party is convinced to have a correct and objective representation of the case).

[32] Note that c can be interpreted as the ratio of the court fee to the amount at stake, which we have normalized to 1

if $z_\Pi > \frac{1}{2}$—and expects to pay the entire court fee in case the defendant wins. Similarly for the defendant.[33]

$$\text{English rule in the devergent-prior model :}$$
$$P_E^V(z_\Pi) = \begin{cases} z_\Pi - c & \text{if} \quad z_\Pi < \frac{1}{2} \\ z_\Pi - \frac{c}{2} & \text{if} \quad z_\Pi = \frac{1}{2} \\ z_\Pi & \text{if} \quad z_\Pi > \frac{1}{2} \end{cases}$$
$$D_E^V(z_\Delta) = \begin{cases} z_\Delta & \text{if} \quad z_\Delta < \frac{1}{2} \\ z_\Delta + \frac{c}{2} & \text{if} \quad z_\Delta = \frac{1}{2} \\ z_\Delta + c & \text{if} \quad z_\Delta > \frac{1}{2} \end{cases}$$

Ignoring the cases in which the parties' signals are exactly equal to $\frac{1}{2}$, which have mass zero, we have three cases: if the parties signals are on the same side of the threshold $\frac{1}{2}$, then litigation occurs if $z_\Pi - z_\Delta > c$, as under the American rule; if $z_\Pi < \frac{1}{2} < z_\Delta$, then the parties never litigate irrespective of c as under the American rule;[34] finally, if $z_\Delta < \frac{1}{2} < z_\Pi$, then the parties always litigate irrespective of c, while under the American rule they settle some of the time.[35]

In the divergent-prior model, the litigation rate under the English rule is higher than under the American rule as the former exacerbates the effects of mutual optimism. The difference with our model comes from the fact that in the divergent-prior model a party fails to consider that the other party faces similarly improved prospects in case of victory and hence will adapt his or her bidding strategy. This is important because the cost of a more aggressive settlement posture is an increased probability of trial. This probability, however, also depends on the other party's posture. In our model, the parties take into account each other's strategies and hence fully appreciate the costs of more daring settlement bids. In contrast, in the divergent-prior model, a party blindly responds to his or her signal and does not act strategically.

4 The amount of litigation

Straightforward inspection of the bid functions derived in the previous section reveals that, as the court fee c increases, the plaintiff's demand decreases and the defendant's offer increases. The parties strategic postures converge and settlement becomes easier. Instead, if the merits of the case q increase, both the plaintiff's demand and the defendant's offer increase. Therefore, we expect the merits of the case not to affect the probability with which a case goes to trial. A change in the fee-shifting rule t does not shift the bids but changes the threshold at which a party shifts to a higher bid in Cases 2, 3 and 4A; in Case 4B this threshold depends exclusively on c and q. Thus, also the fee-shifting rule should not affect the probability that a case goes to trial. We now formalize these intuitions.

Litigation occurs if $P(z_\Pi) > D(z_\Delta)$. By substituting the various bid functions, this condition can be

[33] This representation of the English rule differs from the existing literature because we allow the award to determine fee-shifting; we do so in order to keep the framework of analysis aligned to our model. Consequently, while the literature finds no difference between the English and the American rule in the divergent-prior model with uncertainty about the award (Shavell, 1982), we do. This shows that the difference between uncertainty about the award and uncertainty about the probability of victory is unimportant, what counts is whether fee-shifting depends on the uncertain variable. More importantly, note that our results do not depend on this reinterpretation of the English rule and that they are preserved in a model with uncertainty about the probability of victory, where the literature does find a difference between the English and the American rule while we, again, do not. We examine uncertainty about the probability of victory in Section 7. Visualization of the results is easier in the model with uncertainty about the award while the intuitions are the same; hence we chose to discuss our results in this model.

[34] In this case, the mutual optimism condition is $z_\Pi - z_\Delta > 2c$ under the English rule and $z_\Pi - z_\Delta > c$ under the American rule. These conditions are never satisfied because $z_\Pi < z_\Delta$.

[35] In this case, the mutual-optimism condition for litigation under the English rule is $z_\Pi - z_\Delta > 0$, which in always satisfied because $z_\Pi > z_\Delta$. The condition is $z_\Pi - z_\Delta > c$ under the American rule, which is satisfied only if the difference is large enough.

rewritten in an identical way in all four cases as

$$z_\Pi - z_\Delta > 6c - 1 \tag{3}$$

The litigation condition can be interpreted as a rational equivalent to the mutual-optimism condition, where the divergence comes from asymmetric information. Litigation occurs in the region of the unit square above the settlement line $z_\Pi = z_\Delta + 6c - 1$ in all four cases (see Figures 6, 8, 10, 12) and the probability of litigation—that is, the probability that, given any defendant's signal, the plaintiff's signal is such that the plaintiff's demand is above the defendant's offer—is simply the area of this region:

$$L(c) = \int_0^1 \Pr\left[P(z_\Pi) > D(z_\Delta)\right] dz_\Delta = \int_0^1 \Pr\left[z_\Pi > z_\Delta + 6c - 1\right] dz_\Delta$$

Proposition 6. *The probability of litigation is given by:*

$$L(c) = \begin{cases} 1 - \frac{(6c)^2}{2} & \textit{if} \quad c \leq \frac{1}{6} \\ \frac{(2-6c)^2}{2} & \textit{if} \quad \frac{1}{6} < c \leq \frac{1}{3} \\ 0 & \textit{if} \quad c > \frac{1}{3} \end{cases}$$

In particular, L decreases in c but is independent of q and t.

Proposition 6 confirms well-understood results: the probability of litigation decreases in the cost of litigation and increases in the amount at stake. Our variable c captures the amount of the court fee relative to the amount at stake, which is normalized to 1. Hence c might increase because the court fee increases or because the amount at stake decreases, which is in line with a positive effect of an increase of the amount at stake on the probability of litigation.

Proposition 6 also proves new results: the probability of litigation does not depend on the merits of the case and the fee-shifting rule, although they affect the parties' bid functions. The reason is that the parties' adjust their bidding strategies to changes in q and t, in order to capture the greatest possible share of the joint gains from settlement, which only depend on c. Changes in t and q shift both bids by the same amount or affect the thresholds at which a shift occurs but leave the horizontal distance between the parties' bids unaltered. Independence of q also means that the probability of litigation does not change if we vary the degree to which the parties are asymmetrically informed, as long as the game remain sufficiently balanced ($\frac{1}{3} \leq q \leq \frac{2}{3}$).[36]

5 Characteristics of litigated cases

5.1 Case selection

Although the amount of litigation is not affected by the merits of the case, changes in q do affect the composition of cases that go to trial. We look at the density of the judgment J conditional on the case going to trial.

Proposition 7. *The density of the trial judgment J conditional on litigation is a triangle if $c \leq \frac{1}{6}$ and a tent if $c > \frac{1}{6}$, with vertex at $J = q$.*

[36] Note that this result can be compared to Proposition 3 in Daughety and Reinganum (1994). In their model the degree of asymmetric information is captured by the distance between the good and the bad signal (high damages minus low damages for the plaintiff and high probability of liability minus low probability of liability for the defendant). They find that varying the degree of asymmetric information of the parties affects the probability of litigation.

This density is the same as in Friedman and Wittman (2006) if $q = \frac{1}{2}$. Note that the court fee c directly determines which cases go to trial. The effect of q is due to the fact that it determines the distribution of the parties' signals and, hence, indirectly, puts restrictions on the feasible judgments. Instead, t is irrelevant because the fee-shifting rule does neither affect the parties' choice between settlement and trial nor the court decision on the merits. The modal judgment corresponds to the true merits of the case but, on average, judgments are clearly biased because the court acts in a non-Bayesian way (hence this result would be reversed in a different model).

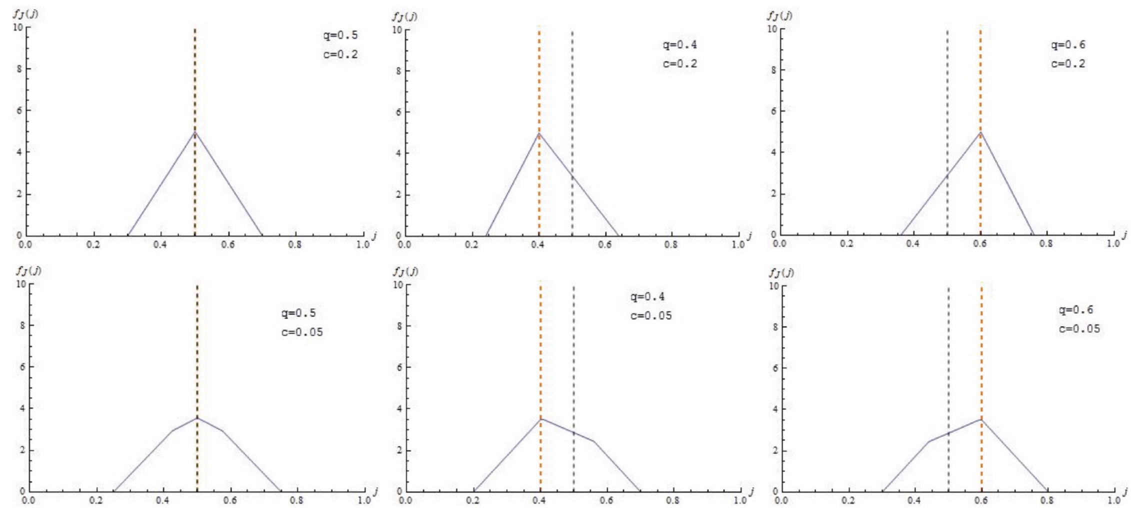


Fig. 14: Density of the judgment conditional on litigation (top: $c \leq \frac{1}{6}$, bottom: $c > \frac{1}{6}$)

5.2 Accuracy: fairness and incentives for primary behavior

Even if fee-shifting does not affect the probability of settlement, it determines the amount at which the parties settle and hence might bring the outcome of a dispute either closer to or further away from the true merits of the case. In so far as the merits of the case q reflect the outcome that is considered just or reflect the proper legal sanction on primary behavior, accuracy is important (Cooter and Rubinfeld, 1989; Bebchuk and Chang, 1996; Katz and Sanchirico, 2012).

To examine the distributional effects of fee-shifting let us define a new variable $G \equiv J + \left(\alpha - \frac{1}{2}\right) c$ that captures both the decision on the merits and fee-shifting. (Note that $G = J$ if the fee is not shifted, that is, with $\alpha = \frac{1}{2}$.) Note that the plaintiff receives $G - \frac{c}{2}$ and the defendant pays $G + \frac{c}{2}$. The settlement amount $S \equiv \frac{p+d}{2}$ includes fee-shifting only implicitly, since we have shown that the parties' bids respond to the possibility that the court will shift the court fee. Ex ante, before the parties collect evidence, the expected outcome of a conflict is:

$$E = \begin{cases} \int_0^{1-6c} \int_0^1 G dz_\Pi dz_\Delta + \int_{1-6c}^1 \left(\int_0^{6c-1+z_\Delta} S dz_\Pi + \int_{6c-1+z_\Delta} G dz_\Pi \right) dz_\Delta & \text{if} \quad c \leq \frac{1}{6} \\ \int_0^{2-6c} \left(\int_0^{6c-1+z_\Delta} S dz_\Pi + \int_{6c-1+z_\Delta} G dz_\Pi \right) dz_\Delta + \int_{2-6c}^1 \int_0^1 S dz_\Pi dz_\Delta & \text{if} \quad c > \frac{1}{6} \end{cases} \tag{4}$$

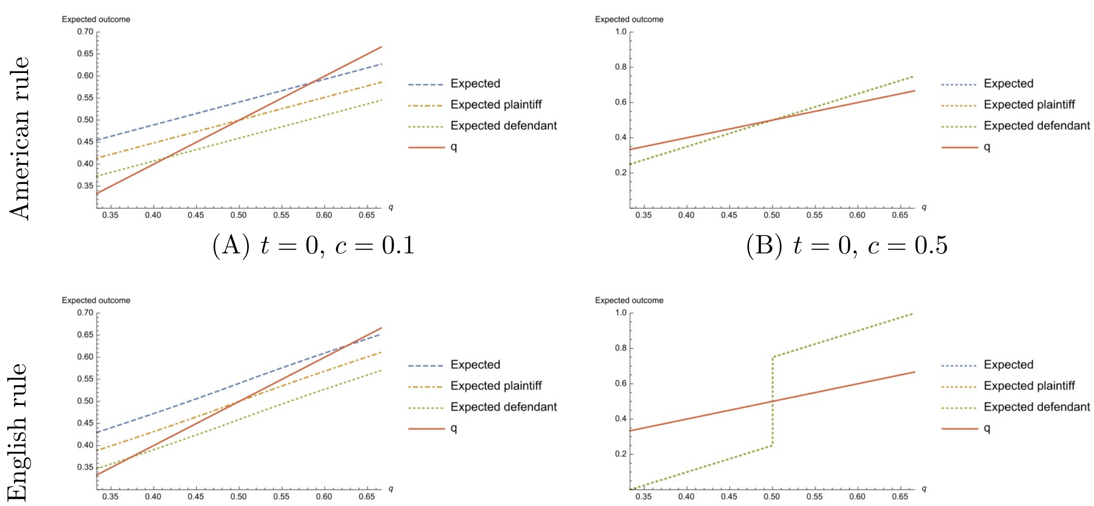
The limits of integration are easily derived from the areas of settlement and litigation in the unit squares in Figures 6, 8, 10, and 12. The two formulations capture two cases: in the first the settlement line is below the diagonal, while in the second the settlement line is above the diagonal. The parties' expected payoffs are readily obtained by subtracting the expected court fee. Therefore, the plaintiff expects to receive $E - L(c)\frac{c}{2}$, while the defendant expects to pay $E + L(c)\frac{c}{2}$. Note that the parties' expected payoffs sum up to zero only when all cases settle. Since $L(c)$ does not depend on t or q, we can focus on the calculation of E.

While being conceptually straightforward, the calculation of the expected outcome is computationally demanding as the variable G depends on the fee-shifting decision α, which in turn depends on the fee-shifting rule t, and hence its specification varies across the four subcases; likewise, since the bids change in the four subcases, the variable S is sensitive to the fee-shifting rule t. These calculations, relegated to the Appendix, allow us to simulate numerically the expected outcome of disputes and obtain two general results.

Ideally, the outcome of adjudication, which is based on the evidence collected by the parties, should be as close as possible to the true merits of the case, which the court does not observe. Therefore, a natural measure of accuracy of the judicial system is the distance between the expected outcome E and the merits q.

Proposition 8. *Whether the American or the English rule produces more accurate outcomes depends on the court fee.*

Visual inspection of Figure 15 shows that if the court fee c is small, the English rule brings the outcome of the case closer to the merits of the case and hence yields more accurate outcomes. Compare graphs A and C. This is due to the fact that fee-shifting tends to correct the court bias against the winning party. If instead the court fee is large, the English rule overshoots on the losing party and polarizes settlement bringing the expected outcome further aways from q than the American rule does. This is more so as q moves away from $\frac{1}{2}$. Note also that the parties' expectations converge in the latter case, since the court fee is so high that all cases settle.[37] From a different angle: increasing court fees make the American rule more accurate (compare A and B) while making the English rule less accurate (compare C and D). Fee-shifting does not seem per se to improve the accuracy of adjudication. The outcome depends heavily on a combination of other factors.



(A) $t = 0$, $c = 0.1$ (B) $t = 0$, $c = 0.5$

(C) $t = 1$, $c = 0.1$ (D) $t = 1$, $c = 0.5$

Fig. 15: Outcomes

Proposition 9. *The optimal fee-shifting rule may be an intermediate rule* $0 < t < 1$.

Specific cases require different fee-shifting rules. Figure 16 shows the level of accuracy of different fee-shifting rules as measured by the square distance between expected outcome and merits. The most accurate fee-shifting rule in this specific case—which involves both litigation and settlement, since $c < \frac{1}{3}$—is neither

[37] Note also that the outcome is always perfectly accurate if $q = \frac{1}{2}$ as in Friedman and Wittman (2006).

the American nor the English rule. An intermediate fee-shifting rule with $t = 0.75$ fares better than the two extremes.

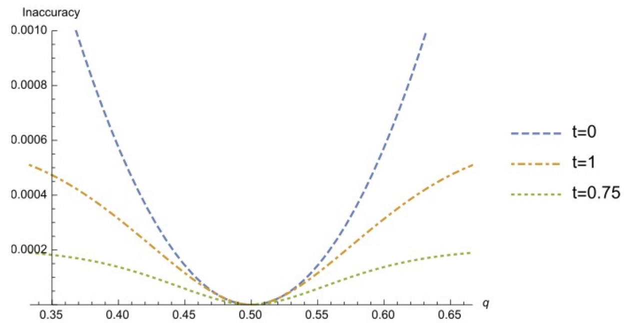


Fig. 16: A case of maximal accuracy with intermediate fee-shifting ($c = 0.25$; inaccuracy= $(E - q)^2$)

5.3 Decisions to file and contest lawsuits

Although this is not part of the model, here we offer some considerations on how fee shifting may affect the plaintiff's filing decision and the defendants decision to contest the plaintiff's claim.[38] Think of a party deciding whether to entrust his or her case to a lawyer and hence formally instate a legal conflict. This decision entails some costs that are incurred before evidence is collected. Therefore, it is instructive to look at the expected value of a suit from an ex ante perspective, that is, before collecting evidence. As Figure 17 shows, fee-shifting, combined with a particularly high court fee, can bring the plaintiff's expected value of litigation and settlement below 0 (the plaintiff might not file) and, symmetrically, the cost for the defendant above 1 (the defendant might not contest the plaintiff's claim). Note that, for simplicity, we have chosen a value of $c > \frac{1}{3}$, so that the plaintiff gains what the defendant pays because all cases settle, yet there is a shift corresponding to fee-shifting which is due to the fact that settlement mimics the outcome of the trial.

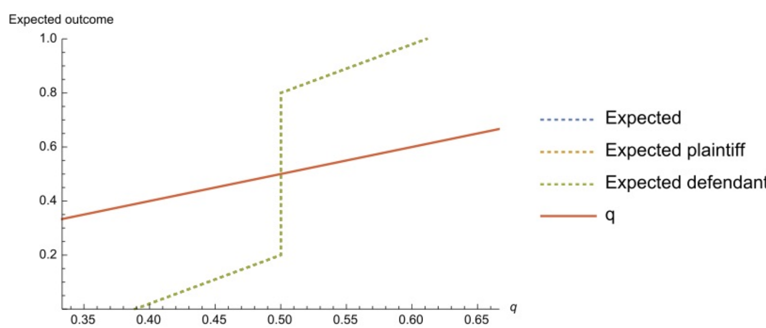


Fig. 17: Expected outcome with $c = 0.6$ and $t = 0.7$

Thus, fee-shifting might restrict filing although it does not affect settlement rates. Cases that are filtered out are those with unequal merits (low q or high q). The larger the fee-shifting parameter t the narrower the range of cases that are not filtered out. This implies that, given the same costs c, courts under the English rule see not only fewer cases but also more balanced cases than courts under the American rule.

[38] For analyses of filing decisions see Shavell (1982); P'ng (1983); Nalebuff (1987).

6 Augmented model: endogenous expenditures on lawyers

In this section we introduce an augmented model in which parties can choose a lawyer to assist them. The lawyer's fee $\lambda > 0$, contrary to the court fee c, is non-refundable under fee-shifting. In virtually all legal systems, lawyers' fees are not fully refundable. The refundable part is usually capped, limited to certain categories of costs or predetermined (Reimann, 2012), so that the refundable part of a lawyer's fee can be considered as a fixed amount and subsumed under c. The variable λ captures the value of the non-refundable lawyer's fee, which to a certain extent can be determined by the parties through their choice of a lawyer. The fee reflects a lawyer's ability to influence the court's interpretation of the evidence. A capable lawyer (high λ) is able to undermine the significance of evidence brought by the other party and boost the weight of his or her own evidence. (We do not model the principal-agent problem that characterizes the lawyer-client relationship.)

As a result of the lawyers' efforts, the court sees the signal θ_Π but interprets it as $\hat{\theta}_\Pi = \beta\theta_\Pi$, where $\beta > 1$ if the plaintiff's lawyer has more weight than the defendant's lawyer (and vice versa when $\beta < 1$). Symmetrically, for the defendant's signal, we have $\hat{\theta}_\Delta = (\theta_\Delta - q)\frac{1-\beta q}{1-q} + \beta q$. Intuitively, we must have $\beta = 1$ if the parties spend the same amounts on lawyers ($\lambda_\Pi = \lambda_\Delta$), that is, with equally able lawyers, neither party is able to sway the court. With equal merits ($q = \frac{1}{2}$), the party with the better lawyer is able to sway the court. If instead the parties' merits differ ($q \neq \frac{1}{2}$), a lawyer's weight in court depends on the combined effect of the merits of the case and the relative abilities of the lawyers. The following simple formulation captures these ideas:

$$\beta = \frac{\lambda_\Pi}{q\lambda_\Pi + (1-q)\lambda_\Delta}$$

For convenience, let us define $\hat{q} \equiv \beta q$ and notice that $\hat{q} \in [0,1]$. Since $\hat{\theta}_\Pi$ and $\hat{\theta}_\Delta$ are linear transformations of θ_Π and θ_Δ, they are uniformly distributed on the intervals $\hat{\theta}_\Pi \in [0,\hat{q}]$ and $\hat{\theta}_\Delta \in [\hat{q},1]$. Therefore, the augmented model preserves the analysis of the previous sections by simply replacing q with $\hat{q}$. Note that this formulation naturally implies that a capable lawyer wins arguments at the margin—that is, in the neighborhood of q—and hence effectively expands the signal space for his or her client.

The timing of the game is now as follows:

Time 0: Choice of lawyer. Both parties jointly observe the quality of the plaintiff's case q and simultaneously decide which lawyer $\lambda > 0$ to hire.

Time 1: Evidence collection. Both parties jointly observe the lawyers' abilities λ_Π and λ_Δ and the distribution of the evidence. The plaintiff privately draws a signal $\hat{\theta}_\Pi \sim U[0,\hat{q}]$; simultaneously, the defendant privately draws a signal $\hat{\theta}_\Delta \sim U[\hat{q},1]$.

Time 2: Settlement negotiations. At the settlement stage, the parties make simultaneous bids as in the basic model.

Time 3: Adjudication and fee-shifting. At trial, the court receives evidence from the parties's lawyers and see the signals $\hat{\theta}_\Pi$ and $\hat{\theta}_\Delta$; based on them, the court adjudicates the case and shifts the court fee as in the basic model.

When choosing their lawyers the parties face the following payoffs:

$$\begin{aligned}\hat{\Pi} &= E(\hat{q}) - L(c)\tfrac{c}{2} - \lambda_\Pi \\ \hat{\Delta} &= E(\hat{q}) + L(c)\tfrac{c}{2} + \lambda_\Delta\end{aligned}$$

The first term is the expected payment that the plaintiff receives, also accounting for settlement and fee-shifting as defined in (4); the second term is the expected court fee; the third term is the lawyer's fee. Note that the second term is independent of q and hence does not affect the choice of lawyer. The plaintiff maximizes $\hat{\Pi}$ and the defendant minimizes $\hat{\Delta}$ so that the first order conditions yield:

$$\lambda_{\Delta} \quad = \quad \frac{1}{E'(\hat{q})}\frac{(q\lambda_{\Pi}+(1-q)\lambda_{\Delta})^2}{q(1-q)} \quad = \quad \lambda_{\Pi}$$

which implies $q = \hat{q}$ and hence:[39]

$$\lambda_{\Pi} = \lambda_{\Delta} = q\,(1-q)\,E'\,(q)$$

The factor $q\,(1-q)$ is increasing in the uncertainty of the outcome of adjudication and is maximal at $q = \frac{1}{2}$. The last factor is nearly constant in q and does depend on c and t, both of which increase it.

Proposition 10. *Expenditures on lawyers*

1. *increase in the uncertainty of the case; that is, increase in q for $q < \frac{1}{2}$ and decrease in q for $q > \frac{1}{2}$;*

2. *increase in the court fee, c;*

3. *increase in fee-shifting, t.*

The last two points can be easily understood by noting that both the court fee and fee-shifting raise the stakes of the litigation game. This dominates the decrease in litigation rates due to increases in c. In addition, an increase in c magnifies the effect of t and vice versa. The presence of discontinuities in the expected payoff functions for certain values of c and t only reinforces these results.

7 Extensions

7.1 Uncertainty about the probability of victory

Our analysis has focused so far on uncertainty about the award. Here we show that our results are valid also in a model with uncertainty about the probability of victory. For instance, the parties may litigate about who owns a disputed asset of value equal to 1; in this case, the case is about whether the plaintiff or the defendant owns the asset—that is, who will receive 1—and not about the amount of the award.

To capture this scenario, we define a new variable J^W which can take the following values:

$$J^W\,(\theta_{\Pi},\theta_{\Delta}) \quad = \quad \begin{cases} \eta J\,(\theta_{\Pi},\theta_{\Delta}) & \text{if} \quad \theta_{\Pi} < 1-\theta_{\Delta} \\ \frac{1}{2} & \text{if} \quad \theta_{\Pi} = 1-\theta_{\Delta} \\ 1-\eta+\eta J\,(\theta_{\Pi},\theta_{\Delta}) & \text{if} \quad \theta_{\Pi} > 1-\theta_{\Delta} \end{cases}$$

If $\eta = 1$ we obtain the previous model with uncertainty about the amount of the award. If $\eta = 0$, the model captures uncertainty over the probability of winning an award of certain value equal to 1. In this case, the judgment can only take three values: 1 when the plaintiff's evidence is stronger than the defendant's evidence and the court assigns ownership of the disputed asset to the plaintiff; 0 when the opposite occurs and the court assigns the asset to the defendant; and, finally, $\frac{1}{2}$ when the asset is split because evidence is indecisive. The greater the plaintiff's signal the more likely it is that the plaintiff wins, and vice versa for

[39] The second order conditions are verified since $E'\,(q)$ is nearly linear and hence $E''\,(q)$ is zero or close to zero. Hence, the dominant term is the second derivative of $\hat{q}$, which has the right sign.

the defendant. When η takes intermediate values we obtain a model with some uncertainty both about the amount at stake and about the probability of victory.

This model includes a discontinuity in the judgment function which adds to the discontinuity in the fee-shifting function. To keep the analysis simple, we focus on the comparison between the American rule ($t = 0$) and the English rule ($t = 1$) when the parties have equal merits ($q = \frac{1}{2}$), as in Section 3.5. The analysis yields step-wise bid functions similar to the basic model.

Proposition 11. *With uncertainty about the probability of victory, the equilibrium bid functions at the settlement stage are:*

American rule with equal merits ($q = \frac{1}{2}$) and uncertainty about the probability of victory:

$$P_A^W(z_\Pi) = \begin{cases} \frac{\eta}{2} - c + \frac{\eta}{3} z_\Pi & \text{if } z_\Pi \leq 3\frac{c}{\eta} \\ 1 - \frac{\eta}{2} - \frac{1}{2}c + \frac{\eta}{3} z_\Pi & \text{if } z_\Pi > 3\frac{c}{\eta} \end{cases}$$

truncated above at $D_A^W(1)$ or below at $D_A^W(0)$

$$D_A^W(z_\Delta) = \begin{cases} \frac{\eta}{6} - c + \frac{\eta}{3} z_\Pi & \text{if } z_\Pi \leq 1 - 3\frac{c}{\eta} \\ 1 - \frac{5\eta}{6} - \frac{1}{2}c + \frac{\eta}{3} z_\Pi & \text{if } z_\Pi > 1 - 3\frac{c}{\eta} \end{cases}$$

truncated above at $P_A^W(1)$ or below at $P_A^W(0)$

English rule with equal merits ($q = \frac{1}{2}$) and uncertainty about the probability of victory:

$$P_E^W(z_\Pi) = \begin{cases} \frac{\eta}{2} - \frac{3}{2}c + \frac{\eta}{3} z_\Pi & \text{if } z_\Pi \leq 3\frac{c}{\eta} \\ 1 - \frac{\eta}{2} - \frac{1}{2}c + \frac{\eta}{3} z_\Pi & \text{if } z_\Pi > 3\frac{c}{\eta} \end{cases}$$

truncated above at $D_E^W(1)$ or below at $D_E^W(0)$

$$D_E^W(z_\Delta) = \begin{cases} \frac{\eta}{6} - \frac{1}{2}c + \frac{\eta}{3} z_\Pi & \text{if } z_\Pi \leq 1 - 3\frac{c}{\eta} \\ 1 - \frac{5\eta}{6} - \frac{2}{3}c + \frac{\eta}{3} z_\Pi & \text{if } z_\Pi > 1 - 3\frac{c}{\eta} \end{cases}$$

truncated above at $P_E^W(1)$ or below at $P_E^W(0)$

Note that, as expected, with $\eta = 1$ the parties' bid functions are as in the basic model in Section 3.5.[40] Similarly, if we set $\eta = 1$, the mutual optimism condition for litigation is the same as the basic model:

$$z_\Pi - z_\Delta > \frac{6}{\eta}c - 1 \tag{5}$$

Our findings shed new light on the question whether fee-shifting affects the probability of litigation in different ways depending on whether uncertainty revolves around the amount of damages rather than the probability of being found liable. While previous literature supports this result (Katz and Sanchirico, 2012, p. 15), the mutual optimism condition in 5 shows that this is not the case in a model of balanced two-sided asymmetric information, where the difference in how well the parties are informed is not too large. The mutual optimism condition does not depend on the fee-shifting rule for any level of η, that is, also when we allow (some) uncertainty about the probability of victory. It is worth stressing that we have allowed fee-shifting to depend on the judgment also in the model with uncertainty about the amount of the award (contrary to what is common in the literature), thereby staking the deck against our main claim.

[40] As we discuss in the Appendix, the equilibria break down for some values of η and c for the same reasons that cause a breakdown in the basic model for values of q below $\frac{1}{3}$ and above $\frac{2}{3}$. This however does not affect our results: in all those cases in which our pure-strategy equilibria hold, there is no difference in litigation rates between the American and the English rule.

7.2 Fee-shifting based on the margin of victory

To model endogenous fee-shifting we have so far fixed the notion of victory at $\frac{1}{2}$ and we have maintained that, with sufficiently precise evidence, if $J < \frac{1}{2}$ the plaintiff pays the court fee, while if $J > \frac{1}{2}$ the defendant pays the court fee. If $J = \frac{1}{2}$ there is no scope for fee-shifting. Bebchuk and Chang (1996) offer a model of endogenous fee-shifting based on a more general formulation of the margin of victory. In this model, if $J < \frac{m}{2}$ the plaintiff pays the court fee, while if $J > \frac{2-m}{2}$ the defendant pays the court fee; if $\frac{m}{2} \leq J \leq \frac{2-m}{2}$ there is no scope for fee-shifting.

In this section, we show that our results remain valid in this model. For simplicity and to keep our framework as close as possible to Bebchuk and Chang (1996) we omit to consider the precision of the evidence, which was an important variable in our previous analysis. We need to redefine the variable α:

$$\alpha_m^M(\theta_\Pi, \theta_\Delta) = \begin{cases} 0 & \text{if} \quad \theta_\Pi + \theta_\Delta < m \\ \frac{1}{2} & \text{if} \quad m \leq \theta_\Pi + \theta_\Delta \leq 2-m \\ 1 & \text{if} \quad \theta_\Pi + \theta_\Delta > 2-m \end{cases}$$

where $m \in [0,1]$ is the margin-of-victory threshold, with $m = 0$ for the American rule and $m = 1$ for the English rule. After normalizing the signals we have:

$$\alpha_m^M(z_\Pi, z_\Delta) = \begin{cases} 0 & \text{if} \quad z_\Pi q + z_\Delta (1-q) + q < m \\ \frac{1}{2} & \text{if} \quad m \leq z_\Pi q + z_\Delta (1-q) + q \leq 2-m \\ 1 & \text{if} \quad z_\Pi q + z_\Delta (1-q) + q > 2-m \end{cases}$$

Although fee-shifting is governed by a different formula, the structure of the game is the same as in the basic model and yields similar step-wise bid functions.

Proposition 12. *With fee-shifting based on the margin of victory, the equilibrium bid functions at the settlement stage are:*

$$P^M(z_\Pi) = \begin{cases} \ldots \\ \ldots \\ \ldots \end{cases}$$

truncated above at $D_M(1)$ *or below at* $D_M(0)$

$$D^M(z_\Delta) = \begin{cases} \ldots \\ \ldots \\ \ldots \end{cases}$$

truncated above at $P_M(1)$ *or below at* $P_M(0)$

The main result concerning the irrelevance of fee-shifting for the probability of settlement remains valid...

8 Conclusion

We have introduced a new model of litigation with two-sided asymmetric information and endogenous fee-shifting. Crucially, the decision to shift the litigation costs is different from the judgment on the merits of the case and is based on the quality of the evidence submitted by the parties. We have demonstrated that, although the parties might be asymmetrically informed to different degrees, if their positions are balanced, fee-shifting does not affect the settlement rate.

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Appendix

A Equilibrium bid functions

Equilibrium bids are derived starting from each party's expected payoff. The plaintiff chooses the optimal settlement demand $p = P(z_\Pi)$ as a function of the normalized signal z_Π by maximizing the expected gain Π; similarly, the defendant chooses the optimal settlement offer $d = D(z_\Delta)$ as a function of the signal z_Δ by minimizing the expected cost Δ, where:

$$\begin{aligned}\Pi &= \tfrac{1}{2}\int_{D^{-1}(p)}^{1}[p + D(x)]\,dx + \tfrac{1}{2}\int_{0}^{D^{-1}(p)}[qz_\Pi + x(1-q) + q - 2(1-\alpha(x, z_\Pi))c]\,dx \\ \Delta &= \tfrac{1}{2}\int_{0}^{P^{-1}(d)}[P(y) + d]\,dy + \tfrac{1}{2}\int_{P^{-1}(d)}^{1}[qy + z_\Delta(1-q) + q + 2\alpha(y, z_\Delta)c]\,dy\end{aligned}$$

The first integral in Π and Δ is the expected amount for which the parties settle, while the second integral is the expected trial outcome. Following Friedman and Wittman (2006), we assume that the bid function is linear and monotonically increasing in the signals. Therefore, the inverses and the derivatives of the bid functions can be explicitly calculated. Note that:

$$\begin{array}{lllll} P\left(P^{-1}(d)\right) = d & , & \int_{0}^{P^{-1}(d)} dy = P^{-1}(d) & , & \frac{dP^{-1}(d)}{dd} = \frac{1}{P'(P^{-1}(d))} \\ D\left(D^{-1}(p)\right) = p & , & \int_{0}^{D^{-1}(p)} dx = D^{-1}(p) & , & \frac{dD^{-1}(p)}{dp} = \frac{1}{D'(D^{-1}(p))} \end{array} \tag{6}$$

The linear bid functions take the following functional form:

$$\begin{array}{lllll} p & = & P_j(z_\Pi) & = & e_j + f_j z_\Pi \\ d & = & D_j(z_\Delta) & = & a_j + b_j z_\Delta \end{array} \tag{7}$$

where e_j, f_j, a_j, b_j are parameters and j refers to the case (or subcase) under analysis. We can also write the following:

$$\begin{array}{lll} z_\Pi = P_j^{-1}(p) = \frac{p - e_j}{f_j} & , & P_j'(z_\Pi) = f_j \\ z_\Delta = D_j^{-1}(d) = \frac{d - a_j}{b_j} & , & D_j'(z_\Delta) = b_j \end{array} \tag{8}$$

A.1 Case 1

Since Case 1 implies $\alpha = \frac{1}{2}$, the payoffs are:

$$\begin{aligned}\Pi_1 &= \tfrac{1}{2}\int_{D^{-1}(p)}^{1}[p + D(x)]\,dx + \tfrac{1}{2}\int_{0}^{D^{-1}(p)}[qz_\Pi + x(1-q) + q - c]\,dx \\ \Delta_1 &= \tfrac{1}{2}\int_{0}^{P^{-1}(d)}[P(y) + d]\,dy + \tfrac{1}{2}\int_{P^{-1}(d)}^{1}[qy + z_\Delta(1-q) + q + c]\,dy\end{aligned}$$

The first derivatives of the parties' payoffs with respect to their bids are:

$$
\begin{aligned}
\frac{d\Pi_1}{dp} &= -\tfrac{1}{2}\tfrac{dD^{-1}(p)}{dp}\left[p + D\left(D^{-1}(p)\right)\right] \\
&\quad + \tfrac{1}{2}\int_{D^{-1}(p)}^{1} dx + \tfrac{1}{2}\tfrac{dD^{-1}(p)}{dp}\left[qz_\Pi + (1-q)\,D^{-1}(p) + q - c\right] \\
\frac{d\Delta_1}{dd} &= \tfrac{1}{2}\tfrac{dP^{-1}(d)}{dd}\left[P\left(P^{-1}(d)\right) + d\right] \\
&\quad + \tfrac{1}{2}\int_{0}^{P^{-1}(d)} dy + \tfrac{1}{2}\left(-\tfrac{dP^{-1}(d)}{dd}\left[qP^{-1}(d) + z_\Delta(1-q) + q + c\right]\right)
\end{aligned}
$$

By using (6) we can simplify the derivatives above and derive the following first order conditions (second order conditions are easily verified):

$$
\begin{aligned}
\tfrac{1}{D'(D^{-1}(p))}\left[qz_\Pi + (1-q)\,D^{-1}(p) + q - c - 2p\right] + \left(1 - D^{-1}(p)\right) &= 0 \\
\tfrac{1}{P'(P^{-1}(d))}\left[2d - qP^{-1}(d) - z_\Delta(1-q) - q - c\right] + P^{-1}(d) &= 0
\end{aligned}
$$

By using (7) and (8) we obtain:

$$
\begin{aligned}
\tfrac{1}{b_1}\left[qz_\Pi + (1-q)\tfrac{p-a_1}{b_1} + q - c - 2p\right] + \tfrac{b_1 - p + a_1}{b_1} &= 0 \\
\tfrac{1}{f_1}\left[2d - q\tfrac{d-e_1}{f_1} - z_\Delta(1-q) - q - c\right] + \tfrac{d-e_1}{f_1} &= 0
\end{aligned}
$$

We can then solve for p and d and obtain:

$$
\begin{aligned}
p &= \left(\tfrac{b_1}{3b_1-(1-q)}\right)\left(b_1 + a_1 - (1-q)\tfrac{a_1}{b_1} + q - c\right) + q\left(\tfrac{b_1}{3b_1-(1-q)}\right)z_\Pi \\
d &= \left(e_1 - q\tfrac{e_1}{f_1} + q + c\right)\left(\tfrac{f_1}{3f_1-q}\right) + (1-q)\left(\tfrac{f_1}{3f_1-q}\right)z_\Delta
\end{aligned}
$$

Note that, by linearity, we have $q\left(\frac{b_1}{3b_1-(1-q)}\right) = f_1$ and $(1-q)\left(\frac{f_1}{3f_1-q}\right) = b_1$, which implies $b_1 = f_1 = \frac{1}{3}$. We can then similarly derive a_1 and e_1 and obtain the following equilibrium bid functions for Case 1:

$$
\begin{aligned}
P_1(z_\Pi) &= \tfrac{1}{2} - \left(\tfrac{5}{2} - 3q\right)c + \tfrac{1}{3}z_\Pi \\
D_1(z_\Delta) &= \tfrac{1}{6} + \left(3q - \tfrac{1}{2}\right)c + \tfrac{1}{3}z_\Delta
\end{aligned}
$$

We first look at the case where $c < \frac{1}{6}$. The plaintiff's demand function is above the defendant's offer function. Any plaintiff's demand above the maximum defendant's offer $D_1(1)$ will result in litigation. Moreover, conditional on litigation, the outcome is unaffected by how much the plaintiff demands. Hence, a plaintiff demand function truncated above at $D_1(1)$ is the essentially unique plaintiff's bid if it is a best response to the defendant's offer. It is easy to see that this is the case given the way in which we have derived the bid functions. Similarly, the defendant's offer is truncated below at $P_1(0)$.

Next suppose that $c \geq \frac{1}{6}$ so that the defendant offer function overlaps with or is above the plaintiff's demand function. The plaintiff will not demand less than the minimum defendant's offer $D_1(0)$ so that the plaintiff's demand must be truncated below at that level. Given that $P_1(6c-1) = D_1(0)$, the plaintiff will demand $D_1(0)$ if $z_\Pi \leq 6c - 1$ and will demand $P_1(z_\Pi)$ if $z_\Pi > 6c - 1$. In the latter case, we know that the defendant's offer is a best response to the plaintiff's demand. In the former case, we need to verify that, even when $z_\Delta = 0$, the defendant will still want to settle with a plaintiff demanding $D_1(0)$. The defendant would clearly be worse off by raising the offer. If the defendant were to reduce the offer, then there would be a trial. The defendant's expected cost at trial is:

$$
\frac{1}{2}\left((1-q)\,0 + q\left[\frac{1}{2}(6c-1)\right] + q + c\right) = \frac{1}{4}q + c\left(\frac{3}{2}q + \frac{1}{2}\right)
$$

where the term in square brackets is the expected value of z_Π given that $z_\Pi \leq 6c - 1$. The defendant's expected cost at trial is greater than (or equal to) the expected cost of settlement $\frac{1}{6} + c\left(3q - \frac{1}{2}\right)$ if $c \geq \frac{1}{6}$ and $q \leq \frac{2}{3}$, thus we have a best response. A similar argument shows that the defendant's offer is truncated above at $P_1(1)$.

A.2 Case 2

In Case 2, fee-shifting is $\alpha(z_\Delta) = 0$ if $z_\Delta < \frac{t-q}{1-q}$ and $\alpha(z_\Delta) = \frac{1}{2}$ if $z_\Delta \geq \frac{t-q}{1-q}$. Therefore, the defendant's payoff directly depends on the defendant's signal, which in turn reflects fee-shifting. Since the defendant can perfectly anticipate fee-shifting, the defendant will offer $\overline{D}(z_\Delta)$ if z_Δ is above the threshold and $\underline{D}(z_\Delta)$ if z_Δ is below the threshold. Let $\underline{d} \equiv \sup\left\{\underline{D}\left(\frac{t-q}{1-q}\right)\right\}$ and $\overline{d} \equiv \overline{D}\left(\frac{t-q}{1-q}\right)$, so that $\underline{D}^{-1}(\underline{d}) = \frac{t-q}{1-q} = \overline{D}^{-1}(\overline{d})$.

In turn, the plaintiff matches the defendant's low offer with a low demand $\underline{P}(z_\Pi)$ and the defendant's high offer with a high demand $\overline{P}(z_\Pi)$, thereby indirectly adjusting the demand to fee-shifting. Accordingly, $\underline{P}^{-1}(\underline{d}) = \overline{P}^{-1}(\overline{d})$ is the value of the plaintiff's signal where the plaintiff's demand shifts. Note that if $z_\Pi < \underline{P}^{-1}(\underline{d})$, the plaintiff goes to trial only with a defendant offering $\underline{D}$ while might settle with both types of defendants. If instead $z_\Pi \geq \overline{P}^{-1}(\overline{d})$ the plaintiff might to trial with both types of defendant while settling only with $\overline{D}$. An analogous argument applies to the defendant. Accordingly, the payoffs can be written as follows:

$$\Pi_2 = \begin{cases} \frac{1}{2}\int_{\frac{t-q}{1-q}}^{1}\left[p + \overline{D}(x)\right]dx + \frac{1}{2}\int_{\underline{D}^{-1}(p)}^{\frac{t-q}{1-q}}\left[p + \underline{D}(x)\right]dx \\ \quad + \frac{1}{2}\int_{0}^{\underline{D}^{-1}(p)}\left[qz_\Pi + x(1-q) + q - 2c\right]dx & \text{if } z_\Pi < \underline{P}^{-1}(\underline{d}) \\ \frac{1}{2}\int_{\overline{D}^{-1}(p)}^{1}\left[p + \overline{D}(x)\right]dx + \frac{1}{2}\int_{\frac{t-q}{1-q}}^{\overline{D}^{-1}(p)}\left[qz_\Pi + x(1-q) + q - c\right]dx \\ \quad + \frac{1}{2}\int_{0}^{\frac{t-q}{1-q}}\left[qz_\Pi + x(1-q) + q - 2c\right]dx & \text{if } z_\Pi \geq \overline{P}^{-1}(\overline{d}) \end{cases}$$

$$\Delta_2 = \begin{cases} \frac{1}{2}\int_{0}^{\underline{P}^{-1}(d)}\left[\underline{P}(y) + d\right]dy + \frac{1}{2}\int_{\underline{P}^{-1}(d)}^{1}\left[qy + z_\Delta(1-q) + q\right]dy & \text{if } z_\Delta < \frac{t-q}{1-q} \\ \frac{1}{2}\int_{0}^{\underline{P}^{-1}(\underline{d})}\left[\underline{P}(y) + d\right]dy + \frac{1}{2}\int_{\overline{P}^{-1}(\overline{d})}^{\overline{P}^{-1}(d)}\left[\overline{P}(y) + d\right]dy \\ \quad + \frac{1}{2}\int_{\overline{P}^{-1}(d)}^{1}\left[qy + z_\Delta(1-q) + q + c\right]dy & \text{if } z_\Delta \geq \frac{t-q}{1-q} \end{cases}$$

Since at the margin a defendant offering $\overline{D}$ settles with a plaintiff demanding $\overline{P}$ and a defendant offering $\underline{D}$ settles with a plaintiff offering $\underline{P}$, we can essentially proceed as in Case 1 for each pair of bids. We can calculate the first order conditions (second order conditions are verified), from which we obtain the bid functions and the truncations as before. From the bid function we can then explicitly calculate the values of $\underline{d}$, $\overline{d}$, $\overline{P}^{-1}(\overline{d})$, and $\underline{P}^{-1}(\underline{d})$. As in Case 1, it is easy to verify that a player's truncated bid is the best response to the other player's truncated bid.

A.3 Case 3

In Case 3, fee-shifting is $\alpha(z_\Pi) = \frac{1}{2}$ if $z_\Pi \leq \frac{1-t}{q}$ and $\alpha(z_\Pi) = 1$ if $z_\Pi > \frac{1-t}{q}$. Case 3 is the mirror image of Case 2, with fee-shifting dependent on the plaintiff's signal. Let $\underline{p} \equiv \underline{P}\left(\frac{1-t}{q}\right)$ and $\overline{p} \equiv \inf\left\{\overline{P}\left(\frac{1-t}{q}\right)\right\}$ so that $\underline{P}^{-1}\left(\underline{p}\right)=\frac{1-t}{q}=\overline{P}^{-1}\left(\overline{p}\right)$. In this case, $\underline{D}^{-1}\left(\underline{p}\right)=\overline{D}^{-1}\left(\overline{p}\right)$ is the signal value at which the defendant's offer shifts. Case 3 is governed by the following payoff functions:

$$\Pi_3=\begin{cases}\frac{1}{2}\int\limits_{\underline{D}^{-1}(\underline{p})}^{1}\left[p+\overline{D}(x)\right]dx+\frac{1}{2}\int\limits_{\underline{D}^{-1}(p)}^{\overline{D}^{-1}(\overline{p})}\left[p+\underline{D}(x)\right]dx \\ \qquad +\frac{1}{2}\int\limits_{0}^{\underline{D}^{-1}(p)}\left[qz_{\Pi}+x\left(1-q\right)+q-c\right]dx & \text{if}\quad z_{\Pi}\leq\frac{1-t}{q}\\ \frac{1}{2}\int\limits_{\overline{D}^{-1}(p)}^{1}\left[p+\overline{D}(x)\right]dx+\frac{1}{2}\int\limits_{0}^{\overline{D}^{-1}(p)}\left[qz_{\Pi}+x\left(1-q\right)+q\right]dx & \text{if}\quad z_{\Pi}>\frac{1-t}{q}\end{cases}$$

$$\Delta_3=\begin{cases}\frac{1}{2}\int\limits_{0}^{\underline{P}^{-1}(d)}\left[\underline{P}(y)+d\right]dy+\frac{1}{2}\int\limits_{\underline{P}^{-1}(d)}^{\frac{1-t}{q}}\left[qy+z_{\Delta}\left(1-q\right)+q+c\right]dy \\ \qquad +\frac{1}{2}\int\limits_{\frac{1-t}{q}}^{1}\left[qy+z_{\Delta}\left(1-q\right)+q+2c\right]dy & \text{if}\quad z_{\Delta}\leq\underline{D}^{-1}(\underline{p})\\ \frac{1}{2}\int\limits_{0}^{\frac{1-t}{q}}\left[\underline{P}(y)+d\right]dy+\frac{1}{2}\int\limits_{\frac{1-t}{q}}^{\overline{P}^{-1}(d)}\left[\overline{P}(y)+d\right]dy \\ \qquad +\frac{1}{2}\int\limits_{\overline{P}^{-1}(d)}^{1}\left[qy+z_{\Delta}\left(1-q\right)+q+2c\right]dy & \text{if}\quad z_{\Delta}>\underline{D}^{-1}(\underline{p})\end{cases}$$

We proceed as before to obtain the parties' equilibrium bids.

A.4 Case 4

In Case 4, fee-shifting depends on the signals of both parties:

$$\alpha\left(z_{\Delta},z_{\Pi}\right)=\begin{cases}0 & \text{if}\quad z_{\Delta}<1-\frac{q}{1-q}z_{\Pi}\quad\text{and}\quad z_{\Delta}<\frac{t-q}{1-q}\\ \frac{1}{2} & \text{if}\quad z_{\Delta}=1-\frac{q}{1-q}z_{\Pi}\quad\text{or}\quad\left(z_{\Delta}\geq\frac{t-q}{1-q}\text{ and }z_{\Pi}\leq\frac{1-t}{q}\right)\\ 1 & \text{if}\quad z_{\Delta}>1-\frac{q}{1-q}z_{\Pi}\quad\text{and}\quad z_{\Pi}>\frac{1-t}{q}\end{cases}$$

In order to define the parties' payoffs, it is convenient to distinguish between two subcases, $q<\frac{1}{2}$ and $q>\frac{1}{2}$, corresponding to the last two graphs of Figure ??. (The case with $q=\frac{1}{2}$ is an identical limit case of the two subcases and can be omitted without loss of generality). These two subcases are different in terms of a relative informational advantage of either party. When $q<\frac{1}{2}$, if $z_{\Delta}\leq\frac{1-2q}{1-q}$ the defendant knows that $\alpha=0$ irrespective of the plaintiff's signal. Conversely, when $q>\frac{1}{2}$, if $z_{\Pi}>\frac{1-q}{q}$ the plaintiff knows that $\alpha=1$. Otherwise, fee-shifting depends on both parties' signals. We start by defining the relevant thresholds at which fee-shifting changes, which in turn define the parties' information sets.

When $q<\frac{1}{2}$, the parties' information sets are:

Plaintiff	
$z_{\Pi}\leq\frac{1-t}{q}$	$\alpha\in\left\{0,\frac{1}{2}\right\}$
$z_{\Pi}>\frac{1-t}{q}$	$\alpha\in\{0,1\}$

Defendant	
$z_{\Delta}\leq\frac{1-2q}{1-q}$	$\alpha=0$
$\frac{1-2q}{1-q}<z_{\Delta}<\frac{t-q}{1-q}$	$\alpha\in\{0,1\}$
$z_{\Delta}\geq\frac{t-q}{1-q}$	$\alpha\in\left\{\frac{1}{2},1\right\}$

The defendant's bid function will respond both to the defendant's signal thresholds and to the expected plaintiff's bidding behavior. Thus, the defendant's bid will shift upward at the following levels of z_{Δ}: $\frac{1-2q}{1-q}$, $\frac{t-q}{1-q}$ and $D^{-1}\left(P\left(\frac{1-t}{q}\right)\right)$, where the latter is the value of the defendant's signal such that the defendant's offer matches the shift in the plaintiff's request. Similarly, the plaintiff's bid will shift upward at the following levels of z_Π: $P^{-1}\left(D\left(\frac{1-2q}{1-q}\right)\right)$, $P^{-1}\left(D\left(\frac{t-q}{1-q}\right)\right)$ and $\frac{1-t}{q}$. There are three ways in which these six thresholds can be ordered:

(Ia) $P^{-1}(D(\frac{1-2q}{1-q})) \leq P^{-1}(D(\frac{t-q}{1-q})) \leq \frac{1-t}{q}$ and $\frac{1-2q}{1-q} \leq \frac{t-q}{1-q} \leq D^{-1}(P(\frac{1-t}{q}))$

(Ib) $P^{-1}(D(\frac{1-2q}{1-q})) \leq \frac{1-t}{q} \leq P^{-1}(D(\frac{t-q}{1-q}))$ and $\frac{1-2q}{1-q} \leq D^{-1}(P(\frac{1-t}{q})) \leq \frac{t-q}{1-q}$

(Ic) $\frac{1-t}{q} \leq P^{-1}(D(\frac{1-2q}{1-q})) \leq P^{-1}(D(\frac{t-q}{1-q}))$ and $D^{-1}(P(\frac{1-t}{q})) \leq D^{-1}(P(\frac{1-q}{q}) \leq \frac{t-q}{1-q}$

When $q > \frac{1}{2}$, the parties' information sets are:

Plaintiff	
$z_\Pi \leq \frac{1-t}{q}$	$\alpha \in \left\{0, \frac{1}{2}\right\}$
$\frac{1-t}{q} < z_\Pi < \frac{1-q}{q}$	$\alpha \in \{0, 1\}$
$z_\Pi \geq \frac{1-q}{q}$	$\alpha = 1$

Defendant	
$z_\Delta < \frac{t-q}{1-q}$	$\alpha \in \{0, 1\}$
$z_\Delta \geq \frac{t-q}{1-q}$	$\alpha \in \left\{\frac{1}{2}, 1\right\}$

In this case, the defendant's bid will shift upward at the following levels of z_Δ: $D^{-1}\left(P\left(\frac{1-t}{q}\right)\right)$, $D^{-1}\left(P\left(\frac{1-q}{q}\right)\right)$ and $\frac{t-q}{1-q}$. The plaintiff's bid will shift upward at the following levels of z_Π: $\frac{1-t}{q}$, $\frac{1-q}{q}$ and $P^{-1}\left(D\left(\frac{t-q}{1-q}\right)\right)$. The are again three ways in which these thresholds can be ordered:

(IIa) $P^{-1}(D(\frac{t-q}{1-q})) \leq \frac{1-t}{q} \leq \frac{1-q}{q}$ and $\frac{t-q}{1-q} \leq D^{-1}(P(\frac{1-t}{q})) \leq D^{-1}(P(\frac{1-q}{q})$

(IIb) $\frac{1-t}{q} \leq P^{-1}(D(\frac{t-q}{1-q})) \leq \frac{1-q}{q}$ and $D^{-1}(P(\frac{1-t}{q})) \leq \frac{t-q}{1-q} \leq D^{-1}(P(\frac{1-q}{q})$

(IIc) $\frac{1-t}{q} \leq \frac{1-q}{q} \leq P^{-1}(D(\frac{t-q}{1-q}))$ and $D^{-1}(P(\frac{1-t}{q})) \leq D^{-1}(P(\frac{1-q}{q}) \leq \frac{t-q}{1-q}$

For each of these three cases we will write the parties' payoffs and derive the bid functions as we did before. The analysis is more involved than but essentially equivalent to the analysis in the previous cases. Given that each party faces three thresholds, we start working with four-pieces bids. We refer to these four pieces by using double lower-bar, simple lower-bar, simple upper-bar and double upper-bar, starting from the lowest piece. In equilibrium, bids will have at most three pieces and hence we will relabel them at the end of the analysis.

Let us illustrate the construction of the payoff functions in subcase (Ia). When offering $\underline{\underline{D}}(z_\Delta)$, the defendant settles with a plaintiff who has $z_\Pi \leq \underline{\underline{P}}^{-1}\left(\underline{\underline{D}}(z_\Delta)\right)$ and litigates in all other cases. Since the defendant's signal is below $\frac{1-2q}{1-q}$, litigation results in $\alpha = 0$. When offering $\underline{D}$, the defendant settles with a plaintiff who has $z_\Pi \leq \underline{P}^{-1}(\underline{D}(z_\Delta))$ and litigates in all other cases. Since the defendant's signal is below $\frac{t-q}{1-q}$, litigation results in $\alpha = 0$ if $z_\Pi < \frac{1-q}{q}(1-z_\Delta)$ and $\alpha = 1$ if $z_\Pi > \frac{1-q}{q}(1-z_\Delta)$.[41] When offering $\overline{D}(z_\Delta)$, the defendant settles with a plaintiff who has $z_\Pi \leq \overline{P}^{-1}\left(\overline{D}(z_\Delta)\right)$ and litigates in all other cases. Since the defendant's signal is above $\frac{t-q}{1-q}$, litigation results in $\alpha = \frac{1}{2}$ if $z_\Pi \leq \frac{1-t}{q}$ and $\alpha = 1$ if $z_\Pi > \frac{1-t}{q}$. Finally, when offering $\overline{\overline{D}}(z_\Delta)$, the defendant settles with a plaintiff who has $z_\Pi \leq \overline{\overline{P}}^{-1}\left(\overline{\overline{D}}(z_\Delta)\right)$ and litigates in all other cases. Since the defendant's signal is above $\frac{t-q}{1-q}$ and the plaintiff's signal conditional on litigation is $z_\Pi > \frac{1-t}{q}$, litigation results in $\alpha = 1$. A similar reasoning applies to the plaintiff. Based on these observations we can construct the four-part parties' payoff functions in subcase (Ia). An analogous reasoning allows us construct the payoff functions in the other subcases.

Note that the following equalities hold in the following subcases:

(Ia) $\overline{\overline{D}}^{-1}\left(\overline{\overline{P}}\left(\frac{1-t}{q}\right)\right) = \overline{D}^{-1}\left(\overline{P}\left(\frac{1-t}{q}\right)\right)$, $\underline{\underline{P}}^{-1}\left(\underline{\underline{D}}\left(\frac{1-2q}{1-q}\right)\right) = \underline{P}^{-1}\left(\underline{D}\left(\frac{1-2q}{1-q}\right)\right)$ and $\underline{P}^{-1}\left(\underline{D}\left(\frac{t-q}{1-q}\right)\right) = \overline{P}^{-1}\left(\overline{D}\left(\frac{t-q}{1-q}\right)\right)$

[41] If $z_\Pi = \frac{1-q}{q}(1-z_\Delta)$ we have $\alpha = \frac{1}{2}$, but this is an interval of length and can be safely ignored in constructing the integrals.

(Ib) $\overline{D}^{-1}(\overline{P}(\frac{1-t}{q})) = \underline{D}^{-1}(\underline{P}(\frac{1-t}{q})),\ \underline{\underline{P}}^{-1}\left(\underline{\underline{D}}\left(\frac{1-2q}{1-q}\right)\right) = \underline{P}^{-1}\left(\underline{D}\left(\frac{1-2q}{1-q}\right)\right)$ and $\overline{P}^{-1}\left(\overline{D}\left(\frac{t-q}{1-q}\right)\right) = \overline{\overline{P}}^{-1}\left(\overline{\overline{D}}\left(\frac{t-q}{1-q}\right)\right)$

(Ic) $\underline{D}^{-1}(\underline{P}(\frac{1-t}{q})) = \underline{\underline{D}}^{-1}(\underline{\underline{P}}(\frac{1-t}{q})),\ \underline{P}^{-1}\left(\underline{D}\left(\frac{1-2q}{1-q}\right)\right) = \overline{P}^{-1}\left(\overline{D}\left(\frac{1-2q}{1-q}\right)\right)$ and $\overline{P}^{-1}\left(\overline{D}\left(\frac{t-q}{1-q}\right)\right) = \overline{\overline{P}}^{-1}\left(\overline{\overline{D}}\left(\frac{t-q}{1-q}\right)\right)$

(IIa) $\overline{\overline{D}}^{-1}(\overline{\overline{P}}(\frac{1-q}{q})) = \overline{D}^{-1}(\overline{P}(\frac{1-q}{q})), \overline{D}^{-1}(\overline{P}(\frac{1-t}{q})) = \underline{D}^{-1}(\overline{P}(\frac{1-t}{q}))$ and $\underline{\underline{P}}^{-1}\left(\underline{\underline{D}}\left(\frac{t-q}{1-q}\right)\right) = \underline{P}^{-1}\left(\underline{D}\left(\frac{t-q}{1-q}\right)\right)$

(IIb) $\overline{\overline{D}}^{-1}(\overline{\overline{P}}(\frac{1-q}{q})) = \overline{D}^{-1}(\overline{P}(\frac{1-q}{q})), \underline{D}^{-1}(\underline{P}(\frac{1-t}{q})) = \underline{\underline{D}}^{-1}(\underline{\underline{P}}(\frac{1-t}{q}))$ and $\underline{P}^{-1}(\underline{D}(\frac{t-q}{1-q})) = \overline{P}^{-1}(\overline{D}(\frac{t-q}{1-q}))$

(IIc) $\overline{D}^{-1}(\overline{P}(\frac{1-q}{q})) = \underline{D}^{-1}(\underline{P}(\frac{1-q}{q})), \underline{D}^{-1}(\underline{P}(\frac{1-t}{q})) = \underline{\underline{D}}^{-1}(\underline{\underline{P}}(\frac{1-t}{q}))$ and $\overline{P}^{-1}(\overline{D}(\frac{t-q}{1-q})) = \overline{\overline{P}}^{-1}(\overline{\overline{D}}(\frac{t-q}{1-q}))$

The parties's payoff functions in each subcase are:

$$
\text{(Ia) } \Pi_4 = \begin{cases}
\left.\begin{array}{l}
\frac{1}{2}\int\limits_{\overline{\overline{D}}^{-1}\left(\overline{\overline{P}}(\frac{1-t}{q})\right)}^{1}\left[p+\overline{\overline{D}}(x)\right]dx \\
+\frac{1}{2}\int\limits_{\frac{t-q}{1-q}}^{\overline{D}^{-1}(\overline{P}(\frac{1-t}{q}))}\left[p+\overline{D}(x)\right]dx \\
+\frac{1}{2}\int\limits_{\frac{1-2q}{1-q}}^{\frac{t-q}{1-q}}\left[p+\underline{D}(x)\right]dx \\
+\frac{1}{2}\int\limits_{\underline{\underline{D}}^{-1}(p)}^{\frac{1-2q}{1-q}}\left[p+\underline{\underline{D}}(x)\right]dx \\
+\frac{1}{2}\int\limits_{0}^{\underline{\underline{D}}^{-1}(p)}\left[qz_{\Pi}+x\left(1-q\right)+q-2c\right]dx
\end{array}\right\} & \text{if} \quad 0 \leq z_{\Pi} < P^{-1}(D(\frac{1-2q}{1-q})) \\
\left.\begin{array}{l}
\frac{1}{2}\int\limits_{\overline{D}^{-1}(\overline{P}(\frac{1-t}{q}))}^{1}\left[p+\overline{\overline{D}}(x)\right]dx \\
+\frac{1}{2}\int\limits_{\frac{t-q}{1-q}}^{\overline{D}^{-1}(\overline{P}(\frac{1-t}{q}))}\left[p+\overline{D}(x)\right]dx \\
+\frac{1}{2}\int\limits_{\underline{D}^{-1}(p)}^{\frac{t-q}{1-q}}\left[p+\underline{D}(x)\right]dx \\
+\frac{1}{2}\int\limits_{0}^{\underline{D}^{-1}(p)}\left[qz_{\Pi}+x\left(1-q\right)+q-2c\right]dx
\end{array}\right\} & \text{if} \quad P^{-1}(D(\frac{1-2q}{1-q})) \leq z_{\Pi} < P^{-1}(D(\frac{t-q}{1-q})) \\
\left.\begin{array}{l}
\frac{1}{2}\int\limits_{\overline{D}^{-1}(\overline{P}(\frac{1-t}{q}))}^{1}\left[p+\overline{\overline{D}}(x)\right]dx \\
+\frac{1}{2}\int\limits_{\overline{D}^{-1}(p)}^{\overline{D}^{-1}(\overline{P}(\frac{1-t}{q}))}\left[p+\overline{D}(x)\right]dx \\
+\frac{1}{2}\int\limits_{\frac{t-q}{1-q}}^{\overline{D}^{-1}(p)}\left[qz_{\Pi}+x\left(1-q\right)+q-c\right]dx \\
+\frac{1}{2}\int\limits_{0}^{\frac{t-q}{1-q}}\left[qz_{\Pi}+x\left(1-q\right)+q-2c\right]dx
\end{array}\right\} & \text{if} \quad P^{-1}(D(\frac{t-q}{1-q})) \leq z_{\Pi} < \frac{1-t}{q} \\
\left.\begin{array}{l}
\frac{1}{2}\int\limits_{\overline{\overline{D}}^{-1}(p)}^{1}\left[p+\overline{\overline{D}}(x)\right]dx \\
+\frac{1}{2}\int\limits_{1-\frac{q}{1-q}z_{\Pi}}^{\overline{\overline{D}}^{-1}(p)}\left[qz_{\Pi}+x\left(1-q\right)+q\right]dx \\
+\frac{1}{2}\int\limits_{0}^{1-\frac{q}{1-q}z_{\Pi}}\left[qz_{\Pi}+x\left(1-q\right)+q-2c\right]dx
\end{array}\right\} & \text{if} \quad \frac{1-t}{q} < z_{\Pi} < 1
\end{cases}
$$

$$
\text{(Ia) } \Delta_4 = \begin{cases}
\left.\begin{aligned}
&\tfrac{1}{2}\int_{0}^{\underline{\underline{P}}^{-1}(d)} \left[\underline{\underline{P}}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\underline{\underline{P}}^{-1}(d)}^{1} \left[qy+z_\Delta\left(1-q\right)+q\right] dy
\end{aligned}\right\} & \text{if} \quad 0 \le z_\Delta < \frac{1-2q}{1-q} \\
\left.\begin{aligned}
&\tfrac{1}{2}\int_{0}^{\underline{\underline{P}}^{-1}(\underline{\underline{D}}(\frac{1-2q}{1-q}))} \left[\underline{\underline{P}}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\underline{P}^{-1}(\underline{D}(\frac{1-2q}{1-q}))}^{\underline{P}^{-1}(d)} \left[\underline{P}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\underline{P}^{-1}(d)}^{\frac{1-q}{q}(1-z_\Delta)} \left[qy+z_\Delta\left(1-q\right)+q\right] dy \\
&+\tfrac{1}{2}\int_{\frac{1-q}{q}(1-z_\Delta)}^{1} \left[qy+z_\Delta\left(1-q\right)+q+2c\right] dy
\end{aligned}\right\} & \text{if} \quad \frac{1-2q}{1-q} \le z_\Delta < \frac{t-q}{1-q} \\
\left.\begin{aligned}
&\tfrac{1}{2}\int_{0}^{\underline{\underline{P}}^{-1}(\underline{\underline{D}}(\frac{1-2q}{1-q}))} \left[\underline{\underline{P}}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\underline{P}^{-1}(\underline{D}(\frac{1-2q}{1-q}))}^{\underline{P}^{-1}(\underline{D}(\frac{t-q}{1-q}))} \left[\underline{P}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\overline{P}^{-1}(\overline{D}(\frac{t-q}{1-q}))}^{\overline{P}^{-1}(d)} \left[\overline{P}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\overline{P}^{-1}(d)}^{\frac{1-t}{q}} \left[qy+z_\Delta\left(1-q\right)+q+c\right] dy \\
&+\tfrac{1}{2}\int_{\frac{1-t}{q}}^{1} \left[qy+z_\Delta\left(1-q\right)+q+2c\right] dy
\end{aligned}\right\} & \text{if} \quad \frac{t-q}{1-q} \le z_\Delta < D^{-1}(P(\frac{1-t}{q})) \\
\left.\begin{aligned}
&\tfrac{1}{2}\int_{0}^{\underline{\underline{P}}^{-1}(\underline{\underline{D}}(\frac{1-2q}{1-q}))} \left[\underline{\underline{P}}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\underline{P}^{-1}(\underline{D}(\frac{1-2q}{1-q}))}^{\underline{P}^{-1}(\underline{D}(\frac{t-q}{1-q}))} \left[\underline{P}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\overline{P}^{-1}(\overline{D}(\frac{t-q}{1-q}))}^{\frac{1-t}{q}} \left[\overline{P}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\frac{1-t}{q}}^{\overline{\overline{P}}^{-1}(d)} \left[\overline{\overline{P}}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\overline{\overline{P}}^{-1}(d)}^{1} \left[qy+z_\Delta\left(1-q\right)+q+2c\right] dy
\end{aligned}\right\} & \text{if} \quad D^{-1}(P(\frac{1-t}{q})) < z_\Delta < 1
\end{cases}
$$

$$
\text{(Ib) } \Pi_4 = \begin{cases}
\left.\begin{aligned}
&\tfrac{1}{2}\int_{\frac{t-q}{1-q}}^{1} \left[p + \overline{\overline{D}}(x)\right] dx \\
&+\tfrac{1}{2}\int_{\underline{D}^{-1}(\underline{P}(\frac{1-t}{q}))}^{\frac{t-q}{1-q}} \left[p + \overline{D}(x)\right] dx \\
&+\tfrac{1}{2}\int_{\frac{1-2q}{1-q}}^{\underline{D}^{-1}(\underline{P}(\frac{1-t}{q}))} \left[p + \underline{D}(x)\right] dx \\
&+\tfrac{1}{2}\int_{\underline{\underline{D}}^{-1}(p)}^{\frac{1-2q}{1-q}} \left[p + \underline{\underline{D}}(x)\right] dx \\
&+\tfrac{1}{2}\int_{0}^{\underline{\underline{D}}^{-1}(p)} \left[q z_\Pi + x\,(1-q) + q - 2c\right] dx
\end{aligned}\right\} & \text{if } \quad 0 \le z_\Pi < P^{-1}(D(\tfrac{1-2q}{1-q})) \\
\left.\begin{aligned}
&\tfrac{1}{2}\int_{\frac{t-q}{1-q}}^{1} \left[p + \overline{\overline{D}}(x)\right] dx \\
&+\tfrac{1}{2}\int_{\underline{D}^{-1}(\underline{P}(\frac{1-t}{q}))}^{\frac{t-q}{1-q}} \left[p + \overline{D}(x)\right] dx \\
&+\tfrac{1}{2}\int_{\underline{D}^{-1}(p)}^{\underline{D}^{-1}(\underline{P}(\frac{1-t}{q}))} \left[p + \underline{D}(x)\right] dx \\
&+\tfrac{1}{2}\int_{0}^{\underline{D}^{-1}(p)} \left[q z_\Pi + x\,(1-q) + q - 2c\right] dx
\end{aligned}\right\} & \text{if } \quad P^{-1}(D(\tfrac{1-2q}{1-q})) \le z_\Pi < \tfrac{1-t}{q} \\
\left.\begin{aligned}
&\tfrac{1}{2}\int_{\frac{t-q}{1-q}}^{1} \left[p + \overline{\overline{D}}(x)\right] dx \\
&+\tfrac{1}{2}\int_{1-\frac{q}{1-q}z_\Pi}^{\frac{t-q}{1-q}} \left[p + \overline{D}_{UP}(x)\right] dx \\
&+\tfrac{1}{2}\int_{\overline{D}_{DW}(p)}^{1-\frac{q}{1-q}z_\Pi} \left[p + \overline{D}_{DW}(x)\right] dx \\
&+\tfrac{1}{2}\int_{0}^{\overline{D}_{DW}(p)} \left[q z_\Pi + x\,(1-q) + q - 2c\right] dx
\end{aligned}\right\} & \text{if } \quad \tfrac{1-t}{q} \le z_\Pi < \widehat{z}_\Pi \\
\left.\begin{aligned}
&\tfrac{1}{2}\int_{\frac{t-q}{1-q}}^{1} \left[p + \overline{\overline{D}}(x)\right] dx \\
&+\tfrac{1}{2}\int_{\overline{D}_{UP}^{-1}(p)}^{\frac{t-q}{1-q}} \left[p + \overline{D}_{UP}(x)\right] dx \\
&+\tfrac{1}{2}\int_{1-\frac{q}{1-q}z_\Pi}^{\overline{D}_{UP}(p)} \left[q z_\Pi + x\,(1-q) + q\right] dx \\
&+\tfrac{1}{2}\int_{0}^{1-\frac{q}{1-q}z_\Pi} \left[q z_\Pi + x\,(1-q) + q - 2c\right] dx
\end{aligned}\right\} & \text{if } \quad \widehat{z}_\Pi \le z_\Pi < P^{-1}(D(\tfrac{t-q}{1-q})) \\
\left.\begin{aligned}
&\tfrac{1}{2}\int_{\overline{\overline{D}}^{-1}(p)}^{1} \left[p + \overline{\overline{D}}(x)\right] dx \\
&+\tfrac{1}{2}\int_{1-\frac{q}{1-q}z_\Pi}^{\overline{\overline{D}}^{-1}(p)} \left[q z_\Pi + x\,(1-q) + q\right] dx \\
&+\tfrac{1}{2}\int_{0}^{1-\frac{q}{1-q}z_\Pi} \left[q z_\Pi + x\,(1-q) + q - 2c\right] dx
\end{aligned}\right\} & \text{if } \quad P^{-1}(D(\tfrac{t-q}{1-q})) < z_\Pi < 1
\end{cases}
$$

$$
\text{(Ib)}\ \Delta_4 = \begin{cases}
\left.\begin{aligned}
&\tfrac{1}{2}\int_{0}^{\underline{\underline{P}}^{-1}(d)} \left[\underline{\underline{P}}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\underline{\underline{P}}^{-1}(d)}^{1} \left[qy+z_\Delta\left(1-q\right)+q\right] dy
\end{aligned}\right\} & \text{if} \quad 0 \le z_\Delta < \frac{1-2q}{1-q} \\
\left.\begin{aligned}
&\tfrac{1}{2}\int_{0}^{\underline{\underline{P}}^{-1}\left(\underline{\underline{D}}\left(\frac{1-2q}{1-q}\right)\right)} \left[\underline{\underline{P}}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\underline{P}^{-1}\left(\underline{D}\left(\frac{1-2q}{1-q}\right)\right)}^{\underline{P}^{-1}(d)} \left[\underline{P}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\underline{P}^{-1}(d)}^{\frac{1-q}{q}(1-z_\Delta)} \left[qy+z_\Delta\left(1-q\right)+q\right] dy \\
&+\tfrac{1}{2}\int_{\frac{1-q}{q}(1-z_\Delta)}^{1} \left[qy+z_\Delta\left(1-q\right)+q+2c\right] dy
\end{aligned}\right\} & \text{if} \quad \frac{1-2q}{1-q} \le z_\Delta < D^{-1}(P(\frac{1-t}{q})) \\
\left.\begin{aligned}
&\tfrac{1}{2}\int_{0}^{\underline{\underline{P}}^{-1}\left(\underline{\underline{D}}\left(\frac{1-2q}{1-q}\right)\right)} \left[\underline{\underline{P}}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\underline{P}^{-1}\left(\underline{D}\left(\frac{1-2q}{1-q}\right)\right)}^{\frac{1-t}{q}} \left[\underline{P}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\frac{1-t}{q}}^{\overline{P}_{DW}^{-1}(d)} \left[\overline{P}_{DW}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\overline{P}_{DW}^{-1}(d)}^{\frac{1-q}{q}(1-z_\Delta)} \left[qy+z_\Delta\left(1-q\right)+q\right] dy \\
&+\tfrac{1}{2}\int_{\frac{1-q}{q}(1-z_\Delta)}^{1} \left[qy+z_\Delta\left(1-q\right)+q+2c\right] dy
\end{aligned}\right\} & \text{if} \quad D^{-1}(P(\frac{1-t}{q})) \le z_\Delta < \widehat{z}_\Delta \\
\left.\begin{aligned}
&\tfrac{1}{2}\int_{0}^{\underline{\underline{P}}^{-1}\left(\underline{\underline{D}}\left(\frac{1-2q}{1-q}\right)\right)} \left[\underline{\underline{P}}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\underline{P}^{-1}\left(\underline{D}\left(\frac{1-2q}{1-q}\right)\right)}^{\frac{1-t}{q}} \left[\underline{P}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\frac{1-t}{q}}^{\frac{1-q}{q}(1-z_\Delta)} \left[\overline{P}_{DW}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\frac{1-q}{q}(1-z_\Delta)}^{\overline{P}_{UP}^{-1}(d)} \left[\overline{P}_{UP}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\overline{P}_{UP}^{-1}(d)}^{1} \left[qy+z_\Delta\left(1-q\right)+q+2c\right] dy
\end{aligned}\right\} & \text{if} \quad \widehat{z}_\Delta \le z_\Delta < \frac{t-q}{1-q} \\
\left.\begin{aligned}
&\tfrac{1}{2}\int_{0}^{\underline{\underline{P}}^{-1}\left(\underline{\underline{D}}\left(\frac{1-2q}{1-q}\right)\right)} \left[\underline{\underline{P}}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\underline{P}^{-1}\left(\underline{D}\left(\frac{1-2q}{1-q}\right)\right)}^{\frac{1-t}{q}} \left[\underline{P}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\frac{1-t}{q}}^{\overline{P}^{-1}\left(\overline{D}\left(\frac{t-q}{1-q}\right)\right)} \left[\overline{P}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\overline{\overline{P}}^{-1}\left(\overline{\overline{D}}\left(\frac{t-q}{1-q}\right)\right)}^{\overline{\overline{P}}^{-1}(d)} \left[\overline{\overline{P}}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\overline{\overline{P}}^{-1}(d)}^{1} \left[qy+z_\Delta\left(1-q\right)+q+2c\right] dy
\end{aligned}\right\} & \text{if} \quad \frac{t-q}{1-q} < z_\Delta < 1
\end{cases}
$$

$$
\text{(Ic) } \Pi_4 = \begin{cases}
\left.\begin{array}{l}
\frac{1}{2}\int_{\frac{t-q}{1-q}}^{1}\left[p+\overline{\overline{D}}(x)\right]dx \\
+\frac{1}{2}\int_{\frac{1-2q}{1-q}}^{\frac{t-q}{1-q}}\left[p+\overline{D}(x)\right]dx \\
+\frac{1}{2}\int_{\underline{D}^{-1}(\underline{P}(\frac{1-t}{q}))}^{\frac{1-2q}{1-q}}\left[p+\underline{D}(x)\right]dx \\
+\frac{1}{2}\int_{\underline{\underline{D}}^{-1}(p)}^{\underline{\underline{D}}^{-1}(\underline{\underline{P}}(\frac{1-t}{q}))}\left[p+\underline{\underline{D}}(x)\right]dx \\
+\frac{1}{2}\int_{0}^{\underline{\underline{D}}^{-1}(p)}\left[qz_{\Pi}+x\left(1-q\right)+q-2c\right]dx
\end{array}\right\} & \text{if} \quad 0 \leq z_{\Pi} < \frac{1-t}{q} \\
\left.\begin{array}{l}
\frac{1}{2}\int_{\frac{t-q}{1-q}}^{1}\left[p+\overline{\overline{D}}(x)\right]dx \\
+\frac{1}{2}\int_{\frac{1-2q}{1-q}}^{\frac{t-q}{1-q}}\left[p+\overline{D}(x)\right]dx \\
+\frac{1}{2}\int_{\underline{D}^{-1}(p)}^{\frac{1-2q}{1-q}}\left[p+\underline{D}(x)\right]dx \\
+\frac{1}{2}\int_{0}^{\underline{D}^{-1}(p)}\left[qz_{\Pi}+x\left(1-q\right)+q-2c\right]dx
\end{array}\right\} & \text{if} \quad \frac{1-t}{q} \leq z_{\Pi} < P^{-1}(D(\frac{1-2q}{1-q})) \\
\left.\begin{array}{l}
\frac{1}{2}\int_{\frac{t-q}{1-q}}^{1}\left[p+\overline{\overline{D}}(x)\right]dx \\
+\frac{1}{2}\int_{1-\frac{q}{1-q}z_{\Pi}}^{\frac{t-q}{1-q}}\left[p+\overline{D}_{UP}(x)\right]dx \\
+\frac{1}{2}\int_{\overline{D}_{DW}^{-1}(p)}^{1-\frac{q}{1-q}z_{\Pi}}\left[p+\overline{D}_{DW}(x)\right]dx \\
+\frac{1}{2}\int_{0}^{\overline{D}_{DW}^{-1}(p)}\left[qz_{\Pi}+x\left(1-q\right)+q-2c\right]dx
\end{array}\right\} & \text{if} \quad P^{-1}(D(\frac{1-2q}{1-q})) \leq z_{\Pi} < \widehat{z}_{\Pi} \\
\left.\begin{array}{l}
\frac{1}{2}\int_{\frac{t-q}{1-q}}^{1}\left[p+\overline{\overline{D}}(x)\right]dx \\
+\frac{1}{2}\int_{\overline{D}_{UP}^{-1}(p)}^{\frac{t-q}{1-q}}\left[p+\overline{D}_{UP}(x)\right]dx \\
+\frac{1}{2}\int_{1-\frac{q}{1-q}z_{\Pi}}^{\overline{D}_{UP}^{-1}(p)}\left[qz_{\Pi}+x\left(1-q\right)+q\right]dx \\
+\frac{1}{2}\int_{0}^{1-\frac{q}{1-q}z_{\Pi}}\left[qz_{\Pi}+x\left(1-q\right)+q-2c\right]dx
\end{array}\right\} & \text{if} \quad \widehat{z}_{\Pi} \leq z_{\Pi} < P^{-1}(D(\frac{t-q}{1-q})) \\
\left.\begin{array}{l}
\frac{1}{2}\int_{\overline{\overline{D}}^{-1}(p)}^{1}\left[p+\overline{\overline{D}}(x)\right]dx \\
+\frac{1}{2}\int_{1-\frac{q}{1-q}z_{\Pi}}^{\overline{\overline{D}}^{-1}(p)}\left[qz_{\Pi}+x\left(1-q\right)+q\right]dx \\
+\frac{1}{2}\int_{0}^{1-\frac{q}{1-q}z_{\Pi}}\left[qz_{\Pi}+x\left(1-q\right)+q-2c\right]dx
\end{array}\right\} & \text{if} \quad P^{-1}(D(\frac{t-q}{1-q})) < z_{\Pi} < 1
\end{cases}
$$

$$
\text{(Ic) } \Delta_4 = \begin{cases}
\left.\begin{array}{l}
\frac{1}{2}\int_{0}^{\underline{\underline{P}}^{-1}(d)} \left[\underline{\underline{P}}(y)+d\right] dy \\
+\frac{1}{2}\int_{\underline{\underline{P}}^{-1}(d)}^{1} \left[qy+z_{\Delta}(1-q)+q\right] dy
\end{array}\right\} & \text{if } \quad 0 \leq z_{\Delta} < D^{-1}(P(\frac{1-t}{q})) \\
\left.\begin{array}{l}
\frac{1}{2}\int_{0}^{\frac{1-t}{q}} \left[\underline{\underline{P}}(y)+d\right] dy \\
+\frac{1}{2}\int_{\frac{1-t}{q}}^{\underline{P}^{-1}(d)} \left[\underline{P}(y)+d\right] dy \\
+\frac{1}{2}\int_{\underline{P}^{-1}(d)}^{1} \left[qy+z_{\Delta}(1-q)+q\right] dy
\end{array}\right\} & \text{if } \quad D^{-1}(P(\frac{1-t}{q})) \leq z_{\Delta} < \frac{1-2q}{1-q} \\
\left.\begin{array}{l}
\frac{1}{2}\int_{0}^{\frac{1-t}{q}} \left[\underline{\underline{P}}(y)+d\right] dy \\
+\frac{1}{2}\int_{\frac{1-t}{q}}^{\underline{P}^{-1}\left(\underline{D}\left(\frac{1-2q}{1-q}\right)\right)} \left[\underline{P}(y)+d\right] dy \\
+\frac{1}{2}\int_{\overline{P}^{-1}\left(\overline{D}\left(\frac{1-2q}{1-q}\right)\right)}^{\overline{P}_{DW}^{-1}(d)} \left[\overline{P}_{DW}(y)+d\right] dy \\
+\frac{1}{2}\int_{\overline{P}_{DW}^{-1}(d)}^{\frac{1-q}{q}(1-z_{\Delta})} \left[qy+z_{\Delta}(1-q)+q\right] dy \\
+\frac{1}{2}\int_{\frac{1-q}{q}(1-z_{\Delta})}^{1} \left[qy+z_{\Delta}(1-q)+q+2c\right] dy
\end{array}\right\} & \text{if } \quad \frac{1-2q}{1-q} \leq z_{\Delta} < \widehat{z}_{\Delta} \\
\left.\begin{array}{l}
\frac{1}{2}\int_{0}^{\frac{1-t}{q}} \left[\underline{\underline{P}}(y)+d\right] dy \\
+\frac{1}{2}\int_{\frac{1-t}{q}}^{\underline{P}^{-1}\left(\underline{D}\left(\frac{1-2q}{1-q}\right)\right)} \left[\underline{P}(y)+d\right] dy \\
+\frac{1}{2}\int_{\overline{P}^{-1}\left(\overline{D}\left(\frac{1-2q}{1-q}\right)\right)}^{\frac{1-q}{q}(1-z_{\Delta})} \left[\overline{P}_{DW}(y)+d\right] dy \\
+\frac{1}{2}\int_{\frac{1-q}{q}(1-z_{\Delta})}^{\overline{P}_{UP}^{-1}(d)} \left[\overline{P}_{UP}(y)+d\right] dy \\
+\frac{1}{2}\int_{\overline{P}_{UP}^{-1}(d)}^{1} \left[qy+z_{\Delta}(1-q)+q+2c\right] dy
\end{array}\right\} & \text{if } \quad \widehat{z}_{\Delta} \leq z_{\Delta} < \frac{t-q}{1-q} \\
\left.\begin{array}{l}
\frac{1}{2}\int_{0}^{\frac{1-t}{q}} \left[\underline{\underline{P}}(y)+d\right] dy \\
+\frac{1}{2}\int_{\frac{1-t}{q}}^{\underline{P}^{-1}\left(\underline{D}\left(\frac{1-2q}{1-q}\right)\right)} \left[\underline{P}(y)+d\right] dy \\
+\frac{1}{2}\int_{\overline{P}^{-1}\left(\overline{D}\left(\frac{1-2q}{1-q}\right)\right)}^{\overline{P}^{-1}\left(\overline{D}\left(\frac{t-q}{1-q}\right)\right)} \left[\overline{P}(y)+d\right] dy \\
+\frac{1}{2}\int_{\overline{\overline{P}}^{-1}\left(\overline{\overline{D}}\left(\frac{t-q}{1-q}\right)\right)}^{\overline{\overline{P}}^{-1}(d)} \left[\overline{\overline{P}}(y)+d\right] dy \\
+\frac{1}{2}\int_{\overline{\overline{P}}^{-1}(d)}^{1} \left[qy+z_{\Delta}(1-q)+q+2c\right] dy
\end{array}\right\} & \text{if } \quad \frac{t-q}{1-q} < z_{\Delta} < 1
\end{cases}
$$

$$
\text{(IIa) } \Pi_4 = \begin{cases}
\left.\begin{aligned}
&\tfrac{1}{2}\int_{\overline{\overline{D}}^{-1}(\overline{\overline{P}}(\frac{1-q}{q}))}^{1}\left[p+\overline{\overline{D}}(x)\right]dx \\
&+\tfrac{1}{2}\int_{\overline{D}^{-1}(\overline{P}(\frac{1-t}{q}))}^{\overline{D}^{-1}(\overline{P}(\frac{1-q}{q}))}\left[p+\overline{D}(x)\right]dx \\
&+\tfrac{1}{2}\int_{\frac{t-q}{1-q}}^{\underline{D}^{-1}(\overline{P}(\frac{1-t}{q}))}\left[p+\underline{D}(x)\right]dx \\
&+\tfrac{1}{2}\int_{\underline{\underline{D}}^{-1}(p)}^{\frac{t-q}{1-q}}\left[p+\underline{\underline{D}}(x)\right]dx \\
&+\tfrac{1}{2}\int_{0}^{\underline{\underline{D}}^{-1}(p)}\left[qz_\Pi+x\,(1-q)+q-2c\right]dx
\end{aligned}\right\} & \text{if} \quad 0 \leq z_\Pi < P^{-1}(D(\frac{t-q}{1-q})) \\
\left.\begin{aligned}
&\tfrac{1}{2}\int_{\overline{\overline{D}}^{-1}(\overline{\overline{P}}(\frac{1-q}{q}))}^{1}\left[p+\overline{\overline{D}}(x)\right]dx \\
&+\tfrac{1}{2}\int_{\overline{D}^{-1}(\overline{P}(\frac{1-t}{q}))}^{\overline{D}^{-1}(\overline{P}(\frac{1-q}{q}))}\left[p+\overline{D}(x)\right]dx \\
&+\tfrac{1}{2}\int_{\underline{D}^{-1}(p)}^{\underline{D}^{-1}(\underline{P}(\frac{1-t}{q}))}\left[p+\underline{D}(x)\right]dx \\
&+\tfrac{1}{2}\int_{\frac{t-q}{1-q}}^{\underline{D}^{-1}(p)}\left[qz_\Pi+x\,(1-q)+q-c\right]dx \\
&+\tfrac{1}{2}\int_{0}^{\frac{t-q}{1-q}}\left[qz_\Pi+x\,(1-q)+q-2c\right]dx
\end{aligned}\right\} & \text{if} \quad P^{-1}(D(\frac{t-q}{1-q})) \leq z_\Pi < \frac{1-t}{q} \\
\left.\begin{aligned}
&\tfrac{1}{2}\int_{\overline{\overline{D}}^{-1}(\overline{\overline{P}}(\frac{1-q}{q}))}^{1}\left[p+\overline{\overline{D}}(x)\right]dx \\
&+\tfrac{1}{2}\int_{\overline{D}^{-1}(p)}^{\overline{D}^{-1}(\overline{P}(\frac{1-q}{q}))}\left[p+\overline{D}(x)\right]dx \\
&+\tfrac{1}{2}\int_{1-\frac{q}{1-q}z_\Pi}^{\overline{D}^{-1}(p)}\left[qz_\Pi+x\,(1-q)+q\right]dx \\
&+\tfrac{1}{2}\int_{0}^{1-\frac{q}{1-q}z_\Pi}\left[qz_\Pi+x\,(1-q)+q-2c\right]dx
\end{aligned}\right\} & \text{if} \quad \frac{1-t}{q} \leq z_\Pi < \frac{1-q}{q} \\
\left.\begin{aligned}
&\tfrac{1}{2}\int_{\overline{\overline{D}}^{-1}(p)}^{1}\left[p+\overline{\overline{D}}(x)\right]dx \\
&+\tfrac{1}{2}\int_{0}^{\overline{\overline{D}}^{-1}(p)}\left[qz_\Pi+x\,(1-q)+q\right]dx
\end{aligned}\right\} & \text{if} \quad \frac{1-q}{q} < z_\Pi < 1
\end{cases}
$$

$$
\text{(IIa) } \Delta_4 = \begin{cases}
\left.\begin{aligned}
&\tfrac{1}{2}\int_{0}^{\underline{\underline{P}}^{-1}(d)} \left[\underline{\underline{P}}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\underline{\underline{P}}^{-1}(d)}^{\frac{1-q}{q}(1-z_\Delta)} \left[qy+z_\Delta(1-q)+q\right] dy \\
&+\tfrac{1}{2}\int_{\frac{1-q}{q}(1-z_\Delta)}^{1} \left[qy+z_\Delta(1-q)+q+2c\right] dy
\end{aligned}\right\} & \text{if} \quad 0 \le z_\Delta < \frac{t-q}{1-q} \\[2ex]
\left.\begin{aligned}
&\tfrac{1}{2}\int_{0}^{\underline{\underline{P}}^{-1}\left(\underline{\underline{D}}\left(\frac{t-q}{1-q}\right)\right)} \left[\underline{\underline{P}}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\underline{P}^{-1}\left(\underline{D}\left(\frac{t-q}{1-q}\right)\right)}^{\underline{P}^{-1}(d)} \left[\underline{P}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\underline{P}^{-1}(d)}^{\frac{1-t}{q}} \left[qy+z_\Delta(1-q)+q+c\right] dy \\
&+\tfrac{1}{2}\int_{\frac{1-t}{q}}^{1} \left[qy+z_\Delta(1-q)+q+2c\right] dy
\end{aligned}\right\} & \text{if} \quad \frac{t-q}{1-q} \le z_\Delta < D^{-1}(P(\frac{1-t}{q})) \\[2ex]
\left.\begin{aligned}
&\tfrac{1}{2}\int_{0}^{\underline{\underline{P}}^{-1}\left(\underline{\underline{D}}\left(\frac{t-q}{1-q}\right)\right)} \left[\underline{\underline{P}}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\underline{P}^{-1}\left(\underline{D}\left(\frac{t-q}{1-q}\right)\right)}^{\frac{1-t}{q}} \left[\underline{P}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\frac{1-t}{q}}^{\overline{P}^{-1}(d)} \left[\overline{P}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\overline{P}^{-1}(d)}^{1} \left[qy+z_\Delta(1-q)+q+2c\right] dy
\end{aligned}\right\} & \text{if} \quad D^{-1}(P(\frac{1-t}{q})) \le z_\Delta < D^{-1}(P(\frac{1-q}{q}) \\[2ex]
\left.\begin{aligned}
&\tfrac{1}{2}\int_{0}^{\underline{\underline{P}}^{-1}\left(\underline{\underline{D}}\left(\frac{t-q}{1-q}\right)\right)} \left[\underline{\underline{P}}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\underline{P}^{-1}\left(\underline{D}\left(\frac{t-q}{1-q}\right)\right)}^{\frac{1-t}{q}} \left[\underline{P}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\frac{1-t}{q}}^{\frac{1-q}{q}} \left[\overline{P}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\frac{1-q}{q}}^{\overline{\overline{P}}^{-1}(d)} \left[\overline{\overline{P}}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\overline{\overline{P}}^{-1}(d)}^{1} \left[qy+z_\Delta(1-q)+q+2c\right] dy
\end{aligned}\right\} & \text{if} \quad D^{-1}(P(\frac{1-q}{q}) < z_\Delta < 1
\end{cases}
$$

$$
\text{(IIb) } \Pi_4 = \begin{cases}
\left.\begin{aligned}
&\tfrac{1}{2}\int_{\overline{\overline{D}}^{-1}(\overline{\overline{P}}(\frac{1-q}{q}))}^{1} \left[p+\overline{\overline{D}}(x)\right]dx \\
&+\tfrac{1}{2}\int_{\frac{t-q}{1-q}}^{\overline{D}^{-1}(\overline{P}(\frac{1-q}{q}))} \left[p+\overline{D}(x)\right]dx \\
&+\tfrac{1}{2}\int_{\underline{D}^{-1}(\underline{P}(\frac{1-t}{q}))}^{\frac{t-q}{1-q}} \left[p+\underline{D}(x)\right]dx \\
&+\tfrac{1}{2}\int_{\underline{\underline{D}}^{-1}(p)}^{\underline{\underline{D}}^{-1}(\underline{\underline{P}}(\frac{1-t}{q}))} \left[p+\underline{\underline{D}}(x)\right]dx \\
&+\tfrac{1}{2}\int_{0}^{\underline{\underline{D}}^{-1}(p)} \left[qz_\Pi+x\,(1-q)+q-2c\right]dx
\end{aligned}\right\} & \text{if} \quad 0 \le z_\Pi < \frac{1-t}{q} \\
\left.\begin{aligned}
&\tfrac{1}{2}\int_{\overline{\overline{D}}^{-1}(\overline{\overline{P}}(\frac{1-q}{q}))}^{1} \left[p+\overline{\overline{D}}(x)\right]dx \\
&+\tfrac{1}{2}\int_{\frac{t-q}{1-q}}^{\overline{D}^{-1}(\overline{P}(\frac{1-q}{q}))} \left[p+\overline{D}(x)\right]dx \\
&+\tfrac{1}{2}\int_{1-\frac{q}{1-q}z_\Pi}^{\frac{t-q}{1-q}} \left[p+\underline{D}_{UP}(x)\right]dx \\
&+\tfrac{1}{2}\int_{\underline{D}_{DW}^{-1}(p)}^{1-\frac{q}{1-q}z_\Pi} \left[p+\underline{D}_{DW}(x)\right]dx \\
&+\tfrac{1}{2}\int_{0}^{\underline{D}_{DW}^{-1}(p)} \left[qz_\Pi+x\,(1-q)+q-2c\right]dx
\end{aligned}\right\} & \text{if} \quad \frac{1-t}{q} \le z_\Pi < \widehat{z}_\Pi \\
\left.\begin{aligned}
&\tfrac{1}{2}\int_{\overline{\overline{D}}^{-1}(\overline{\overline{P}}(\frac{1-q}{q}))}^{1} \left[p+\overline{\overline{D}}(x)\right]dx \\
&+\tfrac{1}{2}\int_{\frac{t-q}{1-q}}^{\overline{D}^{-1}(\overline{P}(\frac{1-q}{q}))} \left[p+\overline{D}(x)\right]dx \\
&+\tfrac{1}{2}\int_{\underline{D}_{UP}^{-1}(p)}^{\frac{t-q}{1-q}} \left[p+\underline{D}_{UP}(x)\right]dx \\
&+\tfrac{1}{2}\int_{1-\frac{q}{1-q}z_\Pi}^{\underline{D}_{UP}^{-1}(p)} \left[qz_\Pi+x\,(1-q)+q\right] \\
&+\tfrac{1}{2}\int_{0}^{1-\frac{q}{1-q}z_\Pi} \left[qz_\Pi+x\,(1-q)+q-2c\right]dx
\end{aligned}\right\} & \text{if} \quad \widehat{z}_\Pi \le z_\Pi < P^{-1}(D(\frac{t-q}{1-q})) \\
\left.\begin{aligned}
&\tfrac{1}{2}\int_{\overline{\overline{D}}^{-1}(\overline{\overline{P}}(\frac{1-q}{q}))}^{1} \left[p+\overline{\overline{D}}(x)\right]dx \\
&+\tfrac{1}{2}\int_{\overline{D}^{-1}(p)}^{\overline{D}^{-1}(\overline{P}(\frac{1-q}{q}))} \left[p+\overline{D}(x)\right]dx \\
&+\tfrac{1}{2}\int_{1-\frac{q}{1-q}z_\Pi}^{\overline{D}^{-1}(p)} \left[qz_\Pi+x\,(1-q)+q\right] \\
&+\tfrac{1}{2}\int_{0}^{1-\frac{q}{1-q}z_\Pi} \left[qz_\Pi+x\,(1-q)+q-2c\right]dx
\end{aligned}\right\} & \text{if} \quad P^{-1}(D(\frac{t-q}{1-q})) \le z_\Pi < \frac{1-q}{q} \\
\left.\begin{aligned}
&\tfrac{1}{2}\int_{\overline{\overline{D}}^{-1}(p)}^{1} \left[p+\overline{\overline{D}}(x)\right]dx \\
&+\tfrac{1}{2}\int_{0}^{\overline{\overline{D}}^{-1}(p)} \left[qz_\Pi+x\,(1-q)+q\right]dx
\end{aligned}\right\} & \text{if} \quad \frac{1-q}{q} < z_\Pi < 1
\end{cases}
$$

$$
\text{(IIb) } \Delta_4 = \begin{cases}
\left.\begin{array}{l}
\frac{1}{2}\int_{0}^{\underline{\underline{P}}^{-1}(d)} \left[\underline{\underline{P}}(y)+d\right] dy \\
+\frac{1}{2}\int_{\underline{\underline{P}}^{-1}(d)}^{\frac{1-q}{q}(1-z_\Delta)} \left[qy+z_\Delta\left(1-q\right)+q\right] dy \\
+\frac{1}{2}\int_{\frac{1-q}{q}(1-z_\Delta)}^{1} \left[qy+z_\Delta\left(1-q\right)+q+2c\right] dy
\end{array}\right\} & \text{if} \quad 0 \leq z_\Delta < D^{-1}(P(\frac{1-t}{q})) \\
\left.\begin{array}{l}
\frac{1}{2}\int_{0}^{\frac{1-t}{q}} \left[\underline{\underline{P}}(y)+d\right] dy + \frac{1}{2}\int_{\frac{1-t}{q}}^{\underline{P}_{DW}^{-1}(d)} \left[\underline{P}_{DW}(y)+d\right] dy \\
+\frac{1}{2}\int_{\underline{P}_{DW}^{-1}(d)}^{\frac{1-q}{q}(1-z_\Delta)} \left[qy+z_\Delta\left(1-q\right)+q\right] dy \\
+\frac{1}{2}\int_{\frac{1-q}{q}(1-z_\Delta)}^{1} \left[qy+z_\Delta\left(1-q\right)+q+2c\right] dy
\end{array}\right\} & \text{if} \quad D^{-1}(P(\frac{1-t}{q})) \leq z_\Delta < \widehat{z}_\Delta \\
\left.\begin{array}{l}
\frac{1}{2}\int_{0}^{\frac{1-t}{q}} \left[\underline{\underline{P}}(y)+d\right] dy \\
+\frac{1}{2}\int_{\frac{1-t}{q}}^{\frac{1-q}{q}(1-z_\Delta)} \left[\underline{P}_{DW}(y)+d\right] dy \\
+\frac{1}{2}\int_{\frac{1-q}{q}(1-z_\Delta)}^{\underline{P}_{UP}^{-1}(d)} \left[\underline{P}_{UP}(y)+d\right] dy \\
+\frac{1}{2}\int_{\underline{P}_{UP}^{-1}(d)}^{1} \left[qy+z_\Delta\left(1-q\right)+q+2c\right] dy
\end{array}\right\} & \text{if} \quad \widehat{z}_\Delta \leq z_\Delta < \frac{t-q}{1-q} \\
\left.\begin{array}{l}
\frac{1}{2}\int_{0}^{\frac{1-t}{q}} \left[\underline{\underline{P}}(y)+d\right] dy \\
+\frac{1}{2}\int_{\frac{1-t}{q}}^{\underline{P}^{-1}(\underline{D}(\frac{t-q}{1-q}))} \left[\underline{P}(y)+d\right] dy \\
+\frac{1}{2}\int_{\overline{P}^{-1}(\overline{D}(\frac{t-q}{1-q}))}^{\overline{P}^{-1}(d)} \left[\overline{P}(y)+d\right] dy \\
+\frac{1}{2}\int_{\overline{P}^{-1}(d)}^{1} \left[qy+z_\Delta\left(1-q\right)+q+2c\right] dy
\end{array}\right\} & \text{if} \quad \frac{t-q}{1-q} \leq z_\Delta < D^{-1}(P(\frac{1-q}{q}) \\
\left.\begin{array}{l}
\frac{1}{2}\int_{0}^{\frac{1-t}{q}} \left[\underline{\underline{P}}(y)+d\right] dy \\
+\frac{1}{2}\int_{\frac{1-t}{q}}^{\underline{P}^{-1}(\underline{D}(\frac{t-q}{1-q}))} \left[\underline{P}(y)+d\right] dy \\
+\frac{1}{2}\int_{\overline{P}^{-1}(\overline{D}(\frac{t-q}{1-q}))}^{\frac{1-q}{q}} \left[\overline{P}(y)+d\right] dy \\
+\frac{1}{2}\int_{\frac{1-q}{q}}^{\overline{\overline{P}}^{-1}(d)} \left[\overline{\overline{P}}(y)+d\right] dy \\
+\frac{1}{2}\int_{\overline{\overline{P}}^{-1}}^{1} \left[qy+z_\Delta\left(1-q\right)+q+2c\right] dy
\end{array}\right\} & \text{if} \quad D^{-1}(P(\frac{1-q}{q}) < z_\Delta < 1
\end{cases}
$$

$$
\text{(IIc) } \Pi_4 = \begin{cases}
\left.\begin{aligned}
&\tfrac{1}{2}\int_{\frac{t-q}{1-q}}^{1} \left[p+\overline{\overline{D}}(x)\right] dx \\
&+\tfrac{1}{2}\int_{\overline{D}^{-1}(\overline{P}(\frac{1-q}{q}))}^{\frac{t-q}{1-q}} \left[p+\overline{D}(x)\right] dx \\
&+\tfrac{1}{2}\int_{\underline{D}^{-1}(\underline{P}(\frac{1-t}{q}))}^{\underline{D}^{-1}(\underline{P}(\frac{1-q}{q}))} \left[p+\underline{D}(x)\right] dx \\
&+\tfrac{1}{2}\int_{\underline{\underline{D}}^{-1}(p)}^{\underline{\underline{D}}^{-1}(\underline{\underline{P}}(\frac{1-t}{q}))} \left[p+\underline{\underline{D}}(x)\right] d \\
&+\tfrac{1}{2}\int_{0}^{\underline{\underline{D}}^{-1}(p)} \left[qz_{\Pi}+x\left(1-q\right)+q-2c\right] dx
\end{aligned}\right\} & \text{if} \quad 0 \le z_{\Pi} < \frac{1-t}{q} \\
\left.\begin{aligned}
&\tfrac{1}{2}\int_{\frac{t-q}{1-q}}^{1} \left[p+\overline{\overline{D}}(x)\right] dx \\
&+\tfrac{1}{2}\int_{\overline{D}^{-1}(\overline{P}(\frac{1-q}{q})))}^{\frac{t-q}{1-q}} \left[p+\overline{D}(x)\right] dx \\
&+\tfrac{1}{2}\int_{1-\frac{q}{1-q}z_{\Pi}}^{\underline{D}^{-1}(\underline{P}(\frac{1-q}{q}))} \left[p+\underline{D}_{UP}(x)\right] dx \\
&+\tfrac{1}{2}\int_{\underline{D}_{DW}^{-1}(p)}^{1-\frac{q}{1-q}z_{\Pi}} \left[p+\underline{D}_{DW}(x)\right] dx \\
&+\tfrac{1}{2}\int_{0}^{\underline{D}_{DW}^{-1}(p)} \left[qz_{\Pi}+x\left(1-q\right)+q-2c\right] dx
\end{aligned}\right\} & \text{if} \quad \frac{1-t}{q} \le z_{\Pi} < \widehat{z}_{\Pi} \\
\left.\begin{aligned}
&\tfrac{1}{2}\int_{\frac{t-q}{1-q}}^{1} \left[p+\overline{\overline{D}}(x)\right] dx \\
&+\tfrac{1}{2}\int_{\overline{D}^{-1}(\overline{P}(\frac{1-q}{q}))}^{\frac{t-q}{1-q}} \left[p+\overline{D}(x)\right] dx \\
&+\tfrac{1}{2}\int_{\underline{D}_{UP}^{-1}(p)}^{\underline{D}^{-1}(\underline{P}(\frac{1-q}{q}))} \left[p+\underline{D}(x)\right] dx \\
&+\tfrac{1}{2}\int_{1-\frac{q}{1-q}z_{\Pi}}^{\underline{D}_{UP}^{-1}(p)} \left[qz_{\Pi}+x\left(1-q\right)+q\right] dx \\
&+\tfrac{1}{2}\int_{0}^{1-\frac{q}{1-q}z_{\Pi}} \left[qz_{\Pi}+x\left(1-q\right)+q-2c\right] dx
\end{aligned}\right\} & \text{if} \quad \widehat{z}_{\Pi} \le z_{\Pi} < \frac{1-q}{q} \\
\left.\begin{aligned}
&\tfrac{1}{2}\int_{\frac{t-q}{1-q}}^{1} \left[p+\overline{\overline{D}}(x)\right] dx \\
&+\tfrac{1}{2}\int_{\overline{D}^{-1}(p)}^{\frac{t-q}{1-q}} \left[p+\overline{D}(x)\right] dx \\
&+\tfrac{1}{2}\int_{0}^{\overline{D}^{-1}(p)} \left[qz_{\Pi}+x\left(1-q\right)+q\right] dx
\end{aligned}\right\} & \text{if} \quad \frac{1-q}{q} \le z_{\Pi} < P^{-1}(D(\tfrac{t-q}{1-q})) \\
\left.\begin{aligned}
&\tfrac{1}{2}\int_{\overline{\overline{D}}^{-1}(p)}^{1} \left[p+\overline{\overline{D}}(x)\right] dx \\
&+\tfrac{1}{2}\int_{0}^{\overline{\overline{D}}(p)} \left[qz_{\Pi}+x\left(1-q\right)+q\right] dx
\end{aligned}\right\} & \text{if} \quad P^{-1}(D(\tfrac{t-q}{1-q})) < z_{\Pi} < 1
\end{cases}
$$

$$
\text{(IIc) } \Delta_4 = \begin{cases}
\begin{aligned}
&\tfrac{1}{2}\int_{0}^{\underline{\underline{P}}^{-1}(d)} \left[\underline{\underline{P}}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\underline{\underline{P}}^{-1}(d)}^{\frac{1-q}{q}(1-z_\Delta)} \left[qy+z_\Delta(1-q)+q\right] dy \\
&+\tfrac{1}{2}\int_{\frac{1-q}{q}(1-z_\Delta)}^{1} \left[qy+z_\Delta(1-q)+q+2c\right] dy
\end{aligned} & \text{if} \quad 0 \le z_\Delta < D^{-1}(P(\frac{1-t}{q})) \\[2ex]
\begin{aligned}
&\tfrac{1}{2}\int_{0}^{\frac{1-t}{q}} \left[\underline{\underline{P}}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\frac{1-t}{q}}^{\underline{P}_{DW}^{-1}(d)} \left[\underline{P}_{DW}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\underline{P}_{DW}^{-1}(d)}^{\frac{1-q}{q}(1-z_\Delta)} \left[qy+z_\Delta(1-q)+q\right] dy \\
&+\tfrac{1}{2}\int_{\frac{1-q}{q}(1-z_\Delta)}^{1} \left[qy+z_\Delta(1-q)+q+2c\right] dy
\end{aligned} & \text{if} \quad D^{-1}(P(\frac{1-t}{q})) \le z_\Delta < \widehat{z}_\Delta \\[2ex]
\begin{aligned}
&\tfrac{1}{2}\int_{0}^{\frac{1-t}{q}} \left[\underline{\underline{P}}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\frac{1-t}{q}}^{\frac{1-q}{q}(1-z_\Delta)} \left[\underline{P}_{DW}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\frac{1-q}{q}(1-z_\Delta)}^{\underline{P}_{UP}^{-1}(d)} \left[\underline{P}_{UP}^{-1}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\underline{P}_{UP}^{-1}(d)}^{1} \left[qy+z_\Delta(1-q)+q+2c\right] dy
\end{aligned} & \text{if} \quad \widehat{z}_\Delta \le z_\Delta < D^{-1}(P(\frac{1-q}{q}) \\[2ex]
\begin{aligned}
&\tfrac{1}{2}\int_{0}^{\frac{1-t}{q}} \left[\underline{\underline{P}}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\frac{1-t}{q}}^{\frac{1-q}{q}} \left[\underline{P}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\frac{1-q}{q}}^{\overline{P}^{-1}(d)} \left[\overline{P}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\overline{P}^{-1}(d)}^{1} \left[qy+z_\Delta(1-q)+q+2c\right] dy
\end{aligned} & \text{if} \quad D^{-1}(P(\frac{1-q}{q}) \le z_\Delta < \frac{t-q}{1-q} \\[2ex]
\begin{aligned}
&\tfrac{1}{2}\int_{0}^{\frac{1-t}{q}} \left[\underline{\underline{P}}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\frac{1-t}{q}}^{\frac{1-q}{q}} \left[\underline{P}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\frac{1-q}{q}}^{\overline{P}^{-1}(\overline{D}(\frac{t-q}{1-q}))} \left[\overline{P}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\overline{\overline{P}}^{-1}(\overline{\overline{D}}(\frac{t-q}{1-q}))}^{\overline{\overline{P}}^{-1}(d)} \left[\overline{\overline{P}}(y)+d\right] dy \\
&+\tfrac{1}{2}\int_{\overline{\overline{P}}^{-1}(d)}^{1} \left[qy+z_\Delta(1-q)+q+2c\right] dy
\end{aligned} & \text{if} \quad \frac{t-q}{1-q} < z_\Delta < 1
\end{cases}
$$

Based on these payoff functions, and matching a plaintiff's demand with the defendant's offer with which that plaintiff settles at the margin, we can calculate the first order conditions (second order conditions are verified) in each of the previous subcases as we did for Cases 1, 2 and 3. Subcases (Ib) and (Ic) yield the same first-order conditions and hence can be merged. Similarly for subcases (IIb) and (IIc). From the first order conditions we can calculate the parties' optimal bid functions and give explicit values to the relevant thresholds and to the conditions for the different subcases.

Since the case with $q < \frac{1}{2}$ gives the same bid functions as the case with $q > \frac{1}{2}$, we can disregard this distinction. Hence, we are left with two cases and the condition that distinguishes them (a condition on the ordering of the thresholds) can be conveniently rewritten in terms of the court fee c. We have the following two subcases for Case 4:

(4A) resulting from grouping subcases (Ia) and (IIa), if $c \leq \frac{1}{6}\frac{1-t}{q(1-q)}$

(4B) resulting from grouping subcases (Ib), (Ic), (IIb), and (IIc), if $c > \frac{1}{6}\frac{1-t}{q(1-q)}$

The corresponding bid functions are as indicated in the text. Since the equilibrium bid functions only have two or three pieces (many of the parts of the bid functions yield the same first order conditions), to keep notation simple, in the text we relabel the various pieces of the bid functions as follows: we refer to the first, second (when existing) and third piece respectively with a lower-bar, a dot, and an upper-bar.

+++

Prop 8

Proof. Inspection of Figure 15. □

Prop. 9

Proof. Inspection of Figure 16. □

Prop. 10

Proof. Prove that E is nearly linear in q and that increases in c and t. □