Uncertainty and Risk Aversion in a Dynamic Oligopoly with Sticky Prices*

Edilio Valentini,† Paolo Vitale‡

Preliminary version. Comments are welcome

Abstract

In this paper we propose a dynamic discrete-time model based on a recursive optimization criterion that allows to characterize risk aversion in an oligopolistic framework represented by homogeneous non-storable good, sticky prices and uncertainty. Our model nests the classical dynamic oligopoly model with sticky prices *à la* Fershtman and Kamien (1987) that can be viewed as the continuous-time limit of our base model with no uncertainty and no risk aversion. In particular, we focus on the continuous-time limit of the infinite horizon formulation and show that the optimal production strategy and the consequent equilibrium price are, respectively, directly and inversely related to the degrees of uncertainty and risk-aversion. However, the effect of uncertainty and risk-aversion crucially depends on the price stickiness hypothesis since, when prices can adjust instantaneously, the steady-state equilibrium of our model with uncertainty and risk aversion collapses to Fershtman and Kamien's analogous.

1 Introduction

How do price stickiness, uncertainty and risk aversion affect the equilibrium outcome of a dynamic oligopoly where firms compete over the demand of a homogeneous, non-storable good? This might be a relevant question for electricity markets where end-use consumers are served by few firms selling a good which is perfectly homogeneous and cannot be stored (at least at reasonable costs). Moreover, retail prices in electricity markets adjust only very gradually to changes in market conditions. In fact, wholesale electricity prices change hour

---

*

†DEc - University G. D'Annunzio of Chieti-Pescara - email: valentin@unich.it
‡DEc - University G. D'Annunzio of Chieti-Pescara - email: p.vitale@unich.it

by hour while retail prices change only few times per year (Borenstein and Holland, 2005). According to Bils and Klenow (2004), for instance, in 1994-1996 the average monthly frequency of price changes in the US electricity market was 43.4 percent, corresponding to an average time between price variations of about 1.8 months. Such price stickiness is also a regulatory issue as it may represent an obstacle for having efficient prices, that is prices which are closely tied to variations in the marginal cost of generating electricity (Joskow and Wolfram, 2012).

To provide an answer to the question above, we establish a differential oligopoly game where firms are managed by risk averse entrepreneurs and produce a non-storable good in an uncertain framework characterised by price stickiness. In differential games some variables evolve over time according to a differential (or difference) equation and players conform their strategic and forward-looking behaviors to an optimal control rule (Dockner, Jorgensen, Van Long, and Sorger, 2000). Applications of differential games in economics are widespread and include macroeconomics, international trade and environment (see Turnovsky, Basar, and D'Orey (1988), Dockner and Haug (1990), van der Ploeg and De Zeeuw (1992) and Dockner and Long (1993) among the others). In industrial organization differential games have been fruitfully employed to investigate dynamic oligopolies characterized by some form of cost adjustments (Driskill and McCafferty (1989), Karp and Perloff (1989), Karp and Perloff (1993) and Wirl (2010), among the others) and the first applications to an oligopoly problem with sticky prices are Simaan and Takayama (1978) and Fershtman and Kamien (1987). Both papers employ the same continuous time dynamic duopoly model with identical firms, linear demand functions and quadratic costs. Production and price are, respectively, the control and the state variables and price stickiness is modeled by assuming that price adjusts according to a differential equation that is function of the difference between the current price and the price indicated by the demand function for the currently produced quantities.

We propose a discrete-time model based on a recursive optimization criterion that allows to characterize risk aversion in an oligopolistic framework with sticky prices and uncertainty. Such model nests the classical dynamic oligopoly with sticky prices *à la* Fershtman and Kamien (1987) that can be viewed as the continuous-time limit of our base model with no uncertainty and no risk aversion. We derive the optimal (subgame perfect) production strategy, and the corresponding equilibrium price, to be compared to both open-loop and closed-loop (feedback) Nash equilibria analysed in Fershtman and Kamien (1987). Under open-loop strategy firms decide a production plan at time zero and stick to it forever while under feedback strategy firms adapt their decisions in every instant of time by taking into account the current value of price. Therefore, if a price reduction occurs under the feedback hypothesis, no commitment is possible or credible and each firm increases its production taking into account that all its rivals are doing the same. As a consequence, in Fershtman and Kamien (1987) the steady state level of production arising in a (symmetric) feedback Nash equilibrium is greater than the steady state level of production arising in a (symmetric) open-loop equilibrium and both are greater than the equilibrium level of production of the corresponding static Cournot game. Consequently the feedback equilibrium is characterized by a stationary price which is lower than the stationary price of the open-loop equilibrium that, in turn, is lower than the equilibrium price of static Cournot game. Fershtman and Kamien (1987) show also that, when price adjusts instantaneously, the steady state equilibrium price converges to the static Cournot equilibrium price if firms use open-loop strategies, while it converges to a lower value if firms follow feedback strategies. Therefore, removing price stickiness is not sufficient to allow that a dynamic oligopoly converges to its static counterpart as this convergence requires also that firms can precommit to their initial output strategies. This result is intriguing since open loop strategies are judged less interesting than feedback strategies for the study of dynamic games (Tsutsui and Mino, 1990) because they are generally not subgame perfect[1].

The model developed by Fershtman and Kamien (1987) has been extended in several directions. Dockner (1988), for instance, generalizes it to the case of more than two firms[2] showing that the dynamic oligopoly price converges to the long run (zero profit) competitive price when the number of firms goes to infinity, independent of the assumption of open-loop or feedback strategy. Tsutsui and Mino (1990) introduce the possibility of price ceilings to consider the case of nonlinear feedback strategies finding that, when the price ceiling is not too high, feedback equilibrium prices can be higher than the equilibrium price that arises under the linear feedback strategy assumed by Fershtman and Kamien (1987). Piga (2000) shows that when firms can invest in advertising the nonlinear feedback equilibrium price may be greater than the open-loop equilibrium price while the latter is above the linear feedback equilibrium price. Other extensions include **?** and **?**, who analyze the profitability of horizontal mergers, Cellini and Lambertini (2007),dealing with the case of firms selling differentiated products and Wiszniewska-Matyszkiel, Bodnar, and Mirota (2015), focusing on firms' behavior off the steady-state price path.

To the best of our knowledge differential oligopolistic games with sticky prices have been always analyzed in deterministic contexts and our paper is the first attempt of extending the feedback strategy model of Fershtman and Kamien (1987) with the inclusion of uncertainty and risk aversion. Such extension allows to show that uncertainty and risk aversion affect the steady-state equilibrium as the optimal production strategy and the consequent equilib-

[1] See Cellini and Lambertini (2004) for a short review of the papers showing under what conditions open-loop strategies can be subgame perfect.

[2] A similar extension is also developed by Cellini and Lambertini (2004).

rium price are, respectively, directly and inversely related to the degrees of uncertainty and risk-aversion. However, the impact of uncertainty and risk-aversion on the optimal production strategy crucially depends on the price stickiness hypothesis. In fact, we show that, even in presence of uncertainty and risk-aversion, when prices can adjust instantaneously, the steady state equilibrium collapses to Fershtman and Kamien's analogous.

The rest of the paper is organised as follows. In Section 2 we first introduce uncertainty and risk-aversion in a discrete-time formulation of a market for a non-storable good with sticky prices, then we consider its continuous-time limit and derive several theoretical results and empirical implications. In Section 3 we concentrate on the stationary solution for the infinite horizon formulation and derive important comparative statics result pertaining to the impact of risk-aversion, uncertainty and the number of firms. Finally, Section 4 investigates what happens to the stationary equilibrium of the infinite horizon formulation when time-discounting collapses to zero and when prices become either infinitely sticky or perfectly flexible. The proofs of all results discussed in the paper relegated in a separate Appendix.

2 A Market for a Non-storable Good with Sticky Prices

We start from a discrete-time formulation of a market for a non-storable good with sticky prices which allows to introduce uncertainty and risk-aversion in a simple, intuitive and tractable manner. We then consider its continuous-time limit and derive several theoretical results and empirical implications. The discrete-time formulation is set out so that its continuous-time limit is consistent with that of Fershtman and Kamien (1987). In this way we can unveil the impact of uncertainty and risk-aversion on a market for a non-storable good with sticky prices, comparing our analysis *vis-à-vis* the existing literature on differential games for markets with price-inertia and imperfect competition.

2.1 A Discrete-time formulation

Let us assume that production and consumption take place at equally spaced in time moments between time 0 and time T. These moments are $t_1, \ldots, t_n, t_{n+1}, \ldots, t_N$, where $t_{n+1} = t_n + \Delta$, with Δ some positive interval of time, while t_N coincides with the final date T in which production is interrupted. This value can easily be pushed towards infinity to consider an infinite horizon formulation and consequently study a stationary equilibrium. Period n will correspond to time t_n. The continuous-time limit will be reached when Δ converges to zero. The discrete-time counterpart of the continuous-time formulation for the dynamics of the price of the non-storable consumption good is as follows

$$p_{n+1} \quad = \quad \mu\,\Delta \;+\; (1+a\Delta)\,p_n \;+\; b\Delta x_n \;+\; \epsilon_{n+1}\,, \tag{1}$$

where p_n is the price of the non-storable good at time t_n, Δx_n is the corresponding quantity produced and brought to the market, ϵ_{n+1} is an idiosyncratic shock to its demand function, with $\epsilon_{n+1} \sim N(0, \sigma_\epsilon^2 \Delta)$, while a, b and μ are constants.

The quantity produced and brought to the market Δx_n is the product of the time interval Δ and the output rate/intensity x_n for period n. [*Fershtman and Kamien refer to u_i as firm i's output rate.*] In oligopoly, where M identical firms produce the non-storable good, $\Delta x_n = \Delta u_{1,n} + \Delta u_{2,n} \ldots + \Delta u_{M,n}$, where $\Delta u_{m,n}$ corresponds to the quantity produced by firm m in period n. This is the product of Δ and firm m's output rate/intensity $u_{m,n}$. In order to concentrate on symmetric equilibria we assume the M firms are perfectly symmetrical in that they share the same cost function, while the entrepreneurs which own and run them share the same degree of risk-aversion.[3]

Now, without loss of generality, let us analyze the optimal production strategy of firm 1. As in Fershtman and Kamien (1987) firm 1 is characterized by quadratic production costs. Specifically, in n the intensity of these costs is qu_n^2, where q is a positive constant and where for simplicity we write $u_{1,n} = u_n$. The sale of the non-storable good generates a revenue which is linear in the quantity brought to the market. This implies that the intensity of the firm's revenue in n is $p_n u_n$, while that of the corresponding profits is $p_n u_n - qu_n^2$.

In Fershtman and Kamien (1987) the entrepreneur maximizes the discounted value of the profits her firm generates. In our formulation, as the price at which the firm will be able to sell the quantity of the non-storable good it produces is subject to idiosyncratic shocks, such profits are uncertain. Therefore, we assume the entrepreneur is risk-averse and is endowed with a special form of recursive preferences proposed by Hansen and Sargent (Hansen and Sargent, 1995). In particular in period n, with $n = 1, 2, \ldots, N$, the entrepreneur solves the following recursive optimization

$$\boldsymbol{\mathcal{V}}_n \quad = \quad \min_{u_n} \left\{ \Delta c_n \;+\; \frac{2}{\rho}\,\ln\left(E_n\left[\exp\left(\delta^{\Delta}\,\frac{\rho}{2}\,\boldsymbol{\mathcal{V}}_{n+1} \right) \right] \right) \right\}\,, \tag{2}$$

where ρ (with $\rho > 0$) is a risk-enhancement coefficient, δ (with $0 < \delta < 1$) is a time-discounting factor, Δc_n is the (per-period) cost function, with $c_n = qu_n^2 - p_n u_n$, and $\boldsymbol{\mathcal{V}}_n$ is the value function (with final condition $\boldsymbol{\mathcal{V}}_{N+1} = 0$).

The optimization criterion in (2) accommodates risk-aversion through the curvature of the

[3] With different degrees of risk-aversion on the part of the M entrepreneurs we would not be able to concentrate on symmetric equilibria.

exponential function. As the convexity of $\ln(E[\exp(\delta^{\Delta}\frac{\rho}{2}\boldsymbol{\mathcal{V}}_{n+1})])$ increases with ρ, this coefficient determines the entrepreneur's degree of risk-aversion. Importantly, for $\rho \downarrow 0$, the recursive optimization in (2) converges to $\boldsymbol{\mathcal{V}}_n = \min_{u_n} E_n[\Delta c_n + \delta^{\Delta}\boldsymbol{\mathcal{V}}_{n+1}]$.[4] As this is the Bellman equation a risk-neutral entrepreneur will solve in our formulation, we conclude that our formulation subsumes that of Fershtman and Kamien, when $\rho = 0$, and extends it by allowing for risk-sensitive preferences, when $\rho > 0$.

Exploiting results by Vitale (2017) the following Lemma can be established.

Lemma 1 *When M identical firms operate in the oligopolistic market for the production of the non-storable good, in period n the optimal production strategy of a generic firm is*

$$u_n = \kappa_{p,n}\, p_n + \kappa_{e,n}(\tilde{\pi}_{n+1}\mu\Delta - \tilde{\vartheta}_{n+1}), \ \textit{with} \tag{3}$$

$$\kappa_{p,n} = \frac{\frac{1}{2} - b(1+a\Delta)\tilde{\pi}_{n+1}}{q + M b^2\Delta\tilde{\pi}_{n+1}}, \quad \kappa_{e,n} = -\frac{b}{q + Mb^2\Delta\tilde{\pi}_{n+1}}, \tag{4}$$

$$\tilde{\pi}_{n+1} = \delta^{\Delta}\pi_{n+1}(1-\delta^{\Delta}\rho\,\sigma_{\epsilon}^2\Delta\,\pi_{n+1})^{-1}, \quad \tilde{\vartheta}_{n+1} = \delta^{\Delta}\vartheta_{n+1}(1-\delta^{\Delta}\rho\,\sigma_{\epsilon}^2\Delta\,\pi_{n+1})^{-1}, \tag{5}$$

$$\pi_n = \Delta q\,\kappa_{p,n}^2 - \Delta\,\kappa_{p,n} + \left[(1+a\Delta) + M\,b\,\Delta\,\kappa_{p,n}\right]^2\,\tilde{\pi}_{n+1}, \tag{6}$$

$$\vartheta_n = [1 + (M-1)b\tilde{\pi}_{n+1}\,\Delta\,\kappa_{e,n}][Mb\,\Delta\,\kappa_{p,n} + (1 + a\Delta)](\tilde{\vartheta}_{n+1} - \tilde{\pi}_{n+1}\mu\Delta) \tag{7}$$

and boundary conditions $\pi_{N+1} = 0$ *and* $\vartheta_{N+1} = 0$.

Proof. See the Appendix.

Solving the recursive system of equations (4), (5), (6) and (7) for $n = 1, 2, \ldots, N$, with the boundary conditions $\pi_{N+1} = 0$ and $\vartheta_{N+1} = 0$ is fairly cumbersome and can be achieved only numerically. However, we are interested in investigating what happens when we consider the continuous-time limit, for $\Delta \downarrow 0$.

2.2 The Continuous-time Limit

For $\Delta \downarrow 0$ this discrete-time formulation converges to a continuous-time limit. In particular, the continuous-time counter-part of equation (1) is given by the following expression

$$\frac{dp(t)}{dt} = \mu + ap(t) + bx(t) + \epsilon(t),$$

[4]The proof of these and other results are available on request. See also Vitale (2017).

where $x(t) = u_1(t) + \ldots + u_M(t)$. Now suppose we define $s = -a$, $\beta = b/a$, $\alpha = \mu/s$. We can then write that

$$\frac{dp(t)}{dt} \;=\; s(\alpha \;-\; \beta x(t) \;-\; p(t)) \;+\; \epsilon(t)\,. \tag{8}$$

Importantly, for $M = 2$, $\beta = 1$ and $\epsilon(t) \equiv 0$ we have equation (1.2) in Fershtman and Kamien (1987). Given our formulation the condition that no variation in the price is expected is

$$E_t\left[\frac{dp(t)}{dt}\right] \;=\; 0\,.$$

Following Fershtman and Kamien (1987) it can be said that if this condition is met the good market is in equilibrium, in that the good price corresponds to that which is found in the demand function for that level of production. This condition corresponds to the following expression

$$p(t) \;=\; \alpha \;-\; \beta x(t)\,.$$

This expression represents an inverse demand function which would prevail in a static model in which prices are fully flexible. In the dynamic model prices are sticky. When a production decision is taken, the good price does not reach immediately its equilibrium value. However, let $p^*(t) \equiv \alpha \;-\; \beta x(t)$ be such a price. Substituting it out in equation (8) we find that

$$\frac{dp(t)}{dt} \;=\; -\,s\,(p(t) \;-\; p^*(t)) \;+\; \epsilon(t)\,, \tag{9}$$

which unveils a mean-reverting dynamics toward the equilibrium price. The speed of convergence towards the long-run equilibrium is determined by the the coefficient s, whose inverse can then be considered a measure of price stickiness.

Using Lemma 1 it is possible to prove the following Proposition, which characterizes the optimal production strategy of the generic firm in the continuous-time limit.

Proposition 1 *When M identical firms operate in the oligopolistic market for the production of the non-storable good, in t the optimal production strategy of the generic firm is*

$$u(t) \;=\; \kappa(t)\,p(t) + \frac{b}{q}\vartheta(t), \;\; \textit{with} \;\; \kappa(t) \;=\; \frac{1}{q}\left(\frac{1}{2} \;-\; b\pi(t)\right) \tag{10}$$

and $\pi(t)$ and $\vartheta(t)$ satisfying the following differential equations

$$\frac{d\pi(t)}{dt} - \frac{1}{q}\left(b\pi(t) - \frac{1}{2}\right)\left((2M-1)b\pi(t) - \frac{1}{2}\right) + (2a + \ln\delta)\pi(t) + \rho\sigma_\epsilon^2\pi(t)^2 = 0\,, \quad (11)$$

$$\frac{d\vartheta(t)}{dt} + \left(\ln\delta + a + \frac{M}{2}\frac{b}{q}\right)\vartheta(t) - \left((2M-1)\frac{b^2}{q} - \rho\sigma_\epsilon^2\right)\pi(t)\vartheta(t) - \mu\pi(t) = 0\,, \quad (12)$$

with boundary conditions $\pi(T) = 0$ and $\vartheta(T) = 0$.

Proof. See the Appendix.

Solving the two differential equations in Proposition 1 is involved. In particular, an explicit solution exists only for the former and hence numerical procedures are called for to describe the dynamics of the equilibrium presented in Proposition 1. However, in the infinite horizon formulation, where the final date T is pushed forward to infinite, we easily characterize a stationary equilibrium in which

$$\kappa(t) = \bar{\kappa}\,, \quad \pi(t) = \bar{\pi} \quad \text{and} \quad \vartheta(t) = \bar{\vartheta}\,.$$

3 Comparative Statics

3.1 Risk-aversion and Uncertainty

As we concentrate on the stationary solution for the infinite horizon formulation, we have an important result pertaining to the impact of risk-aversion and the volatility of the demand shocks.

Proposition 2 *For any parametric constellation, for a larger ρ and/or larger σ_ϵ, the production strategy of the oligopolistic firm in the infinite horizon formulation is more aggressive in that $\bar{\kappa}$ is larger.*

Proof. See the Appendix.

This Proposition posits that the firm will select a more aggressive production strategy when more risk-averse and when more uncertain about future shocks to the demand function.

Intuitively the stationary price is decreasing in the risk-adjustment coefficient ρ as, *ceteris paribus*, the M firms produce larger quantities of the non-storable good. Such intuition is confirmed by the following Proposition.

Proposition 3 *For any parametric constellation p^* is decreasing in ρ, the coefficient of risk-aversion, and σ_ϵ, the volatility of demand shocks.*

Proof. See the Appendix.

3.2 The Number of Firms

An other interesting and apparently counter-intuitive result pertains to the impact of the number of firms in the oligopolistic market. This is presented in the following Proposition.

Proposition 4 *For any parametric choice $\bar{\kappa}$ is increasing in M, the number of firms in the market.*

Proof. See the Appendix.

4 The Relation Between Static and Dynamic Formulations

It is interesting to investigate what happens to the stationary equilibrium of the infinite horizon formulation when time-discounting collapses to zero ($\delta \downarrow 0$) and when prices become either infinitely sticky or perfectly flexible ($s \downarrow 0$ and $s \uparrow \infty$ respectively). In this respect we have some interesting conclusions.

Firstly, for $\delta \downarrow 0$ the stationary equilibrium of the dynamic formulation converges to the equilibrium of the static model with price-taker firms, as suggested by the following Proposition.

Proposition 5 *When the time-discounting factor falls to zero the stationary equilibrium with M firms converges to the expected price, $E_t[p_t]$, of the static equilibrium with price-taker firms, in that*

$$\lim_{\delta \downarrow 0} p^* \;=\; p_{comp}\,, \quad \textit{with} \quad p_{comp} \;=\; \alpha \frac{2q}{M\beta + 2q}\,.$$

Proof. See the Appendix.

For $M = 2$ this limit coincides with the competitive equilibrium of the static formulation of the model presented by Fershtman and Kamien (1987) when $c = 0$.

Clearly, this result is not surprising at all. As δ collapses to zero firms do not take into account future profit opportunities and since prices are sticky they just behave as price-taker agents.

A graphical representation of this intuitive, and in some sense obvious, result is reported for the monopolistic case in the top two panels of Figure 1. Here the coefficient $\bar{\kappa}$ and the stationary price p^* of the dynamic model are plotted against δ and compared to the reference values for the static formulation. Interestingly, we see that for $\delta > 0$ the coefficient $\bar{\kappa}$ is smaller in the dynamic version and hence as the firm produces more slowly the equilibrium price is larger. The difference stems from the fact that in the dynamic model the firm in time t takes into account the impact of its current production decision on future prices and profits and optimally decides to restrain its production. However, when $\delta \downarrow 0$, as the management's concern for future profits vanquishes, its optimal production decision collapses to that of the static formulation with price-taking behavior. In fact, for $\delta \downarrow 0$ $\bar{\kappa} \to \kappa_{\text{comp}}$, where $\kappa_{\text{comp}} = \frac{1}{2q}$ is the coefficient κ of the static formulation with price-taking behavior.

Similar conclusions are reached when s collapses to zero. Indeed, the following Proposition holds.

Proposition 6 *When prices become infinitely sticky ($s \downarrow 0$), the stationary equilibrium converges to the expected price, $E_t[p_t]$, of the static equilibrium with M price-taker firms, in that*

$$\lim_{s \downarrow 0} p^* \; = \; p_{comp} \, .$$

Proof. See the Appendix.

A graphical representation of this result for the monopolistic case is also reported in Figure 3, in the bottom panels. Here the coefficient $\bar{\kappa}$ and the stationary price p^* of the dynamic model are plotted against $(1/s)$, the degree of price stickiness, and compared to the reference values for the static formulation with both strategic and price-taking behavior. Even in this case, for $s \downarrow 0$, as the production decision in t does not impact future prices, the optimal production decision collapses to that of static formulation with price-taking behavior. However, for $s > 0$, as current decisions affect future prices, the management's concern for future profits leads it to slow its production, as witnessed by the smaller coefficient $\bar{\kappa}$ in the dynamic version. Interestingly, we have the following general result.

Proposition 7 *When prices become perfectly flexible ($s \uparrow \infty$), the stationary price converges to a limit value which exceeds the expected price, $E_t(p_t)$, of the static equilibrium with M price-taker firms, in that*

$$\lim_{s \uparrow \infty} p^* \; > \; p_{comp} \, .$$

Proof. See the Appendix.

To qualify this and other results we had better distinguish between monopoly and oligopoly.

4.1 Monopoly

Corollary 1 *For $M = 1$, when prices become perfectly flexible ($s \uparrow \infty$), the stationary price converges to a limit value which corresponds to the expected price, $E_t(p_t)$, of the static equilibrium with* 1 *strategic monopolist, in that*

$$\lim_{s \uparrow \infty} p^* \; = \; p_{strat} \, .$$

Indeed in the static formulation with one strategic monopolist the expected price, $E_t(p_t)$, in equilibrium is $p_{\text{strat}} = \alpha\left(\frac{\beta+2q}{2(\beta+q)}\right)$. This value coincides with the limit behavior of the stationary price for the dynamic model, p^*, when there is only one firm. Such result is confirmed by the bottom panels in Figure 3. Here for $(1/s) \downarrow 0$ we see that the dynamic model coefficient $\bar{\kappa}$ increases. However, it does not converge to the static coefficient with a strategic monopolist, $\kappa_{\text{strat}} = \frac{1}{\beta+2q}$. To see this let $W = (1 + \frac{2q}{\beta})$. Then, notice that for $M = 1$, $\kappa_{\text{stra}} = \frac{1}{\beta+2q} = \frac{1}{2q}(1-g)$, where $g = 1/W$. In addition, for $M = 1$, $\lim_{s\uparrow\infty} b\bar{\pi} = \frac{1}{2}[W - \sqrt{W^2-1}]$, which we denote with l. Since $\lim_{s\uparrow\infty}\bar{\kappa} = \frac{1}{2q}(1-l)$ and $g > l$ we find that $\lim_{s\uparrow\infty}\bar{\kappa} > \kappa_{\text{strat}}$.

Interestingly, even if $\lim_{s\uparrow\infty}\bar{\kappa} > \kappa_{\text{stra}}$ the stationary price of the dynamic problem converges to the expected price of the static equilibrium with a strategic firm. This is because in the dynamic model the production function contains the extra term $\frac{b}{q}\vartheta$ which for $s\uparrow$ converges to the negative value $\frac{\alpha}{\beta}\left(1 - \frac{W}{(W^2-1)^{1/2}}\right)$. All in all, for $s\uparrow\infty$ $\kappa \to \kappa_{\text{dyn}} > \kappa_{\text{stra}}$ and $p^* \to p_{\text{stra}}$, i.e. the value corresponding to the expected price, $E_t(p_t)$, of the static equilibrium with a strategic firm.

In general for $0 < s < \infty$ we conjecture that the following inequalities hold,

$$p_{\text{comp}} < \quad p^* \quad < p_{\text{stra}}\,, \tag{13}$$

$$\kappa_{\text{comp}} > \quad \kappa \quad > \kappa_{\text{stra}}\,, \tag{14}$$

so that in the general case, for s positive but finite, the stationary equilibrium lies in between the two extremes of the strategic and competitive static equilibria. This is clearly borne out by the bottom panels of Figure 3.

4.2 Oligopoly

Corollary 2 *For $M > 1$, $q = 1/2$ and $\beta = 1$, when prices become perfectly flexible ($s\uparrow\infty$), the stationary price converges to a limit value which corresponds to a weighted average of the expected price, $E_t(p_t)$, of the static equilibrium with M price-taker and with M strategic firms, in that there exists $\omega \in (0,1)$ such that*

$$\lim_{s\uparrow\infty} p^* = \omega\, p_{strat} + (1-\omega)\, p_{comp}\,.$$

To see this consider that the expected price, $E_t(p_t)$, of the static equilibrium with M strategic firms is equal to $p_{\text{strat}} = \alpha\,\frac{2}{2+M}$, which can also be written as $p_{\text{strat}} = (1+S)p_{\text{comp}}$, where now $p_{\text{comp}} = \alpha\,\frac{1}{1+M}$, for $S = M/(2+M)$. As it can be established in this case that

$\left(\frac{M + (M-1)\Lambda}{\Lambda(M-1 + M\Lambda)}\right) < 1 + S$ the result is derived.

In particular when, $q = 1/2$ and $\beta = 1$, $p_{\text{strat}} = \frac{1}{2}\alpha$ and $p_{\text{comp}} = \frac{1}{3}\alpha$, while $\omega = \frac{2\sqrt{2/3}}{1+2\sqrt{2/3}}$ which coincides with the corresponding formula presented by Fershtman and Kamien (1987) when $c = 0$,

$$\lim_{s\uparrow\infty} p^* = \frac{p_{\text{comp}} + 2\frac{\sqrt{2}}{\sqrt{3}}\, p_{\text{strat}}}{1 + 2\frac{\sqrt{2}}{\sqrt{3}}}.$$

Indeed, for $M = 2$ and $q = 1/2$ and $\beta = 1$, when $s \uparrow \infty$ our formulation collapses to the stochastic analogue of the static model discussed by Fershtman and Kamien (1987).[5]

One the one hand, this result is reassuring in that it implies that our formulation is consistent with Fershtman and Kamien's and that our analysis can be considered an extension of theirs where the focus is on the impact of risk-aversion, uncertainty and time-horizon on the production decisions of individual firms in a oligopolistic market. Furthermore, it shows how the impact of risk-aversion and uncertainty on the optimal production strategies of oligopolistic firms crucially hinges on the stickiness of the good price. Only when prices adjust slowly to their long-run equilibrium values the attitude of the firms' management and their uncertainty on the dynamics of future prices affect their production decisions. Indeed, when $s \uparrow \infty$ neither $\bar{\pi}$, nor $\bar{\vartheta}$ (and hence $\bar{\kappa}$) are affected by either ρ or σ_ϵ^2 as the equilibrium price, $p(t) = \alpha - \beta x(t)$, is immediately reached and uncertainty over future prices vanquishes.

Figure 2 summarizes the dependence of the steady state equilibrium on δ and $1/s$ for $M = 2$ when q and β take values which differ from those employed by Fershtman and Kamien. In particular the bottom panels refer to the dependence of the stationary price and $\bar{\kappa}$ on the degree of price stickiness, $1/s$. Results are in line with those outlined for $q = 1/2$ and $\beta = 1$. Specifically they are consistent with Corollary 2.

Propositions 6 and 7 propose some further interesting results. In particular, while in a monopolistic market when prices become perfectly flexible the management of the single firm will not consider the impact of its current production decision on future profits and hence the equilibrium converges to the static equilibrium with strategic behavior, when several firms operate in the market, Cournot competition will induce market participants to produce more moving the equilibrium price towards the competitive price. Once again this feature of the equilibrium, is related to the market structure rather than to uncertainty and attitude towards risk.

[5]To see the correspondence one needs to adjust notation. Thus, our $\bar{\pi}$ and $\bar{\vartheta}$ correspond to their $-\frac{1}{2}K$ and $-\frac{1}{2}E$. Then, as $b = -as$, we see that for $a = 1$, $\beta = 1$ and $q = 1/2$ our formulae for $u(t)$ and the value function $\mathcal{V}$ coincide with theirs for $c = 0$. In addition, for $r = -\ln\delta$ equations (B.2) for $\bar{\pi}$ and (B.4) for $\bar{\vartheta}$ coincide with theirs for K and E.

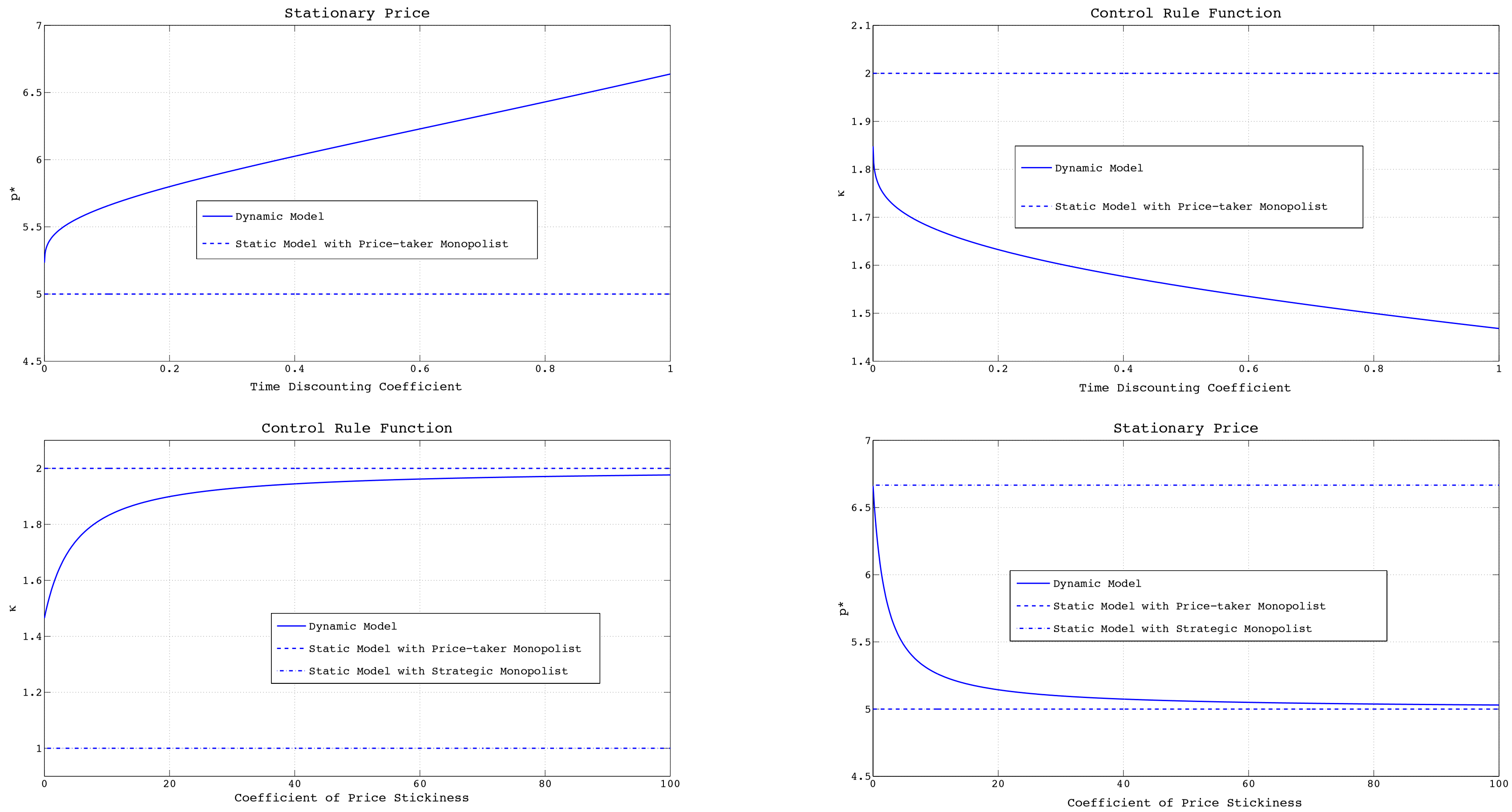

Figure 1: Dependence of p^* and κ on δ (top panels) and $1/s$ (bottom panels) for $M = 1$, $\sigma_\epsilon^2 = 0.1$, $q = 0.25$, $\alpha = 10$, $\beta = 0.5$ and $\rho = 1$. In the top panels $\delta = 0.5$, in the bottom ones $s = 1$

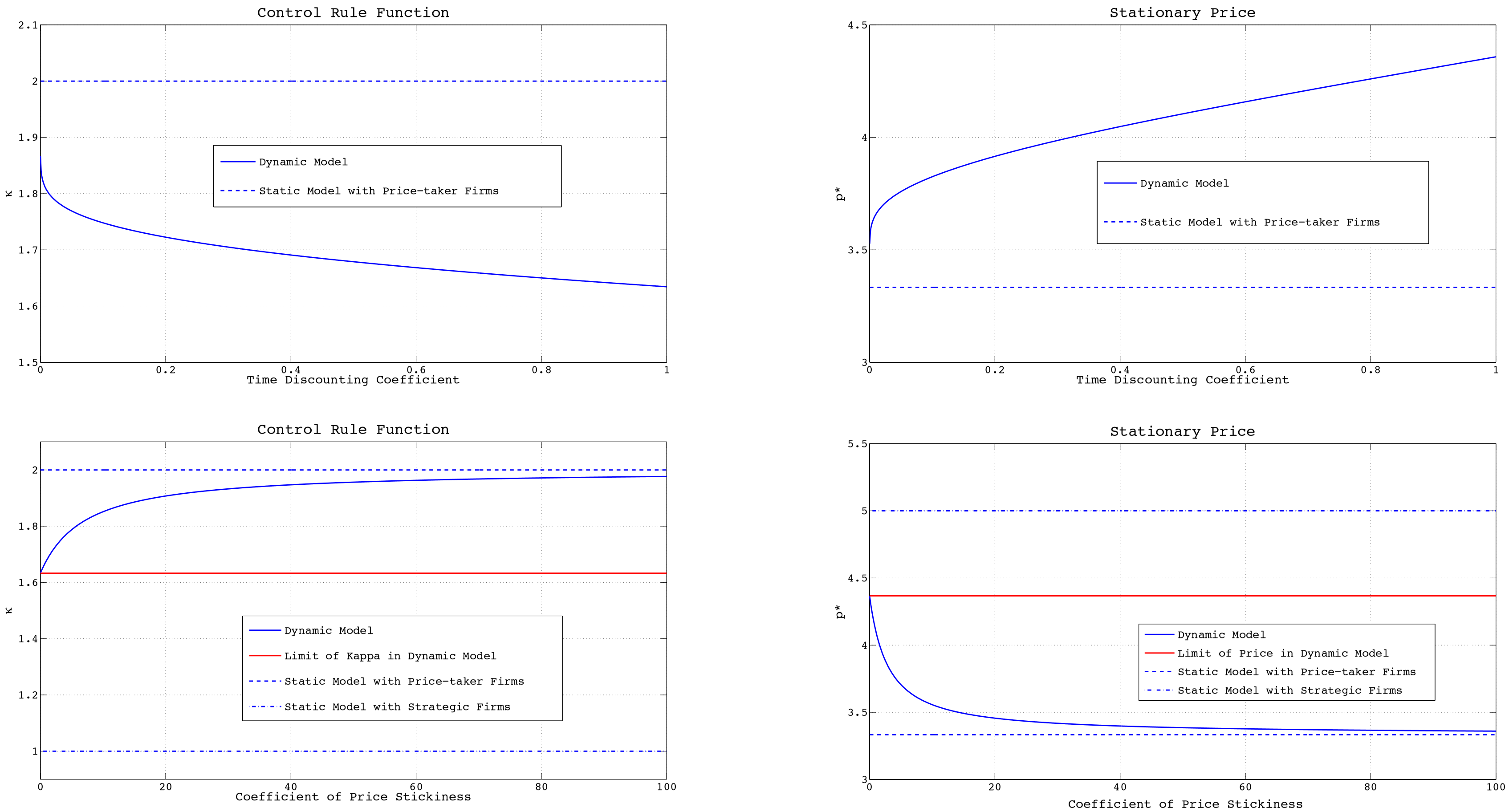

Figure 2: Dependence of p^* and κ on δ (top panels) and $1/s$ (bottom panels) for $M = 2$, $\sigma_\epsilon^2 = 0.1$, $q = 0.25$, $\alpha = 10$, $\beta = 0.5$ and $\rho = 1$. In the bottom panels $\delta = 0.5$, in the top ones $s = 1$

References

BILS, M., AND P. J. KLENOW (2004): "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy*, 112(5), 947–985.

BORENSTEIN, S., AND S. HOLLAND (2005): "Onthe efficiency of competitive electricity markets with time-invariant retail prices," *RAND Journal of Economis*, 36(3), 469–493.

CELLINI, R., AND L. LAMBERTINI (2004): "Dynamic Oligopoly with Sticky Prices: Closed-Loop, Feedback and Open-Loop Solutions," *Journal of Dynamical and Control Systems*, 10, 303–314.

——— (2007): "A Differential Oligopoly Game with Differentiated Goods and Sticky Prices," *European Journal of Operational Research*, 176, 1131–1144.

DOCKNER, E. (1988): "Oll the Relation between Dynamic Oligopolistict Competition and Long-Run Competitive Equilibrium," *European Journal of Political Economy*, 4(1), 47–64.

DOCKNER, E., AND A. HAUG (1990): "Tariffs and quotas under dynamic duopolistic competition," *Journal of International Economics*, 29:, 147.160.

DOCKNER, E. J., S. JORGENSEN, N. VAN LONG, AND G. SORGER (2000): *Differential Games in Economics and Management Science*. Cambridge University Press.

DOCKNER, E. J., AND N. V. LONG (1993): "International Pollution Control: Cooperative versus Noncooperative Strategies," *Journal of Environmental Economics and Management*, 24, 13–29.

DRISKILL, R., AND S. MCCAFFERTY (1989): "Dynamic Duopoly with Adjustment Costs: A Differential Game Approach," *Journal of Economic Theory*, 49, 324–338.

FERSHTMAN, C., AND M. I. KAMIEN (1987): "Dynamic Duopolistic Competition with Sticky Prices," *Econometrica*, 55, 1151–1164.

HANSEN, L., AND T. J. SARGENT (1995): "Discounted Linear Exponential Quadratic Gaussian Control," *IEEE Transactions on Automatic Control*, 40, 968–971.

JOSKOW, P. L., AND C. D. WOLFRAM (2012): "Dynamic Pricing of Electricity," *American Economic Review*, 102(3), 381–385.

KARP, L. S., AND J. M. PERLOFF (1989): "Dynamic Oligopoly in the Rice Export Market," *Review of Economics and Statistics*, 71, 462–470.

——— (1993): "Open-loop and feedback models of dynamic oligopoly," *International Journal of Industrial Organization*, 11, 369–389.

PIGA, C. (2000): "Competition in a Duopoly with Sticky Price and Advertising," *International Journal of Industrial Organization*, 18, 595–614.

SIMAAN, M., AND T. TAKAYAMA (1978): "Game Theory Applied to Dynamic Duopoly Problems with Production Constraints," *Automatica*, 14, 161–166.

TSUTSUI, S., AND K. MINO (1990): "Nonlinear strategies in dynamic duopolistic competition with sticky prices," *Journal of Economic Theory*, 52, 136161.

TURNOVSKY, S. J., T. BASAR, AND V. D'OREY (1988): "Dynamic Strategic Monetary Policies and Coordination in Interdependent Economies," *American Economic Review*, 78(3), 341–361.

VAN DER PLOEG, F., AND A. J. DE ZEEUW (1992): "International Aspects of Pollution Control," *Environmental and Resource Economics*, 2, 117–139.

VITALE, P. (2017): "Pessimistic Optimal Choice for Risk-averse Agents: The Continuous-time Limit," *Computational Economics,*, 49, 17–65.

WIRL, F. (2010): "Dynamic demand and noncompetitive intertemporal output adjustments," *International Journal of Industrial Organization*, 28, 220229.

WISZNIEWSKA-MATYSZKIEL, A., M. BODNAR, AND F. MIROTA (2015): "Dynamic Oligopoly with Sticky Prices: Off-Steady-state Analysis," *Dynamic Games and Applications*, 5, 568–598.

Appendix

Proof of Lemma 1

Suppose firm 1's enterpreneur conjectures that in n firms 2, 3, ..., M will all choose to produce the same quantity Δy_n. In addition, assume that $\boldsymbol{\mathcal{V}}_{n+1} = \pi_{n+1}\, p_{n+1}^2 - 2\vartheta_{n+1} + \nu_{n+1}$, where π_{n+1} and ϑ_{n+1} are some time-variant coefficients. Under this assumption, Lemma 4 in Vitale (2017) shows that solving the recursion in (2) is equivalent to solving the double recursion

$$
\begin{aligned}
F_n(p_n) &= \boldsymbol{\mathcal{L}}\,\widetilde{\boldsymbol{\mathcal{L}}}\, F_{n+1}(p_{n+1}), \text{ where } F_{n+1} \equiv \pi_{n+1}\, p_{n+1}^2 - 2\vartheta_{n+1}, \\
\widetilde{\boldsymbol{\mathcal{L}}}\,\phi(p) &= \max_{\epsilon} \left[\delta\pi(p+\epsilon)^2 - 2\delta\vartheta(p+\epsilon) - \frac{1}{\rho}\frac{1}{\sigma_\epsilon^2\Delta}\epsilon^2 \right] \quad \text{and} \\
\boldsymbol{\mathcal{L}}\,\phi(p) &= \min_{u} \left[\Delta c + \phi\Big(\mu\Delta + (1+a\Delta)\,p + b\Delta\,[u+(M-1)y] \Big) \right].
\end{aligned}
$$

Applying the $\widetilde{\mathcal{L}}$ operator to F_{n+1} we find that $\widetilde{\mathcal{L}}F_{n+1}(p_{n+1}) = \tilde{\pi}_{n+1}\, p_{n+1}^2 - 2\tilde{\vartheta}_{n+1}\, p_{n+1}$ with $\tilde{\pi}_{n+1} = \delta^{\Delta}\pi_{n+1}(1 - \delta^{\Delta}\rho\,\sigma_\epsilon^2\Delta\,\pi_{n+1})^{-1}$, $\tilde{\vartheta}_{n+1} = \delta^{\Delta}\vartheta_{n+1}(1 - \delta^{\Delta}\rho\,\sigma_\epsilon^2\Delta\,\pi_{n+1})^{-1}$ and the second order condition that $\delta^{\Delta}\pi_{n+1} - \frac{1}{\rho}\frac{1}{\sigma_\epsilon^2} < 0$, which will always be satisfied insofar $\pi_{n+1} < 0$.

In applying the $\mathcal{L}$ operator to $F_{n+1}(p_{n+1}) = \tilde{\pi}_{n+1}\, p_{n+1}^2 - 2\tilde{\vartheta}_{n+1}\, p_{n+1}$ we find the following first order condition

$$(q + b^2\Delta\tilde{\pi}_{n+1})\, u_n + \left(b(1+a\Delta)\tilde{\pi}_{n+1} - \frac{1}{2}\right) p_n + (M-1)b^2\Delta\tilde{\pi}_{n+1}y_n + b(\tilde{\pi}_{n+1}\mu\Delta - \tilde{\vartheta}_{n+1}) \;=\; 0$$

and hence the optimal production is

$$u_n \;=\; \frac{\frac{1}{2} - b(1+a\Delta)\tilde{\pi}_{n+1}}{q \,+\, (M-1)b^2\Delta\tilde{\pi}_{n+1}}\, p_n - \frac{(M-1)b\Delta\tilde{\pi}_{n+1}}{q \,+\, (M-1)b^2\Delta\tilde{\pi}_{n+1}}\, y_n - \frac{b(\tilde{\pi}_{n+1}\mu\Delta - \tilde{\vartheta}_{n+1})}{q \,+\, (M-1)b^2\Delta\tilde{\pi}_{n+1}}\,. \quad \text{(A.1)}$$

Crucially, firm 1's conjecture will need to be verified in equilibrium. This is trivially achieved by assuming a symmetric equilibrium in that we posit that $u_n = y_n$. Under such restriction from equation (A.1) it is established that

$$u_n \;=\; \kappa_{p,n}\, p_n \,+\, \kappa_{e,n}(\tilde{\pi}_{n+1}\mu\Delta \,-\, \tilde{\vartheta}_{n+1})\,, \text{ with}$$
$$\kappa_{p,n} \;=\; \frac{\frac{1}{2} \,-\, b(1+a\Delta)\tilde{\pi}_{n+1}}{q \,+\, M\,b^2\Delta\tilde{\pi}_{n+1}} \quad \text{and} \quad \kappa_{e,n} \;=\; -\frac{b}{q \,+\, Mb^2\Delta\tilde{\pi}_{n+1}}.$$

Inserting this expression into the argument of the $\mathcal{L}$ operator it is found that

$$\pi_n \;=\; \Delta q\,\kappa_{p,n}^2 \,-\, \Delta\,\kappa_{p,n} \,+\, \left[(1+a\Delta) \,+\, M\,b\,\Delta\,\kappa_{p,n}\right]^2 \tilde{\pi}_{n+1}\,,$$
$$\vartheta_n \;=\; [1 \,+\, (M-1)b\tilde{\pi}_{n+1}\,\Delta\,\kappa_{e,n}][Mb\,\Delta\,\kappa_{p,n} \,+\, (1 \,+\, a\Delta)](\tilde{\vartheta}_{n+1} \,-\, \tilde{\pi}_{n+1}\mu\Delta)\,. \;\square$$

Proof of Proposition 1

Reconsider the optimal production strategy described in Lemma 1. In the limit, for Δ converging to zero $\frac{(M-1)b\Delta\tilde{\pi}_{n+1}}{q+(M-1)b^2\Delta\tilde{\pi}_{n+1}} \to 0$. In addition, for $\Delta \downarrow 0$, as $\tilde{\pi}_{n+1} \to \pi(t)$ (while $\tilde{\vartheta}_{n+1} \to \vartheta(t)$), $\frac{\frac{1}{2}-b(1+a\Delta)\tilde{\pi}_{n+1}}{q+(M-1)b^2\Delta\tilde{\pi}_{n+1}}$ converges to $\frac{1}{q}\left(\frac{1}{2} - b\pi(t)\right)$, while $-\frac{b(\tilde{\pi}_{n+1}\mu\Delta-\tilde{\vartheta}_{n+1})}{q+(M-1)b^2\Delta\tilde{\pi}_{n+1}}$ converges to $\frac{b}{q}\vartheta(t)$.

Hence, in the limit, the optimal demand function for firm 1in t is

$$u(t) \;=\; \kappa(t)\,p(t) \,+\, \frac{b}{q}\,\vartheta(t)\,, \text{ with } \kappa(t) \;=\; \frac{1}{q}\left(\frac{1}{2} \,-\, b\pi(t)\right),$$

where $\kappa(t) = \lim_{\Delta\downarrow 0}\kappa_{p,n}$, $\frac{b}{q} = \lim_{\Delta\downarrow 0}\kappa_{e,n}$, $\pi(t) = \lim_{\Delta\downarrow 0}\pi_n$ and $\vartheta(t) = \lim_{\Delta\downarrow 0}\vartheta_n$. To identify the limit functions $\pi(t)$ and $\vartheta(t)$, firstly consider that since $[(1+a\Delta) \,+\, M\,b\Delta\,\kappa_n]^2 = (1+2a\Delta) \,+\, 2\,M\,b\,\Delta\,\kappa_n \,+$

$o(\Delta^2)$, it follows that equation (6) can also be written as

$$\pi_n \;=\; \tilde{\pi}_{n+1} \,+\, \Delta(q\,\kappa_{p,n}^2 \,-\, \kappa_{p,n} \,+\, 2a\,\tilde{\pi}_{n+1} \,+\, 2\,M\,b\,\kappa_{p,n}\,\tilde{\pi}_{n+1}) \,+\, o(\Delta^2)\,,$$

where $o(\Delta)$ indicates a term of order Δ or superior. This implies that

$$\frac{\pi_n \,-\, \pi_{n+1}}{\Delta} \;=\; \frac{\tilde{\pi}_{n+1} \,-\, \pi_{n+1}}{\Delta} \,+\, 2a\,\tilde{\pi}_{n+1} + \kappa_{p,n}(q\,\kappa_{p,n} \,-\, 1 \,+\, 2\,M\,b\,\tilde{\pi}_{n+1}) \,+\, o(\Delta)\,.$$

Notice, that it can be established that

$$\kappa_n(q\,\kappa_{p,n} \,-\, 1 \,+\, 2\,M\,b\,\tilde{\pi}_{n+1}) \;=\; -\left(\frac{\frac{1}{2} \,-\, b\tilde{\pi}_{n+1}}{q \,+\, M\,b^2\Delta\tilde{\pi}_{n+1}}\right)\left(q\,\frac{\frac{1}{2} \,-\, (2M-1)\,b\tilde{\pi}_{n+1}}{q \,+\, M\,b^2\Delta\tilde{\pi}_{n+1}}\right) \,+\, o(\Delta)\,.$$

Now, $\lim_{\Delta\downarrow 0}\frac{\pi_n \,-\, \pi_{n+1}}{\Delta} = -\frac{d\pi(t)}{dt}$, $\lim_{\Delta\downarrow 0}\frac{\tilde{\pi}_{n+1} \,-\, \pi_{n+1}}{\Delta} = \ln\delta\,\pi(t) + \rho\sigma_\epsilon^2\pi(t)^2$ and $\lim_{\Delta\downarrow 0}\tilde{\pi}_{n+1} = \lim_{\Delta\downarrow 0}\pi_{n+1} = \pi(t)$. Given the expression above we also see that $\lim_{\Delta\downarrow 0}\kappa_{p,n}(q\,\kappa_{p,n} \,-\, 1 \,+\, 2\,M\,b\,\tilde{\pi}_{n+1}) = -\frac{1}{q}\,(\frac{1}{2} - b\pi(t))(\frac{1}{2} \,-\, (2M-1)\,b\pi(t))$. We conclude that in the limit $\pi(t)$ solves the following differential equation

$$\frac{d\pi(t)}{dt} \,-\, \frac{1}{q}\left(b\pi(t) \,-\, \frac{1}{2}\right)\left((2M-1)b\pi(t) \,-\, \frac{1}{2}\right) + 2a\,\pi(t) \,+\, \ln\delta\,\pi(t) \,+\, \rho\sigma_\epsilon^2\pi(t)^2 \;=\; 0\,.$$

Similarly, equation (7) can also be written as follows

$$\vartheta_n \;=\; \tilde{\vartheta}_{n+1} \,+\, \Delta\Big(a \,+\, M\,b\,\kappa_{p,n} \,+\, (M-1)\,b\,\kappa_{e,n}\,\tilde{\pi}_{n+1} \,+\, o(\Delta)\Big)\,\tilde{\vartheta}_{n+1} \,-\, \Delta(\mu \,+\, o(\Delta))\,\tilde{\pi}_{n+1}\,,$$

so that

$$\frac{\vartheta_n \,-\, \vartheta_{n+1}}{\Delta} \;=\; \frac{\tilde{\vartheta}_{n+1} \,-\, \vartheta_{n+1}}{\Delta} \,+\, \left(a \,+\, M\,b\,\kappa_{p,n} \,+\, (M-1)\,b\,\kappa_{e,n}\,\tilde{\pi}_{n+1}\right)\tilde{\vartheta}_{n+1} \,-\, \mu\tilde{\pi}_{n+1} + o(\Delta))\,.$$

Considering that $\lim_{\Delta\downarrow 0}\frac{\vartheta_n \,-\, \vartheta_{n+1}}{\Delta} = -\frac{d\vartheta(t)}{dt}$, $\lim_{\Delta\downarrow 0}\frac{\tilde{\vartheta}_{n+1} \,-\, \vartheta_{n+1}}{\Delta} = \ln\delta\,\vartheta(t) + \rho\sigma_\epsilon^2\vartheta(t)^2$, $\lim_{\Delta\downarrow 0}\tilde{\vartheta}_{n+1} = \lim_{\Delta\downarrow 0}\vartheta_{n+1} = \vartheta(t)$ and that $\lim_{\Delta\downarrow 0}M\kappa_{p,n} = M\frac{b}{q}(\frac{1}{2} - b\pi(t))$ and $\lim_{\Delta\downarrow 0}(M-1)b\kappa_{e,n}\tilde{\pi}_{n+1} = (M-1)\frac{b^2}{q}\pi(t)$, we conclude that in the limit $\vartheta(t)$ solves the second differential equation

$$\frac{d\vartheta(t)}{dt} + \left(\ln\delta \,+\, a \,+\, \frac{M}{2}\,\frac{b}{q}\right)\vartheta(t) \,-\, \left((2M-1)\frac{b^2}{q} \,-\, \rho\sigma_\epsilon^2\right)\pi(t)\vartheta(t) \,-\, \mu\pi(t) \;=\; 0\,.$$

Proof of Proposition 2.
We rely on a graphical argument. In Figures A.1 and A.2 we show how the determination of $\bar{\pi}$ changes when either ρ or σ_ϵ^2 augments. In Figure A.1 we consider the case in which $(2M-1)\frac{b^2}{q} > \rho\sigma_\epsilon^2$, while in Figure A.2 that in which$(2M-1)\frac{b^2}{q} < \rho\sigma_\epsilon^2$.

Both Figures allow to determine what happens when an increase in ρ and/or in σ_ϵ^2 brings about a reduction in γ ($\gamma = (2M-1)\frac{b^2}{q} - \rho\sigma_\epsilon^2$). In both cases graphical inspection shows that for a larger degree of risk-aversion and/or a larger volatility of the demand shocks the stationary value $\bar{\pi}$, which is always negative, rises. Then, since $\bar{\kappa} = \frac{1}{q}\left(\frac{1}{2} - b\,\bar{\pi}\right)$ and $-b$ is positive we see that $\bar{\kappa}$ is increasing in $\bar{\pi}$. Then, we conclude that for a larger ρ and/or a larger σ_ϵ^2 $\bar{\kappa}$ is larger. □

Proof of Proposition 3.

To establish this result we start by recalling that p^* is the ratio between

$$\left(1 + \frac{M\frac{b^2}{q}\,\bar{\pi}}{\ln\delta + a + \frac{M}{2}\frac{b}{q} - \left((2M-1)\frac{b^2}{q} - \rho\sigma_\epsilon^2\right)\bar{\pi}}\right)\mu \quad \text{and} \quad -a - M\,b\bar{\kappa}\,.$$

Then, we notice that in Proposition 2 we proved that $\bar{\pi}$ and $\bar{\kappa}$ are increasing in ρ and σ_ϵ. In addition, we notice that $-a - Mb\bar{\kappa}$ is increasing in $\bar{\kappa}$. This implies that $-a - Mb\bar{\kappa}$ is increasing in ρ and σ_ϵ. This means that to establish our result we need to prove that the derivatives of the ratio

$$\frac{M\,\frac{b^2}{q}\,\bar{\pi}}{\ln\delta + a + \frac{M}{2}\frac{b}{q} - \left((2M-1)\frac{b^2}{q} - \rho\sigma_\epsilon^2\right)\bar{\pi}}$$

with respect to ρ and with respect to σ_ϵ are negative, so that this ratio is proved to be decreasing in these two parameters. Consider that this ratio can also be written as

$$\frac{G\bar{\pi}}{H - I\bar{\pi}}\,,$$

where $G > 0$ and $H < 0$. The derivative of this ratio wrt ρ (equivalently with respect to σ_ϵ) is

$$\frac{1}{(H - I\bar{\pi})^2}\left[G(H - I\bar{\pi})\frac{d\bar{\pi}}{d\rho} - G\bar{\pi}\left(-I\frac{d\bar{\pi}}{d\rho} - \frac{dI}{d\rho}\,\bar{\pi}\right)\right] = \frac{G}{(H - I\bar{\pi})^2}\left[H\frac{d\bar{\pi}}{d\rho} + \bar{\pi}^2\frac{dI}{d\rho}\right].$$

This expression is negative. In fact, $\frac{G}{(H - I\bar{\pi})^2}$ is positive, while $H\frac{d\bar{\pi}}{d\rho}$ is negative, since H is negative and $\frac{d\bar{\pi}}{d\rho}$ is positive. Finally, $\bar{\pi}^2\frac{dI}{d\rho}$ is negative because clearly $\frac{dI}{d\rho}$ is negative. An identical argument applies to σ_ϵ. This proves that p^* is decreasing in both ρ and σ_ϵ. □

Proof of Proposition 4.

As b is negative, it is sufficient to show that $\bar{\pi}$ is increasing in M. Consider that $\bar{\pi}$ corresponds to

$$\bar{\pi} = -\frac{1}{2}\frac{\lambda}{\gamma} + \frac{1}{2}\frac{\Delta}{\gamma}, \quad \text{where} \quad \Delta = \left[\lambda^2 - \frac{1}{q}\gamma\right]^{1/2}.$$

This can be found considering the intersection between the parabola $\gamma\,\bar{\pi}^2$ and the straight line $-\theta - \lambda\bar{\pi}$. Considering that $\lambda > 0$ and $\theta > 0$, this line presents negative slope and intercept. As for the parabola, it will be convex (concave) if γ is positive (negative).

In Figure A.3 we have a graphical representation of two functions for γ positive. The straight line and the parabola intersect twice, for two negative values of $\bar{\pi}$. Considering that $\bar{\pi}_- = -\frac{1}{2}\frac{\lambda}{\gamma} + \frac{1}{2}\frac{\Delta}{\gamma}$ and $\bar{\pi}_+ = -\frac{1}{2}\frac{\lambda}{\gamma} + \frac{1}{2}\frac{\Delta}{\gamma}$ and that γ and Δ are positive, we conclude that $\bar{\pi}_+$ is the larger value for which the straight line and the parabola intersect. When M augments graphical inspection shows that $\bar{\pi}$ rises. For γ negative, when M rises, graphical inspection does not allow to determine the direction in which $\bar{\pi}$ changes in that the shifts in the straight line and parabola push $\bar{\pi}$ in opposite directions as illustrated by Figure 8. To determine the prevailing effect on $\bar{\pi}$ we need to apply an implicit function argument.

Given that the equation $\gamma\,\bar{\pi}^2 + \lambda\,\bar{\pi} + \theta = 0$, can be written as $F(M, \bar{\pi}) = 0$, so that

$$dF \;=\; \frac{\partial F}{\partial M}\,dM \,+\, \frac{\partial F}{\partial \bar{\pi}}\,d\bar{\pi} \;=\; 0\,,$$

which implies that

$$\frac{d\bar{\pi}}{dM} \;=\; -\,\frac{\partial F}{\partial M}\Big/\frac{\partial F}{\partial \bar{\pi}}\,.$$

Now

$$\frac{d\bar{\pi}}{d\bar{\pi}} \;=\; 2\gamma\,\bar{\pi} \,+\, \lambda \quad \text{and} \quad \frac{\partial F}{\partial M} \;=\; \frac{b}{q}\,\bar{\pi}\;(2b\bar{\pi} \,-\, 1)\;.$$

We consider the sign of such derivative for $\bar{\pi} = \bar{\pi}_+$. Since $\lambda > 0$, for γ negative $\gamma\bar{\pi}_+ > 0$ and hence $\frac{d\bar{\pi}}{dM}$ is positive. In addition, since $\frac{b}{q}\bar{\pi}_+ > 0$, $\frac{\partial F}{\partial \bar{\pi}}$ is negative iff $2b\bar{\pi}_+ < 1$. If this is established then $\frac{d\bar{\pi}}{dM} > 0$. To see that $2b\bar{\pi}_+ < 1$ consider that when γ is negative $\bar{\pi}_+ > -\frac{\theta}{\lambda}$, so that (given that $b < 0$) $b\bar{\pi}_+ < -b\frac{\theta}{\lambda}$. Then, if $-b\frac{\theta}{\lambda}$ is smaller than $1/2$ *at fortiori* $2b\bar{\pi}_+ < 1$. To see that $-b\frac{\theta}{\lambda} < 1/2$ consider that this is equivalent to

$$-\frac{b\theta}{\lambda} \;=\; \frac{\frac{1}{4}\frac{b}{q}}{2a + \ln\delta + M\frac{b}{q}} \;<\; \frac{1}{2} \quad\Leftrightarrow\quad \frac{1}{2}\frac{b}{q} \;>\; 2a + \ln\delta + M\frac{b}{q}\,,$$

which is really the case as a, b and $\ln\delta$ are negative. □

Proof of Proposition 5.

It is sufficient to notice that for $\delta \downarrow 0$ both $\bar{\pi}$ and $\bar{\vartheta}$ converge to zero, so that the optimal production strategy for the M firms is $u(t) = \frac{1}{2q}p(t)$, that is that in the static formulation when the firms' management takes the good price as given. In addition, the stationary price, p^*, converges to $p^* = \alpha\frac{2q}{M\beta+2q}$, which corresponds to the expected good price, $E_t[p_t]$, in the static formulation when the firms' management takes such price as given. □

Proof of Proposition 6.

As before notice that for $s \downarrow 0$ both $\bar{\pi}$ and $\bar{\vartheta}$ converge to zero, so that $u(t) = \frac{1}{2q}p(t)$, while the stationary price converges to $p^* = \alpha\frac{2q}{M\beta+2q}$. □

Proof of Proposition 7.

Let $w = b\bar{\pi}$. For $s \uparrow \infty$, p^* converges to the ratio between

$$\alpha\,\frac{1 \,+\, \frac{M}{2}\frac{\beta}{q} - (M-1)\,\frac{\beta}{q}\,w}{1 \,+\, \frac{M}{2}\frac{\beta}{q} - (2M-1)\,\frac{\beta}{q}\,w} \quad \text{and} \quad \left(1 \,+\, \frac{M}{2}\frac{\beta}{q}\right)\left(\frac{M-1}{2M-1}\right) \,-\, \left(\frac{M}{2M-1}\right)\frac{1}{2}\frac{\beta}{q}\,\Gamma^{1/2}\,,$$

where $\Gamma = \left(M + 2\frac{q}{\beta}\right)^2 - (2M-1)$. This implies that

$$\lim_{s\uparrow\infty} p^* \;=\; \alpha \, \frac{M\left(1 + \frac{M}{2}\frac{\beta}{q}\right) + (M-1)\frac{1}{2}\frac{\beta}{q}\Gamma^{1/2}}{\frac{1}{2}\frac{\beta}{q}\Gamma^{1/2}\left((M-1)\left(1 + \frac{M}{2}\frac{\beta}{q}\right) + M\frac{1}{2}\frac{\beta}{q}\Gamma^{1/2}\right)}.$$

This can be rewritten as

$$\lim_{s\uparrow\infty} p^* \;=\; \alpha \, \frac{2q}{(2q + M\beta)} \left(\frac{M + (M-1)\Lambda}{\Lambda(M-1 + M\Lambda)}\right), \text{ where } \Lambda = \frac{1}{2}\frac{\beta}{q}\frac{\Gamma^{1/2}}{\left(1 + \frac{M}{2}\frac{\beta}{q}\right)}.$$

As $\left(\frac{M + (M-1)\Lambda}{\Lambda(M-1 + M\Lambda)}\right) > 1$ the result is established. □

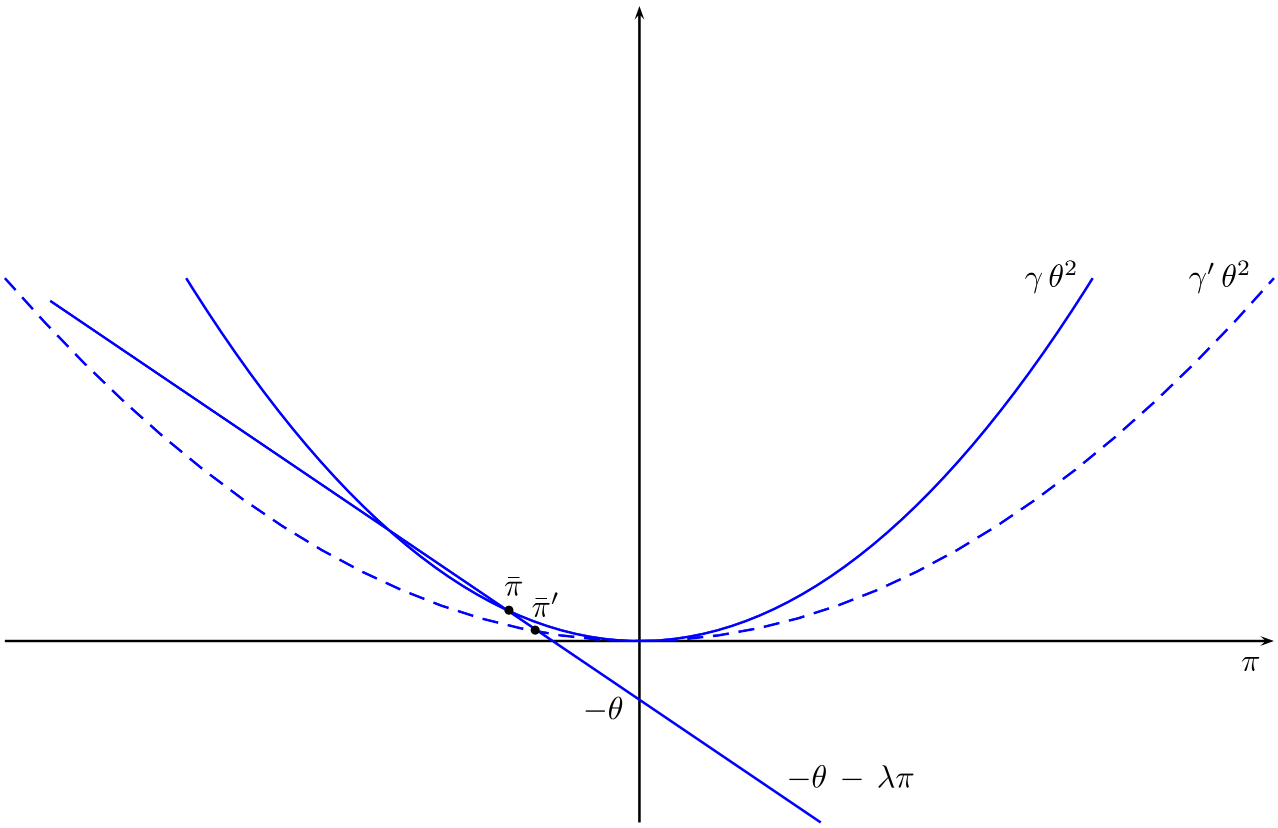

Figure A.1: The determination of $\bar{\pi}$ for $(2M-1)\frac{b^2}{q} > \rho\sigma_\epsilon^2$. For any choice of ρ and σ_ϵ^2 there are two interceptions between the straight line and the parabola. That closer to the origin corresponds to $\bar{\pi}_+ = -\frac{1}{2}\frac{\lambda}{\gamma} + \frac{1}{2}\frac{\Delta}{\gamma}$. When either ρ or σ_ϵ^2 rises, so that γ falls to γ', the parabola moves downward (while the straight line is unaffected given that λ and θ are independent of these two parameters) and the stationary value moves up to $\bar{\pi}'$.

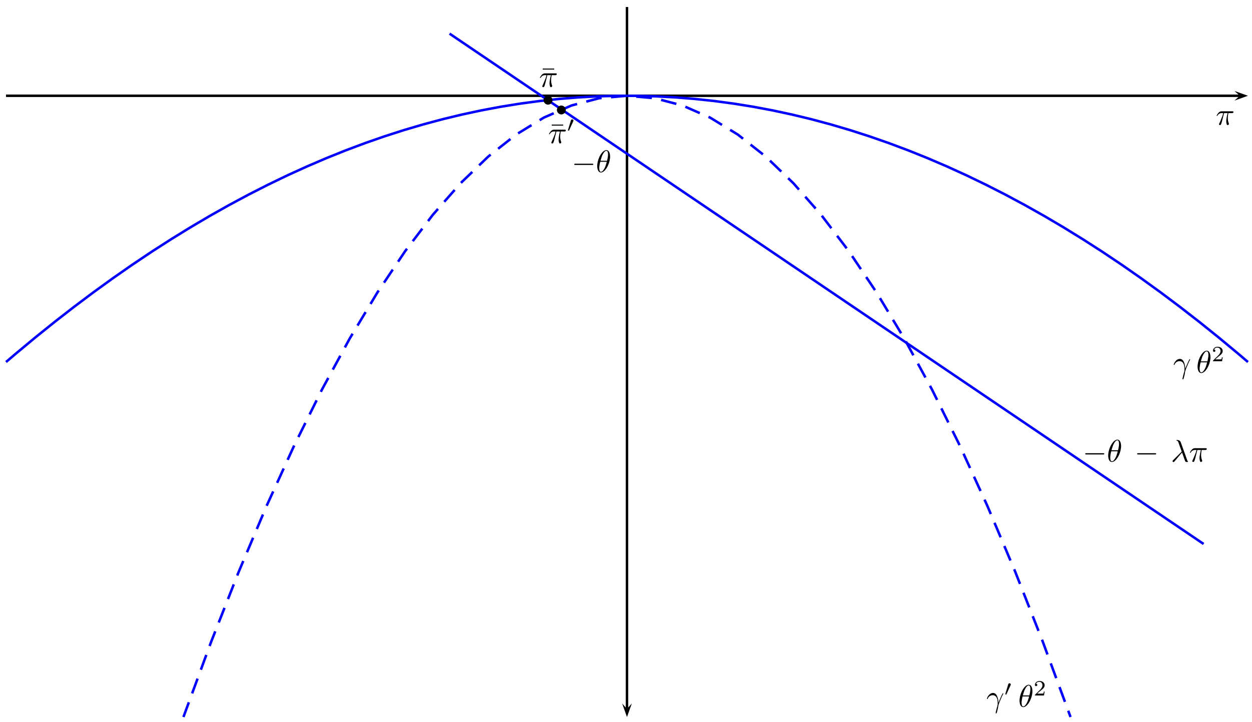

Figure A.2: The stationary value $\bar{\pi}$ for $(2M-1)\frac{b^2}{q} < \rho\sigma_\epsilon^2$. For any choice of ρ and σ_ϵ^2 there are two interceptions between the straight line and the parabola. That smaller one corresponds to $\bar{\pi}_+ = -\frac{1}{2}\,\frac{\lambda}{\gamma} + \frac{1}{2}\,\frac{\Delta}{\gamma}$. When either ρ or σ_ϵ^2 rises, so that γ falls to γ', the parabola moves downward and the stationary value moves up to $\bar{\pi}'$.

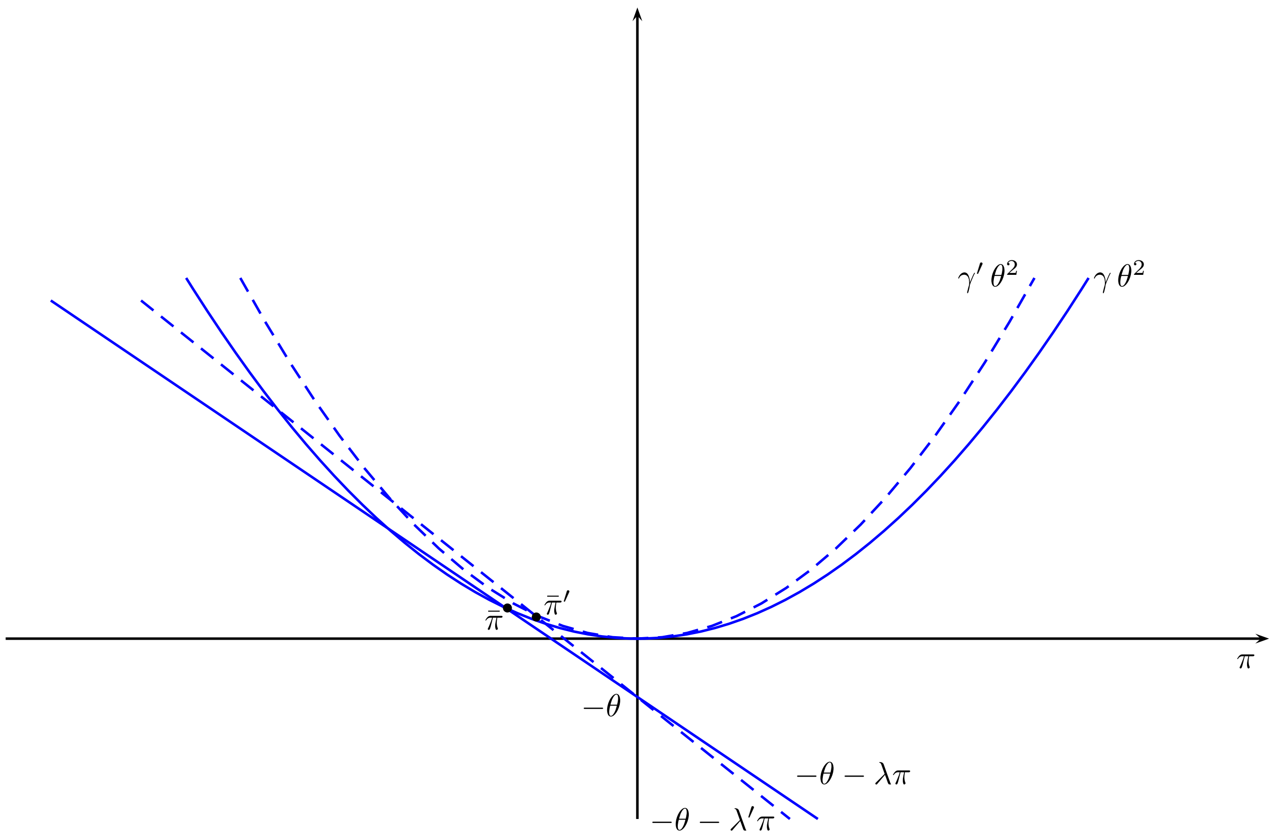

Figure A.3: The stationary value $\bar{\pi}$ for γ positive. For any choice of M there are two interceptions between the straight line and the parabola. That closer to the origin corresponds to $\bar{\pi}_+ = -\frac{1}{2}\frac{\lambda}{\gamma} + \frac{1}{2}\frac{\Delta}{\gamma}$. When M increases λ rises to λ' and γ to γ' , while θ is unaffected. This means that the straight line rotates clockwise, while the parabola moves upward. The stationary value moves up to $\bar{\pi}'$.

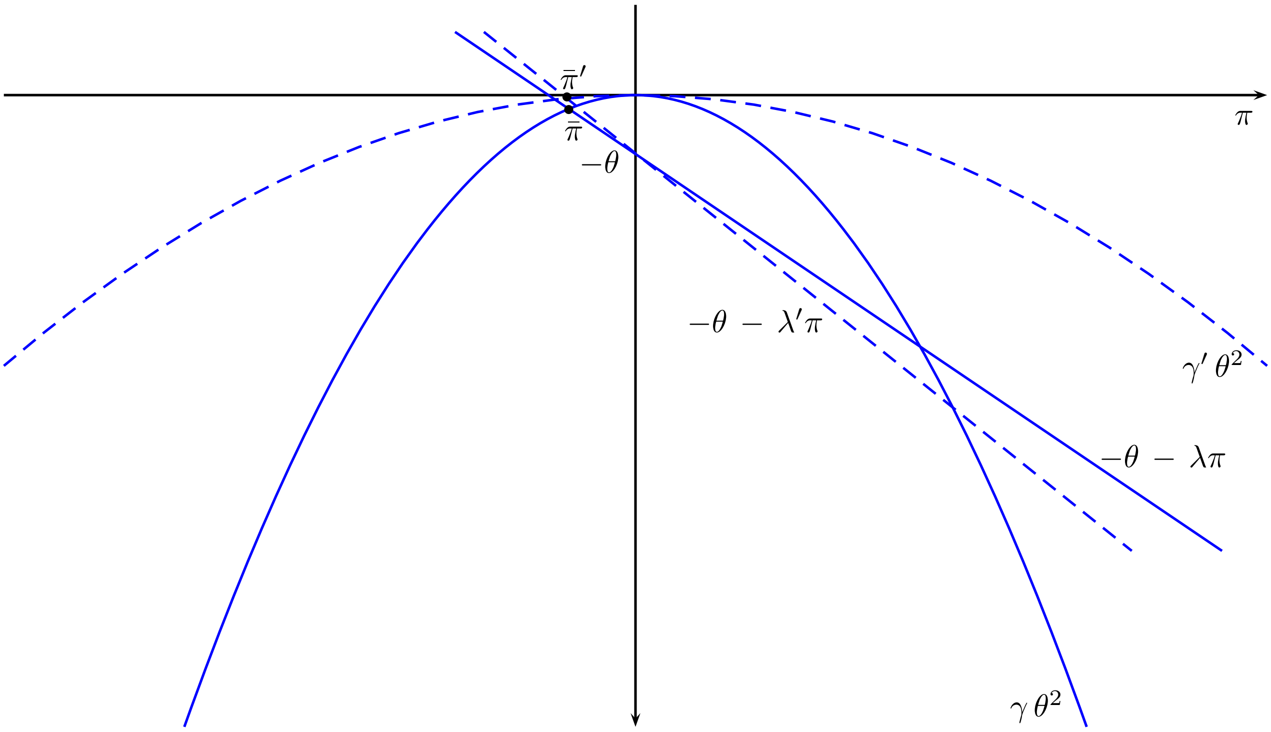

Figure A.4: The stationary value $\bar{\pi}$ for γ negative. For any choice of M there are two interceptions between the straight line and the parabola. One is negative and the other positive. The former corresponds to $\bar{\pi}_+ = -\frac{1}{2}\,\frac{\lambda}{\gamma} + \frac{1}{2}\,\frac{\Delta}{\gamma}$. When M increases λ rises to λ' and γ to γ' , while θ is unaffected. This means that the straight line rotates clockwise, while the parabola moves upward. The stationary value moves up to $\bar{\pi}'$.

Supplementary Material

B.1. Solution to the Differential Equations in Proposition 1.

Consider the former differential equation. We can write it as

$$\frac{d\,\pi(t)}{d\,t} \;=\; h_0 \,+\, h_1\pi(t) \,+\, h_2\pi(t)^2\,,$$

with $h_0 = 1/(4q)$, $h_1 = -(2a + Mb/q + \ln\delta)$, $h_2 = (2M-1)b^2/q - \rho\sigma_\epsilon^2$. This can be transformed into a homogeneous ordinary differential equation of order two,

$$\frac{d^2\,z(t)}{d^2\,t} \;-\; h_1\,\frac{d\,z(t)}{d\,t} \;+\; h_0\,h_2\,z(t) \;\;=\;\; 0\,,\;\text{ with }\;\pi(t) \;=\; -\,\frac{1}{h_2}\,\frac{\frac{d\,z(t)}{d\,t}}{z(t)}\,.$$

Assume then that $z(t) = m\;\exp(\zeta\,t)$. We have a solution of the ODE iff

$$\begin{aligned}\zeta^2\,m\;\exp(\zeta\,t) \;-\; \zeta\,h_1\,m\;\exp(\zeta\,t) \;+\; h_0\,h_2\,m\;\exp(\zeta\,t) \;\;&=\;\; 0\,,\;\text{ ie. iff}\\ m\,\zeta^2 \;-\; m\,h_1\,\zeta \;+\; m\,h_0\,h_2 \;\;&=\;\; 0\,.\end{aligned}$$

This admits two roots equal to $\zeta \;=\; \begin{cases}\zeta_1 \;=\; \frac{1}{2}\,h_1 \,+\, \frac{1}{2}\,\sqrt{\mathcal{D}}\\ \zeta_2 \;=\; \frac{1}{2}\,h_1 \,-\, \frac{1}{2}\,\sqrt{\mathcal{D}}\end{cases}$, with $\mathcal{D} = h_1^2 - 4h_0h_2$. Thus, $z(t) = m_1\;\exp(\zeta_1\,t) + m_2\;\exp(\zeta_2\,t)$. Given that $\pi(t) = -\frac{1}{h_2}\,\frac{\frac{d\,z(t)}{d\,t}}{z(t)}$, we can write that

$$\pi(t) \;\;=\;\; -\,\frac{m_1\,\zeta_1\;\exp(\zeta_1\,t) + m_2\,\zeta_2\;\exp(\zeta_2\,t)}{((2M-1)\frac{b^2}{q} \;-\; \rho\,\sigma_\epsilon^2)\;(m_1\;\exp(\zeta_1\,t) + m_2\;\exp(\zeta_2\,t))}\,.$$

We can impose the terminal condition $\pi(T) = 0$ to find that

$$m_1\,\zeta_1\;\exp(\zeta_1\,T) + m_2\,\zeta_2\;\exp(\zeta_2\,T) \;=\; 0 \;\;\Leftrightarrow\;\; m_2 \;=\; -\frac{\zeta_1}{\zeta_2}\,m_1\;\exp((\zeta_1 - \zeta_2)\,T) \;\;=\;\; -\frac{\zeta_1}{\zeta_2}\,m_1\;\exp(\sqrt{\mathcal{D}}\,T)\,.$$

Re-inserting this expression in that for $\pi(t)$ we find that

$$\begin{aligned}\pi(t) \;\;&=\;\; -\,\frac{1}{((2M-1)\frac{b^2}{q} \;-\; \rho\,\sigma_\epsilon^2)}\,\left(\frac{\zeta_1\;\exp(\zeta_1\,t) \;-\; \zeta_1\;\exp(\sqrt{\mathcal{D}}\,T)\;\exp(\zeta_2\,t)}{\exp(\zeta_1\,t) \;-\; \frac{\zeta_1}{\zeta_2}\;\exp(\sqrt{\mathcal{D}}\,T)\;\exp(\zeta_2\,t)}\right)\\ &=\;\; -\,\frac{\zeta_1}{((2M-1)\frac{b^2}{q} \;-\; \rho\,\sigma_\epsilon^2)}\,\left(\frac{1 \;-\; \exp(\sqrt{\mathcal{D}}\,T)\;\exp(-(\zeta_1 - \zeta_2)\,t)}{1 \;-\; \frac{\zeta_1}{\zeta_2}\;\exp(\sqrt{\mathcal{D}}\,T)\;\exp(-(\zeta_1 - \zeta_2)\,t)}\right)\\ &=\;\; -\,\frac{\zeta_1}{((2M-1)\frac{b^2}{q} \;-\; \rho\,\sigma_\epsilon^2)}\,\left(\frac{1 \;-\; \exp(-\,\sqrt{\mathcal{D}}\,(t-T))}{1 \;-\; \frac{\zeta_1}{\zeta_2}\;\exp(-\sqrt{\mathcal{D}}\,(t-T))}\right)\end{aligned}$$

$$=\;\; -\,\frac{\zeta_1}{((2M-1)\frac{b^2}{q} \;-\; \rho\,\sigma_\epsilon^2)}\,\left(\frac{\exp(\,\sqrt{\mathcal{D}}\,(T-t)) \;-\; 1}{\frac{\zeta_1}{\zeta_2}\;\exp(\sqrt{\mathcal{D}}\,(T-t)) \;-\; 1}\right)\,, \tag{B.1}$$

as $\zeta_1 - \zeta_2 = \sqrt{\mathcal{D}}$. Finally, notice that $\xi_1 = -\frac{1}{2}\left(2a + M\frac{b}{q} + \ln\delta\right) + \frac{1}{2}\sqrt{\mathcal{D}}$ and $\xi_2 = -\frac{1}{2}\left(2a + M\frac{b}{q} + \ln\delta\right) - \frac{1}{2}\sqrt{\mathcal{D}}$, with $\mathcal{D} = \left(2a + M\frac{b}{q} + \ln\delta\right)^2 - \frac{1}{q}\left((2M-1)b^2 - \rho\,\sigma_\epsilon^2\right)$.

B.2. Properties of Stationary Coefficients.

In a stationary equilibrium

$$\kappa(t) = \bar{\kappa}, \quad \pi(t) = \bar{\pi} \text{ and } \vartheta(t) = \bar{\vartheta}.$$

From the differential equation (11) it can be established that $\bar{\pi}$ corresponds to the negative root of the following quadratic equation

$$\gamma\,\bar{\pi}^2 + \lambda\,\bar{\pi} + \theta = 0, \tag{B.2}$$

with $\gamma = \frac{1}{q}(2M-1)\,b^2 - \rho\sigma_\epsilon^2$, $\lambda = -\left(2a + \ln\delta + M\frac{b}{q}\right)$ and $\theta = \frac{1}{4}\frac{1}{q}$. Hence, we can prove some useful results. We start from the following Lemma.

Lemma 2 *For any parametric choice the stationary value $\bar{\pi}$ is negative.*

Proof. Both λ and θ are positive, while the sign of γ depends on ρ, the firms' coefficient of risk-aversion, and σ_ϵ^2 , the volatility of the shocks to the demand function. A graphical representation is provided in Figure B.1 when $(2M-1)\frac{b^2}{q} > \rho\sigma_\epsilon^2$, so that γ is positive.

Solving for the roots of the quadratic equation in $\bar{\pi}$, it is found that

$$\bar{\pi}_{\pm} = -\frac{1}{2}\frac{\lambda}{\gamma} \pm \frac{1}{2}\frac{\Delta}{\gamma}, \quad \text{where} \quad \Delta = \left[\lambda^2 - \frac{1}{q}\gamma\right]^{1/2}.$$

While both $\bar{\pi}_+$ and $\bar{\pi}_-$ are solutions of the quadratic equation which pins down values of $\pi(t)$ compatible with a stationary equilibrium, only the former, $\bar{\pi}_+ = -\frac{1}{2}\frac{\lambda}{\gamma} + \frac{1}{2}\frac{\Delta}{\gamma}$, corresponds to the limit of $\pi(t)$ in equation (11). In fact, inspection of such equation shows that

$$\lim_{t\downarrow-\infty}\pi(t) = \bar{\pi} = -\frac{\xi_2}{\gamma}, \quad \text{where} \quad \xi_2 = \frac{1}{2}\lambda - \frac{1}{2}\Delta.$$

In Figure B.1 the value of $\bar{\pi}$ can be found considering the intersection between the parabola $\gamma\,\bar{\pi}^2$ and the straight line $-\theta - \lambda\bar{\pi}$. Considering that $\lambda > 0$ and $\theta > 0$, this line presents negative slope and intercept. As for the parabola, it will be convex (concave) if $(2M-1)\frac{b^2}{q}$ is larger (smaller) than $\rho\sigma_\epsilon^2$. In the Figure the straight line and the parabola intersect twice, for two negative values of $\bar{\pi}$. Considering that $\bar{\pi}_- = -\frac{1}{2}\frac{\lambda}{\gamma} + \frac{1}{2}\frac{\Delta}{\gamma}$ and $\bar{\pi}_+ = -\frac{1}{2}\frac{\lambda}{\gamma} + \frac{1}{2}\frac{\Delta}{\gamma}$ and that γ and Δ are positive, we conclude that $\bar{\pi}_+$ is the larger value for which the straight line and the parabola intersect. Thus, for γ positive it is found that $\bar{\pi}$ is negative.

In Figure B.2 we show the determination of $\bar{\pi}$ for $(2M-1)\frac{b^2}{q} < \rho\sigma_\epsilon^2$. In this case γ is negative and the parabola is concave. Because now γ is negative $\bar{\pi}_-$ becomes positive and $\bar{\pi}_+$ corresponds to the negative value of $\bar{\pi}$ for which the straight line and the parabola intersect. Even for γ negative it is found that $\bar{\pi}$ is negative. Finally, for $(2M-1)\frac{b^2}{q} = \rho\sigma_\epsilon^2$, $\gamma = 0$ and $\bar{\pi}_+ = -\theta/\lambda$, which is also negative. □

From this we immediately see that if $\pi(t) = \bar{\pi}$ for all t then $\kappa(t) = \bar{\kappa}$ for all t with

$$\bar{\kappa} \;=\; \frac{1}{q}\left(\frac{1}{2} - b\bar{\pi}\right). \tag{B.3}$$

Similarly, from the differential equation (12) it can be concluded that in a stationary equilibrium

$$\bar{\vartheta} \;=\; \frac{\bar{\pi}}{\ln\delta + a + \frac{M}{2}\frac{b}{q} - \left((2M-1)\frac{b^2}{q} - \rho\sigma_\epsilon^2\right)\bar{\pi}}\,\mu\,. \tag{B.4}$$

Another result is presented in the following Lemma.

Lemma 3 *For any parametric constellation the stationary value $\bar{\kappa}$ is positive.*

Proof. Denote $b\bar{\pi}$ with w. This is equal to the smaller root of the quadratic equation

$$A\,w^2 - Bw + \frac{1}{4}C = 0, \quad \text{where}$$

$$A = \left(\frac{2M-1}{q} - \rho\frac{\sigma_\epsilon^2}{b^2}\right), \quad B = \left(\frac{2}{\beta} + \frac{M}{q} + \frac{\ln\delta}{b}\right) \text{ and } C = \frac{1}{q}.$$

Now $w < 1/2$. To verify this inequality notice that it is equivalent to the condition that $\frac{B-\sqrt{B^2-AC}}{A} < 1$. This is equivalent to $B - A < \sqrt{B^2 - AC}$ for $A > 0$ and $B - A > \sqrt{B^2 - AC}$ for $A < 0$. Now, for $A > 0$, $B - A < \sqrt{B^2 - AC} \Leftrightarrow A[A - 2B + C] < 0 \Leftrightarrow A - 2B + C < 0$. For $A < 0$ $B - A > \sqrt{B^2 - AC}$ $\Leftrightarrow A[A - 2B + C] > 0 \Leftrightarrow A - 2B + C < 0$. Now, $A - 2B + C = -\frac{4}{\beta} - \rho\frac{\sigma_\epsilon^2}{b^2} - 2\frac{\ln\delta}{b} - \frac{1}{2}\frac{1}{q}$. This is clearly negative since $\beta, \rho, q > 0$ and $b, \ln\delta < 0$. □

B.3 Positivity of the Steady-state Price.

We establish that the stationary price is positive. This is important because we have not imposed any non-negativity constraint on either $p(t)$ or $u(t)$. Checking that the stationary price is positive will show that such restrictions if imposed would not be binding.

Lemma 4 *For any parametric constellation p^* is positive.*

Proof. In a stationary symmetric equilibrium,

$$\frac{dp(t)}{dt} \;=\; a\,p(t) + M\,b\,u(t) + \mu + \epsilon(t) \;\text{ and }\; u(t) \;=\; \bar{\kappa}\,p(t) + \frac{b}{q}\,\bar{\vartheta}\,.$$

It follows that

$$\frac{dp(t)}{dt} \;=\; -A\,p(t) + M\frac{b^2}{q}\,\bar{\vartheta} + \mu + \epsilon(t)\,, \tag{B.5}$$

with $A = -(a + M\,b\,\bar{\kappa})$. From this it is immediately to derive that the steady-state price is

$$p^* \;=\; \frac{1}{A}\left(M\,\frac{b^2}{q}\,\bar{\vartheta} + \mu\right). \tag{B.6}$$

Substituting the expression for $\bar{\vartheta}$ from equation (B.4) into (B.6) we conclude that

$$p^* \;=\; \frac{1}{A}\left(1 + \frac{M\,\frac{b^2}{q}\,\bar{\pi}}{\ln\delta + a + \frac{M}{2}\,\frac{b}{q} - \left((2M-1)\frac{b^2}{q} - \rho\sigma_\epsilon^2\right)\,\bar{\pi}}\right)\mu\,.$$

Now, considering that $\bar{\pi} = -\xi_2/(\frac{b^2}{q} - \rho\sigma_\epsilon^2)$, where $\xi_2 = -\frac{1}{2}(2a + \frac{b}{p} + \ln\delta) - \frac{1}{2}\sqrt{\mathcal{D}}$, with $\mathcal{D} = (2a + \frac{b}{q} + \ln\delta)^2 - \frac{1}{q}(\frac{b^2}{q} - \rho\sigma_\epsilon^2)$, we can write that

$$\begin{aligned}
&\ln\delta + a + \frac{M}{2}\,\frac{b}{q} - \left((2M-1)\frac{b^2}{q} - \rho\sigma_\epsilon^2\right)\,\bar{\pi} \;= \\
&\ln\delta + a + \frac{1}{2}\,\frac{b}{q} - \left(\frac{b^2}{q} - \rho\sigma_\epsilon^2\right)\,\bar{\pi} + (M-1)\,\frac{b}{q}\,\left(\frac{1}{2} - b\bar{\pi}\right) \;= \\
&\frac{1}{2}\,\left(\ln\delta - \sqrt{\mathcal{D}}\right) + (M-1)\,\frac{b}{q}\,\bar{\kappa}\,,
\end{aligned}$$

which is negative, since $\ln\delta$ and b are negative and $\mathcal{D}$ and $\bar{\kappa}$ are positive. Similarly, $M\frac{b^2}{q}\bar{\pi}$ is negative, while $A = -(a + M\,b\,\bar{\kappa})$ is positive. Combining these three results we conclude that p^* is positive. □

B.4 Positivity of the Steady-State Expected Production.

The following Lemma posits that in the steady state the quantity of the non-storable good produced by any firm is positive.

Lemma 5 *For any parametric constellation the expected quantity produced by the oligopolistic firms in steady state is positive.*

Proof. In steady state the expected quantity produced by a generic firm is $u^* = \bar{\kappa}p^* + \frac{b}{q}\bar{\vartheta}$. Since $\bar{\kappa}$ and p^* are positive, alongside q, while b is negative, if $\bar{\vartheta}$ is negative the result is established. For $\bar{\vartheta}$ positive the term $\frac{b}{q}\bar{\vartheta}$ is negative. Then, to check that u^* is positive consider that $\bar{\vartheta}$ is the ratio between $\bar{\pi}\mu$ and $R = \ln\delta + a + \frac{M}{2}\frac{b}{q} - ((2M-1)\frac{b^2}{q} - \rho\sigma_\epsilon^2)\bar{\pi}$. Since $\bar{\pi}\mu$ is negative, $\bar{\vartheta}$ can be positive insofar R is negative.

Hence, consider that $p^* = \frac{1}{A}\left(\mu + M\frac{b^2}{q}\bar{\vartheta}\right)$, with $A = -(a + Mb\bar{\kappa}) > 0$. Therefore, $u^* = \frac{1}{A}\left(\bar{\kappa}\mu - a\frac{b}{q}\bar{\vartheta}\right)$. Since A is positive , u^* will positive if $\bar{\kappa}\mu - a\frac{b}{q}\bar{\vartheta} > 0$. Given the expression for $\bar{\kappa}$ and A, this is equal to $\frac{\mu}{q}\left[\frac{1}{2} - \left(\frac{a+R}{R}\right)b\bar{\pi}\right]$. Because R and a are negative $0 < \frac{a+R}{R} < 1$. Then, since $b\bar{\pi}$ is positive, we see that $\frac{1}{2} - \left(\frac{a+R}{R}\right)b\bar{\pi} < \frac{1}{2} - b\bar{\pi}$. In the proof of Lemma 3 we have seen that $0 < b\bar{\pi} < \frac{1}{2}$, so that the claim that u^* is positive is established. □

B.5 Stability of the Steady State.

It is interesting to determine the properties of the dynamics of the price of the non-storable good. In this respect we have the following result.

Lemma 6 *For any parametric constellation the steady-state price p^* is stable.*

Proof. In equation (B.5) the coefficient $A = -(a + Mb\bar{\kappa})$ is positive. Then, for $\epsilon(t) \equiv 0$ this dynamic system possesses a steady state for $p^* = \frac{\mu + M\frac{b^2}{q}\bar{\vartheta}}{A}$. In fact, we can define $\hat{p}(t) \equiv p(t) - p^*$, which equivalently can be written as $p(t) = \hat{p}(t) + p^*$. Given that $-Ap^* + M\frac{b^2}{q}\bar{\vartheta} + \mu = 0$, this implies that

$$\frac{dp(t)}{dt} = -A\left(\hat{p}(t) + p^*\right) + M\frac{b^2}{q}\bar{\vartheta} + \mu + \epsilon(t) = -A\,\hat{p}(t) + \epsilon(t))\,.$$

In the end, we conclude that as $A > 0$

$$E_t\left[\frac{dp(t)}{dt}\right] > (<)\,0 \iff \hat{p}(t) < (>)\,0\,,$$

so that the steady state is stable. This is because when $\hat{p}(t)$ is positive, so that the price is above its steady state value, $E_t\left[\frac{dp(t)}{dt}\right]$ is negative, that is the price is expected to fall (and viceversa if $\hat{p}(t)$ is negative). This entails mean reversion of the price to the steady state value. □

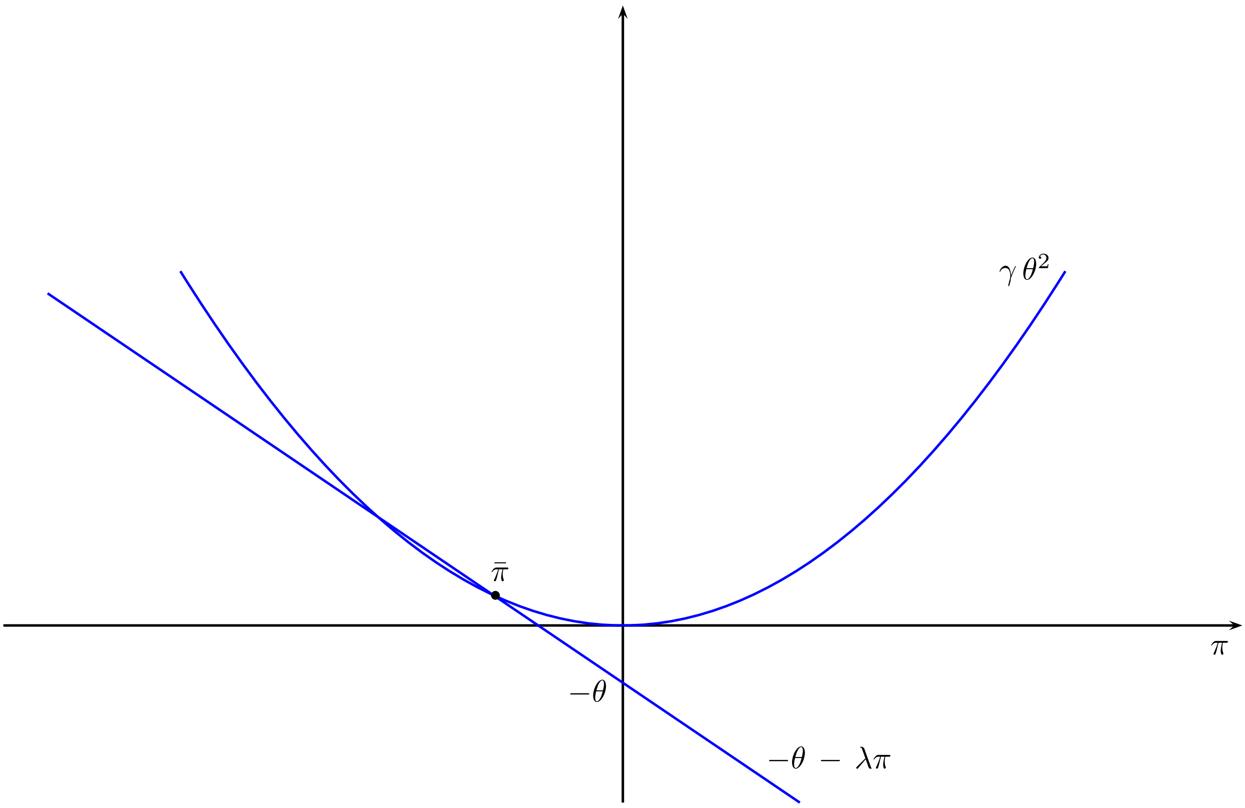

Figure B.1: The determination of $\bar{\pi}$ for $(2M-1)\frac{b^2}{q} > \rho\sigma_\epsilon^2$. For any choice of ρ and σ_ϵ^2 there are two interceptions between the straight line and the parabola. That closer to the origin corresponds to $\bar{\pi}_+ = -\frac{1}{2}\frac{\lambda}{\gamma} + \frac{1}{2}\frac{\Delta}{\gamma}$.

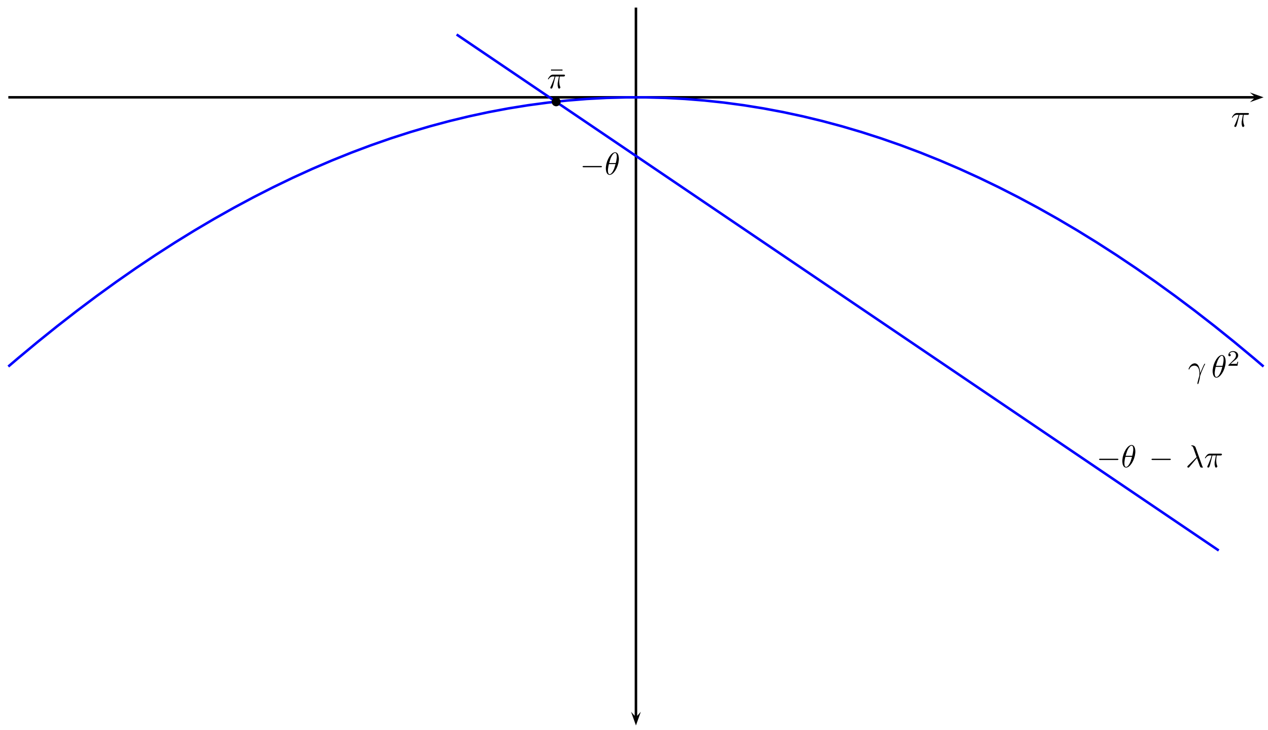

Figure B.2: The determination of $\bar{\pi}$ for $(2M-1)\frac{b^2}{q} < \rho\sigma_\epsilon^2$. For any choice of ρ and σ_ϵ^2 there are two interceptions between the straight line and the parabola. That smaller one corresponds to $\bar{\pi}_+ = -\frac{1}{2}\,\frac{\lambda}{\gamma} + \frac{1}{2}\,\frac{\Delta}{\gamma}$.